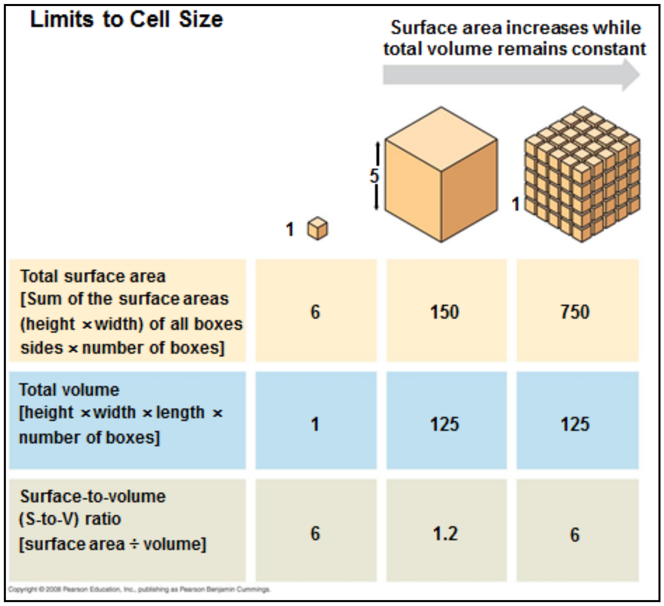
SIZES OF ORGANISMS: THE SURFACE AREA-to-VOLUME RATIO **Name:**______________________________________

Surface area-to-volume ratios affect a biological system's ability to obtain necessary resources or eliminate waste products. As cells increase in volume, the relative surface area decreases and demand for material resources increases; more cellular structures are necessary to adequately exchange materials and energy with the environment. As the surface area increases by a factor of n^2, the volume increases by a factor of n^3 - small cells have a greater surface area relative to volume. These limitations restrict cell size.



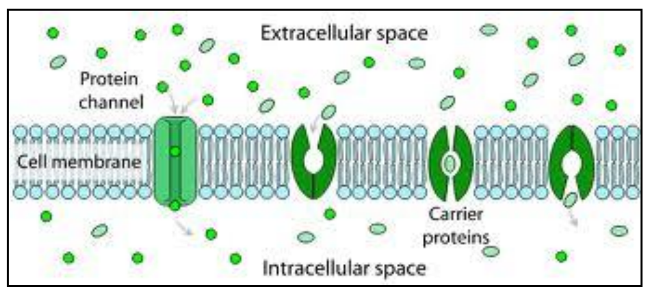
Total surface area [Sum of the surface areas (height × width) of all boxes sides × number of boxes]	6	150	750
Total volume [height × width × length × number of boxes]	1	125	125
Surface-to-volume (S-to-V) ratio [surface area ÷ volume]	6	1.2	6

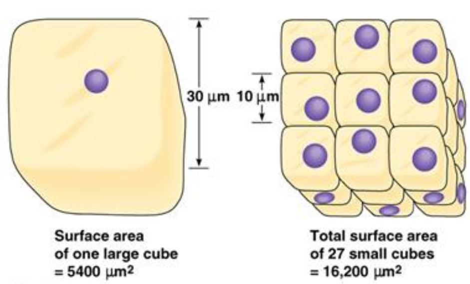
Cube Size	Area of Cube	Volume of Cube	Surface Area to Volume Ratio	Distance of Diffusion	Rate of Diffusion
3cm	54 sq cm	27 cubic cm	2:1	2/5 cm	.07 mm/min
2cm	24 sq cm	8 cubic cm	3:1	2/5 cm	.07 mm/min
1cm	6 sq cm	1 cubic cm	6:1	1/2cm	.08mm/min

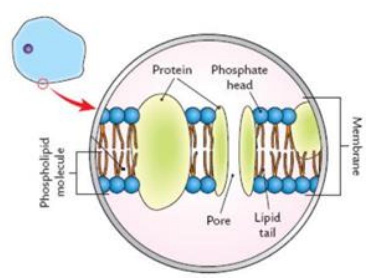
SA to V Ratio and the Plasma Membrane

The surface area of the plasma membrane must be large enough to adequately exchange materials; smaller cells have a more favorable surface area-to-volume ratio for exchange of materials with the environment. **Surface-area-to-volume ratio requires that cells be small:**

- As cells get larger in volume, relative surface area actually decreases
- This limits how large actively metabolizing cells can become
- Cells needing greater surface area use modifications such as folding







This lesson is based on a lesson developed by Dr. Stephen G. Saupe; College of St. Benedict/ St. John's University; Biology Department; Collegeville, MN 56321; ssaupe@csbsju. edu. I found it at http://employees.csbsju.edu/ssaupe/biol327/Lab/sv-ratio.htm

Introduction: Two- and three-dimensional parameters of organisms (i.e., surface area and volume) do not necessarily increase or decrease proportionally to increases or decreases in one-dimensional, or linear, parameters (i.e., length). For example, the greater the diameter of a single-celled organism, the less surface area it has *relative* to its volume. The surface area to volume ratio is a way of expressing the relationship between these parameters as an organism's size changes.

Importance: Changes in the surface area to volume ratio have important implications for limits or constraints on organism size, and help explain some of the modifications seen in larger-bodied organisms.

Question: How is the surface area to volume ratio calculated, and how exactly does it change with changing size? What modifications do larger organisms exhibit to get around this problem?

Variables:

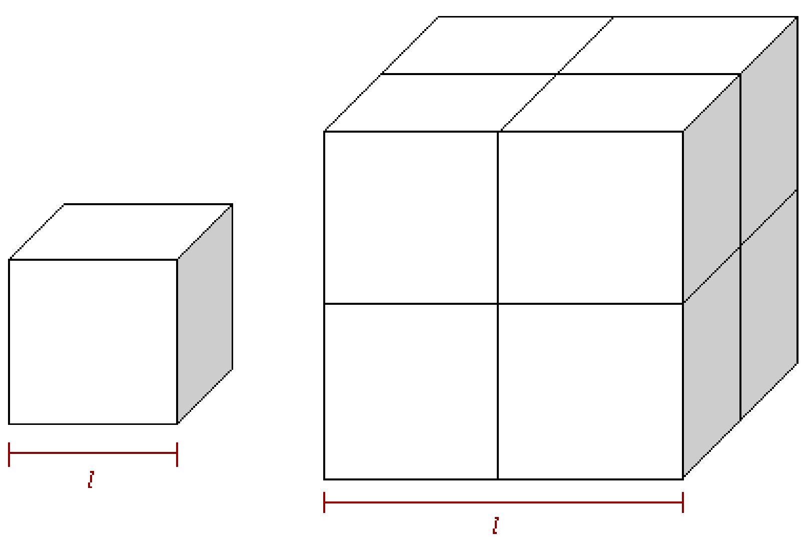
S	surface area (units squared)
V	volume (units cubed)
l	length (units)
r	radius (units)

Methods: For a single-celled organism (or a cell in a multicellular organism's body, for that matter), the surface is a critical interface between the organism/cell and its environment. Exchange of materials often occurs through the process of diffusion, in which dissolved molecules or other particles move from areas of higher concentration to areas of lower concentration (although some exchange is mediated by cellular mechanisms). This type of exchange is a <u>*passive*</u> process, ***and as a result imposes constraints upon the size of a single-celled organism or cell***. ***Materials must be able to reach all parts of a cell quickly, and when volume is too large relative to surface area, diffusion cannot occur at sufficiently high rates to ensure this.***

We'll begin with a reminder of some basic geometric formulae. The surface area and volume of a cube can be found with the following equations:

$$S = 6l^2 \text{ and } V = l^3$$

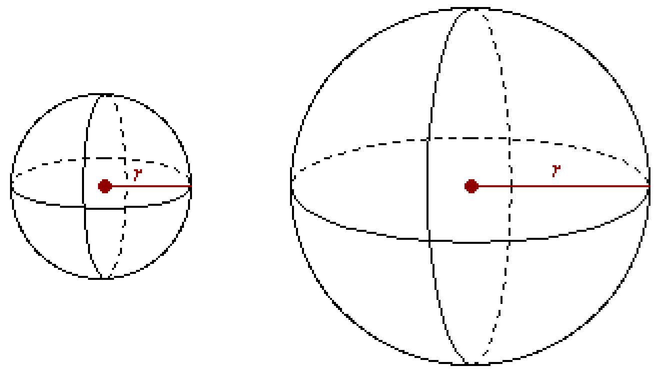
where *S* = surface area (in units squared), *V* = volume (in units cubed), and *l* = the length of one side of the cube.



The equations for the surface area and volume of a sphere are:

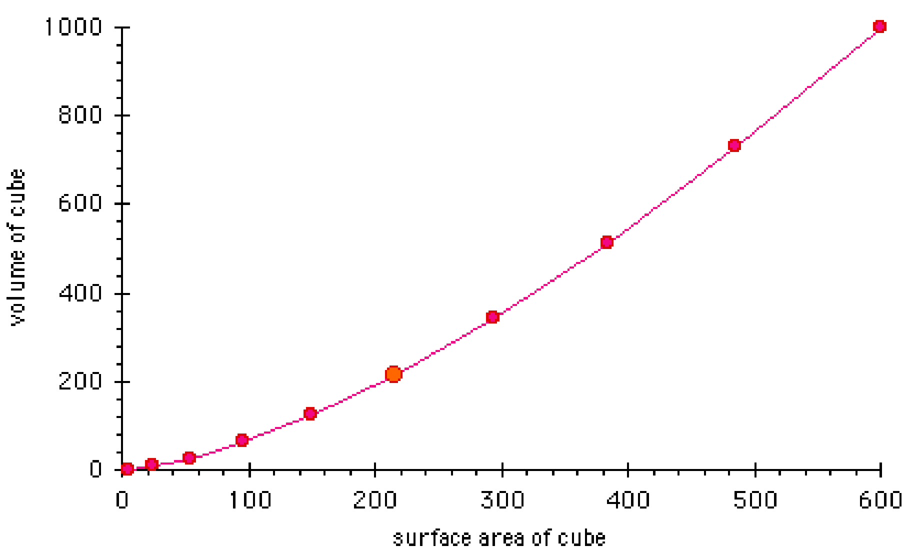
$$S = 4\pi r^2 \text{ and } V = \frac{4\pi r^3}{3}$$

where *r* is the radius of the sphere.



Notice that for any increase, *x* * *l* or *x* * *r*, in length or radius, the increase in surface area is *x* squared (x^2) and the increase in volume is *x* cubed (x^3). For example, when length is doubled (i.e., *x* = 2) surface area is *quadrupled* ($2^2 = 4$) not doubled, and volume is *octupled* ($2^3 = 8$) not tripled. Similarly when length is tripled (*x* = 3) surface area is increased ninefold ($3^2 = 9$) and volume is increased twenty-sevenfold ($3^3 = 27$). The increase in volume is always greater than the increase in surface area. This is true for cubes, spheres, or any other object whose size is increased without changing its shape.

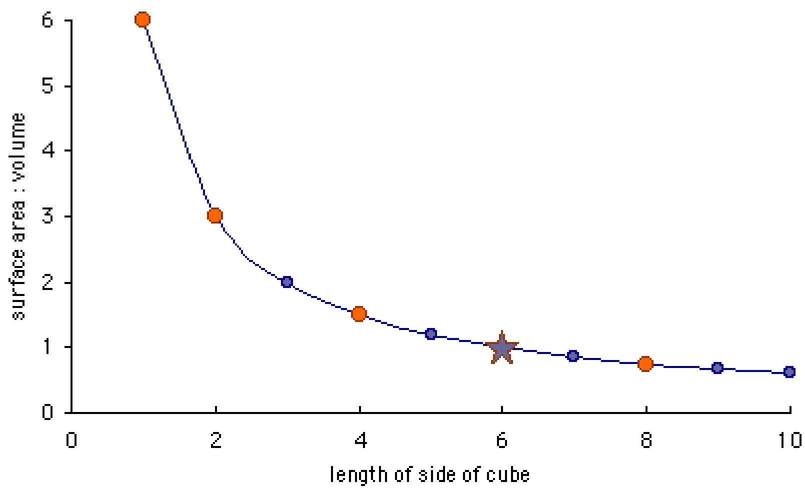
Interpretation: Each point on the graph below represents the surface area and volume for cubes that are increasing by one unit in length, starting with a cube with *l* = 1. The larger orange dot is the size of the cube (*l* = 6) at which surface area and volume have equal values (although the units are different; one is units^2 and one is units^3). For cubes smaller than this, surface area is greater relative to volume than it is in larger cubes (where volume is greater relative to surface area).



Sometimes a graph that shows how the relationship between two variables changes is more instructive. For example, a graph of the *ratio* of surface area to volume, or

$$r_{SV} = \frac{S}{V},$$

clearly illustrates that as the size of an object increases (without changing shape), this ratio decreases. Mathematically, that tells us that the denominator (volume) increases faster relative to the numerator (surface area) as object size increases. The star on the line (at *l* = 6) represents the same point mentioned above: this is the size of the cube where *S* and *V* have equal values, and so the surface area to volume ratio is equal to one.



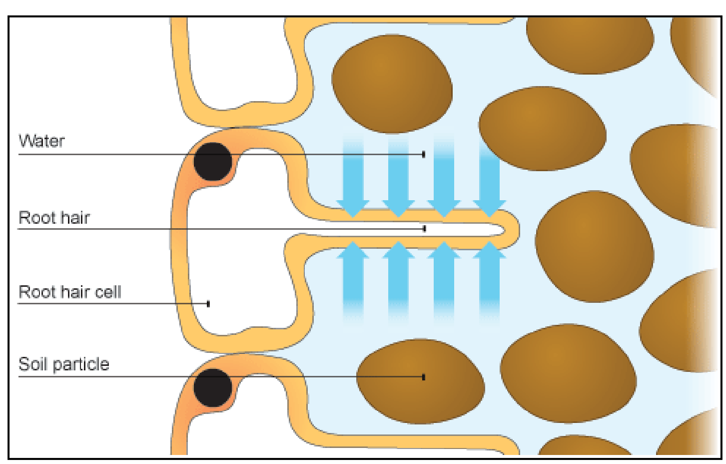
Conclusions: Organisms exhibit a variety of modifications, both physiological and anatomical, to compensate for changes in the surface area to volume ratio associated with size differences. ***One example of this is the higher metabolic rates found in smaller (homeothermic) animals.*** Because of their large surface area relative to volume, small animals lose heat at much higher rates than large animals, and therefore must produce more heat to offset the effects of thermal conductance. Another example is the variety of internal transport systems that have developed in plants and animals for actively moving materials throughout the organism, thus enabling them to circumvent the limits imposed by passive diffusion.

Many organisms have developed structures that actually increase their surface area: the leaves on trees, the microvilli on the lining of the small intestine, root hairs and capillaries, and the convoluted walls of arteries, to name but a few.

Illustrative Example: Root Hairs

An increased surface area to volume ratio means increased exposure to the environment. The higher the SA:Volume ratio for a cell, the more effective the process of diffusion.

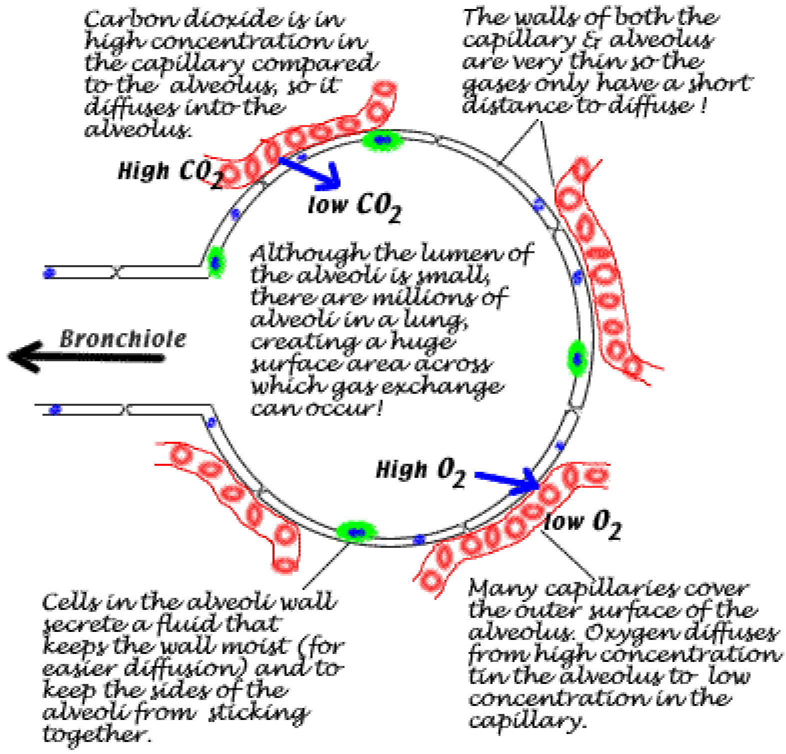
- Root hairs are long, thin hair-like cells that emerge from the root tip to form an important surface over which plants absorb most of their water and nutrients via diffusion.
- They present a large surface area to the surrounding soil, which makes absorbing both water and minerals more efficient using osmosis.



Illustrative Example: Cells of the Alveoli

The ratio between the surface area and volume of cells and organisms has an enormous impact on their biology. Individual organs in animals are often shaped by requirements of surface area to volume ratio.

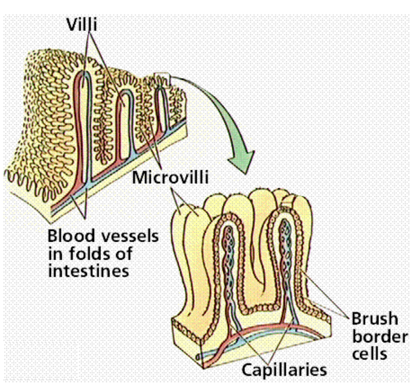
- The numerous internal branchings of the lung and alveoli increase the surface area through which oxygen is passed into the blood and carbon dioxide is released from the blood.
- Human lungs contain millions of alveoli, which together have a surface area of about 100m^2, fifty times that of the skin.

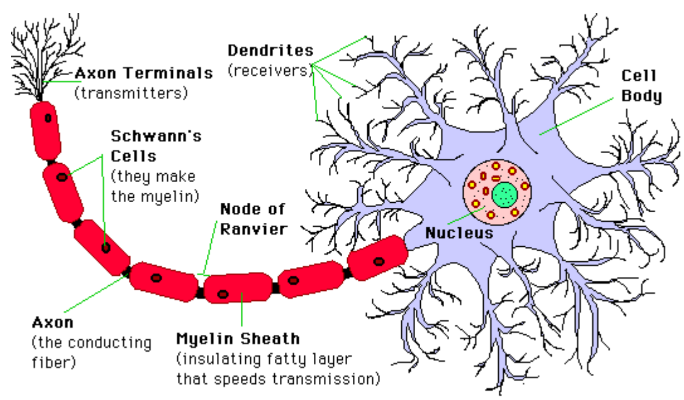


Illustrative Example: Microvilli and Other Cell Types

Large animals require specialized organs (lungs, kidneys, intestines, etc.) that effectively increase the surface area available for exchange processes, and a circulatory system to move material and heat energy between the surface and the core of the organism.

- The intestine has a finely wrinkled internal surface, increasing the area through which nutrients are absorbed by the body.
- A wide and thin cell, such as a nerve cell, or one with membrane protrusions such as microvilli has a greater surface-area-to-volume ratio than a spheroidal one.
- Likewise a worm has proportionately more surface area than a rounder organism of the same mass does.





PRACTICE PROBLEMS

In order for cells to survive, they must constantly exchange ions, gases, nutrients, and wastes with their environment. These exchanges take place at the cell's surface and limit cell growth. These exercises are designed to introduce the concept of surface area, volume, and surface-to-volume ratios (S/V) and their importance in biology.

Surface area is the summation of the areas of the exposed sides of an object and volume is a measure of how much space an object occupies. The surface to volume ratio, or S/V ratio, refers to the amount of surface a structure has relative to its size. To calculate the S/V ratio, simply divide the surface area by the volume. We will examine the effect of size, shape, flattening an object, and elongating an object on surface-to-volume ratios. To perform this function efficiently, there must be an adequate ratio between the cell's volume and its surface area.

I. INFLUENCE OF SIZE ON S/V RATIOS.

The purpose of this exercise is to see how the S/V changes as an object gets larger. We will use a cube to serve as a model cell (or organism). Cubes are especially nice because surface area (length x width x number of sides) and volume (length x width x height) calculations are easy to perform. To calculate the surface-to-volume ratio, divide the surface area by the volume. Complete the table below for a series of cubes of varying size:

Length of Side (mm)	Surface Area (mm^2)	Volume (mm^3)	Surface/Volume Ratio
1mm			
2mm			
3mm			
4mm			
5mm			
6mm			
7mm			
8mm			
9mm			
10mm			

Questions and Analysis:

1. Which cube has the greatest surface area? Volume? S/V ratio?

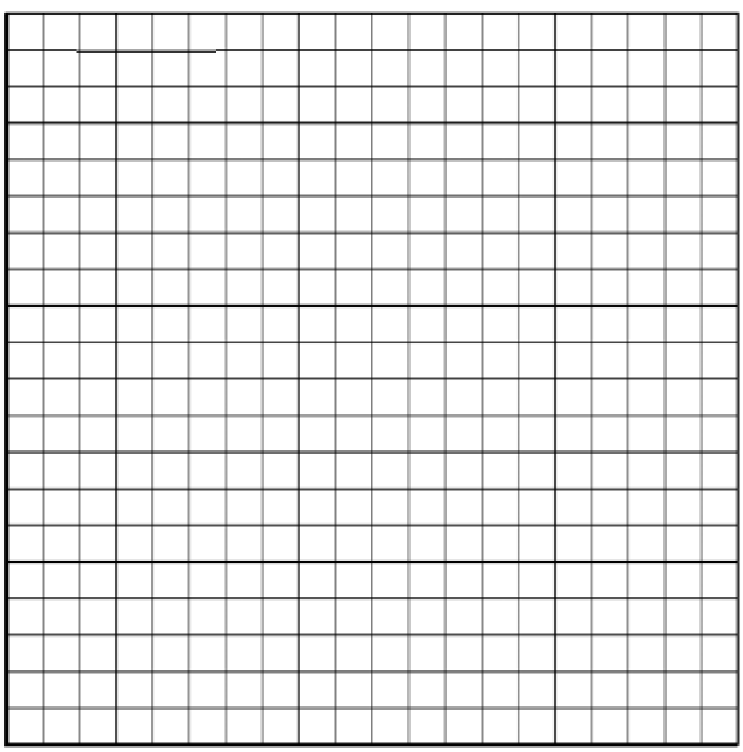
2. What happens to the surface area as the cubes get larger? To the volume?

3. What happens to the S/V ratio as the cubes get larger? Proportionately, which grows faster: surface area or volume? Explain.

4. Which cube has the most surface area in proportion to its volume? Explain.

5. If you cut a cube in half, how does the volume, surface area, and S/V ratio of one of the resultant halves compare to the original?

6. Plot, all on one graph, the following: S/V ratio vs. cube size (length in mm); volume vs. cube size (length in mm); and surface area vs. cube size (length in mm).



II. S/V RATIOS IN FLATTENED OBJECTS:

In this exercise we will explore how flattening an object impacts the surface to volume ratio. Consider a box that is 8 x 8 x 8 mm on a side. Then imagine that we can flatten the box making it thinner and thinner while maintaining the original volume. What will happen to the surface area and S/V ratio as the box is flattened? Complete the table below.

Box No.	Height (mm)	Length (mm)	Width (mm)	SA (mm^2)	Volume (mm^3)	S/V Ratio
1	8	8	8			
2	4	16	8			
3	2	16	16			
4	1	32	16			
5	0.5	32	32			

Questions/ Analysis:

1. Explain why leaves are thin and flat.

2. Why do elephants have large, flat ears?

3. Explain why desert plants generally have smaller leaves.

III. S/V RATIOS IN ELONGATED OBJECTS:

In this exercise we will explore how elongating an object impacts the surface to volume ratio. Consider a box that is 8 x 8 x 8 mm on a side. Then, imagine that we pull on the ends to make it longer and longer while maintaining the original volume. What will happen to the surface area, and S/V ratio as the box is flattened? Complete the table below.

Box No.	Height (mm)	Length (mm)	Width (mm)	SA (mm^2)	Volume (mm^3)	S/V Ratio
1	8	8	8			
2	4	16	8			
3	4	32	4			
4	2	64	4			
5	2	128	2			

Questions/ Analysis:

1. Explain the shape of blood vessels.

2. Explain why roots have “hairs”.

IV. SHAPE AND S/V RATIOS:

In this exercise we will explore the impact of shape on surface to volume ratios. The three shapes given below have approximately the same volume. For each, calculate the volume, surface area, and S/V ratio and complete the table. The last column in the table, “Volume of environment within 1.0 mm of the object” is particularly important. Since the materials that an organism exchanges with its environment come from its immediate surroundings, the greater this volume, the more material that can be exchanged.

Shape	Dimensions (mm)	Volume (mm^3)	Surface Area (mm^2)	S/V Ratio	Volume (mm^3) of the environment within 1.0mm of object?
Sphere	1.2 diameter				
Cube	1x1x1				
Filament	0.1x 0.1x 100				

Note: volume of a sphere = 4/3 (π)r^3 = 4.189r^3. Surface area of sphere = 4(π)r^2 = 12.57 r^2. A filament is a box. To calculate surface area of a box determine the surface area of each face (l x w) and then add them. The volume of a box = l x w x h.

Questions & Analysis:

1. Make a sketch, to scale, of the three objects.

2. Which shape has the greatest surface area? Volume? S/V ratio?

3. If you had to select a package with the greatest volume and smallest surface area, what shape would it be? Explain.

4. Explain why cells divide as they get larger.

5. Explain why the shape of animals is basically “spherical”, whereas plants and fungi are “filamentous”.

6. Given that one way a plant grows is cell elongation. How does the plant’s vacuole help accomplish this task and why does this help solve the S/V ratio conundrum?

7. Based on these activities, what trend do you notice as the size of the cells increases?

8. Why is this important to living cells? What can cells do about it?

9. Interpret the diagrams below – explain why some have “poor” SA:V ratios and some have “better” SA:V ratios. Write your response to the side of each shape.