# Simplex Optimization of Production Mix: A Case of Custard Producing Industries in Nigeria 

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#### Abstract

The study used two custard producing industries, LCI and KFGI as case studies in which time study of their various kinds of labour in custard production as well as the associated sizes of custard and costs were analysed. With the obtained results, a mathematical model was set up using simplex method in which the problem was converted into its standard form of linear programming problem. Simplex method of optimization was used in determining the optimal production proportion and profit margins. The case study of LCI results gave an optimal production mix of $45.8 \%, 39.6 \%$ and $14.6 \%$ for large, medium and small sized custard, respectively, with an increase in profit margin to $49.8 \%$ translating into $¥ 728,142: 00$ for the month of March, 2013. Similarly, KGFI have an optimal production mix of $43.5 \%, 36.5 \%$ and $20 \%$ for large, medium and small sized custard with an increase in profit margin to $51.5 \%$ translating into $\# 905,423.00$ for the month of August, 2013.


Keywords: Custard Production, Simplex Optimization, Optimal Mix, Simulation

## Introduction

Custard powder is a popular dessert item and is often served with egg pudding and jelly crystals. With increasing disposable incomes and changing life-styles, eating habits have also witnessed a definite shift. This phenomenon is no more restricted to the urban elites but it is spreading very fast to other areas as well. Thus, the contemplated products have experienced continuous increase in demand during last few years. Consistent advertising by some established manufacturers have made these products very popular which would help the new entrants provided the product quality is comparable and prices are competitive. Custard industry in Nigeria is one of the fastest food related growing industries in the country. Worthy of note is the fact that most of these industries are faced with the problem of optimizing production cost and the corresponding quantity of the product to meet with the customers demand. They are concerned with the rate of productivity which can be related to the efficiency of the production system. It could equally be seen as a ratio to measure how well an organization (or individual, industry, country) converts input resources (labour, materials, machines, etc.) into goods and services.
Even though there is increase in demand, consumers have become more and more demanding, and the key to firm survival is the recognition of the importance of customer satisfaction. Consequently, companies have been forced to enhance the quality of both their processes and products (Efstratiadis et al., 2000). The focus of this study, the food industry, has also become increasingly multifaceted and competitive in recent years (Chong et al., 2001; Knowles et al., 2004; Spiegel et al., 2006). In this environment, company managers have to deal with a number of problems. Sales are slowing down and operating costs are increasing, while customers are becoming more demanding and selective (Henchion and McIntyre, 2005). Considering the above, this study intends to carry out simplex optimization with a view to determining the optimal production mix for custard producing industries.
In an optimization process, one usually begins with a real life problem, full of details and complexities, some relevant and some not. From this, essential elements are extracted and an algorithm or solution technique to apply to it. In practical problems, the computer will carry out the necessary calculations.


Fig 1: Optimization Process
Chinneck (2000) observes that there is an unavoidable loss of realism as one moves down the diagram in Fig 1 from real world problem to algorithm, model, or solution technique, and finally to computer implementation. After computer implementation, verification takes place. Verification refers to the process of confirming that the simulation model is correctly translated into a computer programme (Okonkwo, 2009). The process of testing and improving a model to increase its validity is commonly referred to as validation (Harvey, 1989). Hence, validation was done to ensure that the model is a true representation to what is obtainable in the real system. Similarly, sensitivity analysis is used to determine how sensitive the model parameters can be when varied, respectively.
Now coming to linear programming model of optimization, Dibua (2004) explained that linear programming model is best applied where a manufacturer wants to develop a production schedule/target and an inventory policy that will satisfy sales demand in future period. Ideally, the schedule and policy will enable the production company to satisfy demand and at the same time minimize the total production and inventory costs". Sonder (2005) observed that linear programming works by searching for the basic feasible solution and ends with the search of optimum solution. This goes to explain simplex method in a more understandable manner. Similarly, Martand (2003) submitted that linear programming could be used to solve the task of production planning and control in production processes which can be seen as highly complex in manufacturing environments while Everette (2003) said that linear programming could be used to provide uninterrupted production by optimizing production processes for efficiency.
Dantzig (1963) formulated a model which satisfactorily represented the technological relations usually encountered in practice. He decided that the ad hoc ground rules had to be discarded and replaced by an explicit objective function. He formulated the planning problem in mathematical terms using a set of axioms. The axioms concerned the relations between two kinds of sets: the first was the set of items being produced or consumed and the second, the set of activities or production processes in which these items would be inputted or outputted in fixed proportions provided these proportions are non-negative multiples of each other. Having seen that the model developed by George B. Dantzig is practically feasible, especially in production outfit such as custard, bread, paint industries, hence the adoption of the model for this study because simplex method technique rests on two concepts viz feasibility and optimality (start with the basic feasible solution or programme and goes ahead to search for optimum solution) and in graphical methods we search only the extreme point solutions. The solution is tested for optimality and if it is optimum, the search is stopped and if the test shows that it is not optimum, a new and better feasible solution is designed and that is guaranteed by the mechanics of the simplex or iterative method because each successive solution is designed only if it is better than each of the previous solutions. Repetition goes on until optimum solution is obtained.

## Steps for Simplex Optimization

To apply the simplex/iterative method, it is necessary to state the problems in the form in which the inequalities in the constraints have been converted to equalities because it is not possible to perform arithmetic calculations upon an inequality. The inequalities in maximization problems are converted to equalities with the aid of slack variables to the left hand side of each inequality. The slack variables in maximizing problems represent any unused capacity in the constraint and its value can take from zero to the maximum of that constraint. Each constraint has its own separate slack variable. The inequalities in the minimization problems are converted into equalities by subtracting one surplus variable.

The simplex method for linear programming model follows the under listed steps:
i) Design the sample problem.
ii) Setup the inequalities describing the problem
iii) Convert the inequalities to equations adding slack variables.
iv) Enter the inequalities in a table for initial basic feasible solutions with all slack variables as basic variables. The table is called simplex table.
v) Compute $C_{j}$ and $P_{j}$ values for this solution where $C_{j}$ is objective function coefficient for variable $j$ and $P_{j}$ represents the decrease in the value of the objective function that will result if one unit of variable corresponding to the column is brought into the solution.
vi) Determine the entering variable (key column) by choosing the one with the highest $\mathrm{C}_{\mathrm{j}}-\mathrm{P}_{\mathrm{j}}$ value.
vii) Determine the key row (outgoing variable) by dividing the solution quantity values by their corresponding solution quantity values by their corresponding key column values and choosing the smallest positive quotient. This means that we compute the ratios for rows where elements in the key column are greater than zero.
viii) Identify the pivot element and compute the values of the key row by dividing all the numbers in the key row by the pivot element. Then change the product mix to the heading of the key column.
ix) Compute the values of the other non-key rows
x) Compute the $P_{j}$ and $C_{j}-P_{j}$ values for this solution.
xi) If the column value in the $\mathrm{C}_{\mathrm{j}}-\mathrm{P}_{\mathrm{j}}$ row is positive, return to step (vi).

If there is no positive $\mathrm{C}_{\mathrm{j}}-\mathrm{P}_{\mathrm{j}}$, then the final solution has been reached

## Linear Programming Model for the Case Studies

Max. $\mathrm{P}=\sum_{j=i}^{n} C_{j} X_{j}$
Subject to the linear constraints
$\sum_{j=i}^{n} a i_{j} X_{j} \leq \mathrm{T}_{\mathrm{i}} ; \mathrm{i}=1,2, \ldots ., \mathrm{m}$
And $X_{j} \geq 0 ; j=1,2$,
Equation 1 can be stated in generalized form with n decision variables and m constraints as follows in this form:

$$
\begin{array}{ll}
\text { Max. } \mathrm{P}= & \mathrm{ax}_{1}+\mathrm{bx} x_{2}+\mathrm{cx}_{3}  \tag{2}\\
& d x_{1}+e x_{2}+f x_{3} \leq T_{1} \\
& g x_{1}+h x_{2}+i x_{3} \leq T_{2} \\
& j x_{1}+k x_{2}+l x_{3} \leq T_{3} \\
& m x_{1}+n x_{2}+p x_{3} \leq T_{4} \\
& q x_{1}+r x_{2}+s x_{3} \leq T_{5} \\
& u x_{1}+v x_{2}+w x_{3} \leq T_{6} \\
& \left(x_{1}, x_{2}, x_{3} \geq 0\right)
\end{array}
$$

Where: $\mathrm{x}_{1}, \mathrm{x}_{2}, \mathrm{x}_{3}$ are the non-basic variables; and $\mathrm{T}_{1}, \mathrm{~T}_{2}, \mathrm{~T}_{3}, \mathrm{~T}_{4}, \mathrm{~T}_{5}, \mathrm{~T}_{6}$ are the total available time.
To solve the mathematical set up model shown above using the simplex method, it requires that the problem be converted into its standard form of linear programing problem:
i. All constraints should be expressed as equations by adding slack variables or surplus variables.
ii. The right-hand side of each constraint should be made non-negative if it is not already. This should be done by multiplying both sides of the resulting constraints by -1 .
iii. The objective function should be of the maximization type.

Equation (1) can be expressed as;
$\operatorname{MaxP}=\sum_{j=i}^{n} C_{j} X_{j}+\sum_{j=i}^{m} O S_{j}$
Subject to the constraints
$\sum_{j=i}^{n} a i_{j} X_{j}+\mathrm{S}_{\mathrm{i}}=\mathrm{T}_{\mathrm{i}} ; \mathrm{i}=1,2, \ldots \ldots \ldots \mathrm{~m}$
And $\mathrm{X}_{\mathrm{j}}, \mathrm{Si}, \geq 0$, for all i and j
Equation (3) can be expressed as;
Thus; Max. $\mathrm{P}=a x_{1}+b x_{2}+c x_{3}+O S_{1}+O S_{2}+O S_{3}+O S_{4}+O S_{5}+O S_{6}$
Subject to the constraints;

$$
\begin{aligned}
& d x_{1}+e x_{2}+f x_{3}+S_{1}+O S_{2}+O S_{3}+O S_{4}+O S_{5}+O S_{6}=T_{1} \\
& g x_{1}+h x_{2}+i x_{3}+O S_{1}+S_{2}+O S_{3}+O S_{4}+O S_{5}+O S_{6}=T_{2} \\
& j x_{1}+k x_{2}+l x_{3}+O S_{1}+O S_{2}+S_{3}+O S_{4}+O S_{5}+O S_{6}=T_{3} \\
& m x_{1}+n x_{2}+p x_{3}+O S_{1}+O S_{2}+O S_{3}+S_{4}+O S_{5}+O S_{6}=T_{4} \\
& q x_{1}+r x_{2}+s x_{3}+O S_{1}+O S_{2}+O S_{3}+S_{4}+O S_{5}+O S_{6}=T_{5} \\
& u x_{1}+v x_{2}+w x_{3}+O S_{1}+O S_{2}+O S_{3}+S_{4}+O S_{5}+O S_{6}=T_{6} \\
& x_{1}, x_{2}, x_{3}, S_{1}, S_{2}, S_{3}, S_{4} S_{5}, S_{6} \geq 0 \text { [non-negative] }
\end{aligned}
$$

where
$\mathrm{x}_{1}, \mathrm{x}_{2}$ and $\mathrm{x}_{3}$ are quantities of the big custards, medium custards and small custards respectively called the nonbasic variables.
$S_{1}, S_{2}, S_{3}$ and $S_{4}$ are the slack variables used to eliminate the inequalities generated in the objective function of the LP model set up.
$\mathrm{P}_{\mathrm{j}}=$ Expected profit to be made after optimization called the total gross amount for outgoing profit.
$\mathrm{C}_{\mathrm{j}}-\mathrm{P}_{\mathrm{j}}=$ Net evaluation row for the objective function of the LP model called decision variable.
$\mathrm{C}_{\mathrm{j}}=$ objective function coefficients
$\mathrm{T}_{1}, \mathrm{~T}_{2}, \mathrm{~T}_{3}, \mathrm{~T}_{4}, \mathrm{~T}_{5}, \mathrm{~T}_{6}=$ Total available time constants
$\mathrm{a}, \mathrm{b}, \mathrm{c}, \mathrm{d}, \mathrm{e}, \mathrm{f}, \mathrm{g}, \mathrm{h}, \mathrm{i}, \mathrm{j}, \mathrm{k}, \mathrm{I}, \mathrm{m} \mathrm{n}, \mathrm{p}, \mathrm{q}, \mathrm{r}, \mathrm{s}, \mathrm{u}, \mathrm{v}$ and w are the process available time constants.

## Application of Simplex Optimization to the Case Studies

Both LCI and KGFI were used as case studies. The operations of the two case studies are quite similar. Hence, detailed explanations of how it was applied to LCI was done while only the essential results were shown in that of KGFI. The official working hours of the LCI factory staff is from 8 am to 5 pm , which is 9 hours. The production period for the batch was assumed to be carried out for 7 hours in a day, while the balance of 2 hours is used for break and down time periods. Also, it was assumed that there was no production after official working hours. The company produces three sizes of custard: the big size custard, the medium size custard and the small size custard. These three sizes of custard require six kinds of labour: premixing, mixing, weighing, sealing, packaging and bagging. The six kinds of labour for the production process do not start at the inception of production. At the first hour of production, some kinds of labour must be completed before others start. Subsequently, they will now begin to go on simultaneously.
After carrying out time study, the average labour time per day for each labour as observed from the factory is as shown in Table 1.

Table 1: Average Processing Time per Day for LCI

| Labour | Average Processing Time (Minutes) |  |  |
| :---: | :---: | :---: | :---: |
|  | $\mathbf{1}^{\text {st }}$ hour | 6 remaining hours | Per day |
| Premixing | 60 | 240 | 280 |
| Mixing | 40 | 220 | 260 |
| Weighing | 20 | 358 | 378 |
| Sealing | 17 | 359 | 376 |
| Packaging | 15 | 360 | 375 |
| bagging | 12 | 360 | 372 |

Similarly average labour time per week (consisting of 5 ordinary days and 1 Saturday) and per month is as shown in Table 2.

Table 2: Average Processing Time of each Labour Per Week

| Labour | Average Processing Time per Week <br> in Minutes | Average Processing Time per Month <br> (Minutes) |
| :--- | :--- | :--- |
| Premixing | 1600 | 6880 |
| Mixing | 1480 | 6360 |
| Weighing | 2148 | 9228 |
| Sealing | 2136 | 9176 |
| Packaging | 2130 | 9150 |
| Bagging | 2112 | 9072 |

The average time required for the production of each of the sizes of custard is as tabulated in Table 3.
Table 3: Average Processing Time of each Labour for each Size of Custard for LCI

| Labour | Large Custard Average <br> Time (Min) | Medium Custard Average <br> Time (Min) | Small Custard Average <br> Time (Min) |
| :---: | :---: | :---: | :---: |
| Premixing | 0.3 | 0.2 | 0.1 |
| Mixing | 0.3 | 0.2 | 0.1 |
| Weighing | 3 | 3 | 3 |
| Sealing | 2 | 2 | 2 |
| Packaging | 2 | 2 | 2 |
| Bagging | 2 | 2 | 2 |

Table 4: Average Processing Time of each Labour for each Size of Custard for KGFI

| Labour | Large Custard Average <br> Time (Min) | Medium Custard <br> Average Time (Min) | Small Custard Average <br> Time (Min) |
| :---: | :---: | :---: | :---: |
| Premixing | 0.3 | 0.2 | 0.1 |
| Mixing | 0.2 | 0.2 | 0.1 |
| Weighing | 2 | 2 | 2 |
| Sealing | 3 | 3 | 3 |
| Packaging | 2 | 2 | 2 |
| Bagging | 3 | 3 | 3 |

## Cost of Different Custard Size and Demand

After cost study of the case studies, the costing of products indicated that LCI and KGFI sells and makes profit from the different sizes of custard as shown in Table 5

Table 5: Prices for Profit for Different Custard Sizes for LCI and KGFI

| Custard Size | Selling Price (n) |  | Profit (N) |  |
| :---: | :---: | :---: | :---: | :---: |
|  | LCI | KGFI | LCI | KGFI |
| Big | 550 | 530 | 100 | 80 |
| Medium | 400 | 380 | 70 | 60 |
| Small | 120 | 130 | 20 | 20 |

LCI and KGFI are faced with a problem of optimization on the quantity of the three sizes of custard to be produced in each production batch for efficient productivity as their present production is not enough to meet demand. Besides, the industry has low profit margin due to inadequate production mix of each size. Louis Carter Industry does not have problem with the selling of produced custard because there is high demand for their products. Hence, the need for a production mix that will enhance profit maximization and cost minimization with a view to meeting their customers demand.
The case study processing time of the various custard sizes, profit made and total available time were transformed into a frame work to enable simplex optimization.

## Transforming Case Study Data and Performing Simplex Optimization

The process time per custard size, profit made and total available time for each of the available labour time of the two cast studies are shown in Table 7 and 8 for LCI and KGFI, respectively.

Table 7: Case Study, Processing Time, Profit and Total Available Time for LCI

| Size of custards | Process Time (Min.) Per custard Size |  |  |  | Profit per |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  | Pre mixing | Mixing | Weighing | Sealing | Packaging | Bagging | custard |
| Large custard $\left(\mathrm{x}_{1}\right)$ | 0.3 | 0.3 | 3 | 2 | 2 | 3 | 100 |
| Medium custard $\left(\mathrm{x}_{2}\right)$ | 0.2 | 0.2 | 3 | 2 | 2 | 3 | 70 |
| small custard $\left(\mathrm{x}_{3}\right)$ | 0.1 | 0.1 | 3 | 2 | 2 | 3 | 20 |
| Total available Time | 280 | 260 | 378 | 376 | 375 | 372 |  |
| (mins/day) |  |  |  |  |  |  |  |

Table 8: Case Study, Processing Time, Profit and Total Available Time for KGFI

| Size of custards | Process Time (Min.) Per custard Size |  |  |  |  | Profit per <br> custard |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  | Pre <br> mixing | Mixing | Weighing | Sealing | Packaging | Bagging |  |
| Large custard ( $\mathrm{x}_{1}$ ) | 0.3 | 0.2 | 3 | 2 | 2 | 3 | 80 |
| Medium custard | 0.2 | 0.2 | 3 | 2 | 2 | 3 | 60 |
| $\left(\mathrm{x}_{2}\right)$ |  |  |  |  |  |  |  |
| small custard $\left(\mathrm{x}_{3}\right)$ | 0.1 | 0.1 | 3 | 2 | 2 | 3 | 20 |
| Total available Time <br> (mins/day) | 240 | 230 | 430 | 428 | 426 | 423 |  |

Calculating Simplex Optimization Using LCI Data
Equation

```
s.t 0.3\mp@subsup{x}{1}{}+0.2\mp@subsup{x}{2}{}+0.1\mp@subsup{x}{3}{}+\mp@subsup{s}{1}{}=280
    0.3\mp@subsup{x}{1}{}+0.2\mp@subsup{x}{2}{}+0.1\mp@subsup{x}{3}{}+\mp@subsup{s}{2}{}=260
    3x
    2x
    2x}+2\mp@subsup{x}{2}{}+2\mp@subsup{x}{3}{}+\mp@subsup{s}{5}{}=37
    3x}+3\mp@subsup{x}{2}{}+3\mp@subsup{x}{3}{}+\mp@subsup{s}{6}{}=37
MaxP = 280x - 260x 2-378x - 376x4-375x 5 - 372x 
P=100x
P-100x - 70x - 20x 
```

Now the frame work is constructed. The coefficients of the problem variables ( $\mathrm{x}_{1}, \mathrm{x}_{2}, \mathrm{x}_{3}$ ) and slack variables $\left(\mathrm{S}_{1}\right.$, $\left.S_{2}, S_{3}, S_{4}, S_{5}, S_{6}\right)$ in the constraints are arranged appropriately and shown in Table 9.

Table 9: Simplex Table Setup for Louis Carter Industry

| $\mathrm{C}_{\mathrm{j}}$ | $100 X_{1}$ | $70 X_{2}$ | $20 X_{3}$ | $S_{1}$ | $S_{2}$ | $S_{3}$ | $S_{4}$ | $S_{5}$ | $S_{6}$ | b |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $S_{1}$ | 0.3 | 0.2 | 0.1 | 1 | 0 | 0 | 0 | 0 | 0 | 280 |
| $S_{2}$ | 0.3 | 0.2 | 0.1 | 0 | 1 | 0 | 0 | 0 | 0 | 260 |
| $S_{3}$ | 3 | 3 | 2 | 0 | 0 | 1 | 0 | 0 | 0 | 378 |
| $S_{4}$ | 2 | 2 | 2 | 0 | 0 | 0 | 1 | 0 | 0 | 376 |
| $S_{5}$ | 2 | 2 | 2 | 0 | 0 | 0 | 0 | 1 | 0 | 375 |
| $S_{6}$ | 3 | 3 | 3 | 0 | 0 | 0 | 0 | 0 | 1 | 372 |
| $\mathrm{P}_{\mathrm{j}}$ | -100 | -70 | -20 | 0 | 0 | 0 | 0 | 0 | 0 |  |

The constant headed by $b$ is included together with the check to provide a check on the numerical calculation as the simplex is developed as shown in the Table 9.

Table 9: Simplex Table Setup

| $\mathrm{C}_{\mathrm{j}}$ | $X_{1}$ | $X_{2}$ | $X_{3}$ | $S_{1}$ | $S_{2}$ | $S_{3}$ | $S_{4}$ | $S_{5}$ | $S_{6}$ | b | Check |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $S_{1}$ | 0.3 | 0.2 | 0.1 | 1 | 0 | 0 | 0 | 0 | 0 | 280 | 933.3 |
| $S_{2}$ | 0.3 | 0.2 | 0.1 | 0 | 1 | 0 | 0 | 0 | 0 | 260 | 866.6 |
| $S_{3}$ | 3 | 3 | 2 | 0 | 0 | 1 | 0 | 0 | 0 | 378 | 126.0 |
| $S_{4}$ | 2 | 2 | 2 | 0 | 0 | 0 | 1 | 0 | 0 | 376 | 188.0 |
| $S_{5}$ | 2 | 2 | 2 | 0 | 0 | 0 | 0 | 1 | 0 | 375 | 187.5 |
| $S_{6}$ | 3 | 3 | 3 | 0 | 0 | 0 | 0 | 0 | 1 | 372 | 124.0 |
| $\mathrm{P}_{\mathrm{i}}$ | -100 | -70 | -20 | 0 | 0 | 0 | 0 | 0 | 0 |  |  |

Step 1
The highest most negative value is chosen as the column -100
Step 2
Divide the value in the $b$ column by the values in the column of step 1 i.e.
$\frac{280}{0.3}=933.3, \frac{260}{0.3}=866.6, \frac{378}{3}=126, \frac{376}{2}=188, \frac{375}{2}=187.5, \frac{372}{3}=124$
Step 3
The intersection of the row and column gives the pivot number
Table 10: First Iteration

| $\mathrm{C}_{\mathrm{j}}$ | $X_{I}$ | $X_{2}$ | $X_{3}$ | $S_{1}$ | $S_{2}$ | $S_{3}$ | $S_{4}$ | $S_{5}$ | $S_{6}$ | b | Check |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $S_{1}$ | 0.3 | 0.2 | 0.1 | 1 | 0 | 0 | 0 | 0 | 0 | 933.3 | 9 |
| $S_{2}$ | 0.3 | 0.2 | 0.1 | 0 | 1 | 0 | 0 | 0 | 0 | 866.6 | 10 |
| $S_{3}$ | 3 | 3 | 2 | 0 | 0 | 1 | 0 | 0 | 0 | 126 | 126 |
| $S_{4}$ | 2 | 2 | 2 | 0 | 0 | 0 | 1 | 0 | 0 | 188 | 188 |
| $S_{5}$ | 2 | 2 | 2 | 0 | 0 | 0 | 0 | 1 | 0 | 187.5 | 187.5 |
| $S_{6}$ | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0.3 | 124 | 124 |
| $\mathrm{P}_{\mathrm{j}}$ | -100 | -25 | 15 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  |

We replace the row with the values obtained as shown in Table 11.

| $\mathrm{C}_{\mathrm{j}}$ | $X_{1}$ | $X_{2}$ | $X_{3}$ | $S_{1}$ | $S_{2}$ | $S_{3}$ | $S_{4}$ | $S_{5}$ | $S_{6}$ | b |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $S_{I}$ | 0 | -0.1 | -0.2 | 1 | 0 | 0 | 0 | 0 | 0 | 242.8 |
| $S_{2}$ | 1 | -0.1 | 0.2 | 0 | 1 | 0 | 0 | 0 | 0 | 222.8 |
| $S_{3}$ | 1 | 0 | -1 | 0 | 0 | 1 | 0 | 0 | 0 | 6 |
| $S_{4}$ | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 128 |
| $S_{5}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 127 |
| $S_{6}$ | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 124 |
| $\mathrm{P}_{\mathrm{j}}$ | 0 | 30 | 80 | 0 | 2 | 0 | 0 | 0 | 30 | 12400 |

We now have a non negative value in our key row hence we stop our iteration process.
$\mathrm{P}_{\text {max }}=12400$

## LCI Custard Production and Cost for March 2013

The data on Table 5 was collected on hourly basis from the factory for a period of one month on a daily basis and from each batch of production in March, 2013 and classified into columns. The quantity of custard produced for various sizes, cost price, selling price and profit made for the month of March 2013 were noted.

Table 5: Daily Quantity and Total Sales of Custard for the Month of March

| Day | Number of mixes | Qty of custard packs produced |  |  | Number of cartoons | Cost price <br> ( $\ddagger$ | Selling price (\#) | Profit (\#) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Large | medium | Small |  |  |  |  |
| Day 1 | 7 | 437 | 0 | 0 | 109 | 152600 | 196200 | 43600 |
| Day 2 | 4 | 0 | 0 | 1333 | 55 | 121000 | 132000 | 11000 |
| Day 3 | 9 | 562 | 0 | 0 | 140 | 168000 | 252000 | 84000 |
| Day 4 | 7 | 0 | 583 | 0 | 145 | 145000 | 203000 | 58000 |
| Day 5 | 8 | 500 | 0 | 0 | 125 | 150000 | 273600 | 123600 |
| Day 6 | 7 | 0 | 583 | 0 | 145 | 145000 | 203000 | 58000 |
| Day 7 | 7 | 437 | 0 | 0 | 109 | 152600 | 196200 | 43600 |
| Day 8 | 4 | 0 | 0 | 1333 | 55 | 121000 | 132000 | 11000 |
| Day 9 | 7 | 437 | 0 | 0 | 109 | 152600 | 196200 | 43600 |
| Day 10 | 8 | 0 | 664 | 0 | 166 | 166000 | 232400 | 66400 |
| Day 11 | 8 | 500 | 0 | 0 | 125 | 150000 | 273000 | 123000 |
| Day 12 | 7 | 437 | 0 | 0 | 109 | 152600 | 196200 | 43600 |
| Day 13 | 6 | 0 | 499 | 0 | 124 | 124000 | 173600 | 49600 |
| Day 14 | 4 | 0 | 0 | 1333 | 55 | 121000 | 132000 | 11000 |
| Day 15 | 8 | 0 | 664 | 0 | 166 | 166000 | 232400 | 66400 |
| Day 16 | 7 | 437 | 0 | 0 | 109 | 152600 | 196200 | 43600 |
| Day 17 | 7 | 437 | 0 | 0 | 109 | 152600 | 196200 | 43600 |
| Day 18 | 6 | 0 | 0 | 1998 | 83 | 182600 | 199200 | 16600 |
| Day 19 | 7 | 437 | 0 | 0 | 109 | 152600 | 196200 | 43600 |
| Day 20 | 3 | 0 | 0 | 999 | 41 | 90200 | 98400 | 8200 |
| Day 21 | 8 | 500 | 0 | 0 | 125 | 150000 | 273600 | 123600 |
| Day 22 | 8 | 500 | 0 | 0 | 125 | 150000 | 273600 | 123600 |
| Day 23 | 7 | 437 | 0 | 0 | 109 | 152600 | 196200 | 43600 |
| Day 24 | 7 | 437 | 0 | 0 | 109 | 152600 | 196200 | 43600 |
| Day 25 | 8 | 500 | 0 | 0 | 125 | 150000 | 273600 | 123600 |
| Day 26 | 4 | 0 | 0 | 1333 | 55 | 121000 | 132000 | 11000 |
| TOTAL | 173 | 6995 | 2993 | 8329 | 2836 | 3794200 | 5255200 | 1461000 |



Fig. 2: LCI Profit and Optimized Profit for the Month of March, 2013 for LCI


Fig. 3: KGFI Profit and Optimized Profit for the Month of August, 2013
Fig 2 and 3 show the case studies profit and optimized profit for LCI and KGFI for the months of March 2013 and August 2013, respectively. For the month specifically analyized, it can be seen that the profit that the company makes someday are greater that the profit they can make when optimized. Specifically, LCI made more profit in days $5,11,21,22$ and 25 while when optimized, more profit is made in the remaining 21 days of the 26 days under consideration, as clearly depicted in Fig 2. Similarly, KGFI made more profits in days 3, 4, 9, 21 and 25 while when optimized made more profit in the remaining 22 days. The total profit made for the 26 days by LCI is One million, four hundred and sixty one thousand naira ( $£ 1,461,000$ ), as against two million one hundred and eighty nine thousand, one hundred and forty two naira ( $\$ 2,189,142$ ), when optimized. By the same token, KGFI has a total profit of One million, seven hundred and fifty eight thousand, one hundred naira ( $\AA 1,758,100$ ), as against two million, six hundred and sixty three thousand, five hundred and twenty three naira only ( $\mathbb{\AA}$ $2,663,523$ ). The marginal profit (extra profit) gained when optimized as shown by LCI and KGFI are seven hundred and twenty eight thousand, one hundred and forty two naira ( $\ddagger 728,142$ ) and nine hundred and five thousand four hundred and twenty three naira ( $¥ 905,423$ ) which indicates that the optimization is highly beneficial.


Fig. 4: LCI Case Study Profit, Optimized Profit and Profit Margin


Fig. 5: KGFI Case Study Profit, Optimized Profit and Profit Margin

Beyond the cost implications, the percentage of custards per batch for each of the three different sizes of custard shown in Fig. 6 and Fig. 7, respectively.


Fig. 6: Percentage Production Mix for LCI


Fig. 7: Percentage Production Mix for KGFI

## Conclusion

In this work, various process time and costs for LCI and KGFI were analyzed. Simplex method of optimization was employed in determining the appropriate production mix and associated total profit for both industries. The case study of LCI results gave an optimal production mix of $45.8 \%, 39.6 \%$ and $14.6 \%$ for large, medium and small sized custard, respectively, with an increase in profit margin to $49.8 \%$ translating into $\# 728,142: 00$ for the month of March, 2013. Similarly, KGFI have an optimal production mix of $43.5 \%, 36.5 \%$ and $20 \%$ for large, medium and small sized custard with an increase in profit margin to $51.5 \%$ translating into $£ 905,423.00$ for the month of August, 2013. This study has been established that simplex method of optimization is a good model for the analysis of appropriate production proportion problem. It gives product mix that maximizes profit and minimizes cost.

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