Power Electronics
Single Phase Controlled Rectifiers

Dr. Firas Obeidat

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Single Phase Controlled Half Wave Rectifier

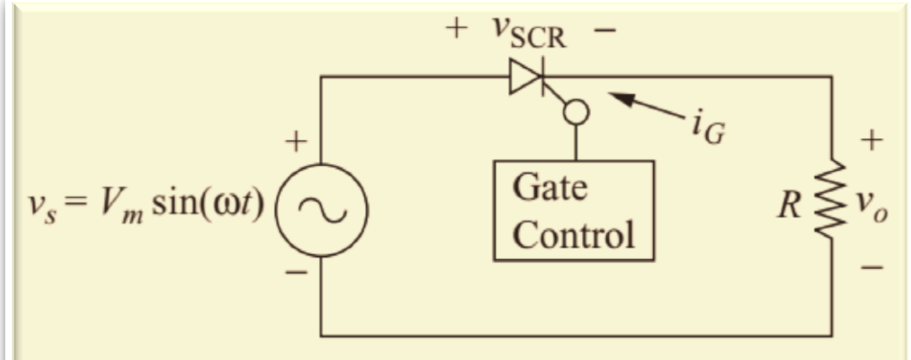
Resistive Load

- A way to control the output of a half-wave rectifier is to use an SCR1 instead of a diode.

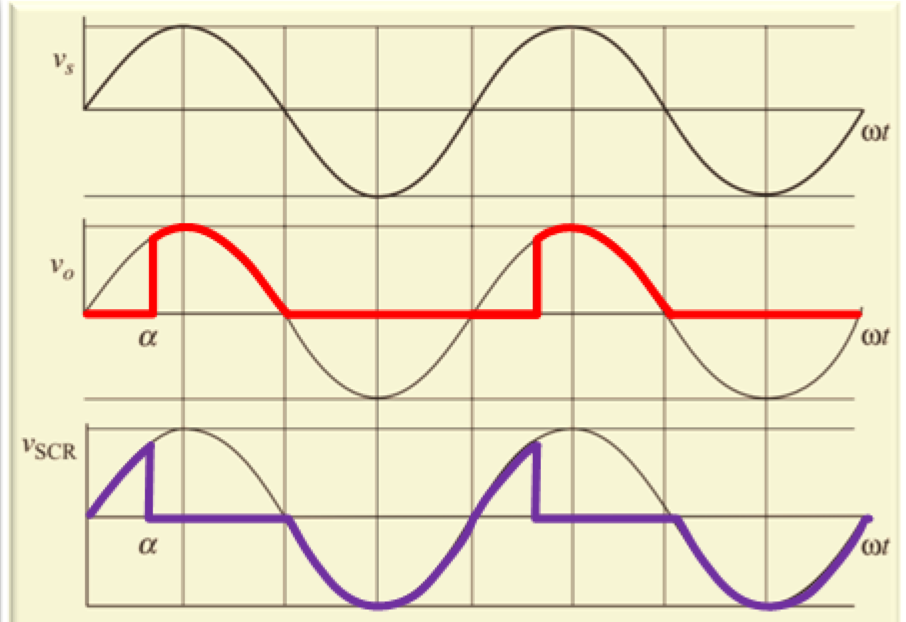
- Two conditions must be met before the SCR can conduct:

1. The SCR must be forward-biased ($v_{SCR} > 0$).
2. A current must be applied to the gate of the SCR.

- The SCR will not begin to conduct as soon as the source becomes positive. Conduction is delayed until a gate current is applied, which is the basis for using the SCR as a means of control. Once the SCR is conducting, the gate current can be removed and the SCR remains on until the current goes to zero.



(a)



(b)

Single Phase Controlled Half Wave Rectifier

Resistive Load

- If a gate signal is applied to the SCR at $\omega t=\alpha$, where $\underline{\alpha}$ is the ***delay (firing or triggering) angle***. The average (dc) voltage across the load resistor and the average (dc) current are

$$V_{dc}=V_o=\frac{1}{2\pi}\int_{\alpha}^{\pi} V_m \sin(\omega t)\, d(\omega t)=\frac{V_m}{2\pi}(1+\cos\alpha)$$

$$I_{dc}=\frac{V_{dc}}{R}=\frac{V_m}{2\pi R}(1+cos\alpha)$$

- The *rms* voltage across the resistor and the *rms* current are computed from

$$V_{rms}=\sqrt{\frac{1}{2\pi}\int_{0}^{2\pi} v_o^2(\omega t)\, d(\omega t)}=\sqrt{\frac{1}{2\pi}\int_{\alpha}^{\pi}[V_m \sin(\omega t)]^2\, d(\omega t)}=\frac{V_m}{2}\sqrt{1-\frac{\alpha}{\pi}+\frac{\sin(2\alpha)}{2\pi}}$$

$$V_{rms}=\frac{V_m}{2}\sqrt{\frac{1}{\pi}(\pi-\alpha+\frac{sin2\alpha}{2})}$$

$$I_{rms}=\frac{V_{rms}}{R}=\frac{V_m}{2R}\sqrt{\frac{1}{\pi}(\pi-\alpha+\frac{sin2\alpha}{2})}$$

- The power absorbed by the resistor is

$$P_{ac}=\frac{{V_{rms}}^2}{R}$$

Single Phase Controlled Half Wave Rectifier

Resistive Load

Example: The single-phase half wave rectifier has a purely resistive load of R and the delay angle is α=π/2, determine: V_{dc}, I_{dc}, V_{rms}, I_{rms}.

$$V_{dc} = \frac{V_m}{2\pi}\left(1 + cos\frac{\pi}{2}\right) = 0.1592V_m$$

$$I_{dc} = \frac{V_m}{2\pi R}\left(1 + cos\frac{\pi}{2}\right) = 0.1592\frac{V_m}{R}$$

$$V_{rms} = \frac{V_m}{2}\sqrt{\frac{1}{\pi}(\pi - \frac{\pi}{2} + \frac{\sin(2\frac{\pi}{2})}{2})} = 0.3536V_m$$

$$I_{rms} = \frac{V_m}{2R}\sqrt{\frac{1}{\pi}(\pi - \frac{\pi}{2} + \frac{\sin(2\frac{\pi}{2})}{2})} = 0.3536\frac{V_m}{R}$$

Single Phase Controlled Half Wave Rectifier

Resistive Load

Example: Design a circuit to produce an average voltage of 40V across a 100Ω load resistor from a 120V$_{rms}$ 60-Hz ac source. Determine the power absorbed by the resistance and the power factor.

$$V_{dc}=\frac{V_m}{2\pi}(1+\cos\alpha)$$

so

$$\alpha=\cos^{-1}\left[V_o\left(\frac{2\pi}{V_m}\right)-1\right]$$

$$=\cos^{-1}\left\{40\left[\frac{2\pi}{\sqrt{2}(120)}\right]-1\right\}=61.2°=1.07\text{ rad}$$

$$V_{rms}=\frac{V_m}{2}\sqrt{1-\frac{\alpha}{\pi}+\frac{\sin(2\alpha)}{2\pi}}$$

$$V_{\text{rms}}=\frac{\sqrt{2}(120)}{2}\sqrt{1-\frac{1.07}{\pi}+\frac{\sin[2(1.07)]}{2\pi}}=75.6\text{ V}$$

$$P_R=\frac{V_{\text{rms}}^2}{R}=\frac{(75.6)^2}{100}=57.1\text{ W}$$

$$I_{rms}=\frac{V_{rms}}{R}=\frac{75.6}{100}=0.756A$$

$$S=V_{s,rms}I_{rms}=120\times0.756=90.72\,VA$$

$$pf=\frac{P_R}{S}=\frac{57.1}{90.72}=0.629$$

Single Phase Controlled Half Wave Rectifier

RL Load

The current is the sum of the forced and natural responses.

$$i(\omega t) = i_f(\omega t) + i_n(\omega t) = \frac{V_m}{Z}\sin(\omega t - \theta) + Ae^{-\omega t/\omega\tau}$$

The constant A is determined from the initial condition $\omega t=\alpha$ $i(\alpha)=0$:

$$i(\alpha) = 0 = \frac{V_m}{Z}\sin(\alpha - \theta) + Ae^{-\alpha/\omega\tau}$$

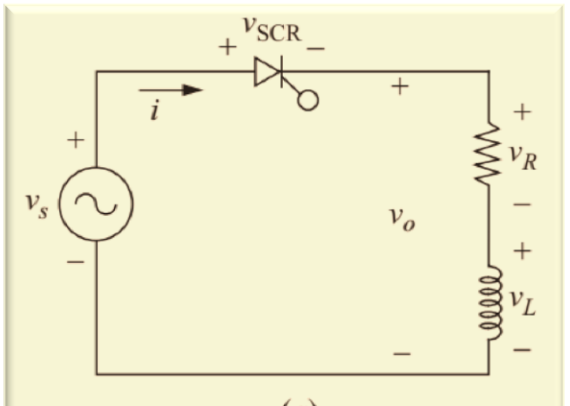
$$A = \left[-\frac{V_m}{Z}\sin(\alpha - \theta)\right]e^{\alpha/\omega\tau}$$

Substituting for A and simplifying,

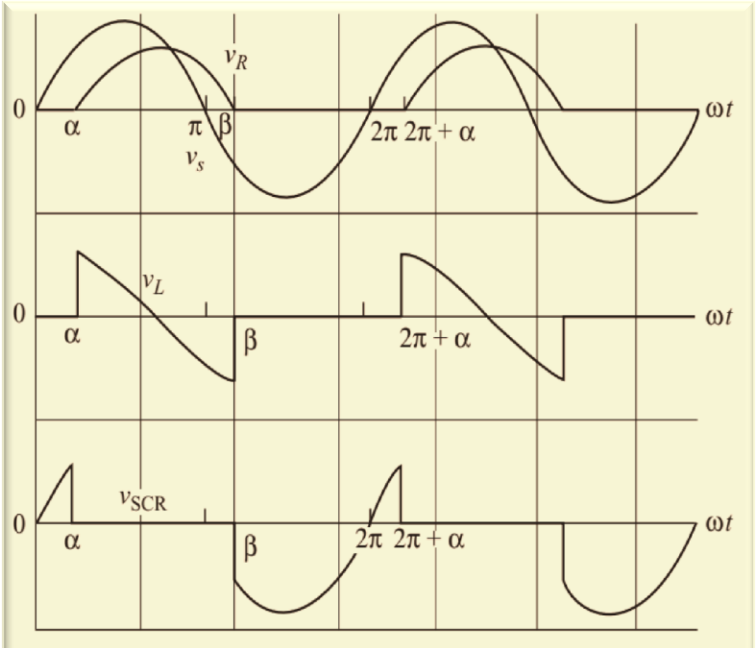
$$i(\omega t) = \begin{cases} \frac{V_m}{Z}\left[\sin(\omega t - \theta) - \sin(\alpha - \theta)e^{(\alpha - \omega t)/\omega\tau}\right] & \text{for } \alpha \le \omega t \le \beta \\ 0 & \text{otherwise} \end{cases}$$

The ***extinction angle β*** is defined as the angle at which the current returns to zero, as in the case of the uncontrolled rectifier. When $\omega t=\beta$

$$i(\beta) = 0 = \frac{V_m}{Z}\left[\sin(\beta - \theta) - \sin(\alpha - \theta)e^{(\alpha - \beta)/\omega\tau}\right]$$



(a)



(b)

Single Phase Controlled Half Wave Rectifier

RL Load

The above equation must be solved numerically for β. The angle (β-α) is called the conduction angle Υ.

The average (dc) output voltage is

$$V_{dc} = V_o = \frac{1}{2\pi}\int_{\alpha}^{\beta} V_m \sin(\omega t)\, d(\omega t) = \frac{V_m}{2\pi}(\cos\alpha - \cos\beta)$$

The average (dc) output voltage is

$$I_{dc} = I_o = \frac{1}{2\pi}\int_{\alpha}^{\beta} i(\omega t)\, d(\omega t)$$

or

$$I_{dc} = I_o = \frac{V_m}{2\pi R}(\cos\alpha - \cos\beta)$$

The *rms* current is computed from

$$I_{\mathrm{rms}} = \sqrt{\frac{1}{2\pi}\int_{\alpha}^{\beta} i^2(\omega t)\, d(\omega t)}$$

Or it can be written as

$$V_{rms} = \sqrt{\frac{1}{2\pi}\int_{\alpha}^{\beta} (V_m \sin\omega t)^2\, d\omega t} = \sqrt{\frac{{V_m}^2}{4\pi}\left(\beta - \alpha - \frac{1}{2}\sin 2\beta + \frac{1}{2}\sin 2\alpha\right)}$$

$$I_{rms} = \frac{V_{rms}}{Z} = \frac{V_{rms}}{\sqrt{R^2 + (\omega L)^2}} = \frac{1}{\sqrt{R^2 + (\omega L)^2}}\sqrt{\frac{{V_m}^2}{4\pi}\left(\beta - \alpha - \frac{1}{2}\sin 2\beta + \frac{1}{2}\sin 2\alpha\right)}$$

Single Phase Controlled Half Wave Rectifier

RL Load

Example: For the circuit of controlled half-wave rectifier with RL Load, the source is 120V$_{rms}$ at 60 Hz, R=20Ω, L=0.04H, and the delay angle is 45°. Determine (a) an expression for i(ωt), (b) the *rms* current, (c) the power absorbed by the load, and (d) the power factor.

(a)

$$V_m = 120\sqrt{2} = 169.7 \text{ V}$$

$$Z = [R^2 + (\omega L)^2]^{0.5} = [20^2 + (377*0.04)^2]^{0.5} = 25.0\ \Omega$$

$$\theta = \tan^{-1}(\omega L/R) = \tan^{-1}(377*0.04)/20) = 0.646 \text{ rad}$$

$$\omega\tau = \omega L/R = 377*0.04/20 = 0.754$$

$$\alpha = 45° = 0.785 \text{ rad}$$

$$i(\omega t) = 6.78 \sin(\omega t - 0.646) - 2.67 e^{-\omega t/0.754} \quad \text{A} \qquad \text{for } \alpha \le \omega t \le \beta$$

(b)

$$I_{\text{rms}} = \sqrt{\frac{1}{2\pi}\int_{0.785}^{3.79}\left[6.78\sin(\omega t - 0.646) - 2.67e^{-\omega t/0.754}\right]^2 d(\omega t)} = 3.26 \text{ A}$$

(c)

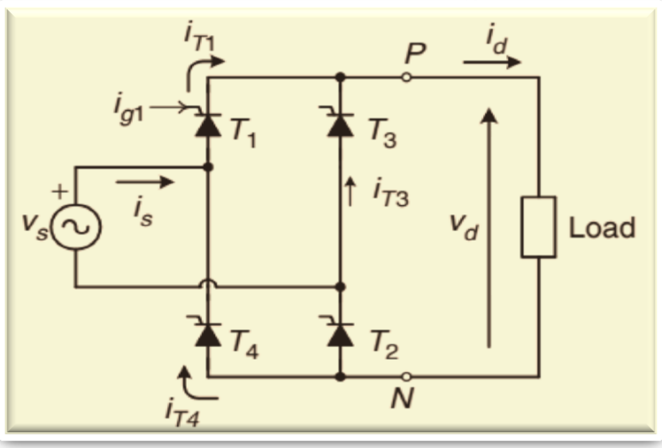
$$P = I_{\text{rms}}^2 R = (3.26)^2(20) = 213 \text{ W}$$

(c)

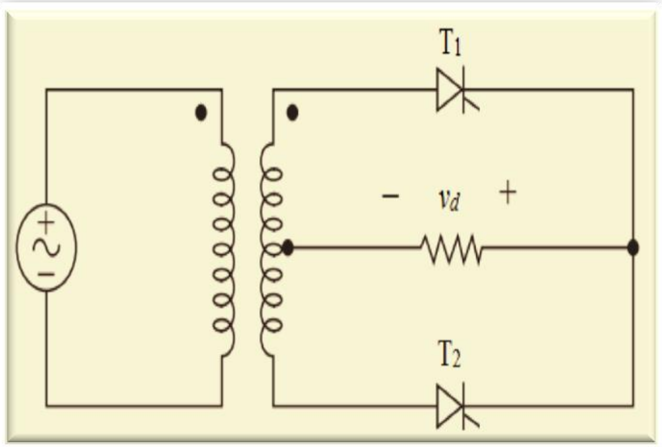
$$\text{pf} = \frac{P}{S} = \frac{213}{(120)(3.26)} = 0.54$$

Single Phase Controlled Full Wave Rectifier

- The first figure shows a fully controlled bridge rectifier, which uses four thyristors to control the average load voltage.
- Thyristors T_1 and T_2 must be fired simultaneously during the positive half wave of the source voltage v_s to allow conduction of current. To ensure simultaneous firing, thyristors T_1 and T_2 use the same firing signal.
- Alternatively, thyristors T_3 and T_4 must be fired simultaneously during the negative half wave of the source voltage.



- For the center-tapped transformer rectifier, T_1 is forward-biased when v_s is positive, and T_2 is forward-biased when v_s is negative, but each will not conduct until it receives a gate signal.



- The delay angle is the angle interval between the forward biasing of the SCR and the gate signal application. If the delay angle is zero, the rectifiers behave exactly as uncontrolled rectifiers with diodes.

Single Phase Controlled Full Wave Rectifier

Resistive Load

The average component of the output voltage and current waveforms are determined from

$$V_{dc}=V_o=\frac{1}{\pi}\int_{\alpha}^{\pi}V_m\sin(\omega t)\,d(\omega t)=\frac{V_m}{\pi}(1+\cos\alpha)$$

$$I_{dc}=\frac{V_{dc}}{R}=\frac{V_m}{\pi R}(1+cos\alpha)$$

The *rms* component of the output voltage and current waveforms are determined from

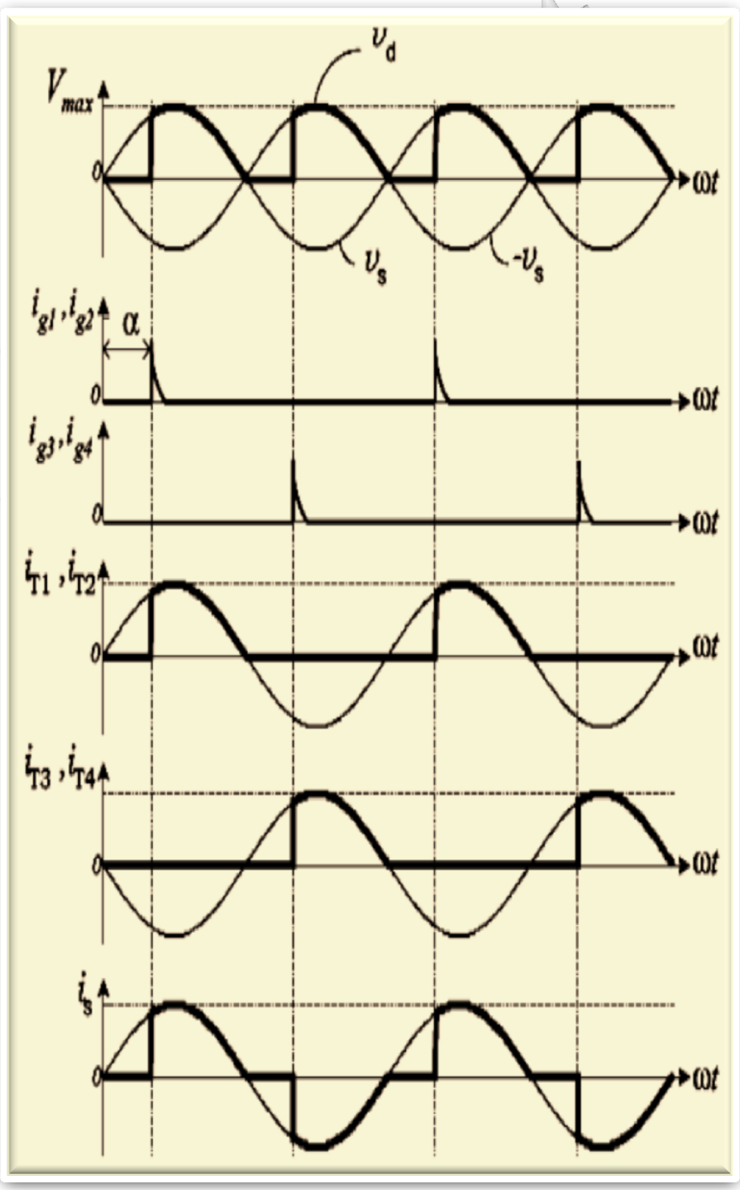
$$V_{rms}=\sqrt{\frac{1}{\pi}\int_{\alpha}^{\pi}(V_m sin\omega t)^2 d\omega t}=V_m\sqrt{\frac{1}{2}-\frac{\alpha}{2\pi}+\frac{\sin(2\alpha)}{4\pi}}$$

$$I_{rms}=\frac{V_{rms}}{R}=\frac{V_m}{R}\sqrt{\frac{1}{2}-\frac{\alpha}{2\pi}+\frac{\sin(2\alpha)}{4\pi}}$$

The power delivered to the load is

$$p=I_{rms}{}^2R$$

The *rms* current in the source is the same as the *rms* current in the load.



Single Phase Controlled Full Wave Rectifier

Resistive Load

Example: The full-wave controlled bridge rectifier has an ac input of 120V_{rms} at 60 Hz and a 20Ω load resistor. The delay angle is 40º. Determine the average current in the load, the power absorbed by the load, and the source voltamperes.

$$V_{dc}=\frac{V_m}{\pi}(1+\cos\alpha)=\frac{\sqrt{2}\,(120)}{\pi}\left(1+\cos 40°\right)=95.4\text{ V}$$

$$I_{dc}=\frac{V_{dc}}{R}=\frac{95.6}{20}=4.77A$$

$$I_{\text{rms}}=\frac{\sqrt{2}(120)}{20}\sqrt{\frac{1}{2}-\frac{0.698}{2\pi}+\frac{\sin[2(0.698)]}{4\pi}}=5.80\text{ A}$$

$$P=I_{\text{rms}}^2R=(5.80)^2(20)=673\text{ W}$$

The *rms* current in the source is also 5.80 A, and the apparent power of the source is

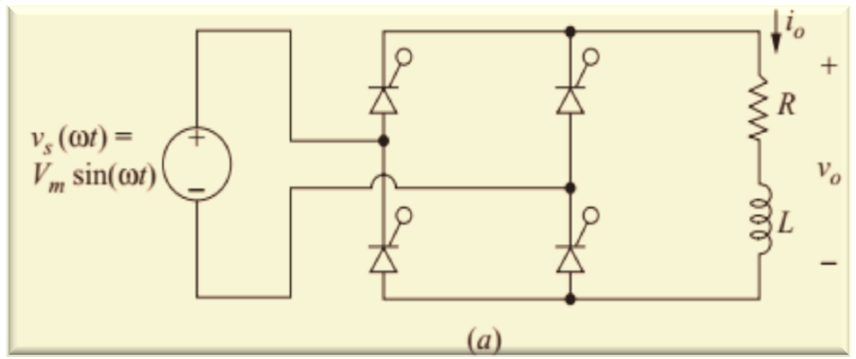
$$S=V_{\text{rms}}I_{\text{rms}}=(120)(5.80)=696\text{ VA}$$

$$\text{pf}=\frac{P}{S}=\frac{672}{696}=0.967$$

Single Phase Controlled Full Wave Rectifier

RL Load, Discontinuous Current

Load current for a controlled full-wave rectifier with an *RL* load (fig. a) can be either continuous or discontinuous.



(*a*)

➢ **For discontinuous current**

1- at ωt=0 with zero load current, SCRs T_1 and T_2 in the bridge rectifier will be forward-biased and T_3 and T_4 will be reverse-biased as the source voltage becomes positive.

2- Gate signals are applied to T_1 and T_2 at ωt=α, turning T_1 and T_2 on. With T_1 and T_2 on, the load voltage is equal to the source voltage.

The output current can be given as

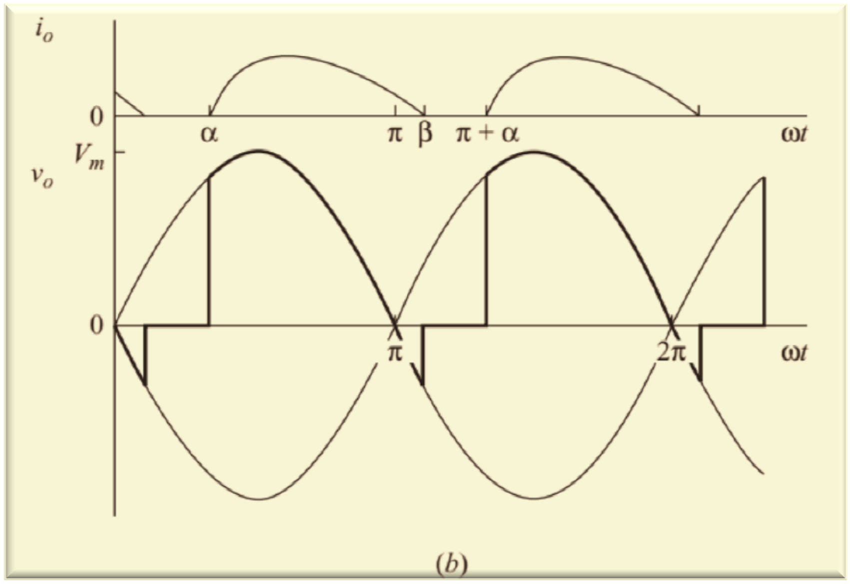
$$i_o(\omega t) = \frac{V_m}{Z}\left[\sin(\omega t - \theta) - \sin(\alpha - \theta)\, e^{-(\omega t - \alpha)/\omega\tau}\right] \quad \text{for } \alpha \le \omega t \le \beta$$

$$Z = \sqrt{R^2 + (\omega L)^2} \qquad \theta = \tan^{-1}\left(\frac{\omega L}{R}\right) \qquad \text{and} \qquad \tau = \frac{L}{R}$$

Single Phase Controlled Full Wave Rectifier

RL Load, Discontinuous Current

The above current function becomes zero at $\omega t=\beta$. If , the current remains at zero until $\omega t=\pi+\alpha$ when gate signals are applied to T_3 and T_4 which are then forward-biased and begin to conduct. This mode of operation is called *discontinuous current* as shown in fig. b.



$\beta<\pi+\alpha \quad \rightarrow \quad$ **Discontinuous current**

Analysis of the controlled full-wave rectifier operating in the discontinuous current mode is identical to that of the controlled half-wave rectifier except that the period for the output current is π rather than 2π rad.

Single Phase Controlled Full Wave Rectifier

RL Load, Discontinuous Current

The average (dc) output voltage is

$$V_{dc} = \frac{1}{\pi}\int_{\alpha}^{\beta} V_m sin\omega t\; d t\omega t = \frac{V_m}{\pi}(cos\alpha - cos\beta)$$

The average (dc) output current is

$$I_{dc} = I_o = \frac{1}{2\pi}\int_{\alpha}^{\beta} i(\omega t)d(\omega t)$$

or

$$I_{dc} = \frac{V_{dc}}{R} = \frac{V_m}{\pi R}(cos\alpha - cos\beta)$$

The *rms* voltage is computed from

$$V_{rms} = \sqrt{\frac{1}{\pi}\int_{\alpha}^{\beta}(V_m sin\omega t)^2\, d\omega t} = \sqrt{\frac{V_m{}^2}{2\pi}\left(\beta - \alpha - \frac{1}{2}\boldsymbol{sin2}\beta + \frac{1}{2}\boldsymbol{sin2}\alpha\right)}$$

The *rms* current is computed from

$$I_{rms} = \sqrt{\frac{1}{2\pi}\int_{\alpha}^{\beta} i^2(\omega t)d(\omega t)}$$

Or it can be written as

$$I_{rms} = \frac{V_{rms}}{Z} = \frac{V_{rms}}{\sqrt{R^2 + (\omega L)^2}} = \frac{1}{\sqrt{R^2 + (\omega L)^2}}\sqrt{\frac{V_m{}^2}{2\pi}\left(\beta - \alpha - \frac{1}{2}\boldsymbol{sin2}\beta + \frac{1}{2}\boldsymbol{sin2}\alpha\right)}$$

Single Phase Controlled Full Wave Rectifier

RL Load, Discontinuous Current

Example: A controlled full-wave bridge rectifier has a source of 120V_{rms} at 60Hz, R=10Ω, L=20mH, and α=60º. Determine (a) an expression for load current, (b) the average load current, and (c) the power absorbed by the load.

$$V_m = \frac{120}{\sqrt{2}} = 169.7 \text{ V}$$

$$Z = \sqrt{R^2 + (\omega L)^2} = \sqrt{10^2 + [(377)(0.02)]^2} = 12.5\ \Omega$$

$$\theta = \tan^{-1}\left(\frac{\omega L}{R}\right) = \tan^{-1}\left[\frac{(377)(0.02)}{10}\right] = 0.646 \text{ rad}$$

$$\omega\tau = \frac{\omega L}{R} = \frac{(377)(0.02)}{10} = 0.754 \text{ rad}$$

$$\alpha = 60° = 1.047 \text{ rad}$$

(a)

$$i_o(\omega t) = 13.6 \sin(\omega t - 0.646) - 21.2e^{-\omega t/0.754} \text{ A} \qquad \text{for } \alpha \le \omega t \le \beta$$

Solving $i_o(\beta) = 0$ numerically for β, $\beta = 3.78$ rad (216°). Since $\pi + \alpha = 4.19 > \beta$, the current is discontinuous, and the above expression for current is valid.

(b)

$$I_{dc} = \frac{1}{2\pi}\int_{\alpha}^{\beta} i(\omega t)\, d(\omega t) = \frac{V_m}{\pi R}(\cos\alpha - \cos\beta) = \frac{169.7}{\pi 10}(\cos 60 - \cos 216) = 7.07A$$

(c)

$$I_{rms} = \frac{1}{\sqrt{R^2 + (\omega L)^2}}\sqrt{\frac{V_m{}^2}{2\pi}\left(\beta - \alpha - \frac{1}{2}\boldsymbol{\sin 2}\beta + \frac{1}{2}\boldsymbol{\sin 2}\alpha\right)}$$

$$I_{rms} = \frac{1}{12.5}\sqrt{\frac{169.7^2}{2\pi}\left(3.78 - 1.047 - \frac{1}{2}\boldsymbol{\sin}(\boldsymbol{2} \times 216) + \frac{1}{2}\boldsymbol{\sin}(\boldsymbol{2} \times 60)\right)} = 8.8 \text{ A}$$

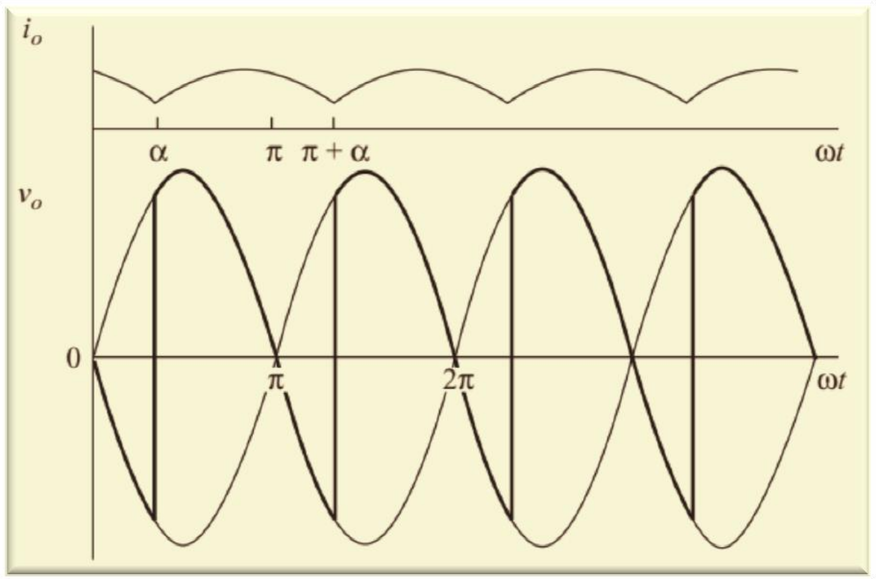
(d)

$$p = I_{rms}{}^2 R = 8.8^2 \times 10 = 774.4 \text{ W}$$

Single Phase Controlled Full Wave Rectifier

RL Load, Continuous Current

- If the load current is still positive at $\omega t=\pi+\alpha$ when gate signals are applied to T_3 and T_4 in the above analysis, T_3 and T_4 are turned ON and T_1 and T_2 are forced OFF.
- The initial condition for current in the second half-cycle is not zero.



In continuous current $\omega t=\pi+\alpha$. The current at $\omega t=\pi+\alpha$ must be greater than zero for continuous-current operation.

$$i(\pi + \alpha) \geq 0$$

$$\sin(\pi + \alpha - \theta) - \sin(\pi + \alpha - \theta)\, e^{-(\pi+\alpha-\alpha)/\omega\tau} \geq 0$$

Using

$$\sin(\pi + \alpha - \theta) = \sin(\theta - \alpha)$$

$$\sin(\theta - \alpha)\left(1 - e^{-(\pi/\omega\tau)}\right) \geq 0$$

Single Phase Controlled Full Wave Rectifier

RL Load, Continuous Current

Solving for α $\alpha \le \theta$

Using $\theta = \tan^{-1}\left(\frac{\omega L}{R}\right)$

$$\boxed{\alpha \le \tan^{-1}\left(\frac{\omega L}{R}\right) \quad \rightarrow \quad \text{Continuous current}}$$

The average (dc) output voltage and current are

$$V_{dc} = \frac{1}{\pi}\int_{\alpha}^{\pi+\alpha} V_m \sin\omega t \, d\omega t = \frac{2V_m}{\pi}\cos\alpha$$

$$I_{dc} = \frac{V_{dc}}{R} = \frac{2V_m}{\pi R}\cos\alpha$$

The *rms* voltage and current are computed from

$$V_{rms} = \sqrt{\frac{1}{\pi}\int_{\alpha}^{\pi+\alpha} (V_m \sin\omega t)^2 \, d\omega t} = \frac{V_m}{\sqrt{2}}$$

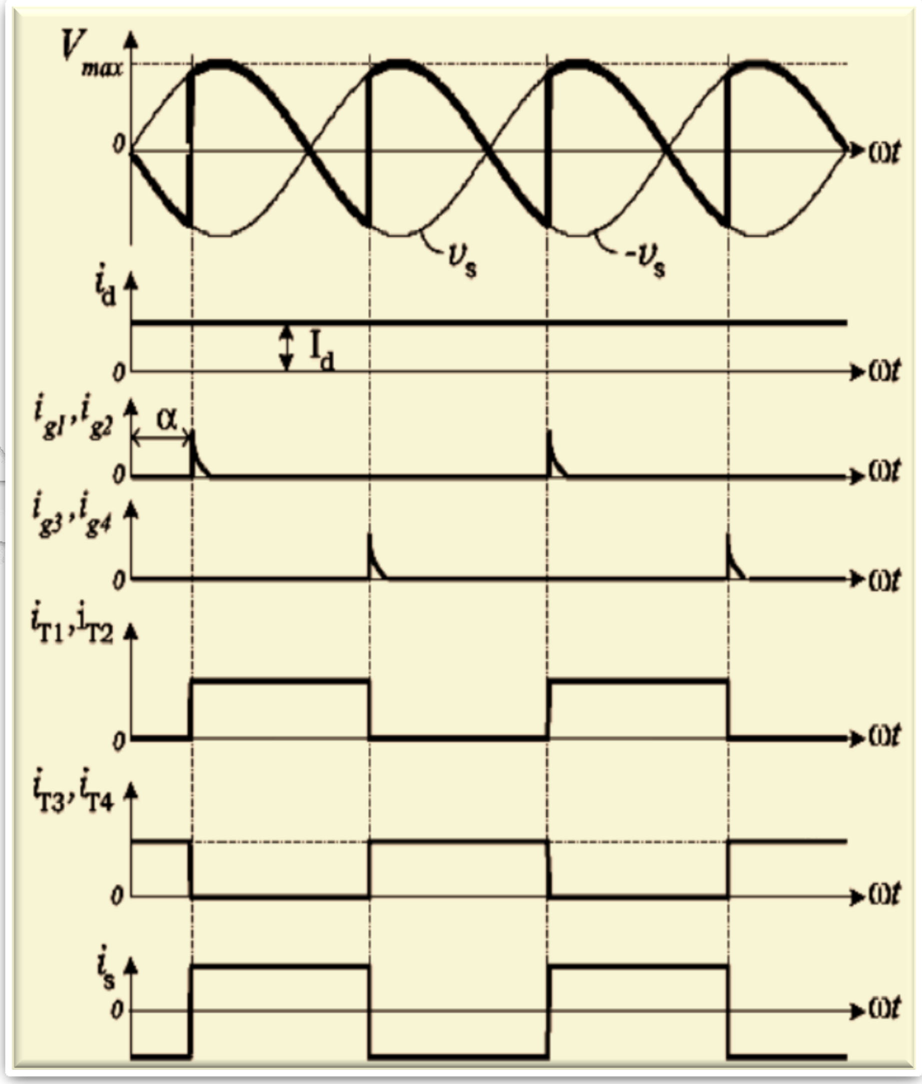
$$I_{rms} = \frac{V_{rms}}{Z} = \frac{V_{rms}}{\sqrt{R^2 + (\omega L)^2}} = \frac{V_m}{\sqrt{2}\sqrt{R^2 + (\omega L)^2}}$$

Single Phase Controlled Full Wave Rectifier

Highly Inductive Load, L>>R

- The behavior of the fully controlled rectifier with resistive-inductive load (with highly inductive load) is shown in the figure. The high-load inductance generates a perfectly filtered current and the rectifier behaves like a current source. With continuous load current, thyristors T_1 and T_2 remain in the ON-state beyond the positive half-wave of the source voltage v_s. For this reason, the load voltage can have a negative instantaneous value.
- The firing of thyristors T_3 and T_4 has two effects:

i) They turn off thyristors T_1 and T_2.
ii) After the commutation they conduct the load current.



Single Phase Controlled Full Wave Rectifier

Highly Inductive Load, L>>R

The average (dc) output voltage and current are

$$V_{dc} = \frac{1}{\pi}\int_{\alpha}^{\pi+\alpha} V_m sin\omega t\, d t\omega t = \frac{2V_m}{\pi} cos\alpha$$

$$I_{dc} = \frac{V_{dc}}{R} = \frac{2V_m}{\pi R} cos\alpha$$

The *rms* voltage and current are computed from

$$V_{rms} = \sqrt{\frac{1}{\pi}\int_{\alpha}^{\pi+\alpha} (V_m sin\omega t)^2\, d\omega t} = \frac{V_m}{\sqrt{2}}$$

$$I_{rms} = I_{dc} = I_a$$

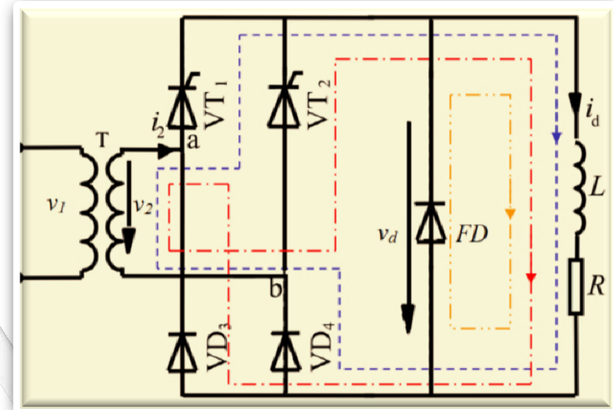
Example: A controlled full-wave bridge rectifier has a source of 120V$_{rms}$ at 60Hz, R=10Ω, L=100mH, and α=60^o. Determine (a) Verify that the load current is continuous. (b) the average load current, and (c) the power absorbed by the load.

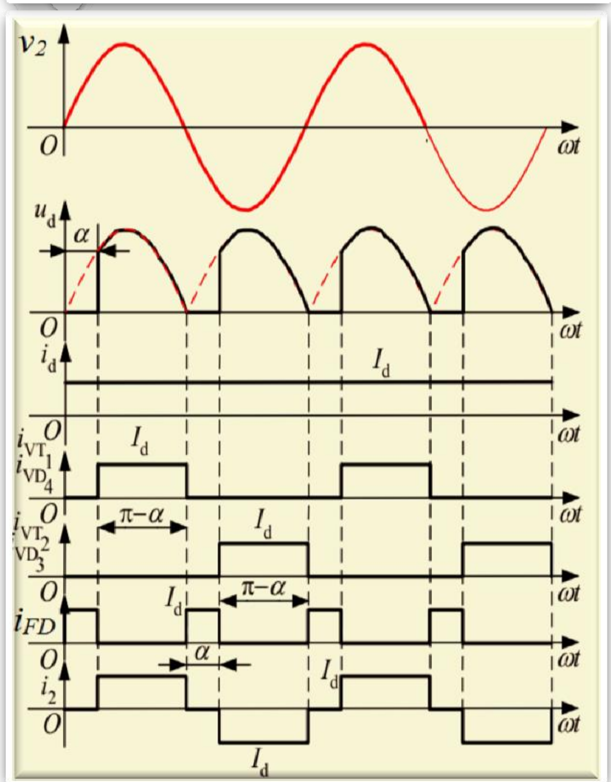
$$\tan^{-1}\left(\frac{\omega L}{R}\right) = \tan^{-1}\left[\frac{(377)(0.1)}{10}\right] = 75°$$

$$\alpha = 60° < 75° \qquad \therefore \text{ continuous current}$$

Single-Phase Bridge Half-Controlled Rectifier

- The rectifier shown in the figure consists of a combination of thyristors and diodes and used to eliminate any negative voltage occurrence at the load terminals. This is because the diode D_{FD} is always activated (forward biased) whenever the load voltage tends to be negative. For one total period of operation of this circuit.





The average (dc) voltage across the load and the average (dc) current are

$$V_{dc} = \frac{1}{\pi}\int_{\alpha}^{\pi} V_m \sin\omega t \, d\omega t = \frac{V_m}{\pi}(1 + \cos\alpha)$$

$$I_{dc} = \frac{V_{dc}}{R} = \frac{V_m}{\pi R}(1 + \cos\alpha)$$

The *rms* component of the output voltage and current waveforms are determined from

$$V_{rms} = \sqrt{\frac{1}{\pi}\int_{\alpha}^{\pi} (V_m \sin\omega t)^2 d\omega t} = V_m\sqrt{\frac{1}{2} - \frac{\alpha}{2\pi} + \frac{\sin(2\alpha)}{4\pi}}$$

$$I_{rms} = \frac{V_{rms}}{R} = \frac{V_m}{R}\sqrt{\frac{1}{2} - \frac{\alpha}{2\pi} + \frac{\sin(2\alpha)}{4\pi}}$$

The power delivered to the load is $p = {I_{rms}}^2 R$


Q
&
A