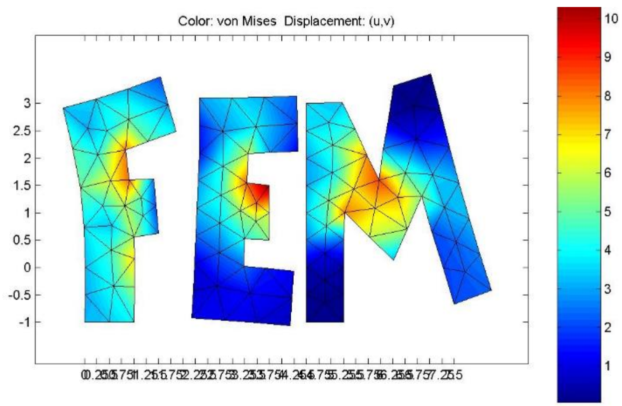
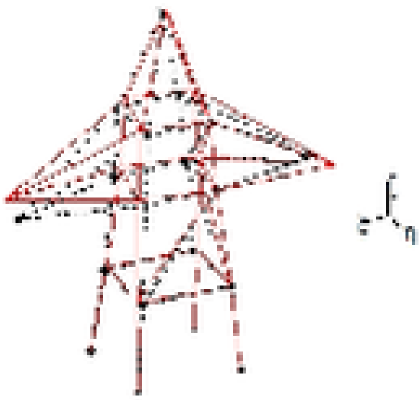
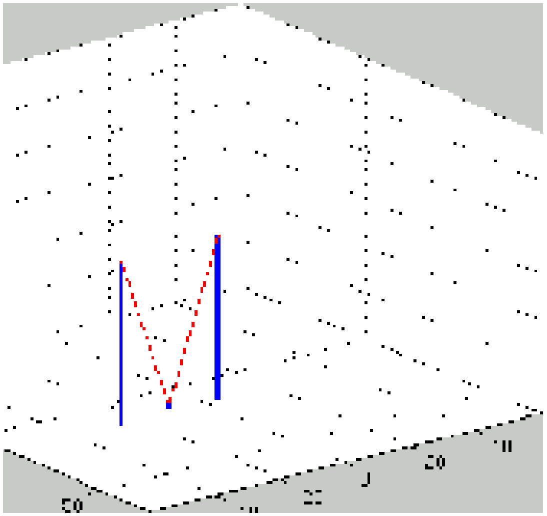
MATLAB as a General Simulation Tool in the Mechanical Engineering Education at Chalmers







[TOUT,YOUT] = ODE23(ODEFUN,TSPAN,Y0)

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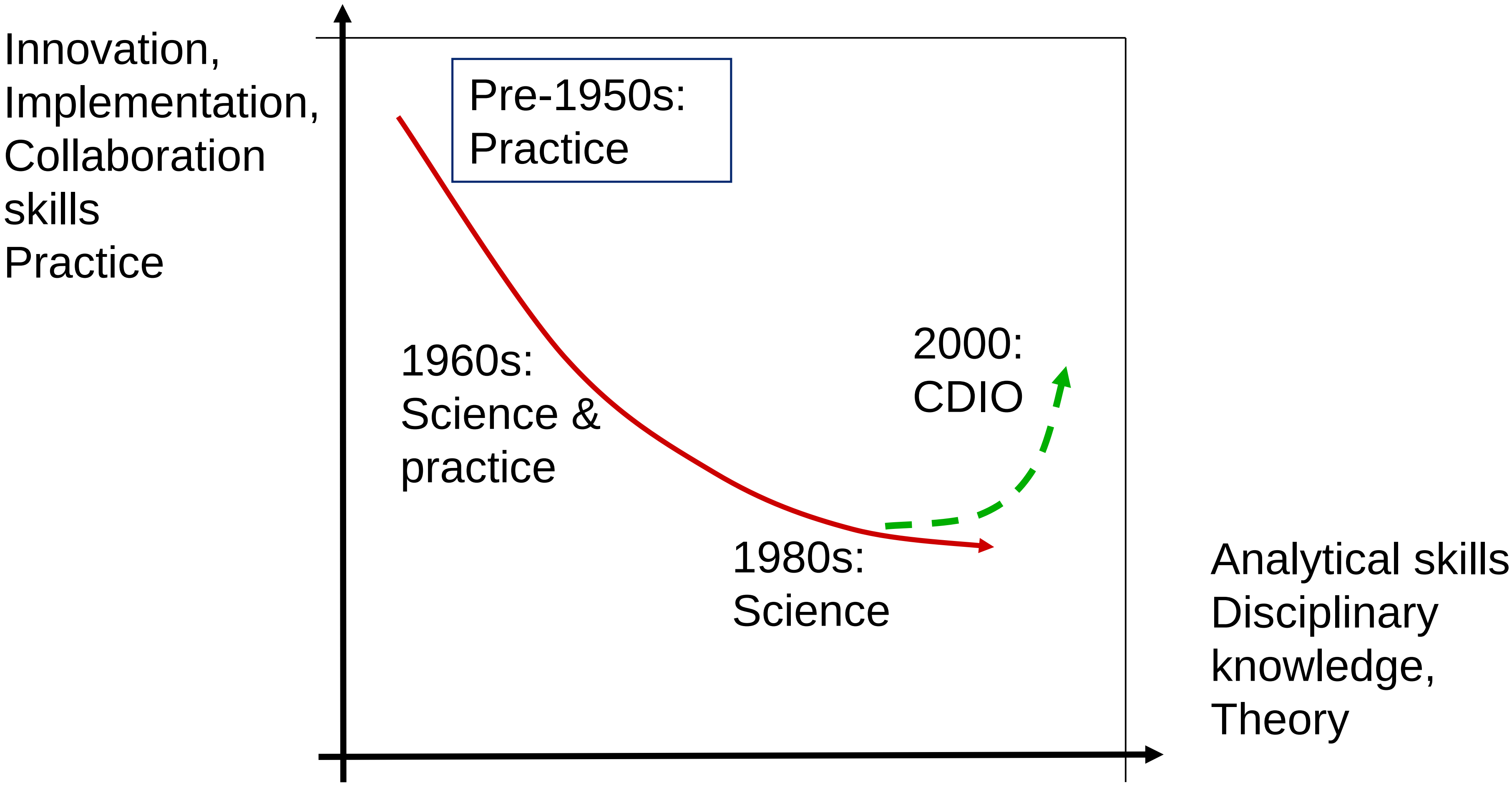
CHALMERS' ME PROGRAM - BASIS

- 5-yrs two cycle program (combined 3-yrs BSc + 2-yrs MSc)
- 500 students on BSc-level + 550 on MSc level incl 200 international students
- Long history of curriculum, pedagogic and learning environment innovations
- Initiator (together with MIT, KTH and LiTH) to the CDIO-initiative
- Integrating general engineering skills and education for sustainable development
- Integrating simulation based math education and computer-based simulations (programming and CAE-tools)
- Early engineering experiences and Multiple design-build-test projects
- Supportive environment with project workspaces, course lab, prototyping workshop and driving simulator
- Globalization

EVOLUTION OF ENGINEERING EDUCATION



Today's and tomorrow's engineers need "both-and"
... Full-detail, synthesis - analysis, disciplinary-interdisciplinary, theory-practice, ...

THE CDIO VISION

An education that stresses the fundamentals, set in the context of Conceiving – Designing – Implementing – Operating systems and products

- A curriculum organised around mutually supporting courses. Rich with student design-build projects
- Integrated approach to learning general engineering skills incl programming
- Featuring active and experiential learning
- Set in both the classroom and a modern learning laboratory/workspace/virtual
- Continuously improved through a robust assessment/evaluation process

Bring forward the role of design and implementation in the education - from paper or computer designs to physical or virtual prototypes

Bridge theory and practice - more authentic, realistic

Improve learning of non-technical skills

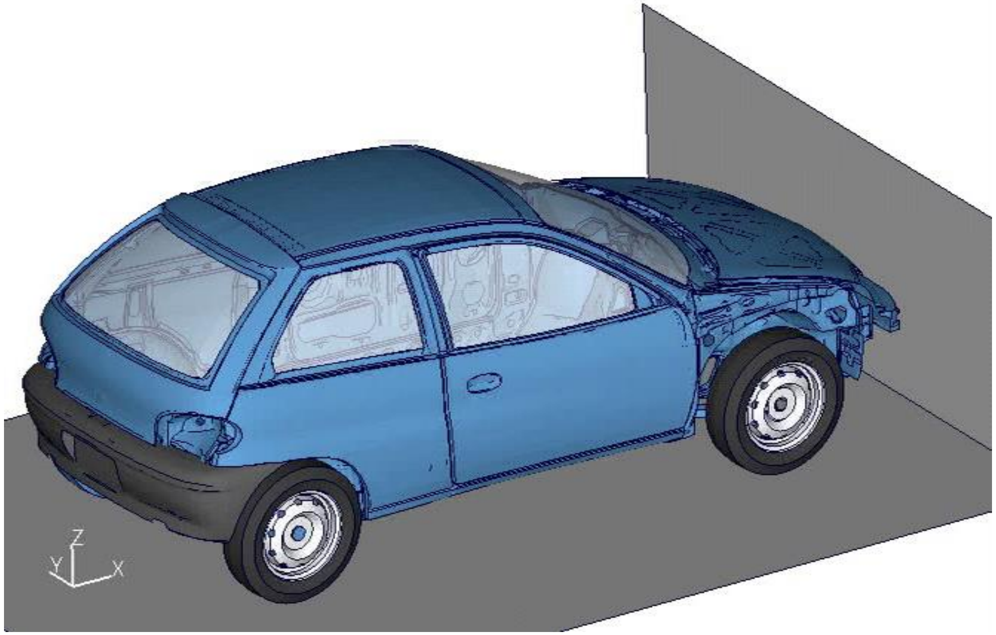
CDIO ENVIRONMENTS

CDIO ENVIRONMENTS

Also need for

- Virtual lab for testing and evaluation
- ICT and programming skills
- Reformed math education

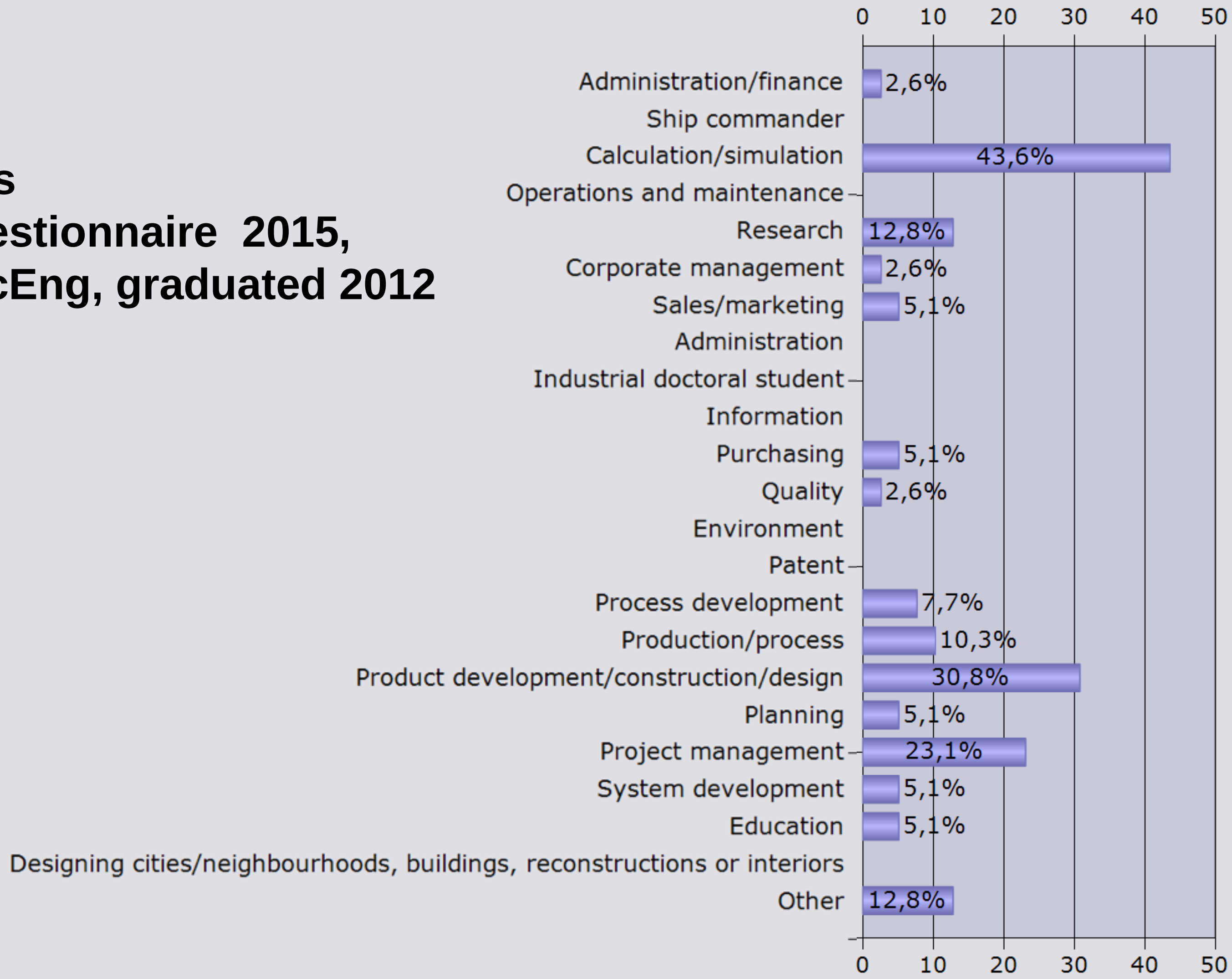
ME ENGINEERS USE A LOT OF ADVANCED MATH



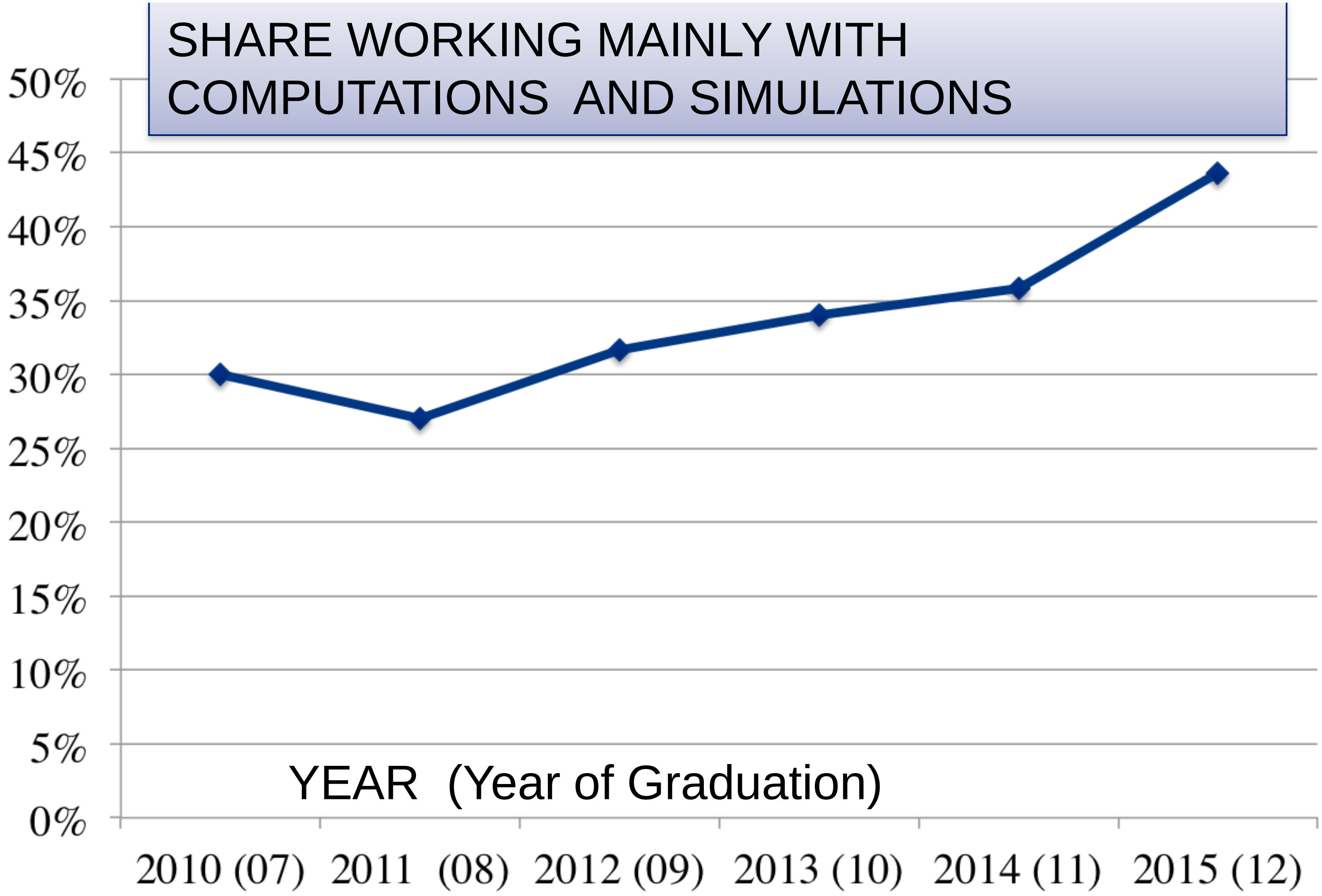
- Simulation driven design
- CAE
- Optimization
- Control
- Industry 4.0
- Internet of things
- Computerization and digitalization
- …..

What do mechanical engineers three years after graduation?

Main Duties
Alumni questionnaire 2015,
MSc in MecEng, graduated 2012



What do mechanical engineers three years after graduation?



CALCULUS: A complete course, Adams

EXERCISES 5.6

Evaluate the integrals in Exercises 1–44. Remember to include a constant of integration with the indefinite integrals. Your answers may appear different from those in the Answers section but may still be correct. For example, evaluating $I = \int \sin x \cos x\, dx$ using the substitution $u = \sin x$ leads to the answer $I = \frac{1}{2}\sin^2 x + C$; using $u = \cos x$ leads to $I = -\frac{1}{2}\cos^2 x + C$; and rewriting $I = \frac{1}{2}\int \sin(2x)\, dx$ leads to $I = -\frac{1}{4}\cos(2x) + C$. These answers are all equal except for different choices for the constant of integration C:
$\frac{1}{2}\sin^2 x = -\frac{1}{2}\cos^2 + \frac{1}{2} = -\frac{1}{4}\cos(2x) + \frac{1}{4}$.

You can always check your own answer to an indefinite integral by differentiating it to get back to the integrand. This is often easier than comparing your answer with the answer in the back of the book. You may find integrals that you can't do, but you should not make mistakes in those you can do because the answer is so easily checked. (This is a good thing to remember during tests and exams.)

1. $\int e^{5-2x}\, dx$
2. $\int \cos(ax+b)\, dx$
3. $\int \sqrt{3x+4}\, dx$
4. $\int e^{2x}\sin(e^{2x})\, dx$
5. $\int \frac{x\, dx}{(4x^2+1)^5}$
6. $\int \frac{\sin\sqrt{x}}{\sqrt{x}}\, dx$
7. $\int x e^{x^2}\, dx$
8. $\int x^2 2^{x^3+1}\, dx$
9. $\int \frac{\cos x}{4+\sin^2 x}\, dx$
10. $\int \frac{\sec^2 x}{\sqrt{1-\tan^2 x}}\, dx$
11. $\int \frac{e^x+1}{e^x-1}\, dx$
12. $\int \frac{\ln t}{t}\, dt$
13. $\int \frac{ds}{\sqrt{4-5s}}$
14. $\int \frac{x+1}{\sqrt{x^2+2x+3}}\, dx$
15. $\int \frac{t\, dt}{\sqrt{4-t^4}}$
16. $\int \frac{x^2\, dx}{2+x^6}$
17. $\int \frac{dx}{e^x+1}$
18. $\int \frac{dx}{e^x+e^{-x}}$
19. $\int \tan x \ln\cos x\, dx$
20. $\int \frac{x+1}{\sqrt{1-x^2}}\, dx$
21. $\int \frac{dx}{x^2+6x+13}$
22. $\int \frac{dx}{\sqrt{4+2x-x^2}}$
23. $\int \sin^3 x \cos^5 x\, dx$
24. $\int \sin^4 t \cos^5 t\, dt$
25. $\int \sin ax \cos^2 ax\, dx$
26. $\int \sin^2 x \cos^2 x\, dx$
27. $\int \sin^6 x\, dx$
28. $\int \cos^4 x\, dx$
29. $\int \sec^5 x \tan x\, dx$
30. $\int \sec^6 x \tan^2 x\, dx$
31. $\int \sqrt{\tan x}\sec^4 x\, dx$
32. $\int \sin^{-2/3} x \cos^3 x\, dx$
33. $\int \cos x \sin^4(\sin x)\, dx$
34. $\int \frac{\sin^3 \ln x \cos^3 \ln x}{x}\, dx$
35. $\int \frac{\sin^2 x}{\cos^4 x}\, dx$
36. $\int \frac{\sin^3 x}{\cos^4 x}\, dx$
37. $\int \csc^5 x \cot^5 x\, dx$
38. $\int \frac{\cos^4 x}{\sin^8 x}\, dx$
39. $\int_0^4 x^3(x^2+1)^{-\frac{1}{2}}\, dx$
40. $\int_1^{\sqrt{e}} \frac{\sin(\pi \ln x)}{x}\, dx$
41. $\int_0^{\pi/2} \sin^4 x\, dx$
42. $\int_{\pi/4}^{\pi} \sin^5 x\, dx$
43. $\int_e^{e^2} \frac{dt}{t\ln t}$
44. $\int_{\frac{\pi^2}{16}}^{\frac{\pi^2}{9}} \frac{2^{\sin\sqrt{x}}\cos\sqrt{x}}{\sqrt{x}}\, dx$

45. Use the identities $\cos 2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$ and $\sin\theta = \cos\left(\frac{\pi}{2} - \theta\right)$ to help you evaluate the following:

$$\int_0^{\pi/2} \sqrt{1+\cos x}\, dx \quad \text{and} \quad \int_0^{\pi/2} \sqrt{1-\sin x}\, dx$$

46. Find the area of the region bounded by $y = x/(x^2+16)$, $y=0$, $x=0$, and $x=2$.
47. Find the area of the region bounded by $y = x/(x^4+16)$, $y=0$, $x=0$, and $x=2$.
48. Express the area bounded by the ellipse $(x^2/a^2) + (y^2/b^2) = 1$ as a definite integral. Make a substitution that converts this integral into one representing the area of a circle, and hence evaluate it.
49. Use the addition formulas for $\sin(x \pm y)$ and $\cos(x \pm y)$ from Section P.6 to establish the following identities:

$$\cos x \cos y = \frac{1}{2}\Big(\cos(x-y) + \cos(x+y)\Big),$$
$$\sin x \sin y = \frac{1}{2}\Big(\cos(x-y) - \cos(x+y)\Big),$$
$$\sin x \cos y = \frac{1}{2}\Big(\sin(x+y) + \sin(x-y)\Big).$$

50. Use the identities established in Exercise 49 to calculate the following integrals:
$\int \cos ax \cos bx\, dx$, $\int \sin ax \sin bx\, dx$,
and $\int \sin ax \cos bx\, dx$.
51. If m and n are integers, show that:
(i) $\int_{-\pi}^{\pi} \cos mx \cos nx\, dx = 0$ if $m \neq n$,
(ii) $\int_{-\pi}^{\pi} \sin mx \sin nx\, dx = 0$ if $m \neq n$,
(iii) $\int_{-\pi}^{\pi} \sin mx \cos nx\, dx = 0$.
52. **(Fourier coefficients)** Suppose that for some positive integer k,

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{k}(a_n \cos nx + b_n \sin nx)$$

BETA Mathematics handbook

31. $\int \frac{\sqrt{ax+b}}{x^n}\, dx = -\frac{1}{b(n-1)}\left(\frac{(ax+b)^{3/2}}{x^{n-1}} + \frac{(2n-5)a}{2}\int \frac{\sqrt{ax+b}}{x^{n-1}}\, dx\right)$

32. $\int \frac{dx}{\sqrt{ax+b}} = \frac{2\sqrt{ax+b}}{a}$

33. $\int \frac{x}{\sqrt{ax+b}}dx = -\frac{2b\sqrt{ax+b}}{a^2} + \frac{2(ax+b)^{3/2}}{3a^2}$

34. $\int \frac{x^2}{\sqrt{ax+b}}dx = \frac{2b^2\sqrt{ax+b}}{a^3} - \frac{4b(ax+b)^{3/2}}{3a^3} + \frac{2(ax+b)^{5/2}}{5a^3}$

35. $\int \frac{x^n}{\sqrt{ax+b}}dx = \frac{2}{a(2n+1)}\left(x^n\sqrt{ax+b} - bn\int \frac{x^{n-1}}{\sqrt{ax+b}}\, dx\right)$

36. $\int \frac{dx}{x^n\sqrt{ax+b}} = -\frac{\sqrt{ax+b}}{(n-1)\,bx^{n-1}} - \frac{(2n-3)a}{(2n-2)b}\int \frac{dx}{x^{n-1}\sqrt{ax+b}} \quad (n \neq 1)$

37. $\int \frac{dx}{x\sqrt{ax+b}} = \begin{cases} \frac{1}{\sqrt{b}}\ln\left|\frac{\sqrt{ax+b}-\sqrt{b}}{\sqrt{ax+b}+\sqrt{b}}\right| & (b>0) \\ \frac{2}{\sqrt{-b}}\arctan\sqrt{\frac{ax+b}{-b}} & (b<0) \end{cases}$

38. $\int \frac{dx}{c+\sqrt{ax+b}} = \frac{2}{a}\left(\sqrt{ax+b} - c\ln\left|c+\sqrt{ax+b}\right|\right)$

39. $\int \frac{\sqrt{ax+b}}{c+\sqrt{ax+b}}dx = \frac{1}{a}\left(ax+b-2c\sqrt{ax+b}+2c^2\ln\left|c+\sqrt{ax+b}\right|\right)$

40. $\int \frac{x}{c+\sqrt{ax+b}}dx =$
$= \frac{1}{a^2}\left(2(c^2-b)\sqrt{ax+b} - c(ax+b) + \frac{2}{3}(ax+b)^{3/2} - 2c(c^2-b)\ln\left|c+\sqrt{ax+b}\right|\right)$

41. $\int \frac{dx}{\sqrt{ax+b}\,(c+\sqrt{ax+b})} = \frac{2}{a}\ln\left|c+\sqrt{ax+b}\right|$

42. $\int \frac{dx}{(ax+b)(c+\sqrt{ax+b})} = \frac{2}{ac}\ln\left|\frac{\sqrt{ax+b}}{c+\sqrt{ax+b}}\right|$

43. $\int \frac{dx}{(c+\sqrt{ax+b})^2} = \frac{2c}{a(c+\sqrt{ax+b})} + \frac{2}{a}\ln\left|c+\sqrt{ax+b}\right|$

44. $\int \frac{\sqrt{ax+b}}{(c+\sqrt{ax+b})^2}dx = \frac{2\sqrt{ax+b}}{a} - \frac{2c^2}{a(c+\sqrt{ax+b})} - \frac{4c}{a}\ln\left|c+\sqrt{ax+b}\right|$

Instead, write a computer program that solves all problems (integrals)

```
f = @(x)(cos(x))^2;
a=-pi;
b=pi;
n=100;
dx=(b-a)/n;
F=0;
for j=1:n
    F=F+f((j+0.5)*dx)*dx;
end
format long
disp(F)
```

$$\int_{-\pi}^{\pi} (\cos(x))^2 \mathrm{d}x$$

3.141592653589793

Mathematics + programming = true

- A computer program solves the "general" problem
- Reduce repetitive exercising to practice more on understanding, problem definition and computations
- Opportunities to practice math and problem solving at a higher level
- Logical and algorithmic thinking, creativity and problem solving
- Requires knowledge of mathematics and programming
- Programming creates an understanding of the digital world and opportunities to solve new problems, create new systems, processes and products

REFORMED SIMULATION BASED MATHEMATICAL EDUCATION

- Launched 2006/2007 and continuously improved,
- New math courses including a basic course in MATLAB programming,
- Integration of mathematics in other fundamental engineering courses, e.g., Mechanics and Automatic Control ('just in time')
- Computer-oriented exercises, assignments and team projects that are used simultaneously in the mathematics courses and in courses of mechanics and solid mechanics,
- Interactive/virtual learning learning environments in math and statistics courses
- Teaching and learning in computer lab. Scheduled classed

CORNERSTONES

- To highlight and clarify modeling, computations, analyses and simulations,
- Full integration of computational aspects (including programming) and symbolic aspects of mathematics,
- Construction of algorithms and writing own programs (programming skills and understanding of mathematics and algorithm construction)
- General equations instead of the simplified special equations whose solutions can be written in elementary functions
- Handle real problems (toolbox)
- Visualization

ME PROGRAMME – INTERGRATED CURRICULUM

YEAR 1

Quarter 1	Quarter 2	Quarter 3	Quarter 4
Programming in MATLAB	**Computer aided engineering**	**Linear algebra**	**Mathematical analysis in several variables**
Introductory mathematics	**Mathematical analysis in a single variable**	**Statics & Solid mechanics**	**Solid mechanics**
Introduction to Mechanical engineering			

- MATLAB programming, numerical solutions and simulations
- Simulation using CAE software (CATIA, ANSYS, ADAMS, FLUENT…)

YEAR 2

Quarter 1	Quarter 2	Quarter 3	Quarter 4
Mechanics - Dynamics 🔴	Machine element 🔵	Thermodynamics and energy technology 🔵	Industrial production and organisation
Material technology	Material and manufacturing technology 🔴 🔵	Integrated design and manufacturing 🔴 🔵	
		Sustainable product development	Engineering economics

🔵 MATLAB programming, numerical solutions and simulations

🔴 Simulation using CAE software (CATIA, ANSYS, ADAMS, FLUENT…)

YEAR 3

Quarter 1	Quarter 2	Quarter 3	Quarter 4
Mechatronics 🔵	**Automatic control** 🔵	**Bachelor diploma project**	
Fluid mechanics 🔴 🔵	**Elective 1**	**Elective 1**	**Mathematical statistics** 🔵

Elective 1

- **Energy conversion** 🔵
- **Finite element method** 🔵
- **Machine design** 🔴 🔵
- **Simulation of production** 🔴

Elective 2

- **Logistics**
- **Sound and vibration** 🔵
- **Object oriented programming**
- **Transforms and differential equations** 🔵
- **Heat transfer** 🔴 🔵

🔵 MATLAB/Simulink programming

🔴 Simulation using CAE software (CATIA, ANSYS, ADAMS, FLUENT…)

YEARS 4 and 5

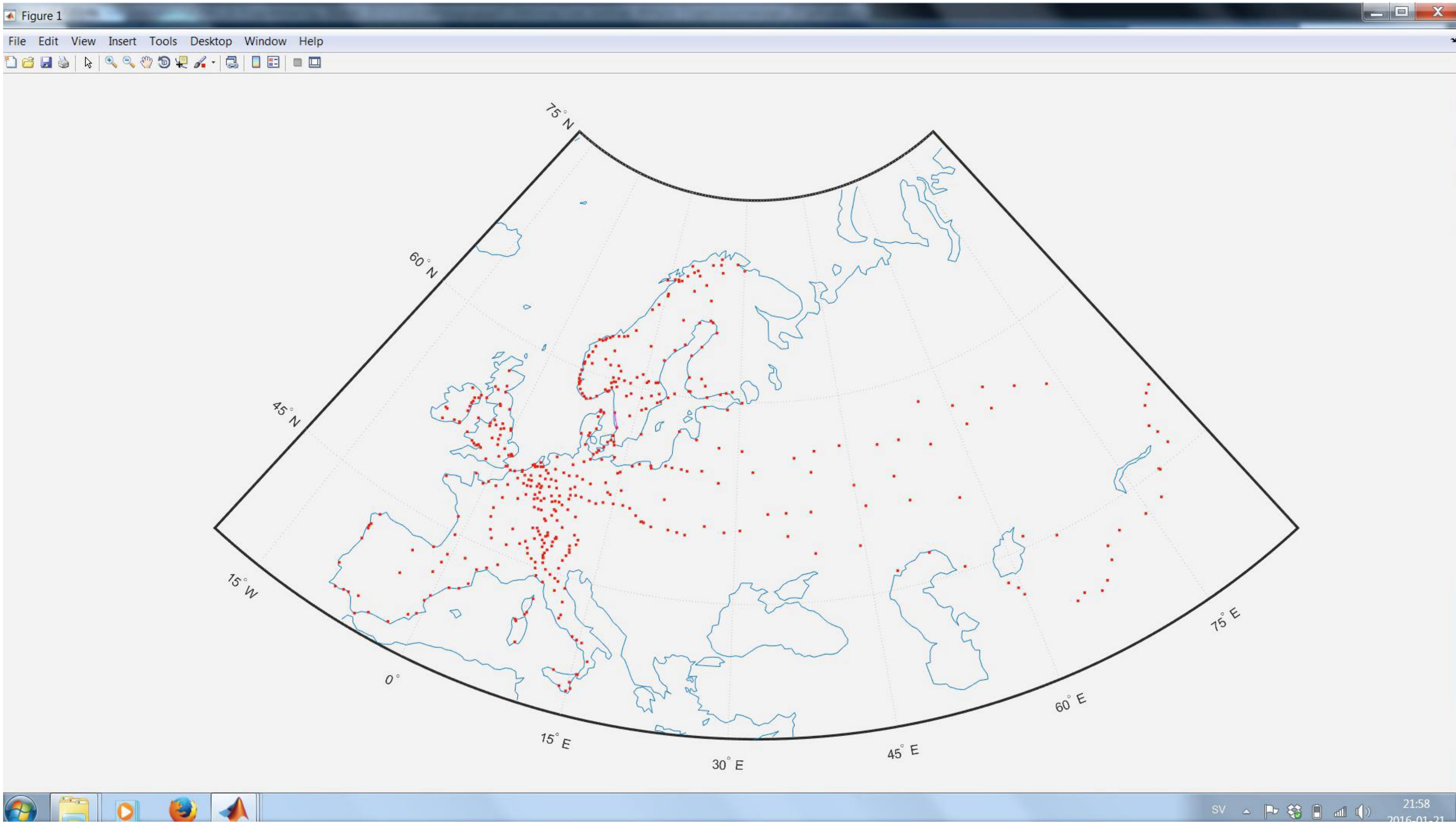
Second cycle, 2 years international master programme. 8 master programmes belong to Mechanical Engineering

- **MSc PROGRAM IN APPLIED MECHANICS**
- **MSc PROGRAM IN AUTOMOTIVE ENGINEERING**
- **MSc PROGRAM IN MATERIALS ENGINEERING**
- **MSc PROGRAM IN NAVAL ARCHITECTUREAND OCEANS ENGINEERING**
- **MSc PROGRAM IN PRODUCT DEVELOPMENT**
- **MSc PROGRAM IN PRODUCTION ENGINEERING**
- **MSc PROGRAM IN SUSTAINABLE ENERGY SYSTEMS**
- **MSc PROGRAM IN TECHNOLOGY, SOCIETY AND THE ENVIRONMENT**

PROGRAMMING IN MATLAB

- Experience shows the need for a separate programming course
- General methodology requires programming
- Why MATLAB?
  - Easy to use and suitable as a first programming environment (cf. edit-compile-execute-debug programming)
  - Easy to visualize
  - Used in all applied courses and in applied research
  - Toolboxes, built-in function and user-submitted libraries
- Aim: *Develop own programs from problem description to working code* (“real programming”)
- 4 programming assignments and final exam in computer lab

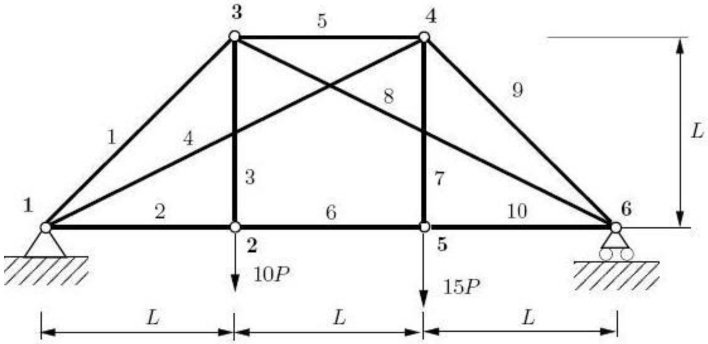
Example: Assignment “Least cost path



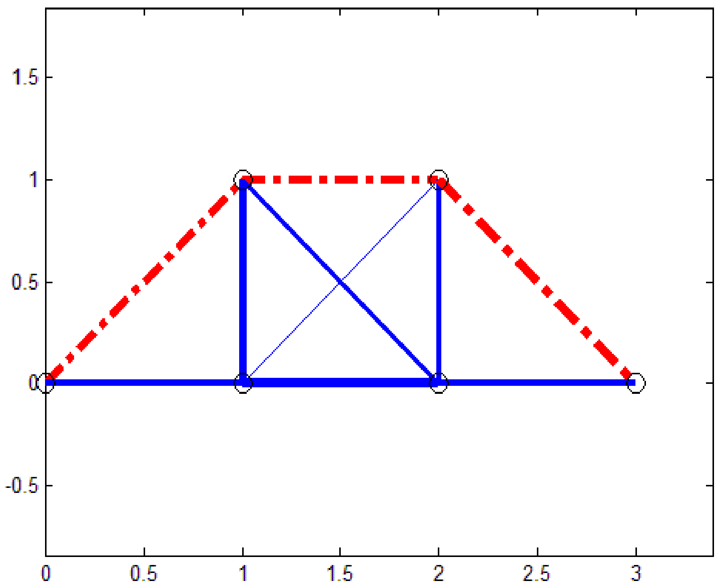
JOINT COMPUTER ASSIGNMENTS, 2 EXAMPLES

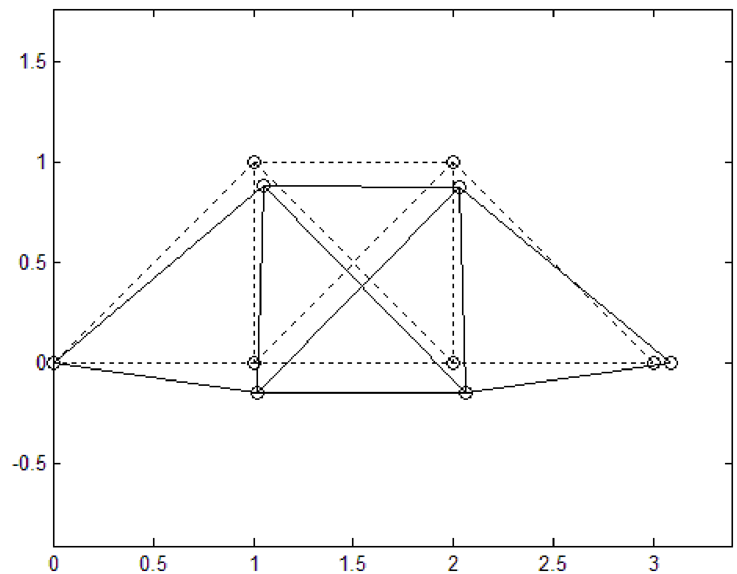
Courses Statics and strength of materials/Linear algebra:

Analysis of elastic truss frame



- Programming: from problem definition to code
- Manage large systems of equations,
- Visualize the stress distribution and deformations and optimization (redesign)
- Introduction to FEM and Structural Mechanics

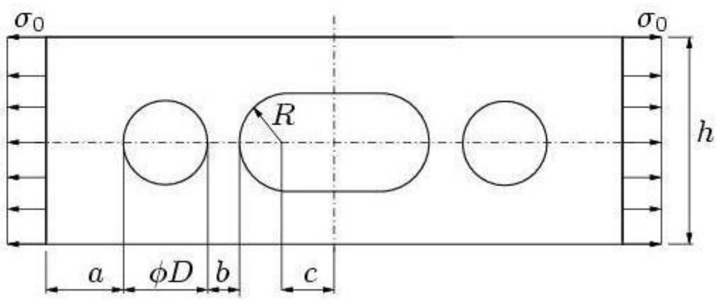


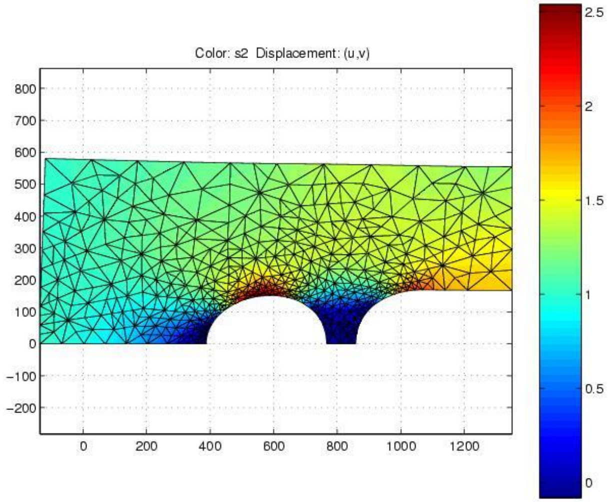


Courses: Mathematical Analysis in Several Variables and Solid Mechanics

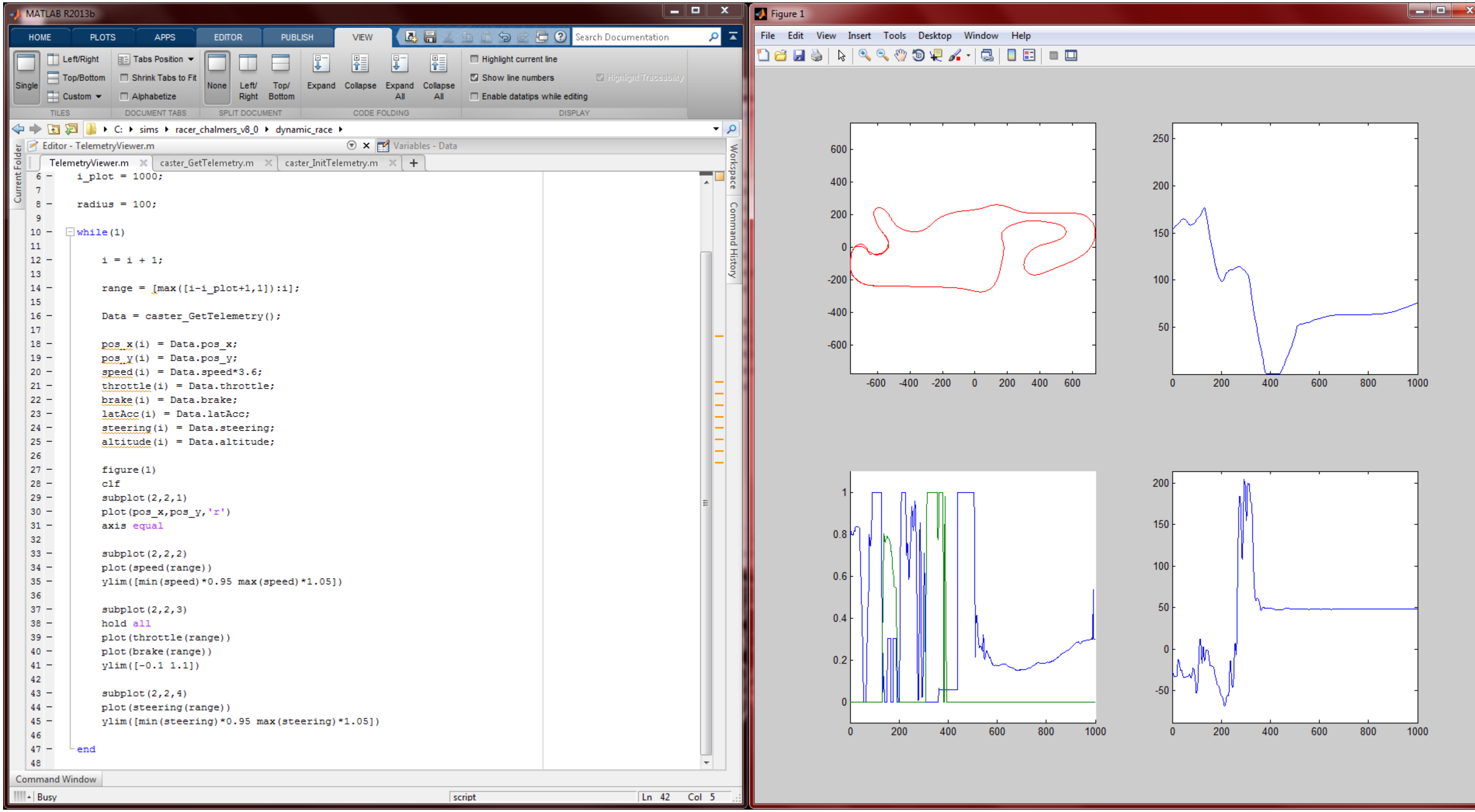
Stress analysis of plane elastic plate with 3 holes

- Develop knowledge about stress distribution and how the stress is increased due to abrupt changes in geometry
- Skills to use the finite element method and introduction to error estimation and adaptive mesh refinement



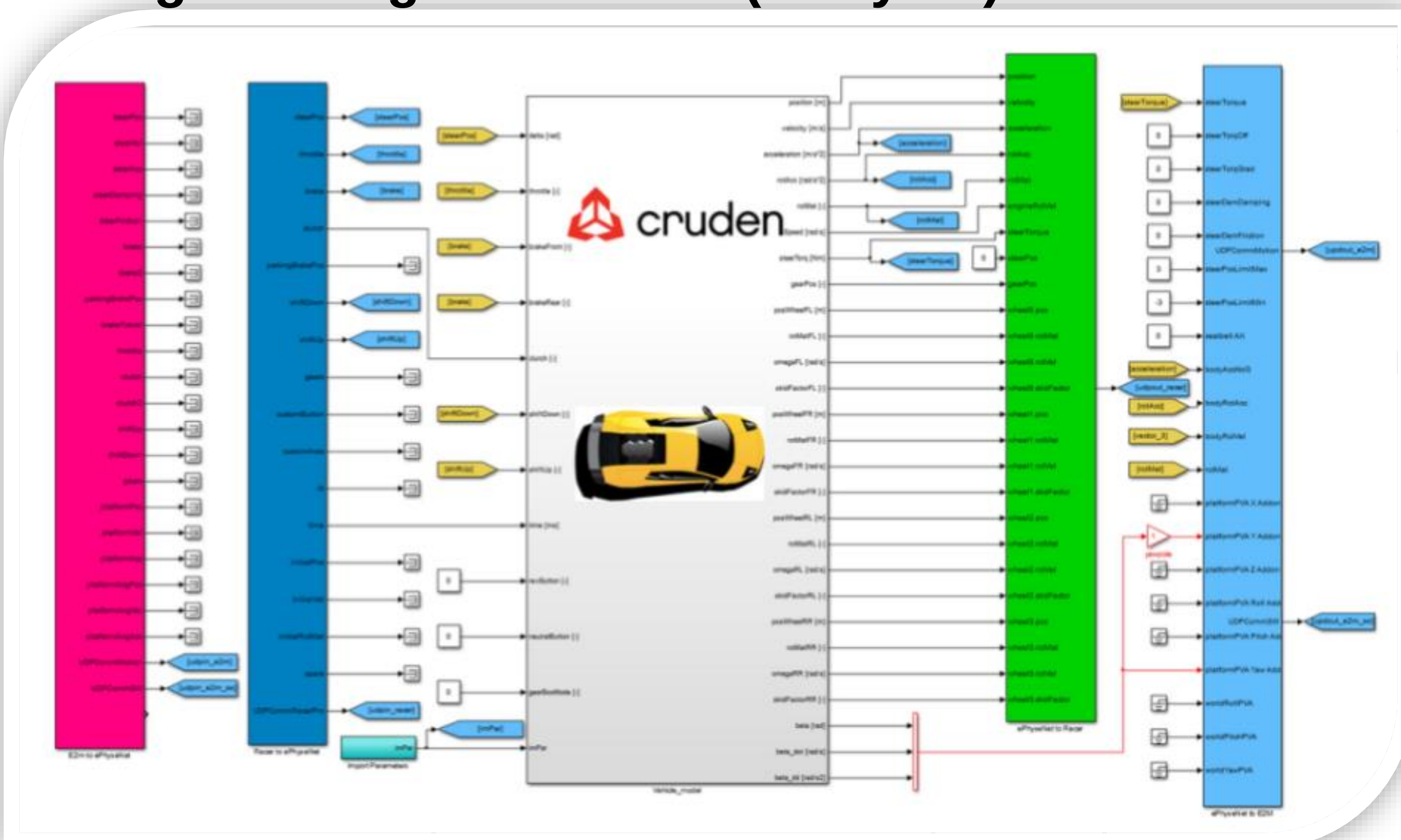


Chalmers ME driving simulator CASTER



Chalmers ME driving simulator – Examples

Programming in MATLAB (1st year)



EVALUATION AND RESULTS

- Main goal that each student should gain knowledge, skills and ability to effectively use computational modelling and simulations in applications has been reached to a large extent
- Programming skills have increased significantly
- Decision making is brought forward in the sense that students consider real systems and structures and solve real problem (reasoning and decision making at a higher level),
- Active learning is emphasized in simulations, open-ended problems and in the virtual/interactive learning environments that are used,
- The Employers claim that the mechanical engineering students have became significantly better prepared for the managing and solving of open-ended problem, carrying out numerical simulations, programming and using modern industrial software.
- Reviewers "Especially encouraged to see CAD and MATLAB integrated as educational tools and the interest to embrace computational mathematics into mechanical engineering courses"

References

- Match course descriptions (learning outcomes, computer assignments, old exams etc) http://www.chalmers.se/sv/institutioner/math/utbildning/grundutbildning-chalmers/arkitekt-och-civilingenjor/maskinteknik/Sidor/default.aspx

- Integration of Computational Mathematics Education in the Mechanical Engineering Curriculum, *Proceedings of 7th International CDIO Conference, Copenhagen, Denmark,* 2011

- A computational mathematics education for students of mechanical engineering, *World Transactions on Engineering and Technology Education*, vol 5, 2006.

Questions?