# Simplified shear provisions of the *AASHTO LRFD Bridge Design Specifications*

Daniel A. Kuchma,
Neil M. Hawkins,
Sang-Ho Kim,
Shaoyun Sun,
and Kang Su Kim

## Editor's quick points

- This paper discusses the findings from National Cooperative Highway Research Program project 12-61, "Simplified Shear Design of Structural Concrete Members."
- Based on an assessment of several leading shear-design methods, a review of field experience, and comparisons with a large experimental database, criteria were developed for simplified provisions.
- The two resulting changes to the *AASHTO LRFD Bridge Design Specifications* shear-design provisions are described.

Published in 1994, the first edition of the American Association of State Highway and Transportation Officials' *AASHTO LRFD Bridge Design Specifications* (AASHTO LRFD specifications)[1] introduced U.S. practitioners to a shear-design procedure differing markedly from that of AASHTO's *Standard Specifications for Highway Bridges* (AASHTO standard specifications).[2] This new procedure, which is based on the modified compression field theory (MCFT)[3] and is termed the sectional design model (SDM), provided a unified approach for the design of both prestressed and nonprestressed concrete members. The procedure in AASHTO LRFD specifications permits design shear stresses that are much greater than those permitted in AASHTO standard specifications. For example, the maximum design shear stress in AASHTO LRFD specifications is 175% higher for 5000 psi (34 MPa) concrete and 250% higher for 10,000 psi (69 MPa) concrete. Unfortunately, the generality of this new procedure was perceived by many as introducing unnecessary complexities, as not being intuitively related to physical behavior, and as being more difficult to understand than the procedure in AASHTO standard specifications.

To address these concerns, the National Cooperative Highway Research Program (NCHRP) undertook project 12-61, "Simplified Shear Design of Structural Concrete Members."[4] The goal of NCHRP project 12-61 was to supplement AASHTO LRFD specifications' method for shear design with procedures that provided direct solutions for transverse and longitudinal shear-reinforcement requirements for concrete structures of common proportions and subjected to customary loading. In this way, the resulting simplified AASHTO LRFD specifications' shear-design provisions would overcome many of the perceived difficulties with the use of the SDM. This paper summarizes the results of NCHRP project 12-61.

## General procedure of the AASHTO LRFD specifications' model

The general procedure of the AASHTO LRFD specifications' SDM for shear (A5.8.3 and A5.8.3.4.2) provides a hand-based shear-design procedure derived from the MCFT. Unlike prior approaches, which focus on expressions for shear strength modified for the effect of other forces, the AASHTO LRFD specifications' procedure accounts for the combined actions of axial load, flexure, and prestressing when designing any section in a member for shear. The nominal shear capacity $V_n$ is taken as the sum of a concrete component $V_c$, a shear-reinforcement component $V_s$, and the transverse component of the prestressing force $V_p$, as shown in Eq. (1).

$$V_n = V_c + V_s + V_p \leq 0.25 f_c' b_v d_v + V_p \tag{1}$$

where

$f_c'$ = concrete compressive strength

$b_v$ = effective web width

$d_v$ = effective shear depth taken as the distance, measured perpendicular to the neutral axis, between the resultants of the tensile and compressive forces due to flexure; it need not be taken to be less than the greater of $0.9d_e$ or $0.72h$

The concrete contribution $V_c$ to the shear capacity is controlled by the value of the coefficient $\beta$ as illustrated by Eq. (2).

$$V_c = \beta\sqrt{f_c'}\, b_v d_v \tag{2}$$

A variable-angle truss model is used to calculate the shear-capacity contribution of the transverse shear reinforcement $V_s$ in accordance with Eq. (3). The angle of the field of diagonal compression $\theta$ is used to determine how many stirrups ($d_v \cot \theta / s$) are included in the transverse tie of the idealized truss.

$$V_s = \frac{A_v f_y d_v \cot(\theta)}{s} \tag{3}$$

where

$A_v$ = area of transverse reinforcement within a distance $s$

$f_y$ = yield strength of reinforcing bars

$d_v$ $\geq 0.9d_e$ or $0.72h$, whichever is greater

$d_e$ = effective depth from extreme compression fiber to the centroid of the tensile force in the tensile reinforcement

$h$ = height of the member

$s$ = spacing of bars of transverse reinforcement

The values for $\beta$ and $\theta$ are obtained from tables. One table is provided in the SDM for $A_v < A_{v,min}$ and another for $A_v \geq A_{v,min}$, where $A_{v,min}$ is the minimum required amount of shear reinforcement in accordance with Eq. (4).

$$A_{v,min} \geq 0.0316\sqrt{f_c'}\,\frac{b_v s}{f_y} \tag{4}$$

where $f_c'$ and $f_y$ are in units of kip/in.[2]

This requirement is about 50% higher than the AASHTO standard specifications' requirements. The designer selects the appropriate row of the table based on the calculated design shear-stress ratio $v/f_c'$ and the appropriate column based on the longitudinal strain $\varepsilon_x$ at mid-depth, which may be taken as one-half of the strain in the longitudinal tension reinforcement $\varepsilon_t$. This strain is equal to the force in the longitudinal tension reinforcement divided by the axial stiffness of the tension reinforcement. As shown in Eq. (5), the effects of all demands on the longitudinal reinforcement are taken into account.

$$\varepsilon_x = \frac{\varepsilon_t}{2} = \frac{\dfrac{M_u}{d_v} + 0.5N_u + 0.5\left(V_u - V_p\right)\cot\theta - A_{ps} f_{po}}{2\left(E_s A_s + E_p A_{ps}\right)} \tag{5}$$

where

$M_u$ = factored moment at section

$N_u$ = factored axial force

$V_u$ = factored shear force at section

$A_{ps}$ = area of prestressing steel on flexural tension side of member at ultimate load

$f_{po}$ = $E_p$ times locked-in difference in strain at ultimate load between the prestressing tendons and the surrounding concrete

$E_p$ = modulus of elasticity of prestressing steel

$E_s$ = modulus of elasticity of reinforcing bars

$A_s$ = area of nonprestressed tension reinforcement on flexural tension side of member at ultimate load

Equation (5) assumes that the member is cracked and therefore only the axial stiffness of the reinforcement needs to be considered when evaluating $\varepsilon_x$. If $\varepsilon_x$ is negative, then the member is uncracked and the axial stiffness of the uncracked concrete needs to be considered in accordance with Eq. (6).

$$\varepsilon_x = \frac{\varepsilon_t}{2} = \frac{\frac{M_u}{d_v} + 0.5N_u + 0.5\left(V_u - V_p\right)\cot\theta - A_{ps}f_{po}}{2\left(E_s A_s + E_p A_{ps} + E_c A_c\right)} \quad (6)$$

where

$E_c$ = modulus of elasticity of concrete

$A_c$ = area of the concrete beneath mid-depth

Alternatively, the designer can conservatively take $\varepsilon_x$ to equal zero if Eq. (5) yields a negative value.

As the longitudinal strain becomes greater, the values for $\beta$ decrease and the values for $\theta$ increase. This means that as the moment and longitudinal strain increase, the magnitudes of both the concrete and shear-reinforcement contributions to shear resistance decrease.

When $A_v < A_{v,min}$, a second table is used. The column by which the designer enters the table is again based on the value of the longitudinal strain at mid-depth $\varepsilon_x$. To determine the row, the spacing of the layers of crack-control reinforcement $s_{xe}$ is used. Equation (7) and Fig. 5.8.3.4.2-2 of the AASHTO LRFD specifications[5] are used to calculate this value.

$$s_{xe} = \frac{1.38 s_x}{0.63 + a_g} \quad (7)$$

$s_x$ = crack-spacing parameter, the lesser of either $d_v$ or the maximum distance between layers of longitudinal-crack-control reinforcement

$a_g$ = maximum aggregate size

In the table, as $s_{xe}$ and $\varepsilon_x$ increase, the value of $\beta$ decreases and the value of $\theta$ increases. The result is that, as the member becomes deeper and the value of the longitudinal strain increases due to flexure, shear, and other effects, the contributions of both the concrete and shear reinforcement decrease.

The MCFT is a behavioral model that can be used to predict the shear stress and shear-strain response of an element subjected to in-plane shear and membrane forces. The theory consists of constitutive, compatibility, and equilibrium relationships that enable determination of the state of stress ($f_x$, $f_y$, $v_{xy}$) in structural concrete corresponding to a specific state of strain ($\varepsilon_x$, $\varepsilon_y$, $\gamma_{xy}$). A full implementation of the MCFT is possible within a two-dimensional continuum-analysis tool, such as that of the program VecTor2.

The derivation of the AASHTO LRFD specifications' hand-based design procedure requires that several simplifications and assumptions be made.[6] The most significant factor is that the distribution of shear stress over the depth of the section is taken as the value at mid-depth as calculated by the MCFT and using the designer-calculated longitudinal strain $\varepsilon_x$ at mid-depth. Additional assumptions are that the shape of the concrete compressive stress-strain response is parabolic with a strain at peak stress of -0.002 and that for members with $A_v \geq A_{v,min}$, the spacing of the cracks is 12 in. (300 mm) and the maximum aggregate size is 0.75 in. (18 mm). AASHTO LRFD specifications permit end regions to be designed for shear based on the calculated shear force at $d_v$, which is the first critical section from the face of the support. While AASHTO LRFD specifications were derived from the MCFT, due to the foregoing significant simplifications and assumptions, the shear capacity determined using the AASHTO LRFD specifications' SDM should not be considered equivalent to the shear capacity calculated by the MCFT.

### Objective and scope

This paper summarizes the work completed in NCHRP project 12-61. The objective of the project was to develop simplified shear-design provisions to complement the comprehensive provisions of the AASHTO LRFD specifications' SDM. The findings from a comparison and evaluation of the principal existing shear-design models are described first, along with the ways in which those findings were used to develop criteria for the simplified AASHTO LRFD specifications shear provisions. The changes to the AASHTO LRFD specifications are then presented along with the evaluation of their effectiveness and likely impact on design practice. This paper concludes with a detailed shear-design example.

## Comparison and evaluation of shear-design provisions

### Discussion of code provisions

The authors reviewed several widely used shear-design methods, and the basis of these methods, for the purpose of identifying differences among these methods as well as positive attributes to be considered for incorporation into the AASHTO LRFD specifications' new simplified shear-design provisions. The methods reviewed were those in ACI 318-02,[7] AASHTO standard specifications,[2] AASHTO's 1979 interim specifications,[8] Canadian Standards Association's (CSA's) *Design of Concrete Structures* (CSA A23.3-94),[9] AASHTO LRFD specifications,[10] CSA's 2004 *Design of Concrete Structures*,[11] Eurocode 2 from 1991,[12] Eurocode 2 from 2002,[13] German code DIN 1045-1,[14] AASHTO's guide specifications for segmental bridges,[15] Japanese specifications for design and construction of concrete structures,[16] and the shear-design approach by Tureyen and Frosch.[17]

All of these design provisions for shear have their roots in the parallel-chord truss model proposed by Ritter[18] and Mörsch.[19,20] In this model, vertical (shear) forces flow to the support along diagonal struts, the vertical components of these forces are lifted up at the base of these struts by vertical transverse reinforcement (stirrups), and the horizontal components are equilibrated by concrete compressive stresses in the top chord and reinforcement tensile stresses in the bottom chord. For any member subjected to a specified shear force, there are four unknowns—diagonal compressive stress, stress in stirrups, stress in longitudinal reinforcement, and angle of diagonal compression—but only three equations of statics to determine those unknowns. This intractable problem of four unknowns and three equations is principally responsible for the large differences among the shear-design methods.

Different design methods use different approaches for calculating the angle of diagonal compression. That angle controls the contribution of the stirrups to the shear resistance $V_s$. It also controls the methods and relationships that appear reasonable for the concrete contribution to shear resistance $V_c$ because that contribution is typically taken as the measured capacity less the calculated contribution from $V_s$. In early versions of the parallel-chord truss model, it was commonly assumed that the angle of diagonal compression was at 45 degrees to the longitudinal axis of the member. When this model was examined by researchers in the United States in the early 1900s,[21–23] it was found to conservatively overestimate capacity by a relatively uniform amount independent of the level of shear reinforcement. Thus, a concrete contribution to shear resistance $V_c$ was added. This contribution was originally taken as a shear stress that was a fraction of the compressive strength times the shear area. Over time, this contribution evolved into the diagonal cracking strength, with the diagonal strength taken as the concrete contribution at ultimate as a matter of convenience and validated by test data.[24] Many expressions have been developed to account for the influence of axial forces, flexure, and prestressing on the diagonal cracking strength as incorporated in ACI 318-02 and the AASHTO standard specifications. In all of these cases, it has been assumed that this concrete contribution is independent of the amount of shear reinforcement provided.

By contrast, it has been common in European design practice to take the angle of diagonal compression $\theta$ as that defined by a plasticity-based approach in which the stirrups are assumed to yield and the diagonal compressive stresses in the concrete are assumed to reach a selected limiting stress. Equations for evaluating $\theta$ can result in values as low as 21.8 degrees. Different expressions for the concrete contribution to shear resistance are used, depending on whether shear reinforcement is provided. In some cases, the concrete contribution to resistance is neglected. The calculated concrete contributions are empirically based or derived from shear-friction models.

The angle of diagonal compression determined by the AASHTO LRFD specifications' method for the ends of prestressed concrete members is commonly 20 degrees to 25 degrees. The use of this angle leads to a much larger contribution from the shear reinforcement, as calculated by Eq. (3), for prestressed concrete members than would be calculated by traditional U.S. design practice. AASHTO LRFD specifications-calculated concrete contribution to shear capacity $V_c$ is the ability of the cracked concrete to carry diagonal tension in the web of a member. This contribution is controlled by the level of longitudinal strain in the member, the reserve capacity of the longitudinal reinforcement at a crack location, and the shear-slip resistance of the concrete. Thus, AASHTO LRFD specifications' concrete contribution is often considered to be controlled by interface shear transfer (aggregate interlock). By contrast to the AASHTO LRFD specifications' approach, Tureyen and Frosch assume that the angle of diagonal compression is 45 degrees and base the concrete contribution on the limiting capacity of the uncracked compression zone.

Based on the review of shear-design provisions, it is concluded that the only similarity in their bases is the parallel-chord truss model. As described, there is a wide variation in how the angle of diagonal compression is evaluated and the basis for its evaluation, with the effect that the contribution of stirrups by one method can be up to three times that calculated by another method. The differences in the calculated concrete contributions to shear resistance, and the basis for this contribution, are just as large.

Key observations from this review, relevant for developing the simplified specifications to the AASHTO LRFD specifications described in this paper, are

- The CSA A23.3-04,[11] AASHTO's 1979 interim specifications,[8] AASHTO LRFD specifications,[1,10] Reineck,[25] Eurocode 2 from 1991 and 2002,[12,13] JSCE specifications,[16] and German code[14] methods all permit the designer to use an angle of diagonal compression $\theta$ flatter than 45 degrees when evaluating the contribution of shear reinforcement to shear capacity.

- AASHTO LRFD specifications, DIN 1045-1, and Eurocode 2 from 1991 and 2002 allow the engineer to design members to support much larger shear stresses than permitted in traditional U.S. design practice.[2,7] Any shear-stress limit is principally intended to guard against diagonal compression failures. For example, the AASHTO LRFD specifications permit members to be designed for shear stresses that can exceed 2.5 times those permitted by the AASHTO standard specifications. In the AASHTO standard specifications, the contribution of the shear reinforcement is limited to $8\sqrt{f_c'}\,b_w d$ ($f_c'$ in psi), where $b_w$ is the width of the web, in order to guard against the member's failing by diagonal crushing of the concrete prior to yielding of the shear reinforcement. According to the MCFT and based on the results of shear tests on plate elements,[26,27] such failure mechanisms do not occur until design shear stresses are in excess of 0.25 $f_c'$. For reinforced concrete members cast with 10 ksi (69 MPa) concrete, this is a 2.5-fold difference. Some state highway authorities are taking advantage of this difference and using prestressed concrete members to replace similar-height steel girders.

- Basing the concrete contribution at ultimate on the diagonal cracking strength gives the designer the ability to check whether a member will be cracked in shear under service-load levels, and this helps in assessing the condition of structures in the field. A survey of designers also found that characterizing the two types of diagonal cracking, web-shear and flexure-shear, as used in ACI 318-02 and the AASHTO standard specifications, was useful for describing shear behavior.

- Provisions differ in their ease of use, ranging from those that use only a couple of basic parameters in simple relationships to those with many terms that require the use of tables and an iterative design procedure. As an example of the latter, AASHTO LRFD specifications' procedure is iterative for design because the longitudinal strain at mid-depth $\varepsilon_x$ is needed to obtain values for $\beta$ and $\theta$. Yet $\varepsilon_x$ is also a function of $\theta$. It is further iterative for capacity evaluation because $\varepsilon_x$, and thus $\beta$ and $\theta$, are functions of $V_u$. Where an AASHTO LRFD specifications procedure is used, this iteration makes it difficult without design aids to evaluate the capacity of an existing structure.

- Provisions differ in their consideration of the influence of shear on longitudinal reinforcement demand. While this influence is directly described in the century-old parallel-chord truss model, many codes, including ACI 318-02 and the AASHTO standard specifications, handle the influence of shear through detailing rules. In the AASHTO LRFD specifications, the demand $T_{min}$ that shear imposes on the longitudinal-reinforcement requirement is directly taken into account by consideration of equilibrium and as given in Eq. (8).

$$T_{min} \geq 0.5N_u + 0.5V_u \cot\theta + \frac{M_u}{d_v} - A_{ps}f_{ps} \qquad (8)$$

where

$f_{ps}$ = stress in prestressing steel at the time for which nominal resistance of member is required

The difference between these two approaches is particularly significant at the ends of simply supported prestressed members where the horizontal component of the diagonal compression force can be large, and yet by AASHTO LRFD specifications, only the developed portion of the strands may be considered to contribute to the required resistance.

## Evaluation of shear-design methods using test database

To evaluate the accuracy of selected shear-design methods, a large experimental database of 1359 beam-test results was assembled and used to calculate the shear-strength ratio ($V_{test}/V_{code}$). This database consisted of 878 reinforced concrete (RC) and 481 prestressed concrete (PC) members.

**Table 1.** Shear resistance measured in testing to shear strength evaluated by design code $V_{test}/V_{code}$ for reinforced concrete and prestressed concrete members

| Member type | | | Reinforced concrete | Prestressed concrete | Reinforced concrete | Prestressed concrete | Reinforced concrete | Prestressed concrete |
|---|---|---|---|---|---|---|---|---|
| With or without $A_v$ | | All | Both | No $A_v$ | With $A_v$ | Both | No $A_v$ | With $A_v$ |
| Number of specimens | | 1359 | 878 | 718 | 160 | 481 | 321 | 160 |
| ACI 318-02 | Mean | 1.44 | 1.51 | 1.54 | 1.35 | 1.32 | 1.38 | 1.21 |
| | COV | 0.371 | 0.404 | 0.418 | 0.277 | 0.248 | 0.247 | 0.221 |
| AASHTO LRFD specifications | Mean | 1.38 | 1.37 | 1.39 | 1.27 | 1.40 | 1.44 | 1.32 |
| | COV | 0.262 | 0.262 | 0.266 | 0.224 | 0.261 | 0.290 | 0.154 |
| CSA | Mean | 1.31 | 1.25 | 1.27 | 1.19 | 1.41 | 1.46 | 1.31 |
| | COV | 0.275 | 0.274 | 0.282 | 0.218 | 0.261 | 0.287 | 0.147 |

Source: *Building Code Requirements for Structural Concrete (ACI 318-02) and Commentary (ACI 318 R-02)*, *AASHTO LRFD Bridge Design Specifications*, and *Design of Concrete Structures* (CSA A23.3-04).
Note: $A_v$ = area of transverse reinforcement; COV = coefficient of variation.

The database was selected to ensure that results for which significant suspected arch action or flexural failures were not included. Most of the RC members had rectangular cross sections and were simply supported using bearings positioned underneath the member. Of these, 718 did not contain shear reinforcement, while 160 did. The PC members consisted of rectangular, T-shaped, and I-shaped sections, and the majority of the members were simply supported, again on bearings positioned underneath the member. Of these, 321 did not contain shear reinforcement, while 160 did. About 80% of both the members in the RC and PC components of the database had depths less than 20 in. (510 mm). Most members were subjected to four-point loading so that there was a region of almost constant shear and a clearly defined shear span length.

**Table 1** presents the resulting mean shear-strength ratios ($V_{test}/V_{code}$) and coefficients of variation (COV) for the ACI 318-02 (also AASHTO standard specifications) method, AASHTO LRFD specifications' method, and CSA A23.3-04. Other methods were also evaluated similarly, and these results are presented in NCHRP report 549.[4] The code-calculated strengths are nominal capacities, and therefore all resistance and strength-reduction factors were set to 1.0.

As a result, the calculated strengths by ACI 318-02 would be equivalent to the calculated strengths by the 16th edition of the AASHTO standard specifications. In this table, the mean and COV are presented for seven segments of the database: all members, all RC members, all RC members without shear reinforcement, all RC members with shear reinforcement, all PC members, all PC members without shear reinforcement, and all PC members with shear reinforcement.

From Table 1, the following observations were made.

- For all of the examined methods, the AASHTO LRFD specifications and CSA methods provided the most accurate estimates of the shear capacity of the members in the database. The means of the strength ratios for both methods were consistent across the different categories of selected members and ranged from 1.19 to 1.46. These methods would therefore be expected to result in conservative designs that made reasonably efficient use of shear reinforcement for the types of members evaluated by this database. The small COVs were particularly impressive for PC members with shear reinforcement: 0.154 and 0.147, respectively, for the AASHTO LRFD specifications and CSA methods.

- The close correlation between the mean shear-strength ratios and COV for the AASHTO LRFD specifications and CSA methods indicated that these two methods should yield similar designs in terms of the amount of required shear reinforcement. As a result, it was concluded that the equations for calculating $\beta$, $\theta$, and $\varepsilon_x$ in CSA A23.3-04, as presented in the next section, are reasonable replacements for the tables and equations for $\varepsilon_x$ in the 2001 edition of the AASHTO LRFD specifications.

- While the overall COV for the ACI 318-02 (also the AASHTO standard specifications) method was markedly greater than the overall COV for the other two methods, this result was principally due to the poor performance of the ACI 318-02 method in predicting the capacity of RC members that do not contain shear reinforcement. Both the mean and COV of the ACI 318-02 method for RC and PC members with shear reinforcement were relatively good. For RC members without shear reinforcement, it should further be noted that, given the measured shear capacity of many of the members, the ACI 318-02 approach would frequently have required designs with minimum shear reinforcement due to the requirement for $A_{v,min}$ when $V_u \geq \varphi V_c/2$.

- For members that only contained close to the traditional ACI 318-02 minimum amount of shear reinforcement ($\rho_v f_y$ = 40 psi to 60 psi [0.28 MPa to 0.41 MPa]), where $\rho_v$ is the ratio of area of vertical shear reinforcement to area of gross concrete area of a horizontal section), the strength ratios ($V_{test}/V_{code}$) were often less than 1.0.[4] Thus, the higher minimum shear reinforcement required by the AASHTO LRFD specifications appears to be necessary.

- Beams that contained large amounts of shear reinforcement were able to support shear stresses close to or greater than the limit of 0.25 $f_c'$ given in the AASHTO LRFD specifications. This result suggests that the upper limit on shear strength in the ACI 318-02 (also AASHTO standard specifications) method may be unduly conservative and that the greater strength limits in the AASHTO LRFD specifications and other codes are more appropriate.

Caution is required when evaluating design provisions with experimental test data because what researchers have tested in laboratories is not representative of the types of members used in practice. Most typical laboratory test members are small (less than 15 in. [380 mm] deep), have rectangular cross sections, do not contain shear reinforcement, are simply supported, are stocky, are loaded by point loads over short shear spans, and are supported on bearings positioned underneath the member. In addition, nearly all members are designed so that shear failures occur near supports. By contrast, bridge members in the field are typically large and continuous, have top flanges, are subjected to uniformly distributed loads, and are often built integrally at their ends into diaphragms or piers. In addition, members in the field need to be designed for shear over their entire length in which there can be a substantial effect of flexure on shear capacity away from simple supports. For these reasons, the fit with experimental test data should only be viewed as one evaluation metric. Another measure, as presented later in this paper, is to compare the code-required amount of shear reinforcement with that determined by numerical methods for design sections that better represent what is used in practice.

### Criteria for proposed simplified provisions

Based on the experiences of practicing bridge engineers, the foregoing review of shear-design methods in codes of practice, the analysis of experimental test data, and a comparison of the required amounts of shear reinforcement for sections in a design database, as will be discussed, the following positive attributes were selected for the simplified provisions.

- They can be directly usable without iteration for the design or capacity rating of a member.
- They have a conceptual basis that is easily understood and can be readily explained by one engineer to another.
- They provide safe and accurate estimates of the members in the experimental database.
- They result in reasonable amounts of shear reinforcement for members in the design database.

It was also desirable that the provisions should help evaluations of the condition of the member under service loads and thereby aid field assessments. It was not considered necessary for the provisions to be unified or as broadly applicable as AASHTO LRFD specifications' general procedure because these new simplified specifications were to be alternate design provisions, not replacements for the general procedure.

## Changes to the AASHTO LRFD specifications

Two changes are now presented. The first change, as incorporated into the fourth edition of the AASHTO LRFD specifications, was the addition of alternative, or simplified, shear-design provisions, which reintroduce basing $V_c$ on the lesser of the calculated web-shear $V_{cw}$ and flexure-shear $V_{ci}$ strength.

With this, a new and more conservative relationship is introduced for $V_{cw}$, and a variable-angle truss model is used for evaluating the contribution of the shear reinforcement with the variable angle based on the angle of predicted diagonal cracking. This alternative is described here as the simplified provision (SIMP). The second change, which shall appear in the 2008 AASHTO LRFD specifications revisions, was the replacement of the current tables for determining $\beta$ and $\theta$, as well as the equation for evaluating $\varepsilon_x$ in the SDM (S5.8.3) by relationships that are equivalent for $\beta$, $\theta$, and $\varepsilon_x$ that were incorporated into the Canadian Standards Association's *Design of Concrete Structures*, CSA A23.3-04.[11] This revised method is henceforth referred to as the CSA method.

### Change 1: SIMP method

The simplified specifications differ from the current AASHTO standard specifications in the expression for web-shear cracking $V_{cw}$, the angle $\theta$ of diagonal compression in the parallel-chord truss model, the maximum shear stress permitted for design, the minimum required amount of shear reinforcement, the evaluation of shear depth, and the requirements for the amount of longitudinal tension reinforcement that must be developed at the face of the support. The expressions for $V_{cw}$ were developed so that they can be applied seamlessly to beams with deformed-bar reinforcement only, with full prestressed reinforcement only, and all combinations of those reinforcement types. A need for seamlessness between RC and PC design was not recognized when the AASHTO standard specifications' provisions for shear were developed because PC and RC were viewed as different materials at the time of development.[24]

**Web-shear cracking strength $V_{cw}$** The estimate of the web-shear cracking force follows directly from Mohr's circle of stress.

$$V_{cw} = f_t\sqrt{1+\frac{f_{pc}}{f_t}}\,b_w d + V_p \tag{9}$$

where

$f_{pc}$ = compressive stress in concrete after all prestress losses have occurred either at centroid of the cross section resisting live load or at the junction of the web and flange when the centroid lies in the flange

$f_t$ = tensile strength of concrete

$d$ = effective member depth

The tensile strength of the concrete $f_t$ can be taken as somewhere between $2\sqrt{f_c'}$ and $4\sqrt{f_c'}$, where $f_c'$ is in psi. A tensile cracking strength close to $4\sqrt{f_c'}$ provides a reasonable estimate of the diagonal cracking strength for the design of the end regions of a fully PC member in which there is no effect of flexure, while a value of $2\sqrt{f_c'}$ is a better estimate of the diagonal cracking strength of an RC member or a PC member with a low level of prestressing. The transition between those two levels is a function of the level of

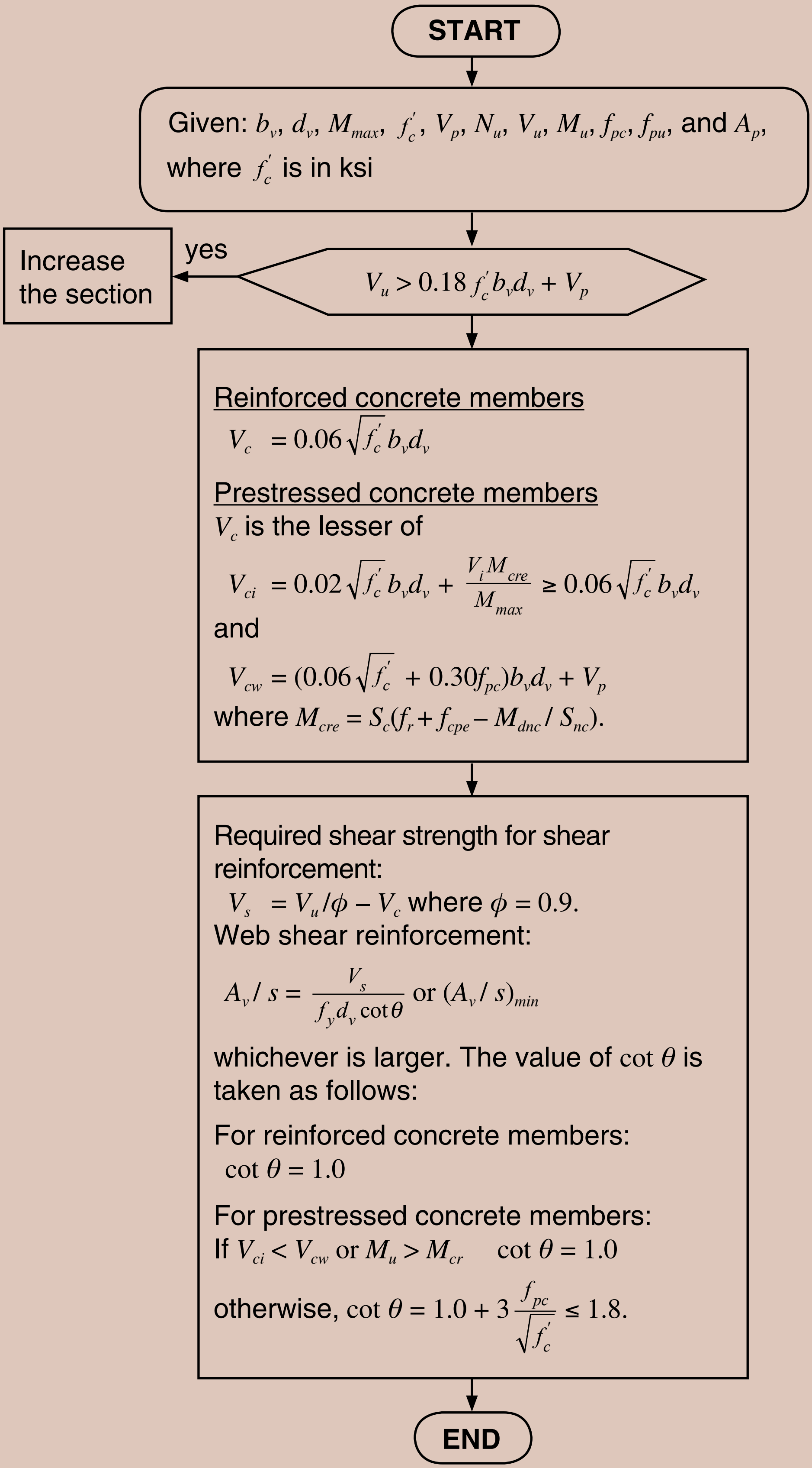



**Figure 1.** This flowchart illustrates the design procedures for the simplified provisions. 1 ksi = 6.895 MPa.

the prestress and the axial load. However, the accuracy of $V_c$ for predicting the cracking load is not the primary issue. Rather, it is the accuracy of $V_c$ for estimating the concrete contribution to the calculated capacity $V_n$.

The concrete contribution $V_c$ must therefore be a lower-bound estimate of the concrete contribution to shear resistance at the ultimate limit state. At this state, the concrete contribution is the sum of the shear carried in the compression zone, the shear carried across diagonal cracks due to shear friction (aggregate interlock), direct tension across diagonal cracks, dowel action, and arch action. Many factors influence the contributions of each of these mechanisms, and attempts to reasonably account for them lead to complicated expressions for $V_c$. Thus, the approach taken in developing this simplified procedure was to use a lower-bound estimate of the diagonal cracking load (Eq. [10]) that when added to the calculated stirrup contribution to shear resistance provides a conservative estimate of the capacity of beams in the test database and, as important, of beams in the design database as discussed in next section of this paper.

$$V_{cw} = (1.9\sqrt{f_c'} + 0.30f_{pc})b_v d_v + V_p \tag{10}$$

where $f_c'$ and $f_{pc}$ are in psi

**Flexure-shear cracking strength $V_{ci}$** The expression for flexure-shear cracking in the AASHTO standard specifications was adopted with minor modifications to account for the use of a different definition for shear depth in Eq. (11).

$$V_{ci} = 0.632\sqrt{f_c'}\, b_v d_v + V_d + \frac{V_i M_{cr}}{M_{max}} \geq 1.9\sqrt{f_c'}\, b_v d_v \tag{11}$$

where

$V_i$ = factored shear force at section due to externally applied loads occurring simultaneously with $M_{max}$

$M_{cr}$ = net cracking moment (= $M_{cre}$)

$M_{max}$ = maximum factored moment at section due to externally applied loads

$f_c'$ is in psi

The sum of the second and third terms is an estimate of the shear force at the time of flexural cracking, while the first term is the increase in shear that has been observed in experiments for the initiating flexural crack to develop into a diagonal crack. Note that the coefficient of the first term has only one significant figure in the fourth edition of the AASHTO LRFD specifications in which $f_c'$ is in ksi.

**Contribution of shear reinforcement $V_s$** The contribution of the shear reinforcement to shear resistance is given by Eq. (3). The angle of shear cracking can be directly calculated by Mohr's circle of stress, as in Eq. (12).

$$\cot\theta = \sqrt{1 + \frac{f_{pc}}{f_t}} \tag{12}$$

When there is no longitudinal precompressive stress $f_{pc}$ or, if flexure-shear cracking governs, cot $\theta = 1$ ($\theta = 45$ degrees). Otherwise, Eq. (13) provides the calculation for cot $\theta$.

$$\cot\theta = 1.0 + 0.095\frac{f_{pc}}{\sqrt{f_c'}} \leq 1.8 \tag{13}$$

where $f_c'$ and $f_{pc}$ are in psi

The complete design procedure for the SIMP is shown in **Fig. 1**.

## Change 2: CSA method

The shear-design provisions in the 1994 *Design of Concrete Structures*[9] were essentially the same as the SDM in the first three editions of the AASHTO LRFD specifications.[1,5,10] In order to simplify the shear-design provisions, the 2004 CSA introduced equations for evaluating $\beta$ and $\theta$ that replaced the use of tables. Furthermore, a new equation for $\varepsilon_x$ was given that assumed $\theta$ was 30 degrees when evaluating the influence of shear on the longitudinal strain $\varepsilon_x$. Change 2 is the adoption of the CSA relationships for $\beta$, $\theta$, and $\varepsilon_x$ as presented in the following. The one significant change is that in place of using the longitudinal strain at mid-depth $\varepsilon_x = \varepsilon_s/2$ in the equations ($\varepsilon_s$ is the strain in the longitudinal tension reinforcement), the modified AASHTO LRFD specifications' method directly uses $\varepsilon_s$.

The factor controlling the contribution of the concrete $\beta$ can be computed from Eq. (14).

$$\beta = \frac{4.8}{(1 + 750\varepsilon_s)} + \frac{51}{(39 + s_{xe})} \tag{14}$$

When the member contains minimum shear reinforcement, $s_{xe}$ is 12 in. (300 mm) and the second term goes to unity. The angle of the diagonal compression field $\theta$ is calculated from Eq. (15).

$$\theta = 29 + 3500\varepsilon_s \tag{15}$$

The longitudinal strain in the longitudinal tension reinforcement $\varepsilon_s$ is computed by Eq. (16).

$$\varepsilon_s = \frac{\frac{M_u}{d_v} + 0.5N_u + V_u - V_p - A_{ps}f_{po}}{E_s A_s + E_p A_{ps}} \tag{16}$$

## Impact of changes on design practice

As previously described, experimental tests alone cannot provide a full assessment of the suitability of provisions because experimental specimens do not always provide good representations of the types of structures built in the field. In order to assess the impact and reliability of the proposed code changes, a design database was produced of 473 cross sections to compare required strengths $\rho_v f_y$ of shear reinforcement for each of these sections by change 1 (SIMP method) and change 2 (CSA method) with three methods (ACI 318-02 [also AASHTO standard specifications], the full general procedure of AASHTO LRFD specifications' SDM, and computer program Response 2000 [R2K]).

It is valuable to compare required amounts of shear reinforcement of the SIMP and CSA methods with the ACI 318-02 method because a large portion of the built infrastructure was designed according to the ACI 318-02 method. The comparison with the general procedure of the AASHTO LRFD specifications' SDM is most useful for assessing the impact of the use of the CSA equations for evaluating $\beta$, $\theta$, and the longitudinal strain over the tabular and iterative general procedure. The comparison with the computer program R2K is useful because it provides a complete implementation of the MCFT for flexural members on which the AASHTO LRFD specifications' SDM is based. In addition, R2K has been proved to provide a more accurate and reliable prediction of the shear capacity of flexural members than any other method.[28] A description of the design database, R2K, and of the results of the comparisons follows.

The database of design cross sections was developed to encompass a range of traditional and possible applications. This includes sections from prestressed and nonprestressed, composite and non-composite, and simply supported and continuous members. For determining the design forces, all of these members were considered to support a uniformly distributed load and were designed for flexure to satisfy the requirements of the AASHTO LRFD specifications.

The sections selected for shear design for simply supported members were located at $d_v$, 0.1$L$, 0.2$L$, 0.3$L$, and 0.4$L$ from the support. The sections selected for shear design for continuous members were located at $d_v$, 0.1$L$, 0.2$L$, 0.3$L$, 0.4$L$, 0.8$L$, 0.9$L$, and ($L$ - $d_v$) from the simple support. In order to obtain a range of design shear-stress levels and *M/V* ratios at each of these sections, each member was designed for multiple span lengths and to support loads that required different levels of flexural reinforcement (50%, 75%, or 100% of the maximum allowable flexural reinforcement). The maximum flexural reinforcement was determined so as to satisfy the maximum reinforcement ratio ($\rho_{max} = 0.75\rho_b$) for all members as well as the requirement

**Table 2.** R2K ratio of required strength of shear reinforcement to required strength by design method

| Reinforcement type | | All | Prestressed concrete members: Simple support | Post-tensioned concrete members: Two-span continuous | Reinforced concrete members: Simple support | Reinforced concrete members: Two-span continuous |
|---|---|---|---|---|---|---|
| Number of cases | | 213 | 120 (38 I-beam, 82 bulb-T beam) | 18 (box girder) | 60 (25 rectangular, 35 T-shape) | 15 (rectangular) |
| ACI 318-02 | Mean | 1.26 | 1.21 | 0.77 | 1.53 | 1.11 |
| | COV | 0.31 | 0.30 | 0.27 | 0.19 | 0.21 |
| | 5% fractile | 25% | 28.6% | 87% | 3.9% | 33% |
| AASHTO LRFD specifications | Mean | 1.42 | 1.64 | 1.06 | 1.22 | 0.88 |
| | COV | 0.37 | 0.35 | 0.14 | 0.18 | 0.23 |
| | 5% fractile | 21% | 11.8% | 34% | 13.3% | 77% |
| CSA | Mean | 1.40 | 1.64 | 0.88 | 1.21 | 0.90 |
| | COV | 0.48 | 0.48 | 0.25 | 0.16 | 0.19 |
| | 5% fractile | 28% | 20.6% | 71% | 12.9% | 72% |
| SIMP | Mean | 1.57 | 1.52 | 1.33 | 1.78 | 1.35 |
| | COV | 0.23 | 0.23 | 0.08 | 0.18 | 0.21 |
| | 5% fractile | 6% | 4.4% | 0% | 0.8% | 11% |

Source: *Building Code Requirements for Structural Concrete (ACI 318-02) and Commentary (ACI 318 R-02)*, *AASHTO LRFD Bridge Design Specifications*, *Design of Concrete Structures* (CSA A23.3-04), and the fourth edition of the *AASHTO LRFD Bridge Design Specifications*.
Note: COV = coefficient of variation; R2K = computer program Response 2000; SIMP = the simplified shear-design provisions added into the fourth edition of the *AASHTO LRFD Bridge Design Specifications*.

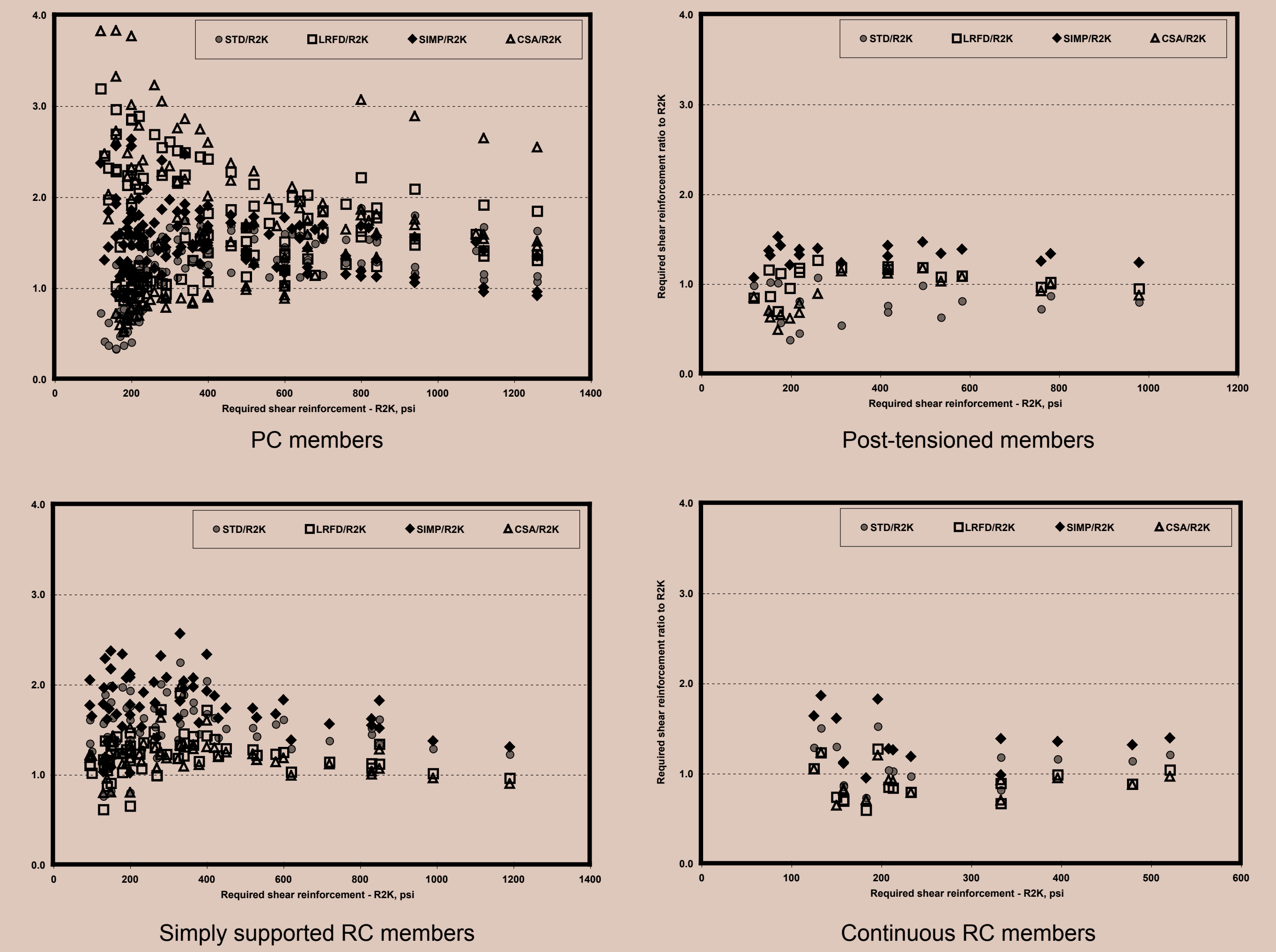



**Figure 2.** These graphs compare the required amount of shear reinforcement for a selection of the design cases.
Note: AASHTO = American Association of State Highway and Transportation Officials; CSA = change that will appear in the 2008 *AASHTO LRFD Bridge Design Specifications* revisions replacing current tables for $\beta$ and $\theta$, as well as the equation for evaluating $\varepsilon_x$ in the sectional design model (S5.8.3); LRFD = *AASHTO LRFD Bridge Design Specifications*; PC = prestressed concrete; R2K = computer program Response 2000; RC = reinforced concrete; SIMP = the simplified shear-design provisions added into the fourth edition of the *AASHTO LRFD Bridge Design Specifications*; STD = AASHTO's *Standard Specifications for Highway Bridges*. 1 psi = 6.895 kPa.

of stress limits for PC members. The former was slightly more conservative than applying the 0.005 limit. This led to some design shear-stress levels that were larger than those commonly seen in current design but that are still admissible by the AASHTO LRFD specifications and are starting to be used in some states. The six different types of members from which the sections were selected were

- a 36-in.-deep (914 mm), simply supported, PC I-beam with 7.5-in.-thick (190 mm) composite slab;
- a 72-in.-deep (1829 mm), simply supported, PC bulb-T girder with 7.5-in.-thick (190 mm) composite slab;
- a 78-in.-deep (1981 mm), two-span, continuous post-tensioned concrete box girder;
- a 36-in.-deep (914 mm), simply supported, rectangular, RC beam;
- a 42-in.-deep (1067 mm), simply supported, T-shaped, RC beam;
- a 36-in.-deep (914 mm), two-span, continuous, RC beam.

R2K is a multilayer sectional analysis tool that can predict the response of a section to the simultaneously occurring actions of axial load, prestressing, moment, and shear. In R2K, the plane section assumption is employed, which imposes a constraint on the distribution of shear stress over the depth of a section. For each layer, an equivalent dual section analysis is performed that uses the MCFT to solve for the angle of diagonal compression, longitudinal stress, and shear stress in each layer.[29] In a typical analysis, the cross section is divided into more than 100 layers. Because this program is based on a general behavioral model (MCFT) and not calibrated by a particular set of beam-test data, it can be expected that R2K provides estimates of the capacity of members in this design database similarly accurate to those for members in experimental test databases. In this use of R2K, the appropriate ratio of $M/V$ and level of prestressing were used as input, and then the amount of shear reinforcement was adjusted until the predicted capacity was equal to $V_u/\varphi$.

**Table 2** and **Fig. 2** summarize the results of the comparisons. Table 2 presents the mean and COV of the ratios of the required strengths $(\rho_v f_y)_{method}$ of shear reinforcement by each of the four design methods to the required strength $(\rho_v f_y)_{R2K}$ determined by R2K. In Table 2, only the results from design cases in which all methods required greater than minimum shear reinforcement are included. In examining the results, the authors were particularly interested in identifying those conditions under which any of the methods were either less conservative—had a lower ratio of $(\rho_v f_y)_{method}/(\rho_v f_y)_{R2K}$—or particularly different from other provisions. For all methods, $\varphi$ was equal to 0.9. From these comparisons, the following observations were made.

- The SIMP method provided the most conservative estimate of the required amount of shear reinforcement with a mean ratio $(\rho_v f_y)_{SIMP}/(\rho_v f_y)_{R2K}$ of 1.57. However, that method also had, at 0.23, the smallest COV of the four design methods. If a normal distribution of data is assumed and a strength-reduction factor of 0.9 is applied, then it would be expected that in only 6% of cases would sections be under-reinforced relative to the amount of shear reinforcement required by R2K. For each of the six design cases, the SIMP provisions were conservative. It should be noted that the SIMP provisions were intentionally selected to be more conservative for sections that were calculated to require only light amounts of shear reinforcement in order to address serviceability and fatigue concerns.[4]

- The ACI 318-02 (also AASHTO standard specifications) method resulted in the lowest mean reinforcement requirement ratio of 1.26. When coupled with a COV of 0.31, this suggests that in 25% of the cases, sections would be under-reinforced relative to the strength of shear reinforcement required by R2K. This method was found to be least conservative for the design of the continuous box beams and somewhat less conservative for the design of the continuous RC beam and bulb-T PC girders.

- The AASHTO LRFD specifications' general procedure and CSA method had similar mean reinforcement requirement ratios for most of the design cases. This result was expected given that the relationships for $\beta$ and $\theta$ for both methods were also derived from the MCFT using the longitudinal strain at mid-depth and similar assumptions as described in the paper by Bentz et al.[30] The CSA method was somewhat less conservative for continuous members. For the general procedure of the AASHTO LRFD specifications' SDM, the mean reinforcement ratio for all members was 1.42, with a COV of 0.37, suggesting that in 21% of cases sections would be under-reinforced relative to the strength of shear reinforcement required by R2K. For the CSA method, the mean reinforcement ratio for all members was 1.40, with a COV of 0.48, suggesting that in 28% of cases sections would be under-reinforced relative to the strength of shear reinforcement required by R2K.

If the results from R2K were perfectly correct, then only the SIMP provisions would closely satisfy the general design philosophy that there should be less than a 5% chance of a design not being conservative. A further evaluation of the SIMP and CSA methods using available experimental test data was presented in two NCHRP reports,[4,28] illustrating the acceptability of these methods for concrete members with strengths up to 18 ksi (124 MPa) and when welded-wire reinforcement is used.

## Design example

A design example is presented to illustrate the use of the SIMP method presented in change 1 and that now appears as the "Simplified Procedure for Prestressed and Nonprestressed Members" in article 5.8.3.4.3 of the fourth edition of the AASHTO LRFD specifications. This design example is of a 100-ft-long (30 m), simply supported, PC girder. **Figures 3** and **4** present details of the elevation and cross section of the example bridge. Figure 4 presents the strand profiles at the end of the girder, at midspan, and in which the location of the strands are shown in an enclosed box. The PC girder contained a total of thirty-two 0.6-in.-diameter (15 mm) strands, two top strands, and thirty bottom strands, among which four strands were draped. The two harping points were located at 0.4*L* from the end supports as shown in Fig. 3. The effective stress in the prestressing steel after all losses $f_{pe}$ was 174 ksi (1200 MPa). The modulus of elasticity of the prestressing steel was 28,500 ksi (196,500 MPa). The minimum specified 28-day strength for the normalweight concrete was 8 ksi (55 MPa) for the girder and 5 ksi (34 MPa) for the deck slab. The girder was designed for a service live load of 2.94 kip/ft (42.9 kN/m). This example presents the shear design by the SIMP procedure at $d_v$ from the center of the support.

### Calculation of shear stress at location $d_v$

The effective depth $d_e$ is calculated as

$$d_e = \frac{A_{ps} f_{py} d_p + A_s f_y d_s}{A_{ps} f_{py} + A_s f_y} = 68.6 \text{ in. (1740 mm)}$$

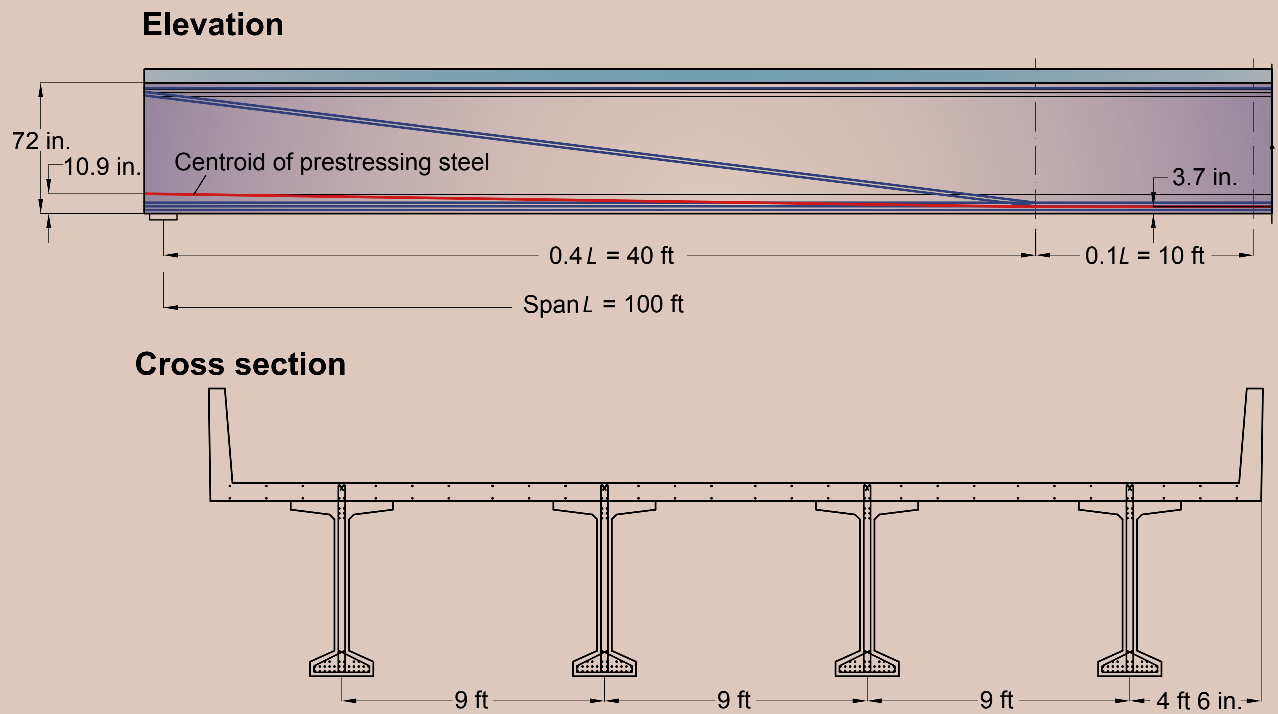



**Figure 3.** The design example's elevation and cross section are shown in detail. 1 in. = 25.4 mm; 1 ft = 0.3048 m.

where

$A_{ps}$ = area of prestressing tendons

$A_s$ = area of nonprestressed reinforcement

$d_p$ = distance from extreme compression fiber to the centroid of the prestressing tendons

$d_s$ = distance from extreme compression fiber to the centroid of the nonprestressed tensile reinforcement

$f_{py}$ = yield strength of prestressing tendons

$f_y$ = yield strength of reinforcing bars

In accordance with the AASHTO LRFD specifications, the effective shear depth $d_v$ is taken as the greater of $0.9d_e$ or $0.72h$:

$$d_v = 0.9d_e = 61.7 \text{ in. (1570 mm)}$$

The design section is located at (61.7 in. + 7.5 in.)/12 = 5.77 ft (1.76 m) from the center of the end support.

Total factored load

$$w_u = 1.25w_{dc} + 1.75w_l$$

$$= [1.25(1.62)] + [1.75(2.94)] = 7.17 \text{ kip/ft (105 kN/m)}$$

where

$w_{dc}$ = unfactored dead load (self-weight only)

$w_l$ = unfactored live load

The factored shear forces at the ends $V_{u1}$ and at midspan $V_{u2}$ can be calculated as

$$V_{u1} = \frac{w_u L}{2} = \frac{7.17(100)}{2} = 386 \text{ kip (1717 kN)}$$

$$V_{u2} = 1.75w_l\left(\frac{L}{8}\right) = \left[1.75(2.94)\right]\left(\frac{100}{8}\right) = 64 \text{ kip (285 kN)}$$

The latter considers that a uniform live load is on half of the span. The shear at 5.77 ft (1.76 m) from the support is calculated as

$$V_u = V_{u1}\left(1-\frac{2x}{L}\right)+V_{u2}\left(\frac{2x}{L}\right)$$

$$= 386\left(1-\frac{2(5.77)}{100}\right)+64\left(\frac{2(5.77)}{100}\right)$$

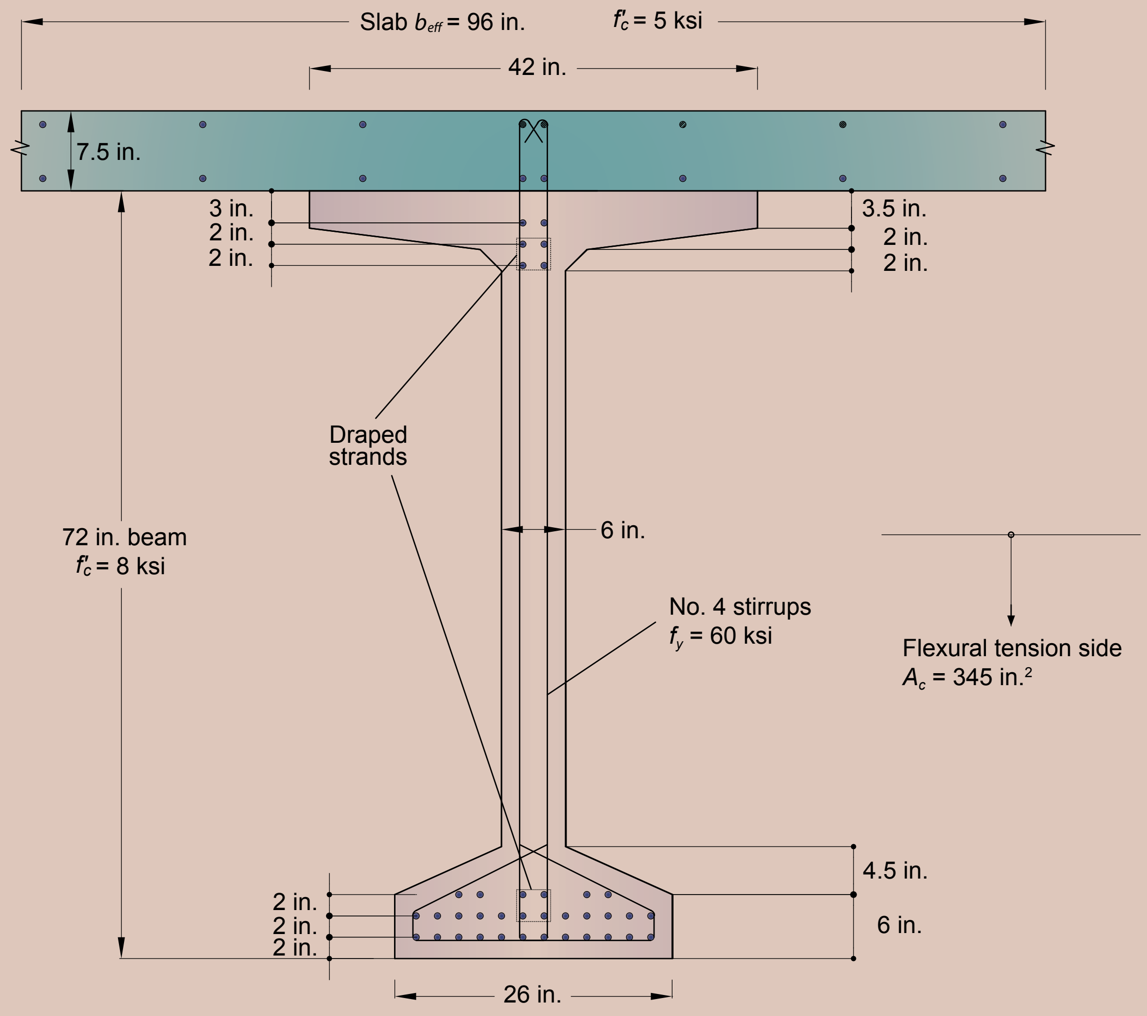



**Figure 4.** This drawing illustrates the material properties and girder geometry for the design example presented in this paper. 1 in. = 25.4 mm; 1 ksi = 6.895 MPa.

= 349 kip (1552 kN)

The shear stress at the design location is calculated by Eq. 5.8.2.9-1[5] as

$$V_u = \frac{\left|V_u - \varphi V_p\right|}{\varphi b_v d_v}$$

$$= \frac{\left|349 - 0.9(19)\right|}{0.9(6)(61.7)} = 0.996 \text{ ksi (6.87 MPa)}$$

which is equivalent to 0.125 $f'_c$.

## Shear design by the simplified procedure (section 5.8.3.4.3)

The flexural-shear cracking strength is evaluated by Eq. (5.8.3.4.3-1), with $f'_c$ expressed in ksi.[5]

$$V_{ci} = 0.02\sqrt{f'_c}\, b_v d_v + V_d + \frac{V_i M_{cre}}{M_{max}} \geq 0.06\sqrt{f'_c}\, b_v d_v$$

where

$M_{cre}$ = moment causing flexural cracking at section due to externally applied loads

The compressive stress in concrete due only to effective prestress $f_{cpe}$ at the extreme tension fiber can be calculated by knowing the eccentricity $e$, which in this case is equal to 22 in. (560 mm).

$$f_{cpe} = \frac{A_{ps} f_{pe}}{A_{nc}} + \frac{\left(A_{ps} f_{pe}\right) e y_t}{I_{nc}}$$

$$= \frac{6.94(174)}{767} + \frac{6.94(174)(22)(36.6)}{545{,}894}$$

= 3.36 ksi (23.2 MPa)

where

$y_t$ = distance from the neutral axis to the extreme tension fiber (bulb-T section only)

$nc$ = subscript referring to noncomposite section (bulb-T section only)

The unfactored dead-load moment acting on the noncomposite section $M_{dnc}$ was calculated as 2662 kip-in. (301 kN-m). Hence, the cracking moment can be calculated as

$$M_{cre} = S_c\left(f_r + f_{cpe} - \frac{M_{dnc}}{S_{nc}}\right)$$

$$= \frac{1,049,353}{53.3}\left(0.24\sqrt{8} + 3.36 - \frac{2662}{14,915}\right)$$

$$= 76,001 \text{ kip-in.} = 6333 \text{ kip-ft (8588 kN-m)}$$

where

$S_c$ = section modulus for the extreme fiber of the composite section

$f_r$ = modulus of rupture of concrete

$S_{nc}$ = section modulus for the extreme fiber of the non-composite section

Having obtained the shear force at section due to unfactored dead load $V_d$ = 73 kip (325 kN) and shear force and moment due to externally applied loads $V_i = V_u - V_d$ = 276 kip (1228 kN) and $M_{max} = M_u - M_d$ = 1940 – 466 = 1494 kip-ft (2026 kN-m), respectively, the flexural-shear cracking strength $V_{ci}$ can be calculated as

$$V_{ci} = 0.02\sqrt{8}(6)(61.7) + 73 + \frac{257(6333)}{1494}$$

$$= 1183 \text{ kip (5262 kN)}$$

The web-shear cracking strength is evaluated by Eq. (5.8.3.4.3-3), with $f_c'$ expressed in ksi.

$$V_{cw} = \left(0.06\sqrt{f_c'} + 0.30 f_{pc}\right) b_v d_v + V_p$$

$$= \left[0.06\sqrt{8} + 0.30(0.84)\right](6)(61.7) + 19$$

$$= 175 \text{ kip (779 kN)}$$

The nominal shear strength provided by the concrete is the lesser of $V_{ci}$ and $V_{cw}$. Therefore, $V_{ci}$ = 175 kip (779 kN).

The resistance required by shear reinforcement can be calculated by Eq. (5.8.3.3-1) as

$$V_s = (V_u / \varphi) - V_c$$

$$= (349 / 0.9) - 175$$

$$= 213 \text{ kip (947 kN)}$$

The proportioning of shear reinforcement is determined as follows. The area of transverse reinforcement within a spacing $s$ is calculated as (using $f_y$ = 60 ksi)

$$\frac{A_v}{s} = \frac{V_s}{f_y d_v \cot\theta}$$

$$= \frac{213}{60(61.7)(1.8)}$$

$$= 0.0320 \text{ in.}^2\text{/in. (0.813 mm}^2\text{/mm)}$$

where

$$\cot\theta = 1.0 + 3\left(f_{pc} / \sqrt{f_c'}\right) \le 1.8$$

$$= 1.0 + 3\left(0.84 / \sqrt{8}\right)$$

$$= 1.89 \Rightarrow \text{use } 1.8$$

Because $V_{ci} > V_{cw}$, Eq. (5.8.3.4.3-4) is used. The required shear reinforcement in equivalent stress form is calculated as

$$\rho_v f_y = \frac{V_s}{b_v d_v \cot\theta}$$

$$= \frac{213(1000)}{6(61.7)(1.8)}$$

$$= 320 \text{ psi (2.21 MPa)}$$

When using no. 4 (13M) double-leg stirrups at a spacing of 12 in. (305 mm),

$$A_v/s = [2(0.20)]/12$$

$$= 0.0333 \text{ in.}^2\text{/in. (0.846 mm}^2\text{/mm)}$$
$$> 0.0320 \text{ in.}^2\text{/in. (0.813 mm}^2\text{/mm)}$$

The provided shear strength by shear reinforcement $V_s$ is evaluated as

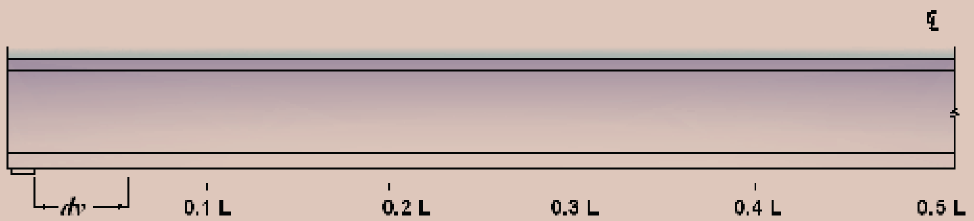
$$V_s = \frac{\left[2(0.20)\right](60)(61.7)(1.8)}{12} = 22 \text{ kip (987 kN)}$$

Eq. (5.8.2.5-1) now checks that the level of provided shear reinforcement is greater than the minimum requirement:

$$(A_v/s)_{min} = 0.0316\frac{\sqrt{f_c'}}{f_y}b_v$$

$$= 0.0316\frac{\sqrt{8}}{60}(6)$$

**Table 3.** Required shear reinforcement by method




| Section | | $d_v$ | $0.1L$ | $0.2L$ | $0.3L$ | $0.4L$ |
|---|---|---|---|---|---|---|
| $d_v$, in. | | 61.7 | 62.6 | 64.4 | 66.3 | 68.2 |
| $V_u$, kip | | 349 | 322 | 257 | 193 | 128 |
| $v/f_c'$ | | 0.125 | 0.113 | 0.086 | 0.061 | 0.037 |
| $M_u$, kip-ft | | 1940 | 3213 | 5713 | 7498 | 8569 |
| SIMP | $\theta$, degrees | 29.1 | 29.1 | 29.1 | 29.1 | 45.0 |
| | $V_c + V_p$, kip | 175 | 180 | 189 | 195 | 145 |
| | $V_s$, kip | 213 | 177 | 96.5 | 19.7 | 0.0 |
| | $\rho_v f_y$, psi | 320 | 262 | 139 | 89.4 | 89.4 |
| AASHTO LRFD specifications | $\varepsilon_x \times 1000$ | -0.098 | -0.057 | 0.177 | 0.690 | 0.945 |
| | $\theta$, degrees | 21.9 | 22.8 | 27.1 | 33.7 | 36.4 |
| | $V_c + V_p$, kip | 117 | 118 | 114 | 104 | 101 |
| | $V_s$, kip | 271 | 239 | 171 | 110 | 41.1 |
| | $\rho_v f_y$, psi | 293 | 268 | 226 | 181 | 89.4 |
| CSA | $\varepsilon_x \times 1000$ | -0.116 | -0.069 | 0.197 | 0.813 | 1.044 |
| | $\theta$, degrees | 28.2 | 28.5 | 30.4 | 34.7 | 36.3 |
| | $V_c + V_p$, kip | 192 | 181 | 134 | 88.3 | 80.6 |
| | $V_s$, kip | 158 | 143 | 124 | 106 | 48.8 |
| | $\rho_v f_y$, psi | 254 | 229 | 209 | 205 | 97.4 |
| AASHTO standard specifications | $\theta$, degrees | 45.0 | 45.0 | 45.0 | 45.0 | 45.0 |
| | $V_c + V_p$, kip | 240 | 245 | 256 | 195 | 124 |
| | $V_s$, kip | 148 | 112 | 29.4 | 19.7 | 18.2 |
| | $\rho_v f_y$, psi | 379 | 284 | 72.2 | 50.0 | 50.0 |

Source: *AASHTO LRFD Bridge Design Specifications*, *Design of Concrete Structures* (CSA A23.3-04), and *Standard Specifications for Highway Bridges*.
Note: SIMP = the simplified shear-design provisions added into the fourth edition of the *AASHTO LRFD Bridge Design Specifications*. 1 in. = 25.4 mm; 1 kip = 4.48 kN; 1 psi = 6.895 kPa.

$= 0.00893$ in.$^2$/in. (0.227 mm$^2$/mm)
$< 0.0333$ in.$^2$/in. (0.846 mm$^2$/mm) OK

It is also checked that the nominal shear resistance is less than AASHTO LRFD specifications' limit.

$V_n = 175 + 222 = 397$ kip (177 kN)

$< 0.25 f_c' b_v d_v + V_p = (0.25)(8)(6)(61.7) + 19$

$= 759$ kip (3376 kN) OK

The shear reinforcements $\rho_v f_y$ required at several locations along the length of this girder were evaluated by the SIMP provisions as well as by the general procedure of the AASHTO LRFD specifications' SDM, the CSA method (change 2), and ACI 318-02 (also AASHTO standard specifications), as shown in **Table 3**.

## Conclusion

- Existing shear-design provisions are based on experimental test data, the equilibrium condition of members in the ultimate limit state, and comprehensive behavioral models for capacity.
- While there is consensus among researchers as to the components that contribute to shear resistance, there is considerable disagreement about the relative magnitude of the contributions, the factors that influence the contributions, and their significance for different design conditions.
- The wide variations in the forms of the shear-design specifications used in different influential codes of practice may result in the amount of shear reinforcement required by one code being two to three times that required by another code for the same section and factored sectional forces.
- It is not possible to make a full assessment of the appropriateness of existing design provisions using the results of experimental test data only because what researchers have tested in laboratories does not provide good representation of the types of structures or the loadings used in the field. To this end, design databases of representative design sections are useful for comparing required strengths $\rho_v f_y$ of shear reinforcement by different methods with each other and with the predictions of numerical methods.
- Simplified shear-design provisions were developed for the AASHTO LRFD specifications that are a modified version of the AASHTO standard specifications. These provisions will usually provide for a more conservative design, while in some situations other influential provisions, such as the general procedure of the LRFD specifications, may not.
- The general procedure of AASHTO LRFD specifications' SDM relies on tables for the evaluation of $\beta$ and $\theta$, the values of which control the contributions of concrete and transverse reinforcement to shear strength as well as the equation for the strain $\varepsilon_x$ at mid-depth. Those tables can be replaced by the equations for the same three quantities in the CSA *Design of Structural Concrete* without a loss in accuracy or conservatism.

## Acknowledgments

The research program described in this paper was sponsored by the National Academy's National Cooperative Highway Research Program as project 12-61, "Simplified Shear Design of Structural Concrete Members." PCI provided additional support for this research. The authors acknowledge the contributions of the other investigators on this project, Karl-Heinz Reineck from the University of Stuttgart in Germany and Robert Mast and Lee Marsh from Berger/ABAM Engineers Inc. in Federal Way, Wash.

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## Notation

$a_g$ = maximum aggregate size

$A_c$ = area of concrete beneath mid-depth

$A_{ps}$ = area of prestressing steel on flexural-tension side of member

$A_s$ = area of nonprestressed tension reinforcement on flexural-tension side of member at ultimate load

$A_v$ = area of transverse reinforcement within a distance $s$

$A_{v,min}$ = area of minimum required transverse reinforcement

$b_v$ = effective web width

$b_w$ = width of web

$d$ = effective member depth

$d_e$ = effective depth from extreme compression fiber to the centroid of the tensile force in the tensile reinforcement

$d_p$ = distance from extreme compression fiber to the centroid of the prestressing tendons

$d_s$ = distance from extreme compression fiber to the centroid of the reinforcing bars

$d_v$ = effective shear depth taken as the distance, measured perpendicular to the neutral axis, between the resultants of the tensile and compressive forces due to flexure; it need not be taken to be less than the greater of $0.9d_e$ or $0.72h$

$e$ = eccentricity

$E_c$ = modulus of elasticity of concrete

$E_p$ = modulus of elasticity of prestressing steel

$E_s$ = modulus of elasticity of reinforcing bars

$f_{cpe}$ = compressive stress in concrete due to effective prestress forces only (after allowance for all prestress losses) at extreme fiber of section where tensile stress is caused by externally applied loads

$f_{pc}$ = compressive stress in concrete after all prestress losses have occurred either at centroid of the cross section resisting live load or at the junction of the web and flange when the centroid lies in the flange

$f_{pe}$ = effective stress in the prestressing steel after losses

$f_{po}$ = $E_p$ times locked-in difference in strain at ultimate load between the prestressing tendons and the surrounding concrete

$f_{ps}$ = stress in prestressing steel at the time for which nominal resistance of member is required

$f_{py}$ = yield strength of prestressing steel

$f_r$ = modulus of rupture of concrete

$f_t$ = tensile strength of concrete

$f_x$ = stress in the $x$-direction

$f_y$ = stress in the $y$-direction; yield strength of reinforcing bars

$f_c'$ = concrete compressive strength

$h$ = height of the member

$L$ = span length

$M_{cr}$ = net cracking moment (= $M_{cre}$)

$M_{cre}$ = moment causing flexural cracking at section due to externally applied loads

$M_{dnc}$ = total unfactored dead-load moment acting on the monolithic or noncomposite section

$M_{max}$ = maximum factored moment at section due to externally applied loads

$M_u$ = factored moment at section

$N_u$ = factored axial force

$s$ = spacing of bars of transverse reinforcement

$s_x$ = crack-spacing parameter, the lesser of either $d_v$ or the maximum distance between layers of longitudinal-crack-control reinforcement

$s_{xe}$ = equivalent crack-spacing parameter $\left(= \dfrac{1.38 s_x}{0.63 + a_g}\right)$

$S_c$ = section modulus for the extreme fiber of the composite section

$S_{nc}$ = section modulus for the extreme fiber of the noncomposite section

$T_{min}$ = minimum tensile capacity required for longitudinal reinforcement on flexural-tension side of member at section

$v$ = shear stress

$v_u$ = factored shear stress

$v_{xy}$ = shear stress in the $x$-$y$ direction

$V_c$ = shear at inclined cracking; nominal shear resistance provided by concrete

$V_{ci}$ = shear at flexure-shear cracking

$V_{code}$ = nominal shear strength of member as evaluated by a specific code method or procedure

$V_{cw}$ = shear at web-shear cracking

$V_d$ = shear force at section due to unfactored dead load

$V_i$ = factored shear force at section due to externally applied loads occurring simultaneously with $M_{max}$

$V_n$ = nominal shear resistance of section considered

$V_p$ = component in the direction of the applied shear of the effective prestressing force

$V_s$ = shear resistance provided by transverse reinforcement

$V_{test}$ = shear resistance measured at ultimate capacity in test

$V_u$ = factored shear force at section

$w_{dc}$ = unfactored dead load (self-weight only)

$w_l$ = unfactored live load

$w_u$ = total factored load

$y_t$ = distance from centroid of cross section to the extreme tension fiber

$\beta$ = factor relating effect of longitudinal strain on the shear capacity of concrete

$\gamma_{xy}$ = shear strain in the $x$-$y$ direction

$\varepsilon_s$ = the strain in the longitudinal tension reinforcement

$\varepsilon_t$ = strain at level of longitudinal reinforcement on tension side of member

$\varepsilon_x$ = longitudinal strain at mid-depth of section; strain in the $x$-direction

$\varepsilon_y$ = strain in the $y$-direction

$\theta$ = angle of inclination of diagonal compressive stress

$\rho_b$ = balanced reinforcement ratio

$\rho_{max}$ = maximum reinforcement ratio = $0.75\rho_b$

$\rho_v$ = ratio of area of vertical shear reinforcement to gross concrete area of a horizontal section

$\varphi$ = resistance factor for shear

## About the authors


Daniel A. Kuchma, PhD, is an assistant professor for the Department of Civil and Environmental Engineering at the University of Illinois at Urbana-Champaign in Urbana, Ill.

Neil M. Hawkins, PhD, is professor emeritus for the Department of Civil and Environmental Engineering at the University of Illinois at Urbana-Champaign.

Sang-Ho Kim is a PhD candidate and research assistant for the Department of Civil and Environmental Engineering at the University of Illinois at Urbana-Champaign.

Shaoyun Sun, PhD, is a researcher for the Department of Civil and Environmental Engineering at the University of Illinois at Urbana-Champaign.

Kang Su Kim, PhD, is an assistant professor for the Department of Architectural Engineering at the University of Seoul in Seoul, South Korea.


## Synopsis


The studies made and the findings from the National Cooperative Highway Research Program (NCHRP) project 12-61, “Simplified Shear Design of Structural Concrete Members,” are discussed. The stated objective of this NCHRP project was to develop practical procedures for the design of shear reinforcement in reinforced and prestressed concrete bridge girders. The motivation was that many bridge designers perceive the general shear-design procedure of the sectional design model in the American Association of State Highway and Transportation Officials’ *AASHTO LRFD Bridge Design Specifications* to be unnecessarily complex and difficult to use. This general procedure unifies the shear design of both prestressed and nonprestressed members and allows the design for much higher shear stresses than is permitted by AASHTO’s *Standard Specifications for Highway Bridges*.

Reviews and evaluations are made of the shear-design methods of several leading codes of practice and influential researchers. Based on this assessment, a review of field experience, and comparisons with a large experimental database, criteria are developed for the simplified provisions, and the two resulting changes to the *AASHTO LRFD Bridge Design Specifications* shear-design provisions are described. The simplified provisions are similar in concept to AASHTO’s *Standard Specifications for Highway Bridges*, contain a new expression for the web-shear cracking capacity, and use a variable-angle truss model to evaluate the contribution of shear reinforcement. An example illustrates the use of the simplified procedure.


## Keywords


Bridge, concrete, girder, LRFD, prestressed, reinforced, shear design, truss model.


## Review policy


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