



Houghton  
Mifflin  
Harcourt

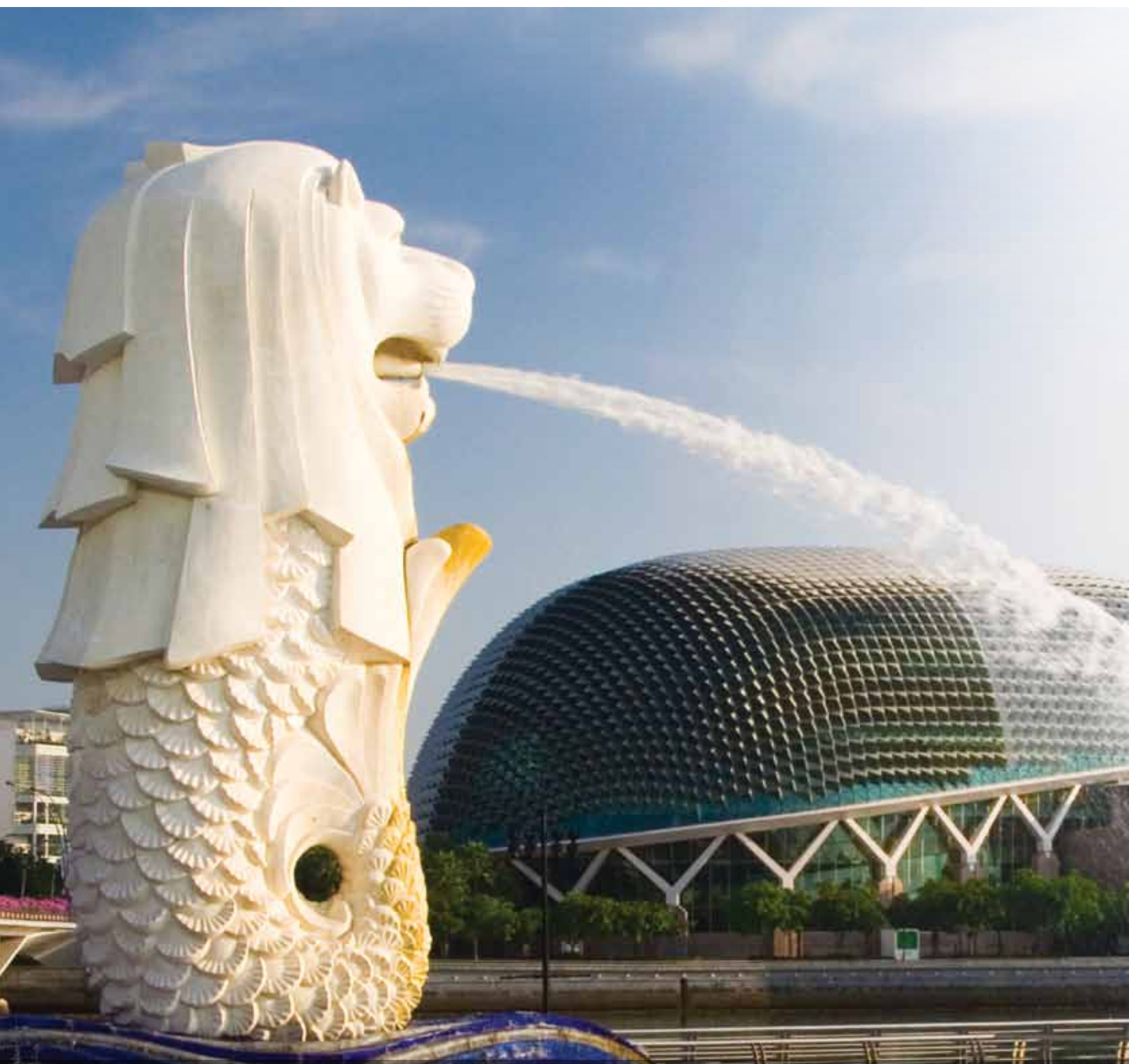
# Math in Focus<sup>®</sup>

Singapore Math  
by Marshall Cavendish

 Marshall Cavendish  
Education

## Singapore Math Research and Efficacy OVERVIEW





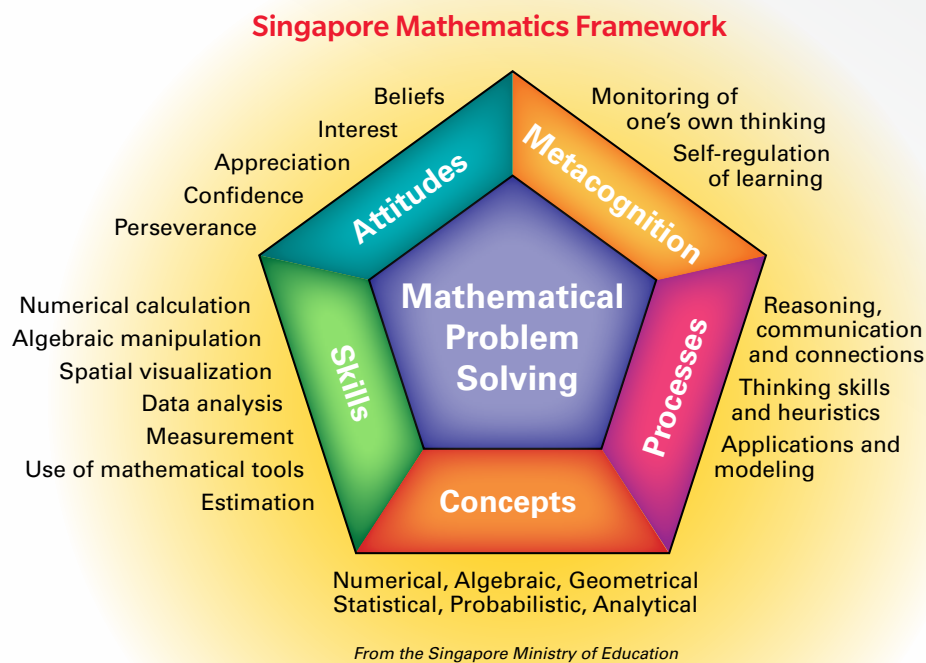
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# What is Singapore Math?

**Math in Focus®:** *Singapore Math* by Marshall Cavendish, for Grades K–8, is the U.S. edition of Singapore’s most widely used curriculum, bringing Singapore’s effective approach to mathematics to U.S. classrooms.

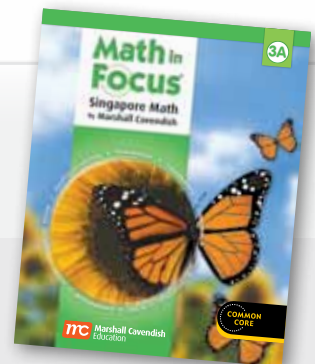
Singapore math emphasizes problem solving and positive attitudes toward mathematics, while focusing on student development of skills, concepts, processes, and metacognition. Students are encouraged to reflect on their thinking and learn how to self-regulate so that they can apply these skills to varied problem-solving activities. The Singapore Ministry of Education uses a pentagon with problem solving in the center to represent its curriculum framework.



This framework aligns with a strong base of foundational research and has also been proven to produce exceptional results both internationally and within the United States.

“Singapore math’s resounding success can be traced to its unique pedagogy which helps students master the concrete, pictorial, and abstract. What we have learned from Singapore has enabled us to form a program that has been proven to improve math score wherever it is implemented.

—Marshall Cavendish, Singapore



# Singapore math is built upon key research-based principles

## Problem Solving

### PRINCIPLES FROM RESEARCH

“Problem solving is a fundamental part of mathematics—and everyday life. The ability to solve problems is both a goal of mathematics—and a tool within mathematics. As such, problem solving should be integrated into all mathematical learning situations.”

—National Council of Teachers of Mathematics, 2000

### TURNING PRINCIPLES INTO SUCCESS WITH MATH IN FOCUS

Problem solving is at the heart of the Singapore math curriculum. Each chapter contains numerous embedded problem-solving situations so that students learn to flexibly apply their mathematical knowledge. Additionally, Put On Your Thinking Cap! problems require students to extend the concepts they have learned to non-routine situations to demonstrate mastery.

Grade 3

### CRITICAL THINKING SKILLS Put On Your Thinking Cap!

#### PROBLEM SOLVING

Rita wrote three 4-digit numbers on a sheet of paper. She accidentally spilled some ink on the paper. Some digits were covered by the ink. Using the clues given, help Rita find the digits covered by the ink.



#### CLUES

The sum of all the ones is 17.

The ones digit of the first number is the greatest 1-digit number. The digit in the tens place of the second number is one more than the digit in the tens place of the first number.

The tens digit of the third number is 4 less than the tens digit of the second number.

#### ON YOUR OWN

Go to Workbook A:  
Put on Your Thinking Cap!  
pages 17-18

32 Chapter 1 Numbers to 10,000

## Conceptual Understanding

### PRINCIPLES FROM RESEARCH

“This **concrete to pictorial to abstract approach** benefits all students but has been shown to be particularly effective with students who have mathematics difficulties, mainly because it moves gradually from actual objects through pictures and then to symbols.” —Jordan, Miller, & Mercer, 1998

### TURNING PRINCIPLES INTO SUCCESS WITH MATH IN FOCUS

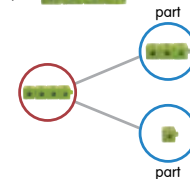
Singapore math emphasizes a concrete to pictorial to abstract pedagogy. Students are first introduced to concepts with concrete manipulatives, which allow them to experience and understand the math they are learning. They then learn to visually represent concepts using models, including number bonds and bar models. Finally, once students have a strong understanding of the concept, they move to the abstract stage where they use symbols, such as numbers and equations, to represent mathematical situations.

Grade 1

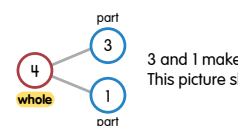
### Learn You can make number bonds with

You can use a number train to make number bonds.

Sam put  into two parts.



How many are in each **part**?



3 and 1 make 4.  
This picture shows a **number bond**.

30 Chapter 2 Number Bonds

## Visualization / Representation

### PRINCIPLES FROM RESEARCH

“For students to understand such mathematical formalisms, we must help them connect these formalisms with other forms of knowledge, including everyday experience, concrete examples, and **visual representations**. Such connections form a conceptual framework that holds mathematical knowledge together and facilitates its retrieval and application.”

—Donovan & Bransford, 2005, p. 364

### TURNING PRINCIPLES INTO SUCCESS WITH MATH IN FOCUS

“Many students who have difficulty grasping abstract mathematical concepts would benefit from visual representations of mathematical ideas. As part of this approach, the Singapore illustrations demonstrate how to graphically decompose, represent, and solve complicated multi-step problems.”

—Ginsburg, et al., 2005, p. xii.

*Math in Focus* teaches students several consistent visual models they can use to make sense of mathematical relationships and solve problems. They include bar models, place-value charts, number bonds, array models, and more. Each model is fully integrated into the program and carried across grade levels, allowing students to build upon them and understand increasingly complex concepts.

**Grade 2**

**Learn** You can use bar models to help you add.

Mandy makes 10 granola bars.  
Aida makes 12 granola bars.  
How many granola bars do they make in all?

$10 + 12 = 22$

They make 22 granola bars in all.

**Check!**  
 $22 - 10 = 12$   
 $22 - 12 = 10$   
 The answer is correct.



Manipulatives play a key role in the concrete to pictorial to abstract development of Singapore math.

## Differentiated Instruction

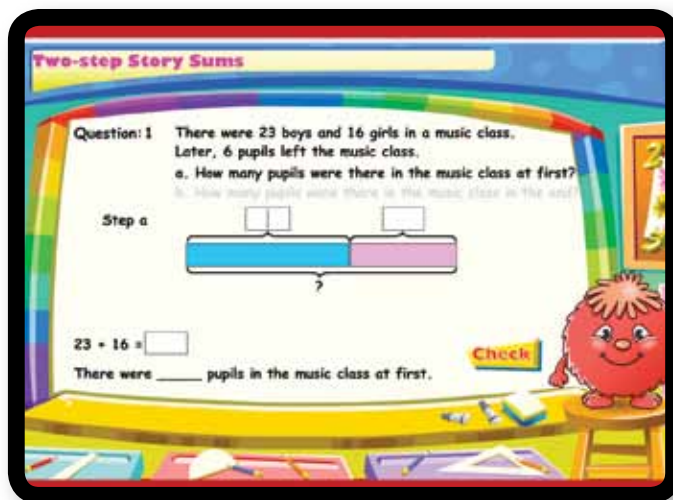
### PRINCIPLES FROM RESEARCH

“Differentiated instruction is an instructional process that has excellent potential to positively impact learning by offering teachers a means to provide instruction to a range of students in today’s classroom situations.”

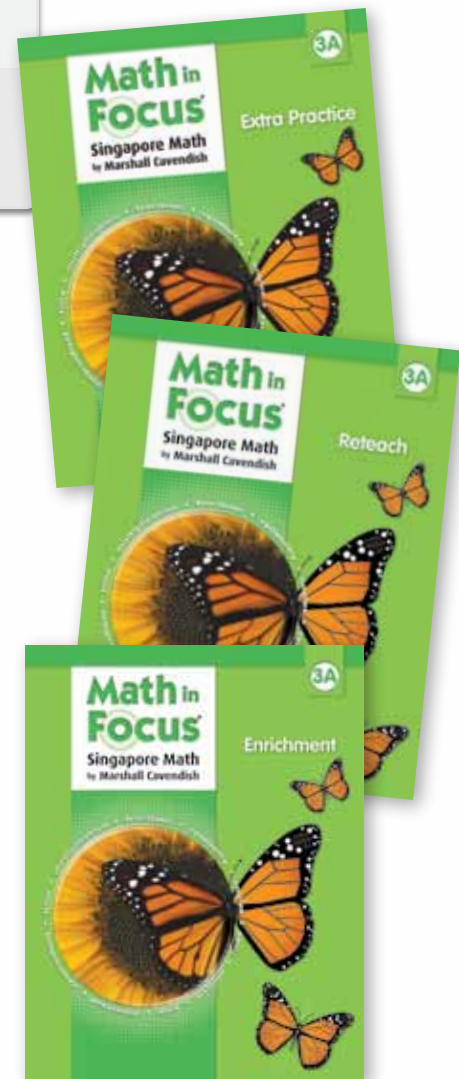
—Hall, Strangman, & Meyer, 2003

### TURNING PRINCIPLES INTO SUCCESS WITH MATH IN FOCUS

*Math in Focus* supports mathematical instruction at a variety of levels to target all learners, from struggling to gifted. Teachers have access to online and print resources including Reteach, Extra Practice, and Enrichment. Students also benefit from colorful, engaging online Interactivities. Additional support is included in the Teacher’s Edition to further differentiate instruction and ensure success for all learners.



Online Student Interactivities bring Singapore math to life with fun, interactive tutorials, activities, and quizzes.



## INTERNATIONAL RESULTS

# Strong research-based foundations lead to exceptional international results

Data from international studies has shown that the United States has been surpassed in its mathematics performance. Meanwhile, Singapore's students have consistently been top performers in international assessments. This is evidenced in Singapore's consistent top performance on the TIMSS and PISA studies.

### Trends in International Math and Science Study (TIMSS)

Since the Trends in International Math and Science Study (TIMSS) began in 1995, Singapore has consistently ranked at the top. The graph displayed here shows the top 15 countries from the 2007 report, with Singapore outperforming the United States by 70 points.

| TIMSS 2007* Grade 4  |            |
|----------------------|------------|
| Hong Kong            | 607        |
| <b>Singapore</b>     | <b>599</b> |
| Japan                | 586        |
| Chinese Taipei       | 576        |
| Kazakhstan           | 549        |
| Russian Federation   | 544        |
| England              | 541        |
| Latvia               | 537        |
| Netherlands          | 535        |
| Lithuania            | 530        |
| <b>United States</b> | <b>529</b> |
| Germany              | 525        |
| Denmark              | 523        |
| Australia            | 516        |
| Hungary              | 510        |

\*<http://nces.ed.gov/timss>

### OECD Programme for International Student Assessment (PISA)

In 2009, Singapore participated in the PISA study, which assesses 15-year-olds in industrialized countries. Singapore was a top-performing nation in math, and while Singapore students scored significantly above the international average, the United States scored below the average, ranking #31 out of 66.

### PISA Mathematics Scale 2009\*\*

|           |                      |            |
|-----------|----------------------|------------|
| 1         | Shanghai-China       | 600        |
| <b>2</b>  | <b>Singapore</b>     | <b>562</b> |
| 3         | Hong Kong-China      | 555        |
| 4         | Korea                | 546        |
| 5         | Chinese Taipei       | 543        |
| 6         | Finland              | 541        |
| 7         | Liechtenstein        | 536        |
| 8         | Switzerland          | 534        |
| 9         | Japan                | 529        |
| 10        | Canada               | 527        |
| 11        | Netherlands          | 526        |
| 12        | Macao-China          | 525        |
| 13        | New Zealand          | 519        |
| 14        | Belgium              | 515        |
| 15        | Australia            | 514        |
| 16        | Germany              | 513        |
| 17        | Estonia              | 512        |
| 18        | Iceland              | 507        |
| 19        | Denmark              | 503        |
| 20        | Slovenia             | 501        |
| 21        | Norway               | 498        |
| 22        | France               | 497        |
| 23        | Slovak Republic      | 497        |
| 24        | Austria              | 496        |
| 25        | Poland               | 495        |
| 26        | Sweden               | 494        |
| 27        | Czech Republic       | 493        |
| 28        | United Kingdom       | 492        |
| 29        | Hungary              | 490        |
| 30        | Luxembourg           | 489        |
| <b>31</b> | <b>United States</b> | <b>487</b> |

# Singapore’s international success paves the way for the Common Core Standards

Because of its consistent success in achieving exceptional student achievement and its foundation on key research-based principles, Singapore math was one of the key models for the Common Core State Standards. According to Achieve, an independent, nonprofit educational foundation:

“Overall, the Common Core State Standards (CCSS) are well aligned to Singapore’s Mathematics Syllabus. Policymakers can be assured that in adopting the CCSS, they will be setting learning expectations for students that are similar to those set by Singapore in terms of rigor, coherence and focus.”

—Achieve\* ([achieve.org/CCSSandSingapore](http://achieve.org/CCSSandSingapore))

The key ideas in the Common Core State Standards parallel the Singapore Mathematics Framework.

## PRINCIPLES FROM THE COMMON CORE

“For over a decade, research studies of mathematics education in high-performing countries have pointed to the conclusion that the mathematics curriculum in the United States must become substantially more focused and coherent in order to improve mathematics achievement in this country.”

—Common Core State Standards, [corestandards.org](http://corestandards.org)

## TURNING PRINCIPLES INTO SUCCESS WITH MATH IN FOCUS

One of the key high-performing countries that the creators of the Common Core State Standards looked at was Singapore, whose mathematics curriculum truly embodies the notion of focused and coherent. Through multi-day lessons and minimal repetition from year to year, students learn concepts in depth to mastery.

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|            | Let’s Practice and Practice and Apply Workbook A: Practice 1  | 10-11 |
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\* Achieve is a bipartisan, nonprofit educational reform organization that partnered with NGA and CCSSO on the Common Core State Standards Initiative.





A model for the  
**COMMON CORE**

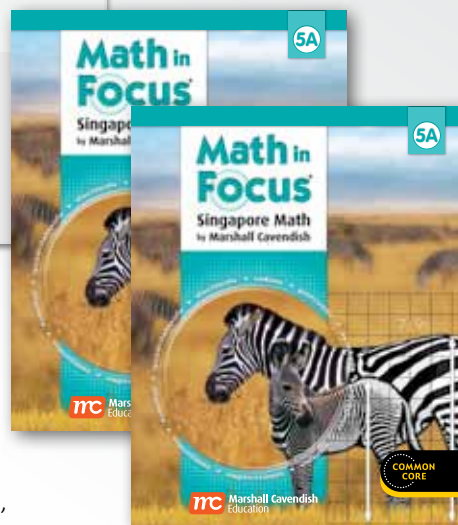
### PRINCIPLES FROM THE COMMON CORE

“The composite standards [of Hong Kong, Korea, and Singapore] have a number of features that can inform an international benchmarking process for the development of K–6 mathematics standards in the U.S. First, the composite standards concentrate the early learning of mathematics on the number, measurement, and geometry strands with less emphasis on data analysis and little exposure to algebra.”

—Common Core State Standards, [corestandards.org](http://corestandards.org)

### TURNING PRINCIPLES INTO SUCCESS WITH MATH IN FOCUS

*Math in Focus* emphasizes number and operations in every grade. The textbook is divided into two books, roughly a semester each. Book A focuses on number and operations while Book B focuses on geometry and measurement, where number concepts are practiced, connected, and applied.



### PRINCIPLES FROM THE COMMON CORE

“One hallmark of mathematical understanding is the ability to justify, in a way appropriate to the student’s mathematical maturity, *why* a particular mathematical statement is true or where a mathematical rule comes from.”

—Common Core State Standards, [corestandards.org](http://corestandards.org)

### TURNING PRINCIPLES INTO SUCCESS WITH MATH IN FOCUS

The Singapore Mathematics Framework and pedagogy emphasize deep understanding, which is demonstrated through consistent opportunities to explain why mathematical concepts work. This is modeled for students throughout *Math in Focus* with thought bubbles, which display pictures of students expressing their understanding. Students then have the opportunity to justify their own understanding through activities such as Math Journals.

Grade 4

READING AND WRITING MATH  
*Math Journal*

Both Andy and Rita think that 0.23 is greater than 0.3.



23 is greater than 3, so 0.23 is greater than 0.3.



23 tenths is greater than 3 tenths, so 0.23 is greater than 0.3.

Do you agree? Why or why not? Explain your answer.

Lesson 7.3 Comparing Decimals 33

# United States students experience exceptional results with Math in Focus

## OLD BRIDGE, NEW JERSEY

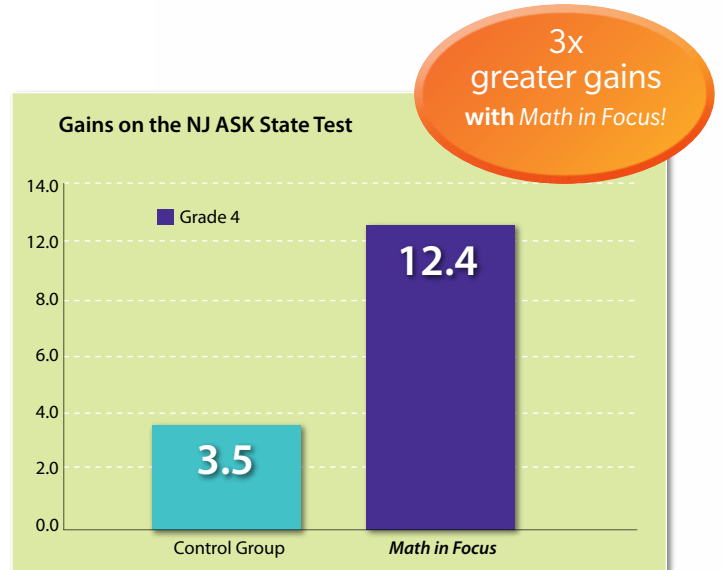
### State Test Score Analysis

An analysis of 2010 state test scores in Old Bridge, NJ sought to determine if *Math in Focus* students experienced greater gains on the state mathematics test (NJ ASK).

### RESULTS

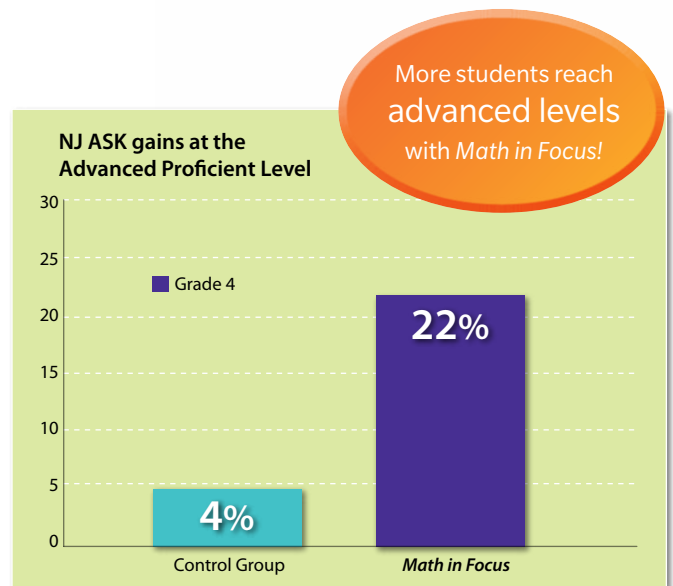
#### 4th-Grade Student Performance on NJ ASK

*Math in Focus* students improved an average of 12.4 points, more than three times the average improvement that occurred across the remaining students in the district in the comparison group.



#### NJ ASK gains at the Advanced Proficient Level

The percentage of 4th-grade students scoring at the highest level, advanced proficient, increased by 22% for the *Math in Focus* group, while the control group increased by only 4%.



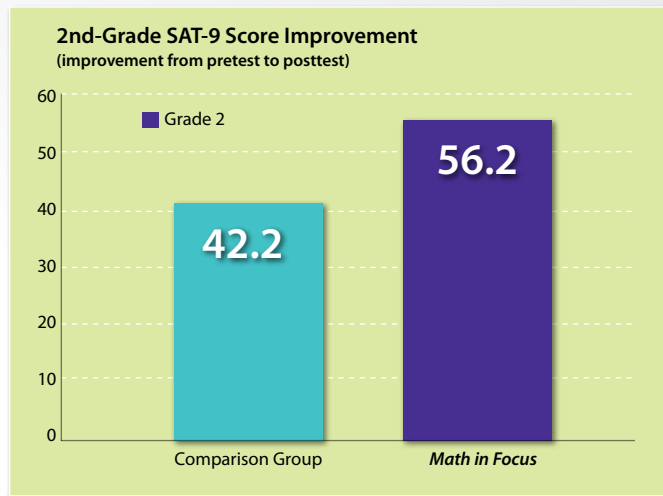
*For full reports, please contact your local Account Manager.*

### Instructional Effectiveness Study Overview

An independent research firm conducted a year-long study that examined 2nd-grade achievement in *Math in Focus* compared to a control group.

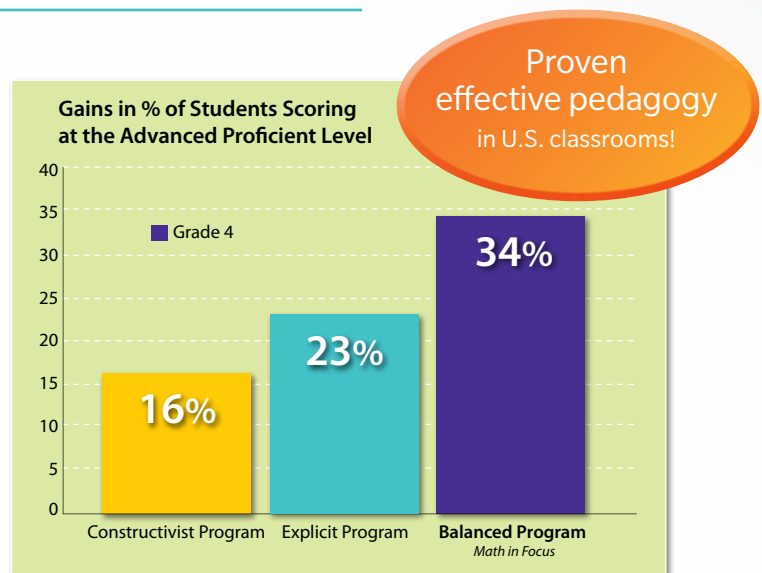
#### RESULTS

At Grade 2, the *Math in Focus* group of students made significant and greater academic gains from pretest to posttest.



### Gains compared to other types of mathematics programs

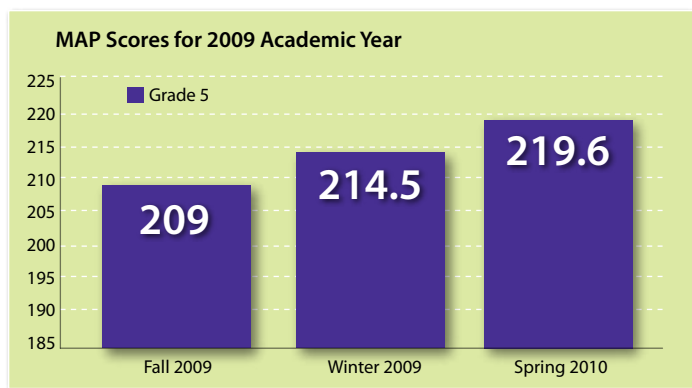
*Math in Focus* student performance on the NJ ASK test was also compared to comparison groups using constructivist and explicit mathematics programs. The three programs were taught in the same schools by similar teachers. The analysis showed that the *Math in Focus*, a program that utilizes a balanced approach, outperformed the students in the alternative programs.



# KENTUCKY

## A longitudinal analysis of *Math in Focus* in Lexington, KY

Longitudinal data analysis was conducted to investigate student math achievement over the course of two consecutive years. The sample was comprised of 272 students from Title 1-eligible schools. Student math performance was measured using the *Measures of Academic Progress* (MAP).



## RESULTS

Statistical analysis indicated that using *Math in Focus* was associated with increased student math performance. This improvement was statistically significant even after controlling for student level variables.

## Lexington, KY Year One KCCT (Kentucky Core Content Test) Highlights

In addition to showing growth in the longitudinal analysis, Lexington students using *Math in Focus*

also showed exceptional results on the Kentucky state test.

Highlights include:

Gains in Proficient and Distinguished ratings for African-American students:

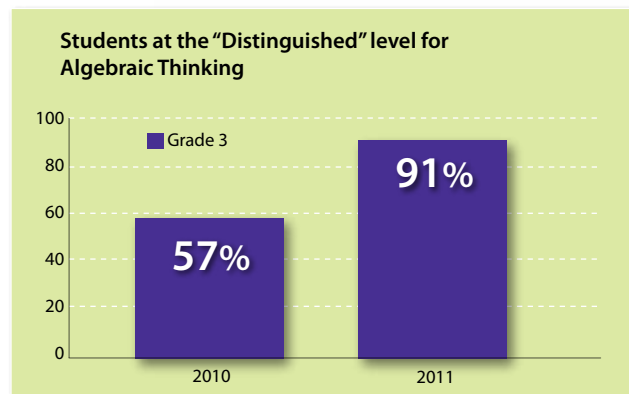
- Deep Springs Elementary – 26% gain at 3<sup>rd</sup> grade
- Breckenridge Elementary – 25% gain at 5<sup>th</sup> grade

Gains in Proficient and Distinguished ratings for Hispanic students:

- Russell Cave Elementary – 26% gain at 3<sup>rd</sup> grade

## Thinklink Benchmark Test Improvement in Marshall County, KY

The Thinklink benchmark test ranks students on four levels: Novice, Apprentice, Proficient, and Distinguished. After using *Math in Focus* in Grade 3 from 2010 to 2011, the Thinklink benchmark test scores showed a significant improvement for *Math in Focus* students, rising from 57% to 91% of students at the Distinguished level for Algebraic Thinking. Distinguished is the highest level that students can achieve.



# MIDDLE SCHOOL

## An analysis of *Math in Focus* Course 1 for Middle School

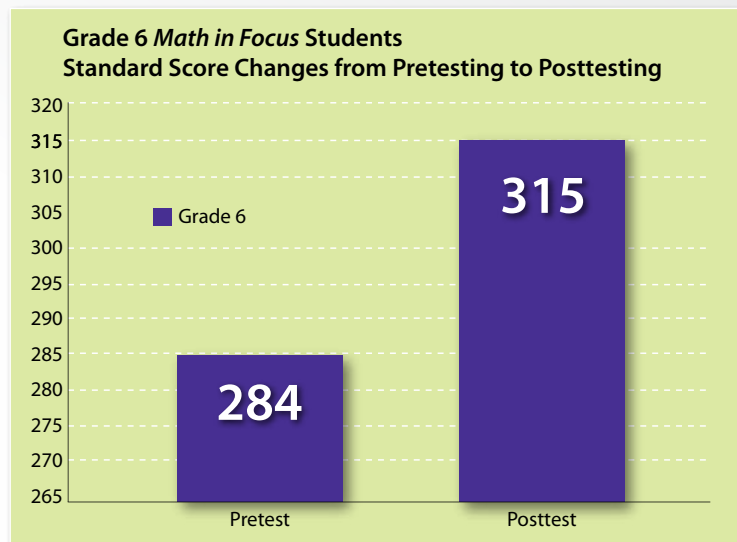
The new *Math in Focus* middle school program has also shown results in U.S. classrooms. Two pilot studies were conducted for *Math in Focus* Course 1, tracking student performance across a 7-week pilot period.

### RESULTS

The analysis showed that the *Math in Focus* classes made significant gains over the course of the 7-week tryout period in both studies.

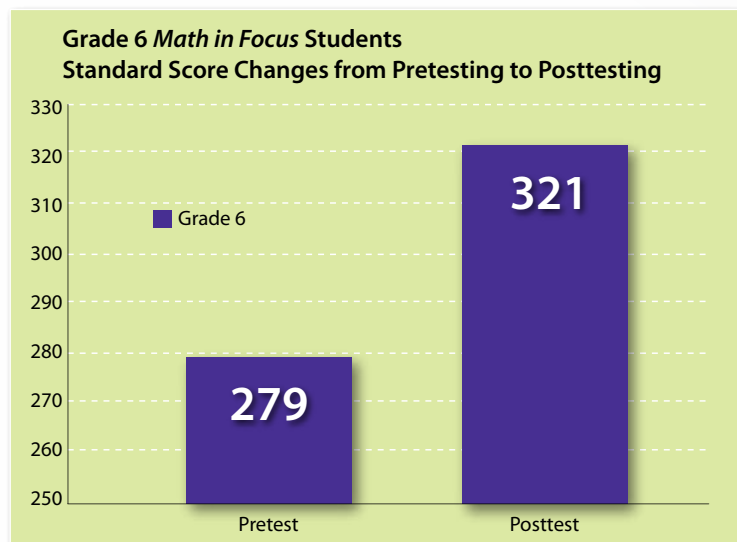
#### Little Rock, AR

Students using *Math in Focus* Course 1 increased 31 standard score points during the trial period.



#### Lexington, KY

Students using *Math in Focus* Course 1 increased 41 standard score points during the trial period.



# Everyone's seeing results thanks to Singapore math!

## Supervisors

"This study is the proof in the pudding that *Math in Focus* is working for our students. Partnering with Houghton Mifflin Harcourt for professional development has allowed our teachers to move from being just comfortable teaching math to reaching a new level of confidence in their math teaching ability. The students, teachers, and parents in our district truly see the benefits of Singapore math."

—Donna Kibbler, Director of Elementary Education, Old Bridge, NJ

## Principals

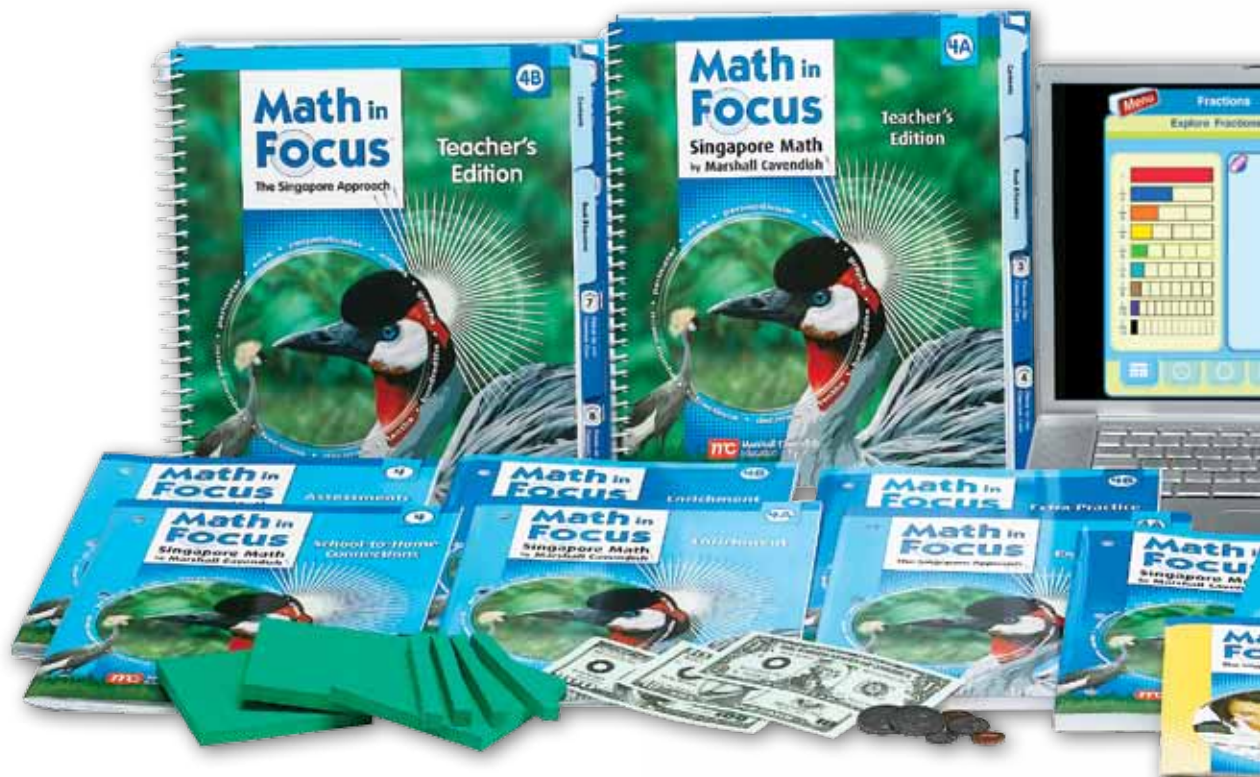
"You should have heard the positive comments from the kids! Mostly, they commented about how much *Math in Focus* made them think, gave them a challenge, but also gave them a few "strategies" to solve the hard problems. The 2<sup>nd</sup> graders were SO excited to be doing multiplication in 2<sup>nd</sup> grade! AMAZING!"

—Nicole Moore, Principal, Shamong, NJ

## Parents

"I'm amazed at my 5<sup>th</sup> grader's problem solving ability after having been in the program for one semester."

—Parent from Rye Neck schools in Westchester County, NY



## Teachers

“*Math in Focus* has helped my students become the most strategic problem solvers I’ve seen in fifteen years of teaching. I’m thrilled with the differentiated support my students receive and the extensive, intuitive resources that allow me to teach with confidence as we implement the Common Core State Standards.”

—Scott Sheets, 5<sup>th</sup>-Grade Teacher, Lexington, KY

## Students

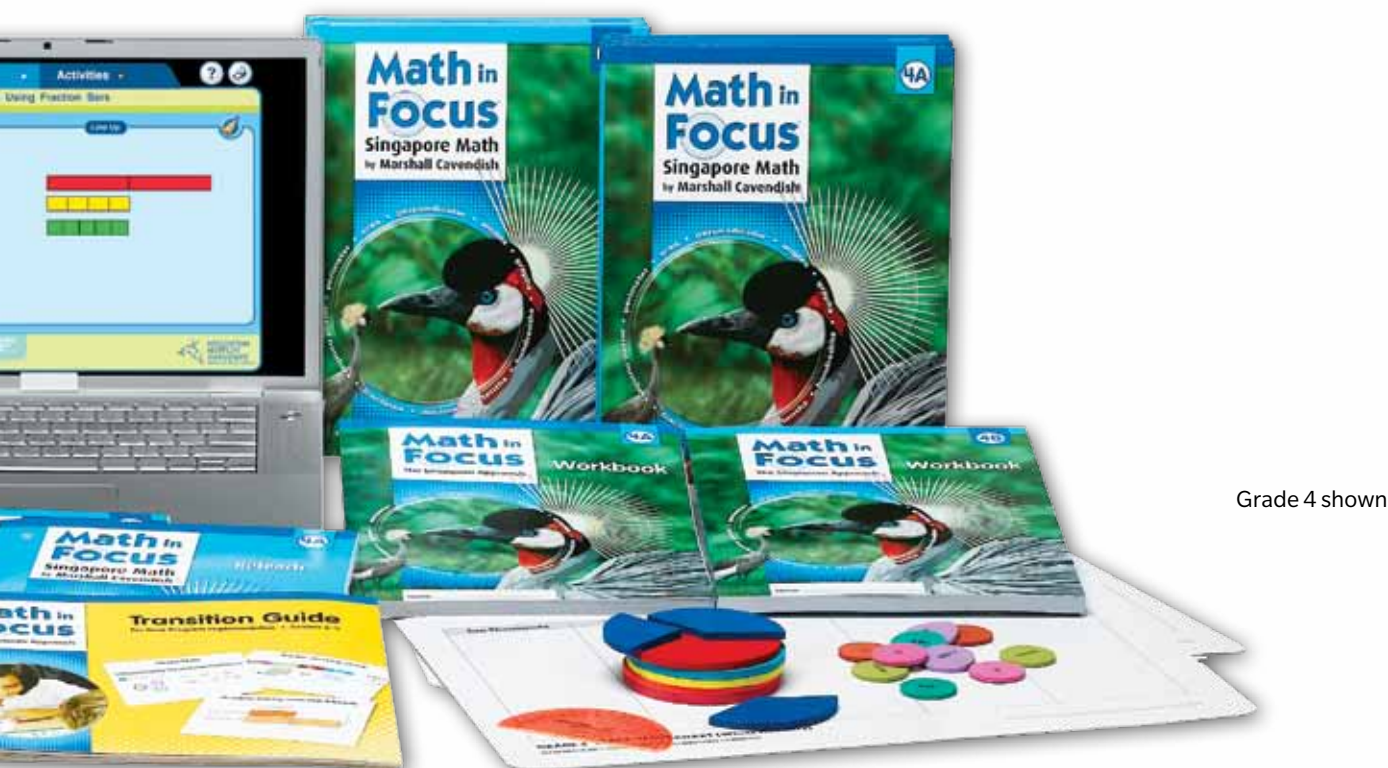
“*Math in Focus* is better because it gives you new and better strategies. If you struggle or forget the strategies, then you can scan your book and pick up extra information to help you. The chapter that gives you the most strategies is *Chapter 5: Bar Modeling*. With this program you are sure to do great on the NJ ASK! Try it! You’ll never know until you try!”

—Michael, 3<sup>rd</sup>-Grade *Math in Focus* Student

## Superintendents

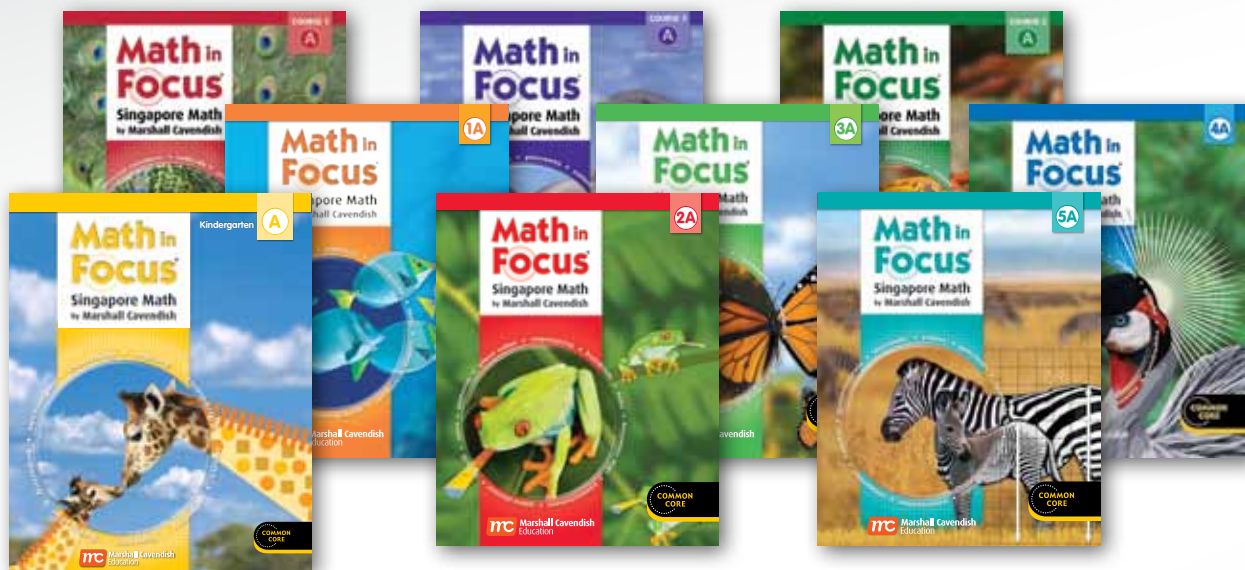
“We have been thrilled with the level of mathematical understanding our students using Singapore math (*Math in Focus*) have achieved. Teachers say it has transformed their teaching. My own son, in 2<sup>nd</sup> grade, is in a *Math in Focus* classroom and is doing a much higher level of math than my older child did in 2<sup>nd</sup> grade with another program. The focus on mastery of number sense, operations, problem solving, and challenging questions correlates perfectly with the direction of the new Common Core State Standards.”

—Karen Cheser, Assistant Superintendent, Boone County Schools, KY



Grade 4 shown

# Experience World-Class Success in Your Classroom!



Grades K-8

[hmheducation.com/singaporemath](http://hmheducation.com/singaporemath)  
800.289.4490

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