9

Simultaneous Equations

$3x + 5y = 14 \ .... \ (1)$
$7x - 2y = 19 \ .... \ (2)$

$x = 3$
$y = 1$

MATHS
CALCULUS
TRIG.

Chapter Contents

Learning Outcomes

Students will be able to:

- Solve linear simultaneous equations using graphs.
- Solve linear simultaneous equations using algebraic methods.
- Use simultaneous equations to solve problems.

Areas of Interaction

Approaches to Learning (Knowledge Acquisition, Problem Solving, Communication, Logical Thinking, IT Skills, Reflection), Human Ingenuity

Investigation 9:01A | Solving problems by 'guess and check'

Consider the following problem.

> A zoo enclosure contains wombats and emus. If there are 50 eyes and 80 legs, find the number of each type of animal.

Knowing that each animal has two eyes but a wombat has 4 legs and an emu has two legs, we could try to solve this problem by guessing a solution and then checking it.

Solution

If each animal has two eyes, then, because there are 50 eyes, I know there must be 25 animals.

If my first guess is 13 wombats and 12 emus, then the number of legs would be $13 \times 4 + 12 \times 2 = 76$.

Since there are more legs than 76, I need to increase the number of wombats to increase the number of legs to 80.

I would eventually arrive at the correct solution of 15 wombats and 10 emus, which gives the correct number of legs ($15 \times 4 + 10 \times 2 = 80$).

Try solving these problems by guessing and then checking various solutions.

1 Two numbers add to give 86 and subtract to give 18. What are the numbers?

2 At the school disco, there were 52 more girls than boys. If the total attendance was 420, how many boys and how many girls attended?

3 In scoring 200 runs, Max hit a total of 128 runs as boundaries. (A boundary is either 4 runs or 6 runs.) If he scored 29 boundaries in total, how many boundaries of each type did he score?

4 Sharon spent $5158 buying either BHP shares or ICI shares. These were valued at $10.50 and $6.80 respectively. If she bought 641 shares in total, how many of each did she buy?
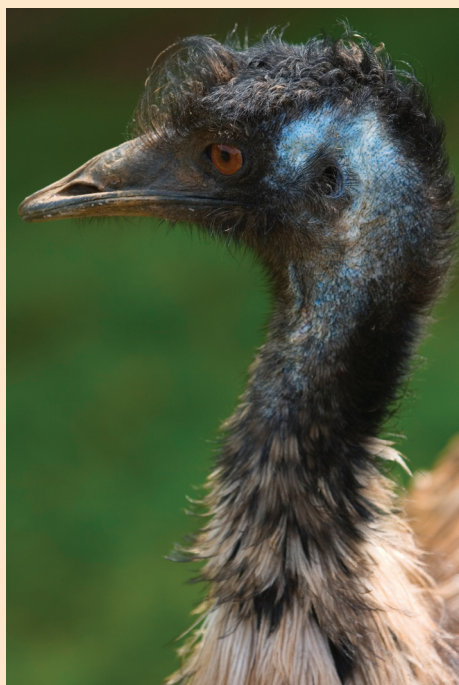
In this chapter, you will learn how to solve problems like those in Investigation 9:01A more systematically. Problems like these have two pieces of information that can be represented by two equations. These can then be solved to find the common or 'simultaneous' solution.

9:01 | The Graphical Method of Solution

If $y = 2x - 1$, find y when:

1 $x = 1$ **2** $x = 0$

3 $x = -1$ **4** $x = -5$
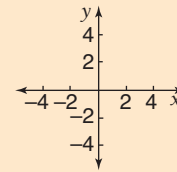
If $x - 2y = 5$, find y when:

5 $x = 0$ **6** $x = 1$

7 $x = 2$ **8** $x = -4$

9 If $3x - y = 2$, complete the table below.

x	0	1	2
y			

10 Copy this number plane and graph the line $3x - y = 2$.

There are many real-life situations in which we wish to find when or where two conditions come or occur together. The following example illustrates this.

worked example

A runner set off from a point and maintained a speed of 9 km/h. Another runner left the same point 10 minutes later, followed the same course, and maintained a speed of 12 km/h. When, and after what distance travelled, would the second runner have caught up to the first runner?
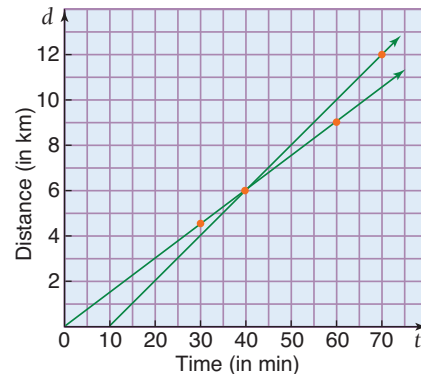
We have chosen to solve this question graphically.

First runner

t	0	30	40	60
d	0	4·5	6	9

Second runner

t	10	30	40	70
d	0	4	6	12

From these tables we can see that the runners meet after 6 km and 40 minutes.

t = time in minutes after the first runner begins
d = distance travelled in kilometres

- From the graph, we can see that the lines cross at (40, 6).
- The simultaneous solution is $t = 40$, $d = 6$.
- The second runner caught the first runner 40 minutes after the first runner had started and when both runners had travelled 6 kilometres.

After the second runner has run for 30 minutes, t = 40.

Often, in questions, the information has to be written in the form of equations. The equations are then graphed using a table of values (as shown above). The point of intersection of the graphs tells us when and where the two conditions occur together.
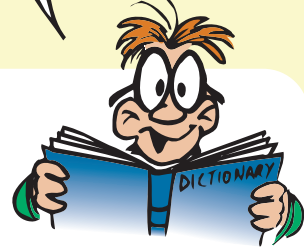
worked example

Solve the following equations simultaneously.

$x + y = 5$

$2x - y = 4$

Solution

You will remember from your earlier work on coordinate geometry that, when the solutions to an equation such as $x + y = 5$ are graphed on a number plane, they form a straight line.
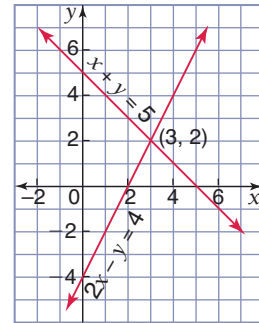
Hence, to solve the equations $x + y = 5$ and $2x - y = 4$ simultaneously, we could simply graph each line and find the point of intersection. Since this point lies on both lines, its coordinates give the solution.

$x + y = 5$

x	0	1	2
y	5	4	2

$2x - y = 4$

x	0	1	2
y	−4	−2	0

- The lines $x + y = 5$ and $2x - y = 4$ intersect at $(3, 2)$. Therefore the solution is:

  $x = 3$

  $y = 2$

> To solve a pair of simultaneous equations graphically, we graph each line. The solution is given by the coordinates of the point of intersection of the lines.

It is sometimes difficult to graph accurately either or both lines, and it is often difficult to read accurately the coordinates of the point of intersection.

Despite these problems, the graphical method remains an extremely useful technique for solving simultaneous equations.

Exercise 9:01

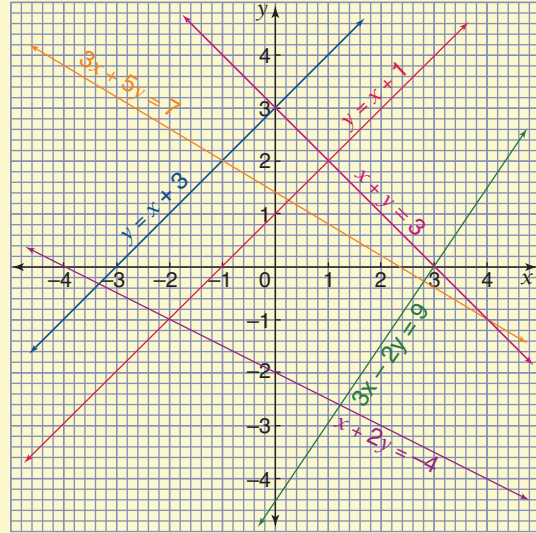**Graphical method
of solution**
1 Graph these lines on the
same number plane and find
where they intersect.
a $y = x + 2$ and $x + y = 2$
b $y = 2x$ and $y = x + 1$

1 Use the graph to write down the solutions to the
following pairs of simultaneous equations.

a $y = x + 1$ **b** $y = x + 1$
    $x + y = 3$ $x + 2y = -4$

c $y = x + 3$ **d** $y = x + 3$
    $3x + 5y = 7$ $x + y = 3$

e $x + y = 3$ **f** $3x - 2y = 9$
    $3x + 5y = 7$ $x + y = 3$

g $y = x + 3$ **h** $y = x + 1$
    $y = x + 1$ $2y = 2x + 2$

Explain why
(g) and (h)
above are
unusual.

2 Use the graph in question **1** to estimate, correct to one decimal place, the solutions of the
following simultaneous equations.

a $y = x + 1$ **b** $y = x + 3$ **c** $3x - 2y = 9$ **d** $3x - 2y = 9$
    $3x + 5y = 7$ $x + 2y = -4$ $x + 2y = -4$ $3x + 5y = 7$

3 Solve each of the following pairs of equations by graphical means. All solutions are integral
(ie they are whole numbers).

a $x + y = 1$ **b** $2x + y = 3$ **c** $x - y = 3$ **d** $3x - y - 2 = 0$
    $2x - y = 5$ $x + y = 1$ $2x + y = 0$ $x - y + 2 = 0$

e $3a - 2b = 1$ **f** $p + 2q = 2$ **g** $3a + 2b = 5$ **h** $p = 6$
    $a - b = 1$ $p - q = -4$ $a = 1$ $p - q = 4$

4 Solve each pair of simultaneous equations by the graphical
method. (Use a scale of 1 cm to 1 unit on each axis.)

a $y = 4x$ **b** $3x - y = 1$ **c** $x = 4y$
    $x + y = 3$ $x - y = 2$ $x + y = 1$

The graphical method
doesn't always give exact
answers.

5 Estimate the solution to each of the following pairs of
simultaneous equations by graphing each, using a scale
of 1 cm to 1 unit on each axis. Give the answers correct
to 1 decimal place.

a $4x + 3y = 3$ **b** $x - y = 2$ **c** $4a - 6b = 1$
    $x - 2y = 1$ $8x + 4y = 7$ $4a + 3b = 4$

6 A car passed a point on a course at exactly 12 noon and maintained a speed of 60 km/h. A second car passed the same point 1 hour later, followed the same course, and maintained a speed of 100 km/h. When, and after what distance from this point, would the second car have caught up to the first car? (*Hint:* Use the method shown in the worked example on page 438 but leave the time in hours.)

7 Mary's salary consisted of a retainer of $480 a week plus $100 for each machine sold in that week. Bob worked for the same company, had no retainer, but was paid $180 for each machine sold. Study the tables below, graph the lines, and use them to find the number, N, of machines Bob would have to sell to have a wage equal to Mary (assuming they both sell the same number of machines). What salary, S, would each receive for this number of sales?

Mary

N	0	4	8
S	480	880	1280

Bob

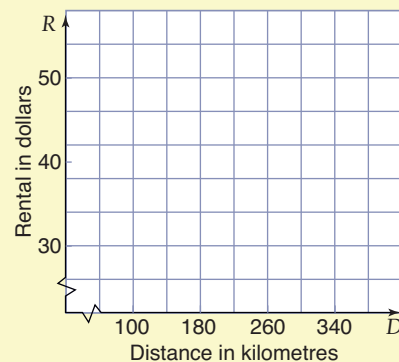| N	0	4	8
S	0	720	1440

N = number of machines
S = salary

8 No Frills Car Rental offers new cars for rent at €38 per day and 50c for every 10 km travelled in excess of 100 km per day. Prestige Car Rental offers the same type of car for €30 per day plus €1 for every 10 km travelled in excess of 100 km per day.

Draw a graph of each case on axes like those shown, and determine what distance would need to be travelled in a day so that the rentals charged by each company would be the same.

9 Star Car Rental offers new cars for rent at $38 per day and $1 for every 10 km travelled in excess of 100 km per day. Safety Car Rental offers the same type of car for $30 per day plus 50c for every 10 km travelled in excess of 100 km per day.

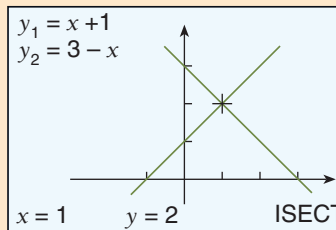Draw a graph of each on axes like those in question **8**, and discuss the results.

Investigation 9:01B | Solving simultaneous equations using a graphics calculator

Using the graphing program on a graphics calculator complete the following tasks.

- Enter the equations of the two lines $y = x + 1$ and $y = 3 - x$. The screen should look like the one shown.

- Draw these graphs and you should have two straight lines intersecting at $(1, 2)$.

- Using the **G-Solv** key, find the point of intersection by pressing the **F5** key labelled **ISCT**.

- At the bottom of the screen, it should show $x = 1$, $y = 2$.

Now press **EXIT** and go back to enter other pairs of equations of straight lines and find their point of intersection.

Graph Func : $y =$
$y_1 = x + 1$
$y_2 = 3 - x$
$y_3 :$
$y_4 :$
$y_5 :$
$y_6 :$

$y_1 = x + 1$
$y_2 = 3 - x$

$x = 1$ $y = 2$ ISECT

Note: You can change the scale on the axes using the **V-Window** option.

Fun Spot 9:01 | What did the book say to the librarian?

Work out the answer to each part and put the letter for that part in the box that is above the correct answer.

Write the equation of:

A line AB   **C** line OB
U line BF   **A** line EB
I the y-axis   **O** line AF
U line OF   **K** line AE
E line CB   **T** the x-axis
T line EF   **N** line OD
Y line CD   **O** line OA

SPOKEN WORD
TITLES

$y = \frac{5}{3}x$

$y = \frac{4}{3}x + 1$

$y = x$

$x = 0$

$y = 0$

$y = 5$

$x = -3$

$y = \frac{1}{3}x + 4$

$y = 3$

$y = -\frac{5}{3}x$

$x = 3$

$y = -\frac{4}{3}x + 1$

$y = -x$

$y = -3$
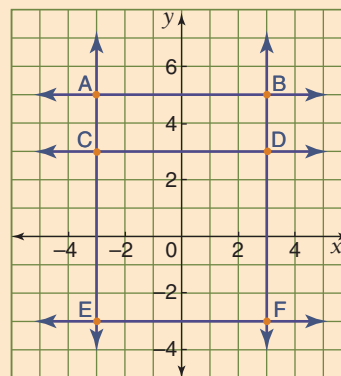
9:02 | The Algebraic Method of Solution

We found in the last section that the graphical method of solution lacked accuracy for many questions. Because of this, we need a method that gives the exact solution. There are two such algebraic methods — the substitution method and the elimination method.

9:02A Substitution method

worked examples

Solve the simultaneous equations:

1 $2x + y = 12$ and $y = 5x - 2$

2 $3a + 2b = 7$, $4a - 3b = 2$

Solutions

When solving simultaneous equations, first 'number' the equations involved.

In this method one pronumeral is replaced by an equivalent expression involving the other pronumeral.

1 $2x + y = 12$ .... ①

$y = 5x - 2$ .... ②

Now from ② we can see that $5x - 2$ is equal to y. If we substitute this for y in equation ①, we have:

$$2x + (5x - 2) = 12$$
$$7x - 2 = 12$$
$$7x = 14$$
$$x = 2$$

So the value of x is 2. This value for x can now be substituted into either equation ① or equation ② to find the value for y:

In ①:
$$2(2) + y = 12$$
$$4 + y = 12$$
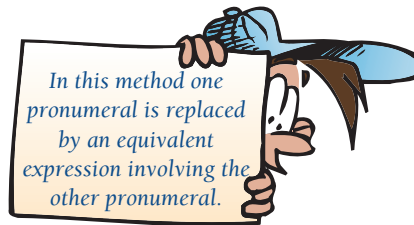$$y = 8$$

In ②:
$$y = 5(2) - 2$$
$$= 10 - 2$$
$$= 8$$

So, the total solution is:
$x = 2$, $y = 8$.

■ To check this answer substitute into equations ① and ②.

2 $3a + 2b = 7$ .... ①

$4a - 3b = 2$ .... ②

Making a the subject of ② gives:

$$a = \frac{2 + 3b}{4}$$

If we substitute this expression for a into equation ①, we get:

Multiply both sides by 4.

$$3\left(\frac{2 + 3b}{4}\right) + 2b = 7$$

$$3(2 + 3b) + 8b = 28$$

$$6 + 9b + 8b = 28$$

$$17b = 22$$

$$b = \frac{22}{17}$$

Substituting this value for b into, say, equation ② gives:

$$4a - 3\left(\frac{22}{17}\right) = 2$$

$$4a - \frac{66}{17} = \frac{34}{17}$$

$$4a = \frac{100}{17}$$

$$a = \frac{25}{17}$$

So the total solution is:

$$a = \frac{25}{17}, b = \frac{22}{17}.$$

■ To check your answer, substitute $a = \frac{25}{17}$, $b = \frac{22}{17}$ in equations ① and ②.

Check each step!

Exercise 9:02A

1 Solve the following pairs of equations using the substitution method.
Check all solutions.

a $x + y = 3$ and $y = 4$
b $x + y = 7$ and $y = x + 3$
c $x + y = -3$ and $y = x + 1$
d $x - y = 5$ and $y = 1 - x$
e $2x + y = 9$ and $y = x - 3$
f $2x + y = 8$ and $y = x - 4$
g $2x - y = 10$ and $y = 10 - 3x$
h $x + 2y = 9$ and $y = 2x - 3$
i $2x + y = 14$ and $x = 6$
j $2x + y = 7$ and $x = y - 4$

2 Use one of each pair of equations to express y in terms of x. Then use the method of substitution to solve the equations. Check all solutions.

a $x + 2y = 4$
   $x - y = 7$

b $2x - 3y = 4$
   $2x + y = 6$

c $x + 2y = 8$
   $x + y = -2$

d $x - y = 2$
   $x + 2y = 11$

e $2x - y = -8$
   $2x + y = 0$

f $x + y = 5$
   $2x + y = 7$

g $x + 2y = 11$
   $2x - y = 2$

h $3x + y = 13$
   $x + 2y = 1$

i $3x + 2y = 2$
   $2x - y = -8$

3 Solve the following simultaneous equations using the substitution method.

a $2x - y = 1$
   $4x + 2y = 5$

b $3a + b = 6$
   $9a + 2b = 1$

c $m - 2n = 3$
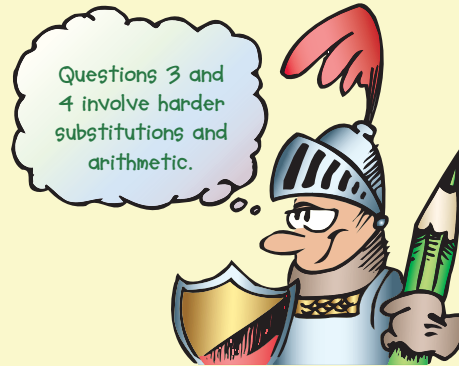 $5m + 2n = 2$

d $4x - 2y = 1$
   $x + 3y = -1$

> Questions 3 and 4 involve harder substitutions and arithmetic.

4 Solve the following pairs of simultaneous equations.

a $2a - 3b = 1$
   $4a + 2b = 5$

b $7x - 2y = 2$
   $3x + 4y = 8$

c $3m - 4n = 1$
   $2m + 3n = 4$

d $2x - 3y = 10$
   $5x - 3y = 3$

9:02B Elimination method

worked examples

Solve each pair of simultaneous equations:

1 $5x - 3y = 20$
    $2x + 3y = 15$

2 $x + 5y = 14$
    $x - 3y = 6$

3 $2x + 3y = 21$
    $5x + 2y = 3$

> ■ In this method, one of the pronumerals is eliminated by adding or subtracting the equations.

Solutions

First, number each equation.

1 $5x - 3y = 20$ .... ①

   $2x + 3y = 15$ .... ②

> ■ You add or subtract the equations, depending upon which operation will eliminate one of the pronumerals.

Now if these equations are 'added', the y terms will be eliminated, giving:

   $7x = 35$
ie $x = 5$

Substituting this value into equation ① we get:
   $5(5) - 3y = 20$
   $25 - 3y = 20$
   $3y = 5$
   $y = \frac{5}{3}$ or $1\frac{2}{3}$.

So the total solution is:

$x = 5, y = 1\frac{2}{3}$.

Check in ①: $5(5) - 3(1\frac{2}{3}) = 20$ (true).

Check in ②: $2(5) + 3(1\frac{2}{3}) = 15$ (true).

2 $x + 5y = 14$ .... ①

$x - 3y = 6$ .... ②

Now if equation ② is 'subtracted' from equation ①, the x terms are eliminated and we get:

$8y = 8$

ie $y = 1$

Substituting this value into ① gives:

$x + 5(1) = 14$

$x + 5 = 14$

$x = 9$

∴ The solution is:

$x = 9, y = 1$.

Check in ①: $9 + 5(1) = 14$ (true).

Check in ②: $9 - 3(1) = 6$ (true).

3 $2x + 3y = 21$ .... ①

$5x + 2y = 3$ .... ②

Multiply equation ① by 2

and equation ② by 3.

This gives:

$4x + 6y = 42$ .... ①*

$15x + 6y = 9$.............. ②*

Now if ②* is subtracted from ①* the y terms are eliminated and we get:

$-11x = 33$

So $x = -3$

Check that the values satisfy both original equations.
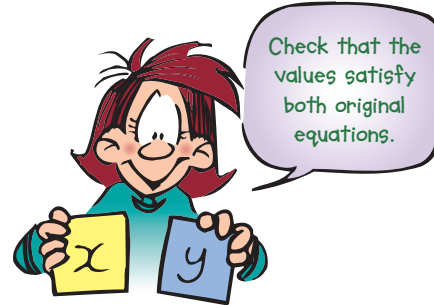
Take one step at a time.

■ *Notice*
To eliminate a pronumeral, the size of the coefficients in each equation must be made the same by multiplying one or both equations by a constant.

Substituting this value into ① gives:

$$2(-3) + 3y = 21$$
$$-6 + 3y = 21$$
$$3y = 27$$
$$y = 9$$

So the solution is $x = -3$, $y = 9$

Check in ①: $2(-3) + 3(9) = 21$ (true).

Check in ②: $5(-3) + 2(9) = 3$ (true).

■ *Note:* In example 3, x could have been eliminated instead of y, by multiplying ① by 5 and ② by 2.

Exercise 9:02B

1 Use the elimination method to solve simultaneously each pair of equations by first adding the equations together.

a	$x + y = 9$	**b**	$x + y = 14$	**c**	$2x + y = 7$
	$x - y = 1$		$2x - y = 1$		$x - y = 2$
d	$x + 2y = 3$	**e**	$3x - 2y = 5$	**f**	$5x - 2y = 1$
	$x - 2y = 7$		$x + 2y = 7$		$3x + 2y = 7$
g	$x + 3y = 10$	**h**	$-x + 2y = 12$	**i**	$3x + y = 11$
	$-x + y = 6$		$x + 2y = -4$		$-3x + 2y = 10$
j	$2x + 7y = 5$	**k**	$5x - 2y = 0$	**l**	$7x + 5y = -3$
	$x - 7y = 16$		$4x + 2y = 9$		$2x - 5y = 21$

2 By first subtracting to eliminate a pronumeral, solve each pair of equations.

a	$2x + y = 5$	**b**	$5x + y = 7$	**c**	$10x + 2y = 2$
	$x + y = 3$		$3x + y = 1$		$7x + 2y = -1$
d	$3x - 2y = 0$	**e**	$5x - y = 14$	**f**	$x - 3y = 1$
	$x - 2y = 4$		$2x - y = 2$		$2x - 3y = 5$
g	$2x + y = 10$	**h**	$2x + 5y = 7$	**i**	$5x - y = 16$
	$x + y = 7$		$2x + y = 5$		$5x - 3y = 8$
j	$6x + y = 13$	**k**	$2x + 5y = 20$	**l**	$7x - 2y = 1$
	$6x - y = 11$		$3x + 5y = 17$		$4x - 2y = 4$

3 Solve these simultaneous equations by the elimination method.

a	$2x + y = 7$	**b**	$x + y = 5$	**c**	$x - y = 12$
	$x - y = -4$		$2x - y = 1$		$2x + y = 3$
d	$3x + 2y = 2$	**e**	$2x + 3y = 13$	**f**	$3x + 4y = -1$
	$x - 2y = -10$		$4x - 3y = -1$		$3x - 2y = -10$
g	$5x + 2y = 1$	**h**	$7x - 3y = 31$	**i**	$8x - 2y = 34$
	$3x - 2y = 7$		$7x + y = -1$		$8x + 4y = 4$

4 After multiplying either, or both of the equations by a constant, use the elimination method to solve each pair of equations.

a $x + y = 7$
$2x + 3y = 17$

b $2x + y = 7$
$x + 2y = 11$

c $5x + y = 12$
$3x + 2y = 10$

d $4x - y = 10$
$x + 3y = 9$

e $4x - y = 6$
$3x + 2y = -1$

f $5x - 2y = -16$
$x + 3y = 7$

g $12x - 3y = 18$
$4x + 2y = 0$

h $3x - 7y = 2$
$9x + 5y = 32$

i $2x + 3y = 8$
$3x + 2y = 7$

j $5x + 2y = 10$
$4x + 3y = 15$

k $5x + 2y = 28$
$3x + 5y = 51$

l $2x + 2y = -2$
$3x - 5y = -19$

m $7x + 3y = 4$
$5x + 2y = 3$

n $2x - 4y = -2$
$3x + 5y = 45$

Use the same setting out as in the examples.

$x + y = 7 \ldots$ ①
$2x + 3y = 17 \ldots$ ②

① × 2

$2x + 2y = 14 \ldots$ ①*
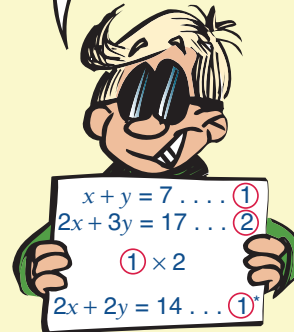
9:03 | Using Simultaneous Equations to Solve Problems

In Chapter 6, we saw how equations could be used to solve problems. Simultaneous equations can also be used to solve problems, often in a much easier way than with only one equation. The same techniques that were met in Chapter 6 also apply here.

These clues will help you solve the problem!

Remember:
• Read the question carefully.
• Work out what the problem wants you to find.
  (These things will be represented by pronumerals.)
• Translate the words of the question into mathematical expressions.
• Form equations by showing how different mathematical expressions are related.
• Solve the equations.
• Finish off with a sentence stating the value of the quantity or quantities that were found.

worked example

Adam is 6 years older than his sister, Bronwyn.
If the sum of their ages is 56 years, find their ages.

Solution

Let Adam's age be x years.
Let Bronwyn's age be y years.
Now, Adam is 6 years older than Bronwyn
$\therefore x = y + 6$ .... ①
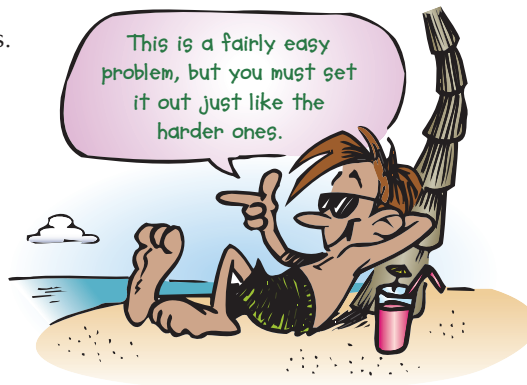*Also*, the sum of their ages is 56 years.
$\therefore x + y = 56$.................... ②
Solving these simultaneously gives:
$x = 31$ and $y = 25$.
$\therefore$ Adam is 31 years old and Bronwyn is 25 years old.

This is a fairly easy problem, but you must set it out just like the harder ones.

Exercise 9:03

1 Form pairs of simultaneous equations and solve the following problems. Let the numbers be x and y.

  a The sum of two numbers is 25 and their difference is 11. Find the numbers.

  b The sum of two numbers is 97 and their difference is 33. Find the numbers.

  c The sum of two numbers is 12, and one of the numbers is three times the other. Find the numbers.

  d The difference between two numbers is 9 and the smaller number plus twice the larger number is equal to 24. Find the numbers.

  e The larger of two numbers is equal to 3 times the smaller number plus 7. Also, twice the larger number plus 5 times the smaller is equal to 69. Find the numbers.
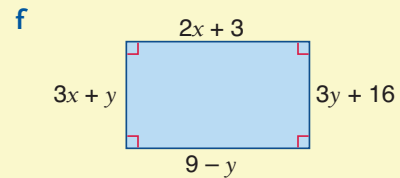
2 In each problem below there are two unknown quantities, and two pieces of information. Form two simultaneous equations and solve each problem.

  a The length of a rectangle is 5 cm more than the width. If the perimeter of the rectangle is 22 cm, find the length and the width.

  b One pen and one pencil cost 57c. Two pens and three pencils cost $1.36. Find the cost of each.

  c If a student's maths mark exceeded her science mark by 15, and the total marks for both tests was 129, find each mark.
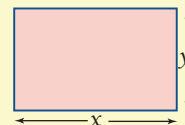
d Six chocolates and three drinks cost $2.85 while three chocolates and two drinks cost $1.65. Find the price of each.

e Bill has twice as much money as Jim. If I give Jim $2.50, he will have three times as much as Bill. How much did Bill and Jim have originally?

3 Form two equations from the information on each figure to find values for x and y.

a

$2x + y$

$x + y$ 7

12

b

$5x - 2y$

7    $x + 2y$

11

c

$3x°$ $(x + y)°$

d

$40°$

$2x°$

$(3x + y)°$

e

$x + 2y$ $3x - 5y$

11

f

$2x + 3$

$3x + y$ $3y + 16$

$9 - y$
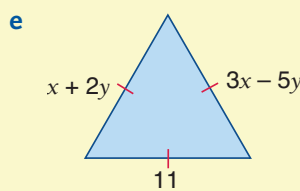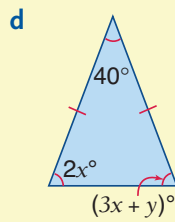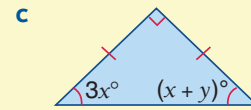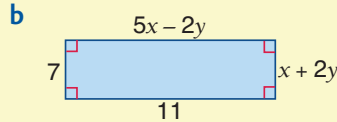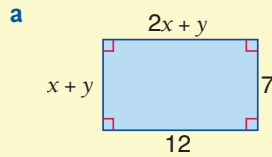
4   **a** A rectangle is 4 cm longer than it is wide. If both the length and breadth are increased by 1 cm, the area would be increased by 18 cm². Find the length and breadth of the rectangle.

b A truck is loaded with two different types of boxes. If 150 of box A and 115 of box B are loaded onto the truck, its capacity of 10 tonnes is reached. If 300 of box A are loaded, then the truck can only take 30 of box B before the capacity of 10 tonnes is reached. Find the weight of each box.

c A theatre has 2100 seats. All of the rows of seats in the theatre have either 45 seats or 40 seats. If there are three times as many rows with 45 seats than those with 40 seats, how many rows are there?

d A firm has five times as many junior workers as it does senior workers. If the weekly wage for a senior is $620 and for a junior is $460, find how many of each are employed if the total weekly wage bill is $43 800.

5 Use graphical methods to solve these.

a Esther can buy aprons for €6 each. She bought a roll of material for €20 and gave it to a dressmaker, who then charged €3.50 for each apron purchased. How many aprons would Esther need to purchase for the cost to be the same as buying them for €6 each.

b Star Bicycles had produced 3000 bicycles and were producing 200 more per week. Prince Bicycles had produced 2500 bicycles and were producing 300 more each week. After how many weeks would they have produced the same number of bicycles?

Technology Applications

9:03 Break-even analysis

Activities

Challenge worksheet 9:03 Simultaneous equations with 3 variables

Reading mathematics 9:03 | Breakfast time

A certain breakfast cereal has printed on the box the information shown here. Examine the figures and answer the questions below.

	30 g alone contains approx.	With $\frac{1}{2}$ cup whole milk contains approx.
Kilojoules	477	837
Calories	114	200
Protein	6·0 g	10·2 g
Fat	0·1 g	5·0 g
Starch and related carbohydrates	17·2 g	17·2 g
Sucrose and other sugars	4·7 g	10·6 g
Total carbohydrate	21·9 g	27·8 g

1 How many grams of this cereal contains 477 kilojoules?

2 How many kilojoules is equivalent to 200 calories?

3 How much milk must be added to give 27·8 g of carbohydrate with 30 g of cereal?

	60 g alone contains		With 1 cup whole milk contains	
	by weight	% of daily allowance	by weight	% of daily allowance
Protein	12·0 g	17	20·4 g	29
Thiamine	0·55 mg	50	0·65 mg	59
Riboflavin	0·8 mg	50	1·2 mg	75
Niacin	5·5 mg	50	5·7 mg	52
Iron	5·0 mg	50	5·1 mg	51
Calcium	38 mg	5	334 mg	48
Phosphorus	94 mg	9	340 mg	34

4 What must be the fat content of $\frac{1}{2}$ cup of milk?

5 How many milligrams of niacin are contained in 60 g of cereal?

6 When 60 g of cereal is added to 1 cup of milk, which mineral has 48% of a person's daily allowance provided?

7 How many milligrams is the total daily allowance of
   a riboflavin?   b calcium?

8 How many grams of cereal alone would be needed to provide 30 g of protein?

Mathematical Terms 9

elimination method
- Solving simultaneous equations by adding or subtracting the equations together to 'eliminate' one pronumeral.

graphical solution
- The solution obtained by graphing two equations in the number plane and observing the point of intersection.
- If the point of intersection is $(3, -2)$, then the solution is $x = 3$ and $y = -2$.

guess and check
- A method of solving problems by guessing a solution and then checking to see if it works. Solutions are modified until the correct solution is found.

simultaneous equations
- When two (or more) pieces of information about a problem can be represented by two (or more) equations.
- These are then solved to find the common or simultaneous solution
  eg The equations $x + y = 10$ and $x - y = 6$ have many solutions but the only simultaneous solution is $x = 8$ and $y = 2$.

substitution method
- Solving simultaneous equations by substituting an equivalent expression for one pronumeral in terms of another, obtained from another equation.
  eg If $y = x + 3$ and $x + y = 7$, then the second equation could be written as $x + (x + 3) = 7$ by substituting for y using the first equation.

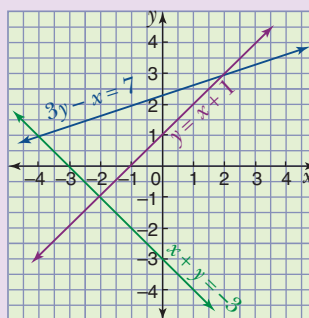## Diagnostic Test 9: | Simultaneous Equations

- These questions reflect the important skills introduced in this chapter.
- Errors made will indicate areas of weakness.
- Each weakness should be treated by going back to the section listed.

	Section
1 Use the graph to solve the following simultaneous equations. **a** $x + y = -3$ $\quad y = x + 1$ **b** $y = x + 1$ $\quad 3y - x = 7$ **c** $3y - x = 7$ $\quad x + y = -3$	9:01

	Section
2 Solve the following simultaneous equations by the substitution method. **a** $y = x - 2$ $\quad$ **b** $x - y = 5$ $\quad$ **c** $4a - b = 3$ $\quad 2x + y = 7$ $\qquad\quad 2x + 3y = 2$ $\qquad\quad 2a + 3b = 11$	9:02A
3 Solve the following simultaneous equations by the elimination method. **a** $2x - y = 3$ $\quad$ **b** $4x - 3y = 11$ $\quad$ **c** $2a - 3b = 4$ $\quad 3x + y = 7$ $\qquad\quad 2x + y = 5$ $\qquad\quad\; 3a - 2b = 6$	9:02B

Chapter 9 | Revision Assignment

1 Solve the following simultaneous equations by the most suitable method.

a $x + y = 3$
$2x - y = 6$

b $4x - y = 3$
$2x + y = 5$

c $4a + b = 6$
$5a - 7b = 9$

d $6a - 3b = 4$
$4a - 3b = 8$

e $a - 3b = 5$
$5a + b = 6$

f $2x - 3y = 6$
$3x - 2y = 5$

g $p = 2q - 7$
$4p + 3q = 5$

h $4x - y = 3$
$4x - 3y = 7$

i $7m - 4n - 6 = 0$
$3m + n = 4$

2 A man is three times as old as his daughter. If the difference in their ages is 36 years, find the age of father and daughter.

3 A theatre can hold 200 people. If the price of admission was $5 per adult and $2 per child, find the number of each present if the theatre was full and the takings were $577.

4 A man has 100 shares of stock A and 200 shares of stock B. The total value of the stock is $420. If he sells 50 shares of stock A and buys 60 shares of stock B, the value of his stock is $402. Find the price of each share.

5 Rectangle A is 3 times longer than rectangle B and twice as wide. If the perimeters of the two are 50 cm and 20 cm respectively, find the dimensions of the larger rectangle.

6 A rectangle has a perimeter of 40 cm. If the length is reduced by 5 cm and 5 cm is added to the width, it becomes a square. Find the dimensions of the rectangle.

7 A canoeist paddles at 16 km/h with the current and 8 km/h against the current. Find the velocity of the current.

Chapter 9 | Working Mathematically

1 You need to replace the wire in your clothes-line. Discuss how you would estimate the length of wire required.
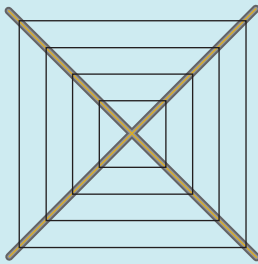
   a On what measurements would you base your estimate?

   b Is it better to overestimate or underestimate?

   c What level of accuracy do you feel is necessary? The diagram shows the arrangement of the wire.

2 What is the last digit of the number 3^{2004}?

3 Two smaller isosceles triangles are joined to form a larger isosceles triangle as shown in the diagram. What is the value of x?

$AB = AC$
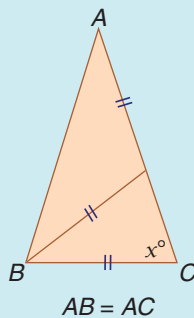
4 In a round-robin competition each team plays every other team. How many games would be played in a round-robin competition that had:

   a three teams?   **b** four teams?

   c five teams?   **d** eight teams?

5 How many different ways are there of selecting three chocolates from five?

6 A school swimming coach has to pick a medley relay team. The team must have 4 swimmers, each of whom must swim one of the four strokes. From the information in the table choose the fastest combination of swimmers.

Name	Back	Breast	Fly	Free
Dixon	37·00	44·91	34·66	30·18
Wynn	37·17	41·98	36·59	31·10
Goad	38·88			
Nguyen	41·15	49·05	39·07	34·13
McCully		43·01		32·70
Grover		43·17		
Harris			37·34	34·44

• What is the fastest medley relay?