The Higher Education Academy Engineering Subject Centre

Basic Engineering

Simple DC Circuits

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The aim of this package is to provide a short self assessment programme for students who want to understand simple circuits and, in particular, how to add resistors in series and parallel.

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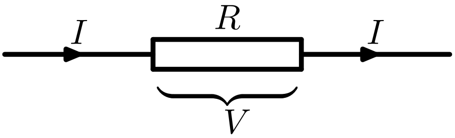
Table of Contents

The full range of these packages and some instructions, should they be required, can be obtained from our web page Mathematics Support Materials.

1. Introduction

Consider the diagram below:



it shows a device or part of an electrical circuit with a potential difference V (S.I. units volt, symbol V) across it and an electric current I (with S.I. unit ampere, symbol A) flowing through it.
The **resistance** R of the device or circuit is *defined* by

$$R = \frac{V}{I}$$

The *S.I. units of resistance* are: volts ampere^{-1}.

Because resistance is an important and frequently used concept, its unit, volts ampere^{-1}, is called an ohm (symbol Ω).

Quiz If a current of $0.045\,A$ is measured to pass through a long wire when a potential difference of $1.5\,V$ is applied to it, what is the resistance of the wire?

(a) $33.3\,\Omega$ (b) $3.33\,\Omega$ (c) $0.0675\,\Omega$ (d) $0.03\,\Omega$

For many conducting materials R is **constant** for a wide range of applied potential differences. This is known as **Ohm's law**:

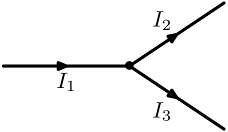
$$\boxed{V = IR}$$

The law states that if the potential difference is, say, doubled then twice as much current will flow through the resistance.

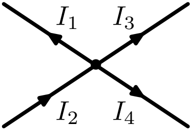
N.B. if in a material, R depends on the current or on the applied potential difference, we say that the material does not obey Ohm's law. In this package it is assumed that all resistors obey Ohm's law.

A fundamental principle in science is that electric charge is conserved. In electric circuits this is expressed by **Kirchoff's law for currents**: the total current that flows into a junction must also flow out of it. In the junction below I_1 flows in and I_2 and I_3 flow out, so Kirchoff's law says:

$$I_1 = I_2 + I_3$$



Quiz Use Kirchoff's law to choose the correct answer for this diagram:



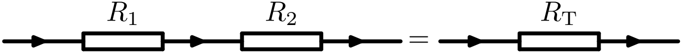
(a) $I_1 + I_2 = I_3 + I_4$ (b) $I_1 + I_4 = I_2 + I_3$

(c) $I_2 = I_1 + I_3 + I_4$ (d) $I_2 + I_4 = I_1 + I_3$

Below we will use these rules to describe combinations of resistors.

2. Resistors in Series

Two resistors connected **in series**, as in the diagram below,



are equivalent to a single resistor with resistance R_T given by

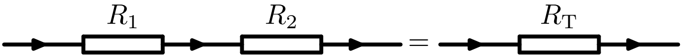
$$\boxed{R_\mathrm{T} = R_1 + R_2}$$

The values of any number of resistors in series can also be added.
Example 1 If two resistors of $R_1 = 3\,\Omega$ and $R_2 = 5\,\Omega$ are added in series, their total resistance is given by:

$$R_\mathrm{T} = 3 + 5 = 8\,\Omega$$

If a current of $4\,A$ flows through this system, then, from Ohm's law ($V = IR$), there is a potential difference of $4 \times 3 = 12\,V$ across R_1 and a potential difference of $4 \times 5 = 20\,\Omega$ across R_2. The total potential difference is thus $12 + 20 = 32\,V$. This is *equivalent* to the same current ($4\,A$) flowing through an $8\,\Omega$ equivalent resistor.

Exercise 1. These exercises refer to the diagram below:



(a) If $R_1 = 6\,\Omega$ and $R_2 = 3\,\Omega$, what is the equivalent resistance R_T?

(b) If $R_1 = 0.08\,\Omega$ and $R_2 = 0.17\,\Omega$, what is the value of R_T?

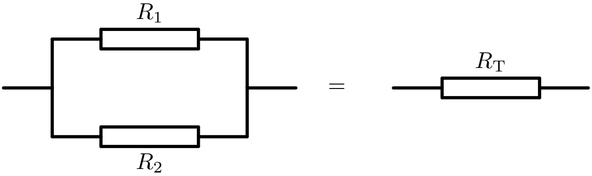
(c) If $R_1 = 0.5\,\Omega$ and $R_2 = 0.2\,\Omega$, what is the value of R_T?

(d) If the equivalent resistance is measured to be $R_T = 27\,\Omega$ and it is known that $R_2 = 15\,\Omega$, what must the value of R_1 be?

(e) If $R_1 = 2\,\Omega$ and $R_2 = 3\,\Omega$, and a current of $3\,A$ is measured to flow through them, what is the potential difference across each of the individual resistors and what is the total potential difference?

3. Resistors in Parallel

Two resistors connected **in parallel**, as in the diagram below,



are equivalent to a single resistor with resistance R_T given by

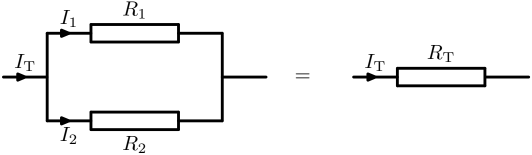
$$\boxed{\frac{1}{R_\mathrm{T}} = \frac{1}{R_1} + \frac{1}{R_2}}$$

A proof of this result is given on the next page.

In the above diagram there are two different ways for current to flow through the circuit, and so the equivalent resistance R_T is **less** than either of R_1 and R_2. This is a general property of resistors in parallel.

Proof:

It is helpful to draw the currents in the diagram:



It is important to realise that the potential difference across the resistors in parallel is the same, call it V. From Ohm's law:

$$V = I_1 R_1 = I_2 R_2 \qquad \therefore \; I_1 = \frac{V}{R_1} \quad \text{and} \quad I_2 = \frac{V}{R_2}$$

This is equivalent to a total resistor R_{T} with a current I_{T} flowing through it:

$$V = I_{\mathrm{T}} R_{\mathrm{T}} \qquad \therefore \; I_{\mathrm{T}} = \frac{V}{R_{\mathrm{T}}}$$

From Kirchoff's law, $I_{\mathrm{T}} = I_1 + I_2$, so from the equations above:

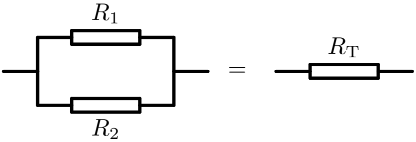
$$\frac{V}{R_{\mathrm{T}}} = \frac{V}{R_1} + \frac{V}{R_2}$$

Cancelling the common factor of V yields the desired result.

Example 2 If resistors of $4\,\Omega$, and $8\,\Omega$ are *added in parallel*, their equivalent (or total) resistance is given by:

$$\begin{aligned}\frac{1}{R_{\mathrm{T}}} &= \frac{1}{4}+\frac{1}{8}\\ &= \frac{2}{8}+\frac{1}{8}=\frac{3}{8} \qquad \textit{see the package on } \textbf{Fractions}\\ \therefore R_{\mathrm{T}} &= \frac{8}{3}\,\Omega\,.\end{aligned}$$

EXERCISE 2.
These exercises refer to the diagram. In each case, calculate the value of whichever of R_1, R_2 or R_T is *not* given.



(a) $R_1 = 6\,\Omega\,,\quad R_2 = 3\,\Omega$ (b) $R_1 = 6\,\Omega\,,\quad R_2 = 4\,\Omega$

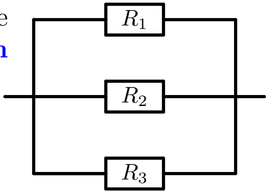
(c) $R_{\mathrm{T}} = 0.04\,\Omega\,,\quad R_2 = 0.2\,\Omega$ (d) $R_1 = 10^4\,\Omega\,,\ R_T = 2.5\times 10^3\,\Omega$

Quiz If two parallel connected resistors have a total resistance of $18\,\Omega$ and one is a $72\,\Omega$ resistor, what is the value of the other?

(a) $90\,\Omega$ (b) $54\,\Omega$ (c) $\frac{72}{5}\,\Omega$ (d) $24\,\Omega$

Any number of resistors in parallel can also be described in this way. Thus **three resistors in parallel** have total resistance:

$$\frac{1}{R_{\mathrm{T}}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$



Quiz If three resistors, $R_1 = 4\,\Omega$, $R_2 = 6\,\Omega$ and $R_3 = 12\,\Omega$, are added in parallel, what is their equivalent resistance?

(a) $22\,\Omega$ (b) $2\,\Omega$ (c) $36\,\Omega$ (d) $\frac{63}{144}\,\Omega$

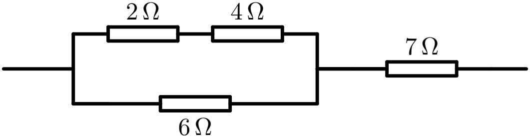
Quiz If four identical resistors, each of resistance $R = 8\,\Omega$, are added in parallel, what is their equivalent resistance?

(a) $2\,\Omega$ (b) $4\,\Omega$ (c) $32\,\Omega$ (d) $0.5\,\Omega$

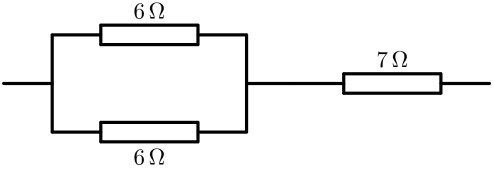
4. Resistor Combinations

The above rules can also be used to calculate the equivalent resistance of combinations of series and parallel resistors. To see how this is done consider the following example.

Example 2 To calculate the equivalent resistance of the following network



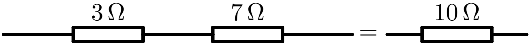
first combine those resistors in the parallel arrangement which are in series (here this is just on the top row):



Then add the resistors in parallel:

$$\frac{1}{R_{\mathrm{T}}} = \frac{1}{6} + \frac{1}{6} = \frac{1}{3} \quad \therefore R_{\mathrm{T}} = 3\Omega.$$

Now we have reduced the network to two resistors in series:

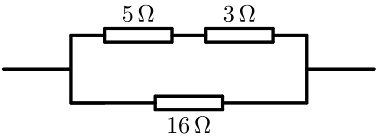


So the initial network is equivalent to a resistor of $10\,\Omega$.

In summary, the **general procedure** is:

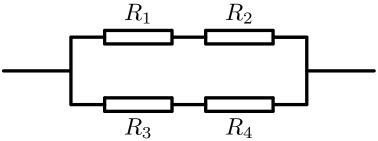
- first add up the series connected resistors that are part of a parallel arrangement;
- then calculate the equivalent resistance for all these equivalent parallel resistors (the network should then have the form of a series of resistors in series);
- finally add all these series resistors to obtain the total equivalent resistance.

Quiz What is the equivalent resistance of the network below?



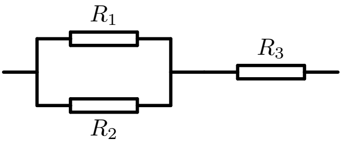
(a) $24\,\Omega$ (b) $\frac{16}{3}\,\Omega$ (c) $\frac{3}{16}\,\Omega$ (d) $2\,\Omega$

Quiz If $R_1 = 5\,\Omega$, $R_2 = 3\,\Omega$, $R_3 = 8\,\Omega$ and $R_4 = 4\,\Omega$, what is the equivalent resistance of the network below?



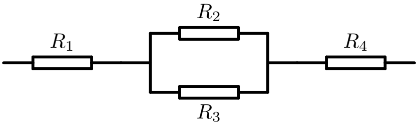
(a) $20\,\Omega$ (b) $\frac{211}{144}\,\Omega$ (c) $\frac{27}{5}\,\Omega$ (d) $\frac{24}{5}\,\Omega$

Quiz If $R_1 = 100\,\Omega$, $R_2 = 300\,\Omega$ and $R_3 = 40\,\Omega$, what is the equivalent resistance of the network below?



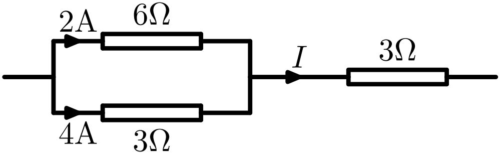
(a) $240\,\Omega$ (b) $115\,\Omega$ (c) $\dfrac{3004}{10}\,\Omega$ (d) $40.75\,\Omega$

Quiz If $R_1 = 5 \times 10^3\,\Omega$, $R_2 = 30\,\Omega$, $R_3 = 60\,\Omega$ and $R_4 = 400\,\Omega$, what is the equivalent resistance of the network below?



(a) $5490\,\Omega$ (b) $990\,\Omega$ (c) $920\,\Omega$ (d) $5420\,\Omega$

5. Final Quiz



Begin Quiz **All questions refer to the above circuit diagram.**

1. What is the current I?

(a) $\frac{6}{5}$ A (b) 6 A (c) 5 A (d) 1 A

2. What is the equivalent resistance of the two parallel resistors?

(a) $2\,\Omega$ (b) $1\,\Omega$ (c) $\frac{5}{6}\,\Omega$ (d) $\frac{6}{5}\,\Omega$

3. What is the total potential difference?

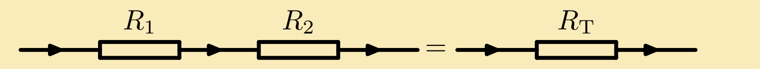
(a) 30 V (b) 22 V (c) $\frac{30}{13}$ V (d) $\frac{15}{4}$ V

End Quiz Score: Correct

Solutions to Exercises

Exercise 1(a)

If two resistors of $R_1 = 6\,\Omega$ and $R_2 = 3\,\Omega$ are added in series, as shown in the picture below



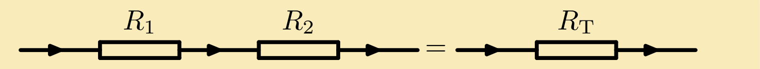
the equivalent resistance is given by:

$$R_\mathrm{T} = R_1 + R_2 = (6+3)\,\Omega = 9\,\Omega\,.$$

Click on the **green** square to return

Exercise 1(b)

If two resistors of $R_1 = 0.08\,\Omega$ and $R_2 = 0.17\,\Omega$ are added in series, as shown in the picture below



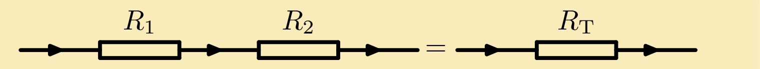
their total resistance R_T is :

$$R_\mathrm{T} = R_1 + R_2 = (0.08 + 0.17)\,\Omega = 0.25\,\Omega\,.$$

Click on the **green** square to return

Exercise 1(c)

If two resistors of $R_1 = 0.5\,\Omega$ and $R_2 = 0.2\,\Omega$ are added in series, as shown in the picture below

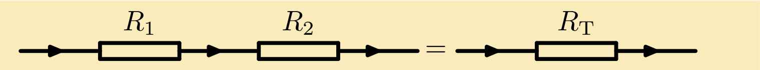


their total resistance R_{T} is :

$$R_{\mathrm{T}} = R_1 + R_2 = (0.5 + 0.2)\,\Omega = 0.7\,\Omega\,.$$

Click on the **green** square to return $\square$

Exercise 1(d)
If an unknown resistor, R_1, and a resistor $R_2 = 15\,\Omega$, are added in series, as shown in the picture below



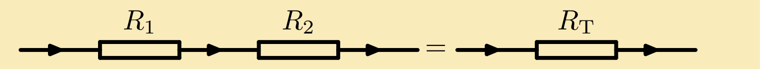
and if the equivalent resistance is $R_\mathrm{T} = 27\,\Omega$ then $R_\mathrm{T} = R_1 + R_2$. Therefore the resistance R_1 is found from the equation

$$R_1 = R_\mathrm{T} - R_2 = (27 - 15)\,\Omega = 12\,\Omega\,.$$

Click on the **green** square to return □

Exercise 1(e)

If a current of $3\,A$ flows through the system shown in the picture below



then, from Ohm's law ($V = IR$), there is a potential difference of

$$V_1 = IR_1 = 3\,A \times 2\,\Omega = 6\,V$$

across R_1 and a potential difference of

$$V_2 = IR_2 = 3\,A \times 3\,\Omega = 9\,V$$

across R_2. The total potential difference is thus

$$V_T = V_1 + V_2 = (6 + 9)\,V = 15\,V\,.$$

N.B. This is equivalent to the same current ($3\,A$) flowing through an equivalent resistor $R_{\mathrm{T}} = R_1 + R_2 = (2 + 3)\,\Omega = 5\,\Omega$.

Click on the **green** square to return $\square$

Exercise 2(a)

If two resistors $R_1 = 6\,\Omega$ and $R_2 = 3\,\Omega$ are added in parallel, the total resistance R_T is determined through the equation:

$$\begin{aligned}\frac{1}{R_\mathrm{T}} &= \frac{1}{6} + \frac{1}{3}\\ &= \frac{1}{6} + \frac{2}{6}\\ &= \frac{3}{6} = \frac{1}{2}\,,\end{aligned}$$

therefore $R_\mathrm{T} = 2\,\Omega\,.$

Click on the **green** square to return

□

Exercise 2(b)

If two resistors $R_1 = 6\,\Omega$ and $R_2 = 4\,\Omega$ are added in parallel, the total resistance R_{T} is determined through the equation:

$$\begin{aligned}\frac{1}{R_{\mathrm{T}}} &= \frac{1}{6} + \frac{1}{4}\\ &= \frac{2}{12} + \frac{3}{12}\\ &= \frac{5}{12}\,,\end{aligned}$$

therefore $R_{\mathrm{T}} = \dfrac{12}{5}\,\Omega = 2.4\,\Omega\,.$

Click on the **green** square to return

□

Exercise 2(c)
If two resistors, one of unknown resistance R_1, and another of $R_2 = 0.2\,\Omega$ are added in parallel, and the total resistance is $R_{\mathrm{T}} = 0.04\,\Omega$ then the resistance of R_1 is determined through the equation:

$$\begin{aligned}\frac{1}{R_1} &= \frac{1}{R_{\mathrm{T}}} - \frac{1}{R_2} \\ &= \frac{1}{0.04} + \frac{1}{0.2} \\ &= \frac{100}{4} - \frac{10}{2} = 25 - 5 = 20\,,\end{aligned}$$

therefore $R_1 = \dfrac{1}{20}\,\Omega = 0.05\,\Omega\,.$

Click on the **green** square to return $\square$

Exercise 2(d)

If two resistors, one of $R_1 = 10^4\,\Omega$, and another of unknown resistance R_2 are added in parallel, and the total resistance is $R_\mathrm{T} = 2.5 \times 10^3\,\Omega$ then the resistance of R_1 is determined through the equation:

$$\begin{aligned} \frac{1}{R_2} &= \frac{1}{R_\mathrm{T}} - \frac{1}{R_1} \\ &= \frac{1}{2.5 \times 10^3} - \frac{1}{10^4} = \frac{10}{2.5} \times 10^{-4} - 1 \times 10^{-4} \\ &= (4-1) \times 10^{-4} = 3 \times 10^{-4}\,, \end{aligned}$$

therefore $R_2 = \dfrac{1}{3 \times 10^{-4}}\,\Omega = \dfrac{1}{3} \times 10^4\,\Omega = 3.3 \times 10^3\,\Omega\,.$

Click on the **green** square to return □

Solutions to Quizzes

Solution to Quiz:

If a potential difference of $1.5\,V$ is applied to a wire and an electric current of $0.045\,A$ is measured to pass through it, the resistance R of the wire is given by

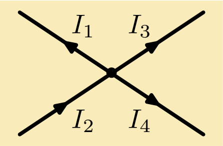
$$R = \frac{V}{I} = \frac{1.5\,V}{0.045\,A} = \frac{1}{0.03}\,\Omega = 33.3\,\Omega\,.$$

In this calculation the resistance is calculated in units of **ohms**

$$\Omega = \text{volts} \times \text{ampere}^{-1}\,.$$

End Quiz

Solution to Quiz:



According to **Kirchoff's law for currents**: the total current that flows into a junction must flow out of it. In the junction shown in the picture I_2 flows in and I_1, I_3 and I_4 flow out, therefore:

$$I_2 = I_1 + I_3 + I_4\,.$$

End Quiz

Solution to Quiz:

If two parallel connected resistors have a total resistance of $R_{\mathrm{T}} = 18\,\Omega$ and one resistor is $R_1 = 72\,\Omega$, then the resistance of R_2 is found from the equation:

$$\begin{aligned}\frac{1}{R_2} &= \frac{1}{R_{\mathrm{T}}} - \frac{1}{R_1} \\ &= \frac{1}{18} - \frac{1}{72} = \frac{4}{72} - \frac{1}{72} \\ &= \frac{3}{72} = \frac{1}{24}\,,\end{aligned}$$

therefore $R_2 = 24\,\Omega\,.$

End Quiz

Solution to Quiz:
If three resistors, $R_1 = 4\,\Omega$, $R_2 = 6\,\Omega$ and $R_3 = 12\,\Omega$, are added in parallel, then the total resistance R_{T} is determined through the equation:

$$\begin{aligned}\frac{1}{R_{\mathrm{T}}} &= \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \\ &= \frac{1}{4} + \frac{1}{6} + \frac{1}{12} \\ &= \frac{3}{12} + \frac{2}{12} + \frac{1}{12} \\ &= \frac{6}{12} = \frac{1}{2}\,,\end{aligned}$$

therefore $R_{\mathrm{T}} = 2\,\Omega\,.$

End Quiz

Solution to Quiz:
If four identical resistors, each of resistance $R = 8\,\Omega$, are added in parallel, then the total resistance R_T is determined via the equation:

$$\begin{aligned}\frac{1}{R_\mathrm{T}} &= \frac{1}{R}+\frac{1}{R}+\frac{1}{R}+\frac{1}{R}=\frac{4}{R}\\ &= \frac{4}{8}=\frac{1}{2}\,,\end{aligned}$$

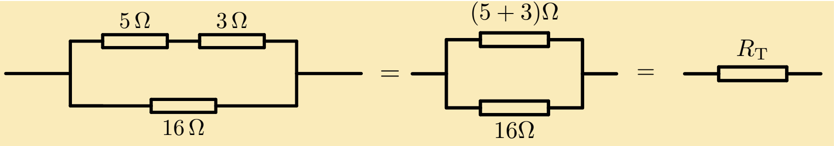
therefore $R_\mathrm{T} = 2\,\Omega$.

N.B. If n identical resistors are combined in parallel, then the total resistance is

$$R_\mathrm{T} = \frac{R}{n}\,.$$

End Quiz

Solution to Quiz:
We have to calculate the equivalent resistance of the network below

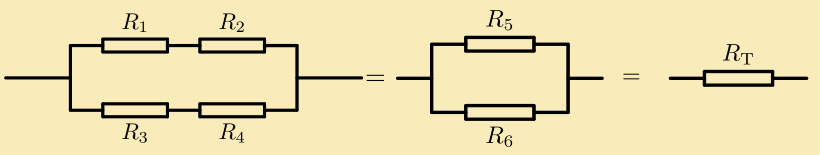


Combining in parallel the resistance $R_1 = (5+3)\,\Omega = 8\,\Omega$ obtained in the first step with the resistance $R_2 = 16\,\Omega$, one can calculate the total resistance via

$$\begin{aligned}\frac{1}{R_T} &= \frac{1}{8} + \frac{1}{16}\\ &= \frac{2}{16} + \frac{1}{16} = \frac{3}{16}\,.\end{aligned}$$

Thus $R_T = \dfrac{16}{3}\,\Omega$. End Quiz

Solution to Quiz:
The calculation of the equivalent resistance of the network is shown in the picture below

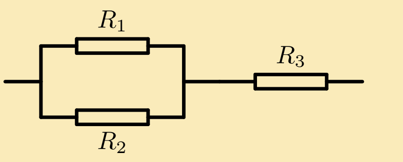


First combine the resistors in series. From $R_1 = 5\,\Omega$ and $R_2 = 3\,\Omega$, we have $R_5 = (5+3)\,\Omega = 8\,\Omega$ and similarly from $R_3 = 8\,\Omega$ and $R_4 = 4\,\Omega$, $R_6 = (8+4)\,\Omega = 12\,\Omega$. Now the equivalent resistors R_5 and R_6 are in parallel, so R_T is found from the equation

$$\frac{1}{R_\mathrm{T}} = \frac{1}{R_5} + \frac{1}{R_6} = \frac{1}{8} + \frac{1}{12} = \frac{3+2}{24} = \frac{5}{24}\,,$$

so $R_\mathrm{T} = \dfrac{24}{5}\,\Omega$. End Quiz

Solution to Quiz:



To find the equivalent resistance of this network, we first calculate the equivalent resistance R_{12} of $R_1 = 100\,\Omega$ and $R_2 = 300\,\Omega$ in parallel

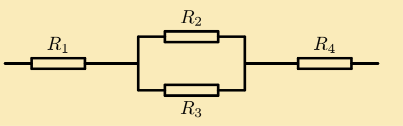
$$\begin{aligned}\frac{1}{R_{12}} &= \frac{1}{R_1} + \frac{1}{R_2} \\ &= \frac{1}{100} + \frac{1}{300} = \frac{3+1}{300} = \frac{4}{300} = \frac{1}{75}\,,\end{aligned}$$

so $R_{12} = 75\,\Omega$. Now R_{12} and $R_3 = 40\,\Omega$ are in series, therefore the total resistance is

$$R_{\mathrm{T}} = R_{12} + R_3 = (75 + 40)\,\Omega = 115\,\Omega\,.$$

End Quiz

Solution to Quiz:



To find the total resistance of the network shown in the picture, first calculate the equivalent resistance R_{23} of $R_2 = 30\,\Omega$ and $R_3 = 60\,\Omega$ in parallel

$$\begin{aligned}\frac{1}{R_{23}} &= \frac{1}{R_2} + \frac{1}{R_3} \\ &= \frac{1}{30} + \frac{1}{60} = \frac{2}{60} + \frac{1}{60} = \frac{3}{60} = \frac{1}{20}\,,\end{aligned}$$

so $R_{23} = 20\,\Omega$. Now this resulting resistor R_{23} and $R_1 = 5 \times 10^3\,\Omega$ and $R_4 = 400\,\Omega$ are all three in series, so the total resistance is

$$R_{\mathrm{T}} = R_1 + R_{23} + R_4 = (5000 + 20 + 400)\,\Omega = 5420\,\Omega\,.$$

End Quiz