Simple pendulum and properties of simple harmonic motion, virtual lab

Purpose

1. Understand simple harmonic motion (SHM).

2. Study the position, velocity and acceleration graphs for a simple harmonic oscillator (SHO).

3. Study SHM for (a) a simple pendulum; and (b) a mass attached to a spring (horizontal and vertical).

Introduction

Any motion that repeats itself at definite intervals of time is said to be a periodic motion. If the motion carries the system back and forth, then the motion is said to be oscillatory (or vibrating). A special type of oscillatory motion is called simple harmonic motion (SHM). A system undergoing SHM is said to be a simple harmonic oscillator (SHO). All simple harmonic motions are periodic motions. But not all periodic motions are simple harmonic. For example, the orbit of the earth about the sun is a periodic motion, but it is not SHM. Whereas, the oscillatory motion of a simple pendulum is a SHM, and since it repeats the motion in definite intervals of time called the period, *T*, it a periodic motion. The precise definition of a simple harmonic motion is that the net force, $\vec{F}$ on the simple harmonic oscillator has a magnitude that is proportional to the displacement from some equilibrium position, $\overrightarrow{\Delta x}$. The force also has a direction that always points towards the equilibrium position and is therefore known as a "restoring" force:

$$\vec{F}_{\text{restoring}} = -c\,\overrightarrow{\Delta x}. \qquad (1)$$

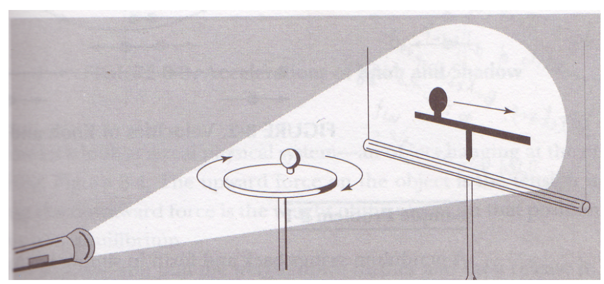

Figure 1: SHM is the projection of uniform circular motion. (From Robert L. Lehrman, *Physics The Easy Way*, 3rd edition, ch. 8, Barron's, 1998)

Any elastic object (like a stretched rubber band, or a stretched guitar string, or a mass attached to a spring, etc…) obeys Hooke's law, for which the constant "c" in the above equation is the spring constant, k. Such objects will undergo simple harmonic motion for small displacements and when ignoring dissipative forces like friction.

Simple harmonic motion can be represented mathematically by the projection of a uniform circular motion on the x axis (or y axis), see Fig. 1. Uniform circular motion is not SHM, but the *projection* of uniform circular motion is SHM.[1]

Since simple harmonic motion can be represented as the projection of uniform circular motion, it can be shown that, the displacement of the SHO in one direction can be written (see Fig. 2) as:

$$x = A\sin(wt + \phi). \qquad (2)$$

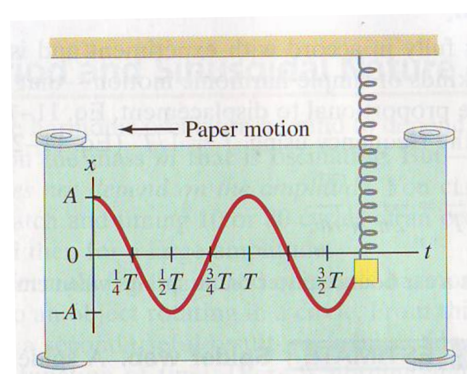

Figure 2: Displacement of a SHO as a function of time. (From Douglas C. Giancoli, *Physics: Principles with Applications,* 6th edition, Ch. 11, Pearson Prentice Hall, 2005)

Notice that the displacement x of a simple harmonic oscillator is

[1] For a simulation of the projection circular motion into component SHMs, see: https://www.animations.physics.unsw.edu.au/jw/SHM.htm#projection

sinusoidal (a sine or cosine function). A is the amplitude of the oscillation (the maximum displacement or the radius of the corresponding uniform circular motion) and ϕ is the phase constant (or phase angle). A and ϕ are determined by the initial displacement and the initial velocity of the oscillator.

If the displacement of the oscillator is as given in Eq. (2), then the velocity of the oscillator is:

$$v = v_{max} \cos(\omega t + \phi), \qquad (3)$$

and the acceleration of the oscillator is

$$a = a_{max} sin(\omega t + \phi), \qquad (4)$$

where $v_{max} = \omega A$, and $a_{max} = -\omega^2 A$, where ω is the angular speed of the corresponding uniform circular motion. Since ω is constant (as required by ***uniform*** circular motion), and one cycle takes an angle of 2π in a time of one period time, T, then $\omega = \frac{2\pi}{T}$. T is the period – the time taken for one complete oscillation. Comparing the expressions above for displacement and acceleration we get

$$a = -\omega^2 x. \qquad (5)$$

This last equation is equivalent to the statement of SHM in Eq. (1), since force and acceleration are related by Newton's 2nd Law.

For the simple pendulum, $F_{\text{restoring}} = -mg sin\theta$ where θ is the angle made by the string to the vertical (see Fig. 3). The negative sign is again because $F_{\text{restoring}}$ opposes any increase in θ. The motion of a simple pendulum is simple harmonic in the limit the mass of the string is negligible compared to the mass of the pendulum bob (the metal sphere attached to the string), and that the string does not stretch (inextensible).

For a small displacement angle, θ, $-mg sin\theta \simeq -mg\theta$. But $F = ma$ (Newton's 2nd Law), and using Eq. (5) above and Fig. 3,

$$-mg\,\theta = -m\omega^2 x.$$

Since $x = L\theta$, we have that $-mg\theta = -m\,\omega^2 L\theta$ and so:

$$T = 2\pi\sqrt{L/g}. \qquad (6)$$

Can you derive this last step for T?

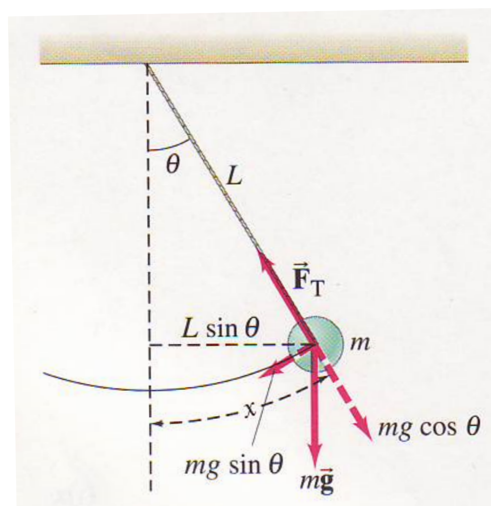

Figure 3: A simple pendulum. (From Douglas C. Giancoli, *Physics: Principles with Applications,* 6th edition, Ch. 11, Pearson Prentice Hall, 2005)

Running the experiment **The data sheets are on page 6**

Part 1, Dependence of time period, T, on the length of the pendulum

1) Open the simulator: https://phet.colorado.edu/sims/html/pendulum-lab/latest/pendulum-lab_en.html Click on *introduction*. Notice that the mass is set to 1 kg. Adjust the length L of the string to 0.7 m. There is no friction and gravity is due to that of the Earth. Click the stop watch at the bottom left of the screen. Do not start it yet. Drag the hanging mass a small angle (5^{o}) and release it. When the pendulum is at maximum displacement, start the stop watch and count for 5 cycles, then as soon as you reach 5 cycles, stop the stopwatch. Record the total time, and divide it by 5 to obtain an "experimental" value of the oscillation period (why)? Now, calculate the expected period of the oscillation using Eq. (6). Compare this value to your measured value.

2) Repeat step 1 for L = 1 m.

3) With L = 1 m, repeat the simulation, but with a different initial angle, $\theta_o = 10^{o}$.

> a) Does the period, T depend on the initial angle?
>
> b) Can you explain physically what happens? (Hint: think of the restoring force, and also the distance the bob travels).

4) Repeat step 3 with the value of the mass changed to 0.5 Kg.

> a) Does the period, T change?
>
> b) Can you explain physically what happens when the mass m is decreased? (Hint: think of the restoring force and also the inertia).

Part 2: Position, velocity and acceleration of the simple pendulum

1) Open a new simulator: https://www.myphysicslab.com/pendulum/pendulum-en.html
2) Select *Tme Graph* in the top menu. Click the *pause* symbol under the graph, then click *clear graph*, then select *angle acceleration* in line 3. Click the *play symbol* to start the oscillation. The green graph represents the angle, the red graph represents the angle velocity, and the blue graph represents the angular acceleration. After a few cycles, pause the simulator.
    a) What is the magnitude of the phase difference (the difference in angle) between the angle velocity graph (red) and the angle graph (green)?
    b) What is the phase difference between the angular acceleration graph (blue) and the angle graph (green)?
    c) Explain your answers to questions a and b? (Hint: think in terms of the motion of the pendulum at its max displacement points and as it passes the equilibrium point).

Part 3: A mass on a spring on a horizontal frictionless table

As stated in the Introduction, for a mass attached to a spring, $F_{\text{restoring}} = -k\,\Delta x$ where k is called the spring constant (or stiffness). Accordingly, $F = -k\Delta x = ma = -m\,\omega^2\Delta x$ and so we arrive at

$$T = 2\pi\sqrt{m/k} \qquad (7)$$

1) Open a new simulator: http://physics.bu.edu/~duffy/HTML5/mass_on_spring_graphs.html You can see a mass attached to a spring on a horizontal, frictionless table. Use the default attached mass, m = 2 kg and spring stiffness, k = 2 N/m. Play the simulation and notice the graph of the position.

a) What is its form?

b) Is it sinusoidal?

c) Does this confirm SHM? (Hint: See Fig. 2, in the Introduction).

2) Measure the Period T, from the graph. Using Eq. (7), calculate the theoretical period T $= 2\pi\sqrt{m/k}$. Compare the two values.

3) Reset the graph and click the velocity graph. What is the phase difference between the position and the velocity graph?

4) Reset the graph and click the acceleration graph. What is the phase difference of the acceleration from the position graph?

5) What do you expect to happen to the period if m is increased? Explain in terms of physics. (Hint: think in terms of inertia).

6) What do you expect to happen to the period if k is increased? Explain in terms of physics. (Hint: Think in terms of $F_{restoring}$).

7) Now increase m in the simulator to 3 kg. What happens? Was your expectation correct? Measure the period and compare it to the theoretical value, which you should calculate using Eq. (7).

8) Set m back to 2 kg, and increase k to 3 N/m, in the simulator and check your prediction for the effect on the oscillation of increasing k. Was your expectation correct? Measure the period and compare it again to the theoretical value.

Part 4: A mass attached to a vertical spring

When a mass is attached to a *vertical* spring and the mass is in equilibrium (not moving), then

$$mg = k\Delta x \qquad (8)$$

where Δx is the vertical, downward displacement. If we measure Δx we can calculate k.

If the mass is pulled a little more displacement so that the spring is stretched and the system is set in oscillation motion, then it undergoes simple harmonic motion, SHM. See Fig. 2 given in the Introduction.

1) Open a new simulator https://phet.colorado.edu/sims/html/masses-and-springs/latest/masses-and-springs_en.html
2) Click on *Intro*. Pause the simulator, using the pause button at the lower right of the page. Keep all the default settings. We will call the left spring (the one towards your left hand), spring 1 and we will only use spring 1. Grab the stopwatch that is on the right part of the simulator, and drag it to the left of the page, next to spring 1. Check the box for *Equilibrium Position* at the top right of the screen. This will display the equilibrium position for the mass when it is attached to the spring. Also, check the box for *Natural position*. This displays the position when the mass is not attached to the spring. Notice that the damping is set to 0 (no energy is lost, meaning no energy is dissipated, so the oscillations are sustained).
3) With the simulator paused, attach the 100 grams mass to spring , and using the simulator ruler, measure Δx, the displacement between the normal position and the equilibrium position.
4) Then using Eq. (8) above, calculate k.
5) Now attach the 100 g mass to spring 1. Play the simulator. Using the stopwatch, measure the time for 10 complete cycles, (starting from down, lowest position, and then up, then hitting down again: is an example of one complete cycle). Calculate the measured period by dividing the total time by 10. We will call this value T_1. Pause the simulator. Remove the 100 grams mass from the hook of the spring.
6) Now attach an unknown mass to the same spring, the blue medium one, in place of the 100g and repeat the previous step to measure the period in this instance. We will call this value T_2. Using T_1 and T_2, calculate the value of the unknown blue mass. Note that here you do not need the value of k. Why? (Hint: Think of dividing the expressions for T_1 and T_2). Show your work and your answer here. (You can then copy it to your lab report)
7) With the hanging mass still set to 100 g, what will happen if the initial stretch of the spring with the mass (to set it into oscillation) is a little larger, i.e. larger amplitude, A, (not too much larger, so as not to deform the spring)?
    a) what do you expect to happen to the period, T? Now, try it (use the mouse to drag the mass for a little larger initial displacement, they play.
    b) Measure the period
    c) Does period, T depend on amplitude, A?
    d) Explain in terms of physics. (Hint: think in terms of the restoring force and also the distance that needs to be travelled for one complete cycle).

Data sheet

Name: Group: Date experiment performed:

Part 1:

Step 1) L= 0.7 m: T calculated= T measured=

Step 2) L = 1 m: T calculated= T measured=

Answer for step 3) a : Answer for step 3) b:

Answer for step 4) a : Answer for step 4) b:

Part 2:

Answer for step 2) a : Answer for step 2) b: Answer for step 2) c:

Part 3:

Answer for step 1) a : Answer for step 1) b: Answer for step 1) c:

Step 2) T measured from graph= T calculated (show the eqn.)=

Step 3) answer:

Step 4) answer:

Answer for step 5): Answer for step 6:

Answer for step 7):

Answer for step 8) : T measured= T calculated=

Part 4:

Step 4) k calculated=

Step 5) T_1= Step 6) T_2= Calculation of unknown mass (show your work):

Step 7) a: b: c: d: