# SKILL PRICES, OCCUPATIONS, AND CHANGES IN THE WAGE STRUCTURE FOR LOW SKILLED MEN 

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#### Abstract

This paper studies the effect of the change in demand for occupations on wages for low skilled men. We develop an equilibrium model of occupational assignment in which workers have multidimensional skills that are exploited differently across different occupations. We allow for a rich specification of technological change which has heterogenous effects on different occupations and different parts of the skill distribution. We estimate the model combining four datasets: (1) O*NET, to measure skill intensity across occupations, (2) NLSY79, to identify life-cycle supply effects, (3) CPS (ORG), to estimate the evolution of skill prices and occupations over time, and (4) NLSY97 to see how the gain to specific skills has changed and to identify change in preferences. We have three main findings. First, the reallocation away from manual jobs towards services and changes in the wage structure were driven by demand factors while the supply of skills, selection into different occupations, and changes in preferences across cohorts played lesser role. Second, frictions play a crucial role in preventing wages in traditional blue collar occupations from falling substantially relative to other occupations. Finally, while we see an increase in the payoff to interpersonal skills over time, manual skills are substantially more important than others and still remain so for low educated males.


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## 1 Introduction

Compared to other demographic groups, low-skilled (no college) men have fared poorly in the last 40 years. This group has actually seen their median real wage decrease during this period. During the same time span, there has been a substantial shift in the type of work that this group performs as occupations have moved from more traditional blue collar occupations to service and clerical occupations. This paper tries to understand the relationship between these two trends by investigating the role of the change in occupational composition and the payments to multi-dimensional skills in explaining recent changes in the wage structure for low skilled men. From a policy perspective, if our goal is to invest in skills to help these men, the occupational trends have implications for which skills have increased most in value.

Answering these questions requires a structural model. We develop a dynamic generalized Roy model in which individuals are endowed with a three dimensional vector of skills: cognitive, manual, and interpersonal. Each period they may direct their search to "desired" occupation but may not be able to work in that occupation due to labor market frictions. Skills evolve on the job, but differently in different occupations. Firms observe the technology and then post vacancies to attract workers. The search markets are segmented by skill and occupations. Once firms and workers are matched, the firms make take it or leave it offers to new or incumbent workers without knowing whether these offers will be accepted.

One of the biggest challenges for this type of model is identification. We formally show a simplified model can be identified. There are a number of different identification problems. The first, which is ubiquitous in this literature, is the age-cohort-time identification problem. It renders it impossible to perfectly separate wage changes within an occupation into the part due to changes in prices versus changes in composition without assumptions. If cohort and age effects are completely unrestricted, there is always a distribution of skills that can reconcile any hedonic pricing equation. This is, of course, a feature of any analysis that follows different cohorts over time, not just a problem in our paper. We address the age-cohort-time effect by assuming that the underlying initial skill level is identical across cohorts, conditional on the probability of going to college. We also assume the human capital accumulation technology does not fundamentally differ across cohorts. We allow cohort preferences to vary in a tightly parameterized way. We use a revealed preference argument contrasting the cohorts of the NLSY79 with those of the NLSY97. We use our model of human capital accumulation to estimate the age effect. This same set of assumptions also addresses the second main identification challenge: separating supply from demand. Identification of the dynamic supply of skill comes from the NLSY79 in which we have a
long panel of workers who face changing wages.
The third main challenge is identification of the wage process which come from three places. First, a crucial part of our study uses O*NET to estimate the skill intensity of each occupation. Second, we follow Deming (2017) by using the contrast between the NLSY79 and NLSY97 to measure the increasing importance of social skill. Third, once we identify the supply of worker skill as a function of prices from the panel structure of the NLSY79, we can use the CPS to recover the prices and also the aggregate supply of skill to the population. We provide an identification argument of a stylized version of our model to formally justify this approach.

While we do need to make some assumptions to estimate our structural model, the advantage is that the resulting estimated model is rich and allows us to say a number of things about the wage structure for low skilled men.

First, we are able to estimate the changes in the hedonic pricing equation over time. We see skill prices falling for the median skilled worker in all occupations but rising for relatively high skilled workers in those occupations. Prices rise for the lowest workers in some professions, but fall in others.

Second, one pattern that we found is that many of the occupations that are expanding actually see relatively large declines in skill prices. We can not reconcile this trend with a frictionless model. We incorporate three different potential explanations for the weak relationship between employment and wages into our model. First, it could be that selection effects are strong and wages evolutions are not in line with price changes. We do find empirical evidence of some selection effects but they are not large enough to explain the lack of relative convergence of wages. Second, it could be that preferences have changed over time: services occupations became more popular than manual ones. Contrasting the two NLSY waves, we find that preferences appear extremely stable over time. Our third explanation turns out to be the most important: we find that the reallocation away from manual jobs towards services are driven by demand factors and in particular the rising capital costs in operators occupations. Frictions play an important role in preventing wages in traditional blue collar occupations from falling substantially relative to other occupations.

Finally, we explore the payoff to different skills and how that has changed over time. We find that the importance of interpersonal skills grows over time going from little value at the beginning of the period to substantial returns later. However, manual skills remain the most important. If we were able to boost these skills for low skilled men prior to labor market entry, we could substantially increase their lifetime earnings.

Section 2 discusses the related literature while Section 3 describes the data and presents some motivating facts. Section 4 presents the model that we use to explain them. Section 5 discusses identification while Section 6 describes the estimation strategy. Section 7 presents the estimation results and Section 8 discusses the determinants of the changes in occupational composition. Section 9 examines the change in the payoff to different skills. We conclude in Section 10.

## 2 Related Literature

This paper is related to large literatures on skill-biased technological change, human capital, and on structural models that try to address these issues. A full survey of all of these literatures is beyond our scope but we briefly name some key papers.

There is a very large literature on changes in the wage structure, a seminal paper is Katz and Murphy (1992) and surveys/overviews include Katz and Autor (1999), Dinardo and Card (2002), Goldin and Katz (2009), and Acemoglu and Autor (2011). Of particular relevance to us in this literature is the importance of occupations. There are two threads that focus on occupations.

The first is the polarization of the labor market: the simultaneous growth of the share of employment in high wage occupations and low wage occupations. This has been discussed in a large number of papers and a full survey is beyond our scope. Key ones are Autor et al. (2003), Autor et al. (2006), Acemoglu and Autor (2011), Autor and Dorn (2013), Goos et al. (2014), and Cortes (2016). Beaudry et al. (2016) highlight that this trend largely ends in 2000 after which we see a decrease in demand for cognitive skill. Michel et al. (2013) and Hunt and Nunn (2022) are critical of some aspects of this literature arguing that it does not explain many features of the wage distribution. Since polarization is not our focus, this is not a first order concern for our results. Using a model-based approach, we estimate how these recent patterns are related to trends in different skill prices and we examine the consequences for the wage structure. We also differ from much of this literature in focusing on occupations directly and then using our three types of skills rather than focusing on routineness (or complexity which Caines et al., 2017 argue is important).

The second thread is papers that use decompositions to look at occupations. The fact that there is a lot of variation within occupations goes back at least to Slichter (1950). Using a variance decomposition, Juhn et al. (1993) show that much of the rise in wage inequality can be explain with an increased returns to unobserved ability. While Juhn et al. (1993) describes
a method and says it could be used for occupations, they only show results for industries. Quite a few papers have used similar types of decompositions based on occupations or tasks since. Examples include Lemieux (2006), Alsalam et al. (2006), Kim and Sakamoto (2008), Mouw and Kalleberg (2010), Scotese (2013), and Burstein et al. (2019). The main findings of these papers is that within occupation variation tends to be most important in both levels and trends in inequality, but the relevant importance of occupations varies across the papers. Our counterfactual differ from these in quite a few ways. We focus on low skilled men, our main focus is on wage levels rather than inequality and we assess the role played by different skills.

A few papers adopt various approaches to try to separate skill prices from composition effects. The major issue here is separating time, age and cohort effects. Antonczyk et al. (2018) address this problem by assuming separability between age and time effects following MaCurdy and Mroz (1995). They find that cohort effects are small in the U.S. Another approach is a "flat spot" method which assumes there is some point in the lifecycle for which age effects are flat allowing one to separate time effects from cohort effects. This approach was initially used by Heckman et al. (1998) and expanded on by Bowlus and Robinson (2012) and Bowlus et al. (2021). This approach is challenging here as we are trying to identify occupation specific prices and occupation switching is common even late in the life cycle so there is still a selection problem. In a series of papers Lochner and Shin (2014), Lochner et al. (2018), and Lu et al. (2020) develop different models of wages in which skill prices can be identified and estimated from moments of panel data. Gottschalk et al. (2015) estimate return to different skills by focusing on entry level wages and using bounds to account for selection on unobserved variables. Böhm (2020) uses implications of a generalized Roy model and the envelope theorem to estimate skill differences between the different cohorts of the NLSY. Our approach uses various elements of these approaches in different ways. A key assumption is the cohorts are ex-ante identical (conditional on education levels) and as shown in Section 6 we require panel data (NLSY) and then combine it with O*NET and the CPS to obtain identification.

While it does not look specifically at occupations, Charles et al. (2019) is particularly relevant in that the main focus is really on high school men. They argue that a large part of the decrease in labor supply since 2000 was due to decrease in manufacturing, but before 2007 this was masked by the housing boom. They also find a large role for the decline in manufacturing to explain the decrease in wages for low skilled men. This does not contradict our findings because we are looking at different effects. They measure equilibrium effects by
looking across regions. The decline in manufacturing could lead to a substantial decrease in wages for all jobs which is consistent with both their findings and ours. ${ }^{1}$ This suggests that much of the decline that we find in wages within occupations could be due to declining manufacturing wages. It is also an important reminder that our analysis is partial equilibrium as we are not trying to identify the source of decrease in demand.

Another key to identification for us is the contrast between NLSY79 and NLSY97 which we use to identify cohort effects and the returns to different type of skills. Comparing NLSY waves is also used by Altonji et al. (2012), Castex and Dechter (2014), and Deming (2017). Using pre-market measures of skills, Castex and Dechter (2014) find declining returns to cognitive skills while Deming (2017) documents the rising of social skills. We extend this literature by considering the role of manual skills. Using our structural model, we also find an increase in the payoff to interpersonal skills. However, we find that manual skills remain the most important skills for non-college educated men. We will return to Deming (2017) below.

Closest to our approach are papers that estimate equilibrium models of the labor market to understand the skill premium Heckman et al. (1998), the growth of the service sector Lee and Wolpin (2006), changes in the wage structure Johnson and Keane (2013), and the gains from trade Dix-Carneiro (2014) and Traiberman (2019). These papers all assume log wages are additively separable in prices and skills, partly because this equation can be microfounded with an aggregate production that features perfect substitutability across workers given observables (such as education, occupation or experience). We build on this literature by allowing for a flexible non-linear relationship between wages and an index of unobserved skills. This flexibility is key for understanding changes in the wage structure. Our main question is also different from these other papers.

Our methodology is also closely related to structural papers that use the tasks approach to modeling specific human capital. Poletaev and Robinson (2008) and Gathmann and Schönberg (2010) show the importance of tasks as measures of human capital. Sanders and Taber (2012) provide a survey of the evidence. A number of papers use this approach in estimating models of the labor market including Sullivan (2010), Yamaguchi (2012), Sanders (2016), Lindenlaub (2017), Lise and Postel-Vinay (2020) and Guvenen et al. (2020). While they do not explicitly use the task approach, Keane and Wolpin (1997) predates the others and allows for two types of experience that differ by occupation. We differ from these papers

[^0]in a number of ways. The most important one is our focus on understanding changes in the wage structure and labor market trends while they are more interested in the life-cycle.

In an attempt to directly measure the trends in returns to tasks, Atalay et al. (2020) use the text from job ads to construct a new data set of occupational content from 1960 to 2000. They find within-occupation task content shifts are at least as important as employment shifts across occupations. They however focus on the distinction between routine and nonroutine tasks. Cavounidis et al. (2022) use time series from the Dictionary of Occupational Titles to look at changes in occupational skill-content over the 1960, 1970, and 1980 censuses. They also find considerable shifts within occupation. They develops an equilibrium model of the labor market that can explain the patterns in the data.

## 3 Motivating Facts

We use four different datasets. The details about our data construction are described in Appendix A. We need a consistent definition of occupations across these datasets and over time. We use a modified version of the occupation classification of Autor and Dorn (2013) reducing their 15 occupations down to the 8 listed in Table 1 and Figure 1. ${ }^{2}$

First, we use the Outgoing Rotation Group data from the Current Population Survey (ORG CPS), to estimate the evolution of skill prices and occupations over time. Second, we use the National Longitudinal Survey of Youth, 1979 (NLSY79), to identify life-cycle supply effects.

Third, we use $\mathrm{O}^{*}$ NET, to measure skill intensity across occupations. We categorize skills into cognitive, interpersonal and manual. We use factor analysis to reduce these questions to a one dimensional factor for each combination of occupation and skill. ${ }^{3}$ Figure 1 reports the implied skill intensity of each occupation. We have renormalized so that the sum of the skills adds to one for each occupation. Occupations can be characterized into three groups broadly defined. The first two occupations correspond to managerial and clerical occupations and are intensive in both cognitive and inter-personal skills. The service sector is intensive in inter-personal skills and manual skills. The remaining five occupations are intensive in manual skills which is expected since they are associated with blue-collar jobs. Overall, there

[^1]Figure 1: Skill intensity by occupations

is wide dispersion in the type of skills used by different occupations.
Our fourth data set is the National Longitudinal Survey of Youth, 1997 (NLSY97). O*NET measures the skill intensity of occupations at a point in time. To identify within occupation changes, we combine NLSY79 and NLSY97 following Deming (2017). We also use it to identify how preferences vary across cohorts.

To motivate our analysis, we present data on changes in the distribution of log wages over time controlling for age. We examine 20-60 year old males with a high school degree or less. Figure 2 shows the familiar patterns. There are a few things to note. First, and most important, there has been a substantial decline in the median wage over this time period falling by around $0.12 \log$ points. ${ }^{4}$ The story for the $90^{t h}$ quantile is quite different as wages for this group have risen during this time period. The $10^{\text {th }}$ quantile is somewhere in the middle. Wages have fallen, but not by as much as the median. Clearly the patterns are not monotonic over the time period. The wages at every quantile fell through the eighties and early nineties, rose from the mid nineties to early 2000s. The patterns for the different quantiles are quite different during most of the 2000 s, but then all three fell substantially during the great recession and have subsequently recovered.

[^2]Figure 2: Changes in Log Wage Quantiles over Time


At the same time the occupation distribution has been changing considerably over time as can be seen in Table 1.

Table 1: Changes in Occupational Distribution over Time

| Occupation | \% in 1979 | \% in 2017 | Difference |
| :--- | :---: | :---: | :---: |
| Managers | 7.4 | 7.1 | -0.3 |
| Clerical | 10.1 | 11.2 | 1.1 |
| Services | 8.3 | 14.0 | 5.7 |
| Operators | 17.3 | 8.0 | -9.3 |
| Mechanics | 8.5 | 6.1 | -2.3 |
| Construction | 8.5 | 8.9 | 0.4 |
| Precision | 6.8 | 3.5 | -3.3 |
| Transport | 15.0 | 15.8 | 0.8 |
| Not-working | 18.2 | 25.4 | 7.3 |

The most notable changes are the decline in operators, the increase in services and the rise of not-working. It is also important to point out that the operator occupation is not representative of blue collar occupations. Construction and transportation have remained roughly constant and mechanics has had a relatively small fall. Precision production resembles operators and has almost been cut in half. Adding the five blue collar occupations together and conditioning on being employed, the decline has been from $68 \%$ of the workforce
to $57 \%$. The fraction of these workers doing service jobs has risen by about 9 percentage points. While these changes are substantial, we would argue they are not huge. For example, the fraction of these men in the manufacturing sector has fallen by much more. It fell by more than half during this same period so the change in occupational composition is small in comparison. Furthermore, even in 2017 a majority of low skilled workers are employed in blue collar jobs.

Figure 3 presents the changes in mean wages across time for different occupations and the changes in occupation share.

Figure 3: Wages growth and employment growth


We see that most occupations experience decreases in wages. It is also clear that wage patterns are not closely related to the changes in occupation share. For example, clerical workers see quite a large fall in their wages even though it is a growing occupation, and operators see a relatively modest fall in wages even though it is declining faster than any other occupation. This is surprising as the conventional wisdom is that the change in occupations over time has primarily been driven by changes in demand changes. If this were the case, one would expect this pattern to trace out a supply curve and be upward sloping. ${ }^{5}$ We

[^3]view it as a puzzle why operators are shrinking relative to clerical workers despite wages falling more rapidly for the clerical workers. However, these wages patterns cannot directly be interpreted as technology shocks. Wages change for two reasons, because the composition of workers is changing and because skill prices are changing. A major goal of our work is to sort out these differences.

## 4 Model

## Overview

We begin with an overview of the model before we get into the details.

- Market
- The labor market is frictional. Workers who want jobs direct their search to particular occupations which are divided into a continuum of submarkets depending on worker type.
- Workers and firms produce an intermediate good or service which is sold in a competitive market according to a hedonic pricing equation.
- The technology and hedonic pricing equation change over time but are determined outside our model. We estimate them taking them as given within the model using flexible functional forms.
- Workers
- Workers choose occupations-first in whether/where to direct their search and then whether to accept offers or move to non-employment.
- They have multidimensional human capital which evolves depending on the sequence of occupations at which they work.
- Firms
- Pay an up front capital cost (which varies over time) and then potentially search/retain workers.
relative wages satisfies $\left(\frac{w_{1}}{w_{2}}\right)^{\sigma+\eta}=\left(\frac{A_{1}}{A_{2}}\right)^{\sigma-1}$ and equilibrium relative occupation share is $\left(\frac{A_{1}}{A_{2}}\right)^{\frac{\eta(\sigma-1)}{\eta+\sigma}}$. Following a relative demand shock, relative wages changes and relative employment changes have the same sign if $\sigma>-\eta$. Only supply shifts, such as a change in $\eta$, can explain the lack of correlation.
- Make offers to workers without knowing whether the worker will accept them.


## Firm Technology and Market Structure

Let $j=1, . ., J$ index occupations and $j=0$ denotes not working. We use $i$ subscript to denote an individual and $t$ to index time. When a worker with state variable $\mathcal{S}_{i t}$ (defined explicitly below) works in occupation $j$ at time $t$ they produce output that the firm sells at $f_{j t}\left(\mathcal{S}_{i t}\right)$. Note that $f_{j t}$ incorporates both the production function for output and the hedonic pricing equation. The distinction between the two is not relevant for our analysis.

In order to produce this output the firm must pay an up front capital cost $c_{j t}$ before they know whether the position will be filled. They then either try to retain their current worker if they have one or create a vacancy for the job if they do not.

The labor market is organized by submarkets indexed by time $t$, an occupation $j$, and worker type $\mathcal{S}_{i t}$. Matching within a submarket depends on the constant-return-to-scale matching function

$$
M=B S^{\eta} V^{1-\eta}
$$

where $S$ includes all searchers and $V$ is the number of vacancy created. Let $\mu=\frac{V}{S}$ be labor market tightness. We use the notation $\mu_{t j}\left(\mathcal{S}_{i t}\right)$. The probability of filling a vacancy for a firm is $\frac{M}{V}=m(\mu)$. The probability of finding a job in a submarket is $\alpha(\mu)=\frac{M}{S}=\mu m(\mu)$. Each period $t$, any worker can direct their search to at most one submarket.

To retain a worker the firm moves first and makes a take it or leave it offer. The worker then decides whether to accept the offer or not. Importantly the worker has private information about outside opportunities and idiosyncratic tastes, so the firm does not know whether the worker will stay. The firm therefore faces a trade-off between the cost of higher wages and the higher probability of retaining a worker that comes with higher wages.

The wage process for workers who meet a new employer works the same way. The new firm makes a take it or leave it offer and the worker decides whether to take it. The firm in this case knows the current employment status of the worker when they make the offer (as it is part of $\mathcal{S}_{i t}$ ).

We assume workers and firms have rational expectations and perfect foresight about future technology.

## Workers Choices and Preferences

We let $j_{i t}$ denote the occupation in which individual $i$ works at time $t$. The vector of state variables $\mathcal{S}_{i t}$ at time $t$ for individual $i$ is,

$$
\mathcal{S}_{i t} \equiv\left\{\theta_{i t}, a_{i t}, \tau_{i t}, j_{i t-1}, t\right\}
$$

where $\theta_{i t}=\left(\theta_{i t}^{c}, \theta_{i t}^{i}, \theta_{i t}^{m}\right)$ is a vector of general skills composed of cognitive, interpersonal and manual skills. The other state variables are age $a_{i t}$, consecutive tenure in the current occupation $\tau_{i t}$ and last period occupation $j_{i t-1}$. Time $t$ is relevant as it indexes the current and future values of aggregate variables which vary across cohorts (conditional on age). Workers are finite lived and retire at age $A$.

The workers are born with initial endowment of skills $\tilde{\theta}_{i}$. Skills then evolve between periods depending on the occupation of choice. More generally the state variables evolve exogenously and deterministically from the perspective of the worker given the current occupation $j_{i t}$ according to $S^{\prime}$,

$$
\mathcal{S}_{i t+1}=S^{\prime}\left(j_{i t}, \mathcal{S}_{i t}\right)
$$

Individual $i$ with state variables $\mathcal{S}_{i t}$ who searches for a job in occupation $\kappa$ and works in occupation $j$ has flow utility

$$
w\left(j, \mathcal{S}_{i t}\right)+\vartheta_{j y_{i}}+\nu_{i j t}-\chi_{i \kappa t}
$$

where $w\left(j, \mathcal{S}_{i t}\right)$ is the wage they would receive in job $j, \vartheta_{j y}$ are non-pecuniary benefits common to all workers from birth year $y, \nu_{i j t}$ is a taste shifter for an occupation, and $\chi_{i \kappa t}$ is the cost of search. The notation $y_{i} \equiv t-a_{i t}$ denotes the year in which individual $i$ was born.

We let $\kappa=0$ denote no search. We assume

$$
\chi_{i \kappa t}= \begin{cases}\widetilde{\chi}_{i 0 t} & \kappa=0 \\ \bar{\chi}+\widetilde{\chi}_{i \kappa t} & \kappa=1, \ldots, J\end{cases}
$$

The $\widetilde{\chi}_{i \kappa t}$ are i.i.d. and type I extreme value with scale parameter $\sigma_{\chi}$ and the $\nu_{i j t}$ are type I extreme value with scale parameter $\sigma_{\nu}$.

Utility shocks $\nu_{i j t}$ and search cost shocks $\chi_{i \kappa t}$ are not contractible and not known to firms when they make their offers. This leads to inefficient separations.

## Timing

Each period can be broken into three sub-periods.

## Sub-period 1:

- Potential firms decide whether to enter the market and operating firms decide whether to exit.
- All firms that choose to enter or to remain must pay a fixed capital cost $c_{j t}$ for each potential worker.
- The $\widetilde{\chi}_{i \kappa t}$ are revealed to the workers and they decide whether to search for an occupation (only one at a time).
- Firms post take it or leave it wage offers to their current employees. They don't know whether workers will search and get outside offers.


## Sub-period 2:

- Nature reveals the outcome of the matching as determined by the matching function.
- This determines the choice set which will be available to the worker in sub-period 3.
- It also revealed to entry firms whether they were matched with a worker.


## Sub-period 3:

- Upon seeing the outcome of the matching, the entry firm (when relevant) makes take it or leave it wage offers for that period. It knows the previous labor status of the worker but not their $\nu_{i j t}$.
- The $\nu_{i j t}$ are revealed and the agent chooses an option from his choice set given the offered wages.
- Production occurs.

All other state variables including human capital evolve between periods according to $S^{\prime}$.

## Worker Problem

Since the terminal period is simpler than the earlier ones, we present the model for a period prior to the retirement period $a_{i t}<A$. The discount rate is $R$.

We let $\mathcal{B}_{i t}$ be the choice set of jobs available to the worker in sub-period 3. If firms in the previous occupation continue to employ workers of type $\mathcal{S}_{i t}$ it is

$$
\mathcal{B}_{i t}= \begin{cases}\left\{0, j_{i t-1}, \kappa\right\} & \text { successful search } \\ \left\{0, j_{i t-1}\right\} & \text { unsuccessful/no search. }\end{cases}
$$

Workers can always remain in their current occupation if they were employed. They can always choose to become non-employed. In the notation the second element in the choice set is redundant when $j_{t-1}=0$. If firms in their previous occupation chooses not to continue to hire workers of type $\mathcal{S}_{i t}$ then this option does not exist.

Since workers only make decisions in the first and third sub-period we define $V^{1}\left(\mathcal{S}_{i t}, \chi_{i \cdot t}\right)$ and $V^{3}\left(\mathcal{S}_{i t}, \mathcal{B}_{i t}, v_{i \cdot t}\right)$ to be the value functions for sub-periods 1 and 3 respectively where $\mathcal{B}_{i t}$ is the choice set defined above, and $\nu_{i \cdot t}$ and $\chi_{i \cdot t}$ are the vectors of taste shocks and application costs. The $\chi_{i \cdot t}$ are revealed to the worker in sub-period 1 while the $\nu_{i \cdot t}$ are revealed in sub-period 3.

Working backwards within a period,

$$
V^{3}\left(\mathcal{S}_{i t}, \mathcal{B}_{i t}, v_{i \cdot t}\right)=\max _{j \in \mathcal{B}_{i t}}\left\{w\left(j, \mathcal{S}_{i t}\right)+\vartheta_{j y_{i}}+\nu_{i j t}+\frac{1}{1+R} E_{\chi}\left[V^{1}\left(S^{\prime}\left(j, \mathcal{S}_{i t}\right), \chi_{i \cdot t+1}\right)\right]\right\}
$$

where $R$ is the interest rate and the rest of the components have been defined above. The $E_{\chi}$ incorporates expectations over $\chi_{i \cdot t+1}$.

The value function in sub-period 1 is

$$
\begin{aligned}
V^{1}\left(\mathcal{S}_{i t}, \chi_{i \cdot t+1}\right)= & \max \left\{-\chi_{i 0 t}+E_{v} V^{3}\left(\mathcal{S}_{i t},\left\{0, j_{i t-1}, \nu_{i \cdot t}\right\}\right),\right. \\
& \max _{\kappa \in(1, . . J) \backslash\left\{j_{i t-1}\right\}}\left\{-\chi_{i \kappa t}+\left[1-\alpha\left(\mu_{t \kappa}\left(\mathcal{S}_{i t}\right)\right)\right] E_{v} V^{3}\left(\mathcal{S}_{i t},\left\{0, j_{i t-1}\right\}, \nu_{i \cdot t}\right)\right. \\
& \left.\left.+\alpha\left(\mu_{t \kappa}\left(\mathcal{S}_{i t}\right)\right) E_{v} V^{3}\left(\mathcal{S}_{i t},\left\{0, j_{i t-1}, k\right\}, \nu_{i \cdot t}\right)\right\}\right\}
\end{aligned}
$$

The first part of this expression is the option of not searching and the second component is for searching for a job in a different occupation. In this last part notice there are three pieces: the cost of searching, the continuation value for unsuccessful searches, and the continuation value for successful searches. The expectation $E_{v}$ is over values of $\nu_{i \cdot t}$.

## Firm Problem

First consider the problem for a firm that either has a continuing worker or has matched with a potentially new one. We write $\Pi_{j t}\left(\mathcal{S}_{i t}\right)$ as the expected value of discounted income when matched to a worker of type $\mathcal{S}_{i t}$ and where the capital cost $c_{j t}$ has been sunk. Then,
$\Pi_{j t}\left(\mathcal{S}_{i t}\right)=\max _{w} P\left(j_{i t}=j ; w, \mathcal{S}_{i t}\right)\left[f_{j t}\left(\mathcal{S}_{i t}\right)-w+\frac{1}{1+R} E_{t} \max \left\{\Pi_{j t+1}\left(S^{\prime}\left(j, \mathcal{S}_{i t}\right)\right)-c_{j t+1}, 0\right\}\right]$
where $f_{j t}\left(\mathcal{S}_{i t}\right)$ is the value of the output of the worker as defined above and $P\left(j_{i t}=j ; w, \mathcal{S}_{i t}\right)$ is the probability worker stays at the firm given the wage and state variables of the worker. Note that this function $P(\cdot)$ is a complicated object but the pieces are all defined in our discussion of the workers problem. For new workers it is the probability of choosing the new job over the previous job and non-employment. For incumbent workers it depends on the conditional probability of staying conditional on $\mathcal{B}_{i t}$ and integrates this over the possible choice sets $\mathcal{B}_{i t}$. The maximization also takes into account that the firm might not continue the job next period. The key here is that the firm does not observe the $\nu_{i \cdot t}$ so has uncertainty about whether the firm will accept the job or not. This is similar to the models in, for example, Burdett and Mortensen (1998), Card et al. (2018) and Lamadon et al. (2022) as there is a tradeoff in that higher wages makes it more likely to attract and/or keep the worker.

Assuming Nash, the first order conditioning determining wages is

$$
\begin{align*}
P\left(j_{i t}=j ; w\left(j, \mathcal{S}_{i t}\right), \mathcal{S}_{i t}\right)= & {\left[f_{j t}\left(\mathcal{S}_{i t}\right)-w\left(j, \mathcal{S}_{i t}\right)+\frac{1}{1+R} E_{t} \max \left\{\left[\Pi_{j t+1}\left(F\left(j, \mathcal{S}_{i t}\right)\right)-\kappa_{j t+1}, 0\right\}\right]\right] } \\
& \times \frac{\partial P\left(j_{i t}=j ; w\left(j, \mathcal{S}_{i t}\right), \mathcal{S}_{i t}\right)}{\partial w} \tag{1}
\end{align*}
$$

Tightness $\mu_{t j}\left(\mathcal{S}_{i t}\right)$ is pinned down by the free entry condition:

$$
m\left(\mu_{t j}\left(\mathcal{S}_{i t}\right)\right) \Pi_{j t}\left(\mathcal{S}_{i t},\left\{0, j, j_{i t-1}\right\}\right) \leq c_{j t},
$$

with equality if $\mu_{t j}\left(\mathcal{S}_{i t}\right)>0$.

## Parameterization of Hedonic Pricing: $f_{j t}(\cdot)$

In parameterizing $f_{j t}\left(\mathcal{S}_{i t}\right)$ it is useful to break it into two different parts. In the first we define a human capital index

$$
\begin{equation*}
h_{j t}\left(\mathcal{S}_{i t}\right)=\theta_{i t}^{\prime} \beta_{j t}+\sigma(j, \tau) 1\left(j=j_{i t-1}\right), \tag{2}
\end{equation*}
$$

where $\beta_{j t}$ is a vector of skill weights and $\sigma$ is occupation-specific human capital. In practice we estimate the $\beta_{j t}$ using data from $\mathrm{O}^{*}$ NET and from information from the two NSLY data sets. We provide details in section 6 below.

As the second part we parameterize the rest of $f_{j t}(\cdot)$ as

$$
f_{j t}\left(\mathcal{S}_{i t}\right) \equiv \begin{cases}\delta_{j t}+\alpha_{1 j t} h_{j t}\left(\mathcal{S}_{i t}\right) & \text { if } h_{j t}\left(\mathcal{S}_{i t}\right) \leq h_{j}^{*}  \tag{3}\\ \delta_{j t}+\alpha_{1 j t} h_{j}^{*}+\alpha_{2 j t}\left(h_{j t}\left(\mathcal{S}_{i t}\right)-h_{j}^{*}\right) & \text { otherwise }\end{cases}
$$

which is a linear spline (in logs) with a kink point at $h_{j}^{*}$. We choose this specification in order to allow the pricing of high, medium, and low skilled workers to vary differently within occupation. That is, within each occupation, all individuals are affected equally by technology changes through the occupation specific constant $\delta_{j t}$. Depending on the level of his human capital index $h_{j t}\left(\mathcal{S}_{i t}\right)$, an individual sees his skills multiplied by either $\alpha_{1 j t}$ or $\alpha_{2 j t}$ depending on whether his index is below (or above) some threshold $h_{j}^{*}$.

A standard labor demand model in which workers are perfect substitutes within an occupation would yield special case of Equation (3) in which $\alpha_{1 j t}=\alpha_{2 j t}=1, \forall j, t{ }^{6} \quad$ We chose this more general parameterization for two main reasons. First, with the standard formulation, an increase in the within-variance can only be attributed to supply factors or occupational composition. Our more general formulation allows technology to favor some level of human capital more than other. And we will show it is key for understanding changes in the wage structure. Second, Figure 2 shows quite different time-patterns of different wage

[^4]quantiles. We use a more flexible model in an attempt to allow the model to capture these patterns within occupations.

## Evolution of State Variables

Initial human capital $\widetilde{\theta}_{i}$ is drawn from a multivariate normal distribution with variance $\Sigma^{\theta}$ and birth year specific mean equal to

$$
\begin{equation*}
b_{l} \times\left(P_{y_{i}}(\text { College })-P_{1979}(\text { College })\right), \tag{4}
\end{equation*}
$$

where $b_{l}$ is a skill-specific parameter to be estimated and $P_{y}$ (College) is the share of lowskilled men that attended college in cohort $y$. Note that we have normalized the mean of $\widetilde{\theta}_{i}$ to zero in 1979. We estimate $P_{y}$ (College) using the Census and ACS. This accounts for selection on schooling.

The general human capital variables transition takes the form,

$$
\theta_{i t+1}^{l}=d_{0 j_{i t}} d_{1 l} \exp \left[-d_{2}\left(a_{i t}-18\right)\right]+\theta_{i t}^{l}\left(1-d_{3 l}\right)
$$

The individual accumulates general skills at different speed depending on an occupation fixed effect $d_{0 j}$, a skill fixed effect $d_{1 l}$ and potential experience $a_{i t}-18$ according to $\exp \left[-d_{2}\left(a_{i t}-18\right)\right]$. Skills depreciates at rate $d_{3 l}{ }^{7}$

Occupation specific human capital and occupation specific tenure are determined, respectively, by

$$
\begin{aligned}
\sigma(j, \tau) & = \begin{cases}0 & \tau=0 \\
\sigma(j, \tau-1)+\gamma_{0 j} \exp \left(-\gamma_{1} \tau\right) & \tau>0\end{cases} \\
\tau_{i t+1} & =\left(\tau_{i t}+1\right) 1\left(j_{i t}=j_{i t-1}\right),
\end{aligned}
$$

with $\tau_{i t}=0$ at labor market entry. Occupation specific human capital $\sigma$ is a deterministic function of $\tau$. Occupation tenure is reset to zero after a switch to keep the dimension of the state space tractable-otherwise we would need to keep track of each individual entire work history. Stayers get additional occupation-specific tenure through $\gamma_{0 j} \exp \left(-\gamma_{1} \tau\right)$ where the specific human capital profile is concave in $\tau, \gamma_{1}>0$.

[^5]
## 5 Identification

While there are many parameters in our model and many identification issues arise, at a broad level there are three main problems all of which are classic identification problems in econometrics. The first issue is the age/time/cohort effect problem that these are fundamentally collinear. Second is the most classic identification problem in economics: separating supply from demand. The third is the sample selection problem inherent with the Roy (1951) model that occupations are chosen and we only see your wages in those occupations (e.g. Heckman and Honoré, 1990). It is impossible to make progress on the questions addressed in this paper without making some assumptions. In Appendix D we informally show identification in a non-parametric simplified version of our model. In this section we discuss those results focusing on which assumptions we think are the most important for addressing identification.

Our approaches to solving both the first and the second problem are closely related. One assumption that can solve both is that cohorts are ex-ante identical. That is, they would have identical initial human capital and preferences and their human capital accumulation process would be identical. Ex-post, cohorts would differ, but only because they face different markets. This resolves the age/time/cohort effect by assuming no cohort effects. While we acknowledge that this assumption is strong, the age/cohort/time effect is fundamentally unidentified so making progress on these issues requires some strong assumption. This assumption also resolves the supply/demand problem by assuming the driving force of changes across time is changes in demand for workers-the lifecycle supply function does not change. This change in demand could be due to technological change or increasing international trade-we do not take a stand on that.

In practice we do allow changes across cohorts in two limited ways and tightly identified ways. We allow for some change in initial human capital which we imagine happening not because of cohorts per se, but driven by a selection problem. We are only looking at men who did not go to college and the fraction going to college change over time. To address this, we allow the initial distribution of skills to depend on cohort but only through the college attendance rate as specified in equation (4). Note that this does not monotonically increase over time so it can be separated from systematic time and age effects. The second is that we allow preferences to change across cohorts, but force identification to come from a restricted part of the data. Specifically we compare the two cohorts of the National Longitudinal Survey of Youth using a revealed preference approach where we measure the extent to which job to job transitions from different occupations vary across the two cohorts. We do this using
indirect inference so these are explained within the model. In practice we find essentially no evidence of major changes in preferences across cohorts so it does not play a major role in the analysis.

The generalized Roy model is essentially identified from the fact that we have panel data and observe the same workers in different occupations following a similar approach as Bonhomme et al. (2019). Specifically we have 4 dimensions of skill-the three dimensional $\theta$ and occupational tenure. Conditional on the other three, tenure is observable and straightforward to deal with. Note as well that there is no uncertainly in human capital conditional on their initial $\theta$ and employment history so we can think of individuals "types" as determined by their initial human capital. Given that selection depends on $\theta$ and human capital only, the model is generically identified from the panel data in which we observe different workers in different occupations and also how this movement changes with wages.

Appendix D presents a sketch of identification in a stylized version of our model. We do not formally lay out all of the conditions but rather give a basic outline of the argument to make it easier to follow. We simplify the model in some dimensions but complicate it in others by relaxing parametric assumptions to show non-parametric identification.

Here are the key features:

- We show identification in the extreme version of our model: cohorts are ex-ante identical in terms of endowments, preferences, and human capital accumulation process. All changes over time are driven by technology.
- Motivated by Bonhomme et al. (2019) we assume that underlying types defined by initial endowment of $\theta_{i t}$ is discrete. Note that this differs from our model which assumes normal error terms but as in Bonhomme et al. (2019) we do not think it is fundamental for identification but makes it easier to see. ${ }^{8}$
- Wages are observed with i.i.d. median zero measurement error.
- We relax the extreme value assumption on $\vartheta_{j}+\nu_{i j t}$ and allow it to be flexible.
- Similarly, we do not make a parametric assumption about the distribution of $\chi_{i j t}$ but as in the text continue to assume it is iid across time with distribution $G^{\chi}$. We normalize the value of not searching to zero and assume the distribution of the remaining variables has full support.

[^6]- We continue to assume

$$
\begin{equation*}
h_{j t}\left(\mathcal{S}_{i t}\right)=\theta_{i t}^{\prime} \beta_{j t}+\sigma(j, \tau) 1\left(j=j_{i t-1}\right), \tag{5}
\end{equation*}
$$

and that $f_{j t}\left(\mathcal{S}_{j t}\right)=\tilde{f}_{j t}\left(h_{j t}\left(\mathcal{S}_{i t}\right)\right)$ for some monotonically increasing $\tilde{f}_{j t}$, but do not restrict its form.

- Human capital accumulation takes the more general form

$$
\begin{equation*}
\theta_{i t+1}^{l}=g_{l}\left(j_{i t}, a_{i t}\right)+\left(1-d_{l}\right) \theta_{i t} \tag{6}
\end{equation*}
$$

- $\left(\beta_{11}, \ldots, \beta_{J 1}\right)$ is known in the first period.
- We observe a large number of cohorts and follow their occupational choices and wages from labor market entry to retirement.

The basic stages of identification are the following:
Step 1 As we have human capital in our model, we can not follow Bonhomme et al. (2019) directly. However we use the panel data in similar ways. Using a deconvolution argument we show that the distribution of the measurement error is identified. We also show that conditional on a particular cohort, first period of employment, and first initial occupation we can identify the distribution of types, the relevant wages $w\left(j, \mathcal{S}_{i t}\right)$ for the different types in the different states of the world, and the conditional choice probabilities of moving from one occupation to another. What we can not do at this point is connect the labelling across different initial occupations and cohorts. For example, in a one period model for a particular year we can identify the marginal distribution of wages in each occupation but not the joint distribution.

Step 2 We focus on a simple case with two occupations and a lifecycle of three periods. We show that from the wages and conditional choice probabilities we can identify the distribution of occupational tastes, the distribution of search costs, and the discount rate. From the wage setting and differential equations of the firms we can identify the $\operatorname{costs} c_{j t}$, the productivity $f_{j t}\left(\mathcal{S}_{i t}\right)$, as well as market tightness in every market.

Step 3 We show that combining the raw identification of Step 1 with the aspects of the model that are identified in Step 2, we can complete the labeling across different initial occupations and cohorts.

Step 4 The fourth step is more informal. We identify the general productivity $f_{j t}\left(\mathcal{S}_{i t}\right)$ in Steps 1-3, but have not separately identified the distribution of the underlying factors $\theta$. For many counterfactual questions this is not necessary as one could think about this as just a functional form for $f_{j t}$. However, in our counterfactuals we think about changing the initial values of $\theta$ in which case we do need to make this distinction. In our general discussion we argue that it might not technically be point identified with a finite set of time periods-because there are only a finite set of values. However, we discuss how this could yield set identification.

## 6 Estimation

Let $\Lambda$ be the vector of structural parameters. We estimate our model using indirect inference. Indirect inference works by selection of a set of statistics of interest $\hat{\Psi}$ which the model is asked to reproduce. ${ }^{9}$ For an arbitrary value of the vector of parameters to be estimated $\Lambda$, we use the model to generate the target moments $\Psi(\Lambda)$. The parameter estimate $\hat{\Lambda}$ is then derived by searching over the parameter space to find the parameter vector that minimizes the criterion function,

$$
\begin{equation*}
\hat{\Lambda}=\arg \min _{\Lambda}(\hat{\Psi}-\Psi(\Lambda))^{\prime} W(\hat{\Psi}-\Psi(\Lambda)) \tag{7}
\end{equation*}
$$

where $W$ is a weighting matrix. This procedure generates a consistent estimate of $\Lambda$. Before discussing the estimation approach we fill in some details about the econometric specification.

## Pre-set parameters

- $\alpha_{1}$ is normalized to one in clerical occupation in 1979.
- The real interest rate $R$ is set to $5 \%$.
- The elasticity of the matching function with respect to the number of searchers $\eta$ is set to 0.5 .

[^7]
## Measurement/Classification Error

We allow reported wages and occupations to be contaminated by measurement errors. In the simulation, we multiply true wages by $u$ where $\log (u) \sim N\left(0, \sigma_{u}^{2}\right)$ before calculating target moments. Occupations can be misclassified but not-working is always correctly reported. Let $\pi_{t}\left(j_{0}, j_{1}\right)$ be the probability that occupation $j_{0}$ is reported given that the true occupation is $j_{1}$ at time $t$. Formally,

$$
\pi_{t}\left(j_{0}, j_{1}\right)=\operatorname{Pr}\left(j_{i t}^{*}=j_{0} \mid j_{i t}=j_{1}\right), \quad j_{0}, j_{1}=1, \ldots, J
$$

In principle that is $J(J-1)$ additional parameters to be estimated for any given set of control variables. We follow Keane and Wolpin (2001) and assume classification errors are unbiased, e.g. the probability that a person is observed in an occupation is equal to the true probability that he/she chooses that occupation. Formally,

$$
\operatorname{Pr}\left(j_{i t}^{*}=j\right)=\operatorname{Pr}\left(j_{i t}=j\right), \quad j=1, \ldots, J .
$$

Under that assumption, the $\pi_{t}$ are known up to an unknown parameter $E$,

$$
\pi_{t}\left(j_{0}, j_{1}\right)= \begin{cases}(1-E) \operatorname{Pr}\left(j_{i t}=j_{0}\right), & j_{1} \neq j_{0} \\ E+(1-E) \operatorname{Pr}\left(j_{i t}=j_{0}\right) & j_{1}=j_{0}\end{cases}
$$

Test scores are noisy measure of skills before labor market entry. AFQT is a noisy measure of cognitive skills $\tilde{\theta}_{1}$. The measure of social skills constructed by Deming (2017) is a noisy measure of inter-personal skill $\tilde{\theta}_{2}$. The variance of these two measures is denoted by $\tilde{\sigma}_{l}, l=$ 1,2 .

## Specification and Estimation of factor prices $\beta$

Figure 1 presented our estimates of skill weights from $\mathrm{O}^{*}$ NET. Unfortunately $\mathrm{O}^{*}$ NET is not a proper panel so we can not use it alone to estimate changes in skills weights. We augment the $\mathrm{O}^{*}$ NET by using information from the comparison between the NLSY79 and the NLSY97. Specifically we let $\overline{\beta_{j}^{l}}$ be the time-invariant loading factor we estimated from O*NET. We assume it represents the (constant) skill intensity until the end of the period of observation and on. We allow for time trends prior to that year.

To reduce the dimension of the problem, we allow for a trend common to all occupations
$a_{l}$ but that differs across skills. And we allow for a trend that is a function of the observed change in the skill composition of an occupation. If individuals with a high skill level are more represented in an occupation across NLSY waves, it suggests that this occupation became more intense in that skill, ceteris paribus. Formally, let $x_{j l}$ denotes the difference in difference in proportions for each occupation $j$ and test score $l$. It is a difference between above/below median in skill $l$ and a difference across NLSY waves. For $l=1,2$, the trend is $\left(a_{0 l}+a_{1 l} x_{j l}\right)$. For $l=3$, it is calculated as a residual $\beta_{j t}^{3}=1-\left(\beta_{j t}^{1}+\beta_{j t}^{2}\right)$. We then estimate these parameters along with the rest of the structural parameters. To identify these parameters, we use the combination of NLSY waves. We give more details when we present the auxiliary parameters below.

## Algorithm Details

It is in principle possible to estimate the full vector of parameters $\Lambda$ at once but we found that to be computationally prohibitive. Small variations in some parameters can lead some individuals to switch occupations creating discontinuities in the objective function. We also have a large number of parameters. Instead, we develop a sequential algorithm. We first divide both the structural parameters $\Lambda=\left(\Lambda_{1}, \Lambda_{2}, \Lambda_{3}\right)$ and auxiliary parameters $m=\left(m_{1}, m_{2}, m_{3}\right)$ into three groups. We obtain starting values by dividing the estimation algorithm into three iterative steps which we repeat until convergence.

Each step selects a subset of the structural parameters to fit a subset of the auxiliary parameters. Let $\mathcal{J}(\Lambda)$ be simulated data generated by individual optimal decisions given a sequence of shocks and parameters $\Lambda$ and $m_{j}(\mathcal{J}(\Lambda))$ is a set of auxiliary parameters produced from that simulated data. Given $\Lambda^{-1}$ from a previous iteration.

1. Choose $\Lambda_{1}$ to fit $m_{1}\left(\mathcal{J}\left(\Lambda_{1}, \Lambda_{2}^{-1}, \Lambda_{3}^{-1}\right)\right)$
2. Choose $\Lambda_{2}$ to fit $m_{2}\left(\mathcal{J}\left(\Lambda_{1}, \Lambda_{2}, \Lambda_{3}^{-1}\right)\right)$
3. Choose $\Lambda_{3}$ to fit $m_{3}\left(\mathcal{J}\left(\Lambda_{1}, \Lambda_{2}, \Lambda_{3}\right)\right)$

Using NLSY79 moments $m_{1}$, Step 1 estimates the preference and time invariant technology parameters $\Lambda_{1}=\left(\Sigma,\left\{\delta_{j}^{0}, \alpha_{j}^{0}, c_{j}^{0}, \vartheta_{j}^{0}\right\}_{j=1}^{J}, \bar{\chi}, \sigma_{\nu}, \sigma_{\chi}, b, \sigma_{u}, \tilde{\sigma}_{1}, \tilde{\sigma}_{2}, E\right)$ where $\delta_{j}^{0}, \alpha_{j}^{0}$ are initial hedonic prices, $c_{j}^{0}$ are initial capital costs and $\vartheta_{j}^{0}$ are initial non-pecuniary benefits common to all workers. Step 2 estimates life-cycle wage growth parameters $\Lambda_{2}=$ $\left(\left\{d_{0 j}, \gamma_{0 j}\right\}_{j=1}^{J},\left\{d_{1 l}, d_{3 l}\right\}_{l=1}^{L}, \gamma_{1}, d_{2}\right)$, holding fixed individual choice, to fit $m_{2}$. The advantage is that we only need to solve the model at the beginning of this step. We re-solve the
model at each new parameters guess $\Lambda_{3}=\left(\left\{\delta_{j t}, \alpha_{j t}^{1}, \alpha_{j t}^{2}, \beta_{j t}, c_{j t}, \vartheta_{j t}\right\}_{j=1}^{J}, \mu_{c}^{\theta}\right)$ which contains all the time-trend parameters.

These steps and those in the identification section are not directly related, but we give a loose mapping here. We can think of Step 4 in the identification section as not part of estimation per se, but rather as a functional form for the pricing equation $f_{j t}\left(\mathcal{S}_{j t}\right)$. The second set of parameters $\left(\Lambda_{2}\right)$ essentially comes from Step 1 in which we show that we can identify the sequence of wages following entry cohorts. The third set $\left(\Lambda_{3}\right)$ is essentially the very last part of Step 2 in which we discuss identifying costs and productivity across cohorts and in Step 3 where we can match groups across cohorts. The first set of auxiliary parameters $\left(\Lambda_{1}\right)$ correspond to the rest of Steps 1 and 2.

Once this procedure is done, we use these estimates as an initial guess and then estimate the full set of structural parameters together using Equation (7). We find this works very well in practice as the procedure provides excellent starting points so the final stage is relatively quick.

This leaves us with a total of 212 parameters divided into groups of 46,23 and 143.

## Auxiliary Parameters

As mentioned above, we partition the vector of auxiliary parameters $m$ into three vectors defined as follows.
$m_{1}$ contains all the auxiliary parameters that are used to identify the preference and time invariant technology parameters. The data moments are:

- (CPS for NLSY79 cohorts) Quantiles of the wage distribution by occupation and by age.
- (CPS for NLSY79 cohorts) The proportion of individuals choosing each of the $J+1$ occupations by age.
- (NLSY79) Occupation Mobility
- The proportion of occupation-stayers between $t$ and $t+1$ and between $t$ and $t+2$ for each of the $J+1$ occupations in the population and for two different age group.
- The proportion of occupation-switchers moving into each $J+1$ occupation between $t$ and $t+1$ and between $t$ and $t+2$ in the population and for two different age group.
- The transition between each of the $J+1$ occupation between $t$ and $t+1$ and between $t$ and $t+2$ in the population for two different age group.
- The median occupation-specific tenure and the median experience in each of the $J+1$ occupations
- The auto-correlation of wages by age.
- The winning rate of each of the $J$ occupations defined as the proportion of transition to an occupation $j$ from a different occupation $j^{\prime}$ among all transitions between these two occupations.
$m_{2}$ contains all the auxiliary parameters that are used to identify the human capital accumulation parameters. Using NLSY79 cohorts, the moments are
- (CPS) The median wage by occupation and age.
- (NLSY79) The median wage by years of general work experience for each of the $J$ occupations. The median wage by years of occupation-specific experience for each of the $J$ occupations.
- (NLSY79) The auto-correlations of wages in level between $t$ and $t+1$ separately for occupation stayers and occupation switchers.
- (NLSY79) The mean 1-year difference in wages by current occupation, past occupation, and for two different experience group and for two different occupation-specific tenure group.
$m_{3}$ contains all the auxiliary parameters used to identify movement in output, capital costs and preferences.
- (CPS) Quantiles of the wage distribution for each year and for each of the $J$ occupations.
- (CPS) The proportion of individuals choosing each of the $J+1$ occupations by year in the CPS used to identify trends in search frictions.
- (NLSY79 and NLSY97) The test scores coefficients in log-wage linear regressions across NLSY waves and controlling for age and year fixed effects. See Appendix E for details.
- (NLSY79 and NLSY97) The proportion of individuals choosing each of the $J+1$ occupations by test scores and across NLSY waves.
- (NLSY79 and NLSY97) the winning rate of each the $J$ occupations and across NLSY waves.


## 7 Changes in the Wage Structure

This section discusses the estimates of our model. Since we have a lot of parameters, we do not discuss all in detail. The structural parameters related to the life-cycle can be found in Appendix F. In this section we examine the estimated time trends which are our main focus.

## Price Series

Figure 4 summarizes the estimated price series. For each occupation, we graph our estimate of the hedonic pricing equation $f_{j t}(\cdot)$ function at three different points in time. ${ }^{10}$ That is the $X$ axis represents the $h$ variable in equation (3) and we plot the function at the beginning of the period 1979, at the end of the period 2017 and in 1990.

The heterogenous effects of technological changes are apparent. None of the occupations has been positively affected by technological change (or other drivers of the wage structure) throughout their distribution. The 1980s led to a large a decline in the price for all but the most skilled workers in most occupations. This is precisely the period of acceleration of technological change documented by the literature dating back to at least Katz and Murphy (1992). The pricing function has been more stable since 1990 in some occupations. We also see that prices at the bottom have increased relative to prices in the middle within half of the occupations.

Looking at the overall full pattern from 1979 to 2017 we see an increase in the price at the highest level of $h$ for all occupations, a decrease at the median for all occupations, and no clear pattern at the bottom with some declining (such as managers and precision production) and others increasing (such as operators and transportation).

[^8]Figure 4: Price series by decades









Figure 5: Skill intensity by occupations: 1979 and 2017

Skill Intensity in 1979


Skill Intensity in 2017


## Skill Weights

Figure 5 reports skills weights-the $\beta$ in the human capital equation at the beginning and at the end of the period.

The weights at the beginning of the period are identified directly from O'NET. The timetrend is identified primarily from linear wages regressions across NLSY waves reported in Table 2. The weight on inter-personal skills rose significantly over time. It was zero in all occupations except for management and clerical occupations at the beginning of the period. This is driven by the lack of statistical significance of the inter-personal skills coefficients for both wages and the probability of working in the NLSY79. Following the same logic, we find that the loading factor on cognitive skills declined in all occupations. The weight on manual skills is overall stable but displays heterogeneous trends by occupations. It remained fairly stable or rose in manual occupations but declined in management, clerical and services occupations.

## Cohort Effects

The results on cohort effects, $b_{l}$ which is defined in Equation (4) can be found in Table 2. As one would expect, we find a negative selection on cognitive skills on men who do not go to college. That is more people went to college, the endowment in cognitive skills declined for low-skilled men as people with higher cognitive skills are likely to attend college. Interestingly, we find positive selection for manual skills which suggests that more manually able men are less likely to go to college. On this dimension, the increasing college attendance
rate actually leads to a more positively selected group. This is important as manual skills turn out to be the most important for low skilled men as we will discuss below. Finally, there have been little change for inter-personal skills.

Table 2: Skill weights by occupations and cohort effects

|  | Cognitive |  | Inter-personal |  |
| :--- | :---: | :---: | :---: | :---: |
| Manual |  |  |  |  |
| $a_{0 l}$ | $-0.0049(0.0001)$ | $0.0105(0.0001)$ |  |  |
| $a_{1 l}$ | $0.004(0.0001)$ | 0.0067 | $(0.0012)$ |  |
| cohort effects $\left(b_{l}\right)$ | $-0.656(0.04)$ | 0.0545 | $(0.0178)$ | 0.1183 |

## Decomposition of Wage Trends

According to our model, both the supply of skills, technology in the current occupation, and competition for worker services in outside occupations impact the wage distribution. To see the decomposition we repeat Equation 1:

$$
\begin{aligned}
P\left(j_{i t}=j ; w\left(j, \mathcal{S}_{i t}\right), \mathcal{S}_{i t}\right)= & {\left[f_{j t}\left(\mathcal{S}_{i t}\right)-w\left(j, \mathcal{S}_{i t}\right)+\frac{1}{1+R} E_{t} \max \left\{\left[\Pi_{j t+1}\left(F\left(j, \mathcal{S}_{i t}\right)\right)-\kappa_{j t+1}, 0\right\}\right]\right] } \\
& \times \frac{\partial P\left(j_{i t}=j ; w\left(j, \mathcal{S}_{i t}\right), \mathcal{S}_{i t}\right)}{\partial w}
\end{aligned}
$$

Abstracting from the continuation value there are three things that change the wages across time: the skill prices, $f_{j t}(\cdot)$, the distribution of skills themselves, $\mathcal{S}_{i t}$, and outside competition $P\left(j_{i t}=j ; w\left(j, \mathcal{S}_{i t}\right), \mathcal{S}_{i t}\right)$. We now use our model to assess the relative importance of each of these factors over time.

To isolate the effect of outside-options pressure on wages, we assume firm set wages "wrongly" assuming the effect of varying wages on worker occupational choices remained constant over time. That is we keep all the arguments of the function $P\left(j_{i t}=j ; w, \mathcal{S}_{i t}\right)$ fixed to what it was in 1979 conditional on a given wage $w$ and state $\mathcal{S}_{i t}$. Next to separate supply effects from technology, we keep technology fixed-both the hedonic pricing function $f_{j t}(\cdot)$ and the capital costs $c_{j t}$, so that only allow the endowment in each three skills of each cohort to vary over time.

Figure 6 reports the results for the 10th, 50th and 90 th quantiles, normalizing each quantile to zero in 1979. The solid line is the baseline. The dashed line is the counterfactual with the alternative wage-setting (i.e. ignoring changes in $\mathrm{P}(\cdot)$ ). The dotted line is the

Figure 6: Technological Change and Competitive Pressure

counterfactual where only supply changes (i.e. ignoring changes in $P(\cdot)$ and in technology).

Changing the wage setting mechanism has different effects on different parts of the wage distribution.

For the median worker during the 1980s the baseline falls faster than the dashed line. Since the difference between the two is competitive pressure, this shows that outside competition fell over time and therefore wages went down more than their marginal productivity. This is because their outside option worsened. We interpret this as coming from the fact that with automation, machines replaced these workers leading to this decline. This is reflected in our model by a rise in capital intensity, especially in some manual occupations. We discuss this below in Table 4.

The top of the distribution is similar to the middle but less dramatic. Frictions also reinforced the effects of technological change but frictions play a lesser role and the hedonic pricing function is the dominant driving force.

The tenth quantile is different. In this case the dashed line fell faster during the 1980s than the solid line. This results because competitive pressure increased during this period
for workers at the bottom of the skill distribution. It is driven by the rise of Services. Workers in other occupations became less valuable, especially in the 1980s: the hedonic pricing equation decreased at the bottom even more than in the middle. Employers in other occupations had to compensate workers with low skills to prevent them from moving to the increasing number of Service jobs. This effect is not important for workers at either the middle or the top because these are low-paying jobs that did not substantially improve these workers' outside options.

Finally, the supply of skills plays essentially no role in any of the three quantiles. This is seen by the essentially flat dotted line. It is worth recalling that we only allow cohort to have different endowment in each three skills at labor market entry and that differences are a function of the proportion of men entering college for each cohort. This result was therefore expected as college enrollment has been mostly stable for men and the test scores measures of both cognitive and inter-personal skills have very similar distributions across NLSY waves.

## 8 Evolution of Occupational Composition

We saw in Figure 3 in Section 3 that employment and wages evolutions are only weakly related. There are three factors within our model that could potentially reconcile this result:

1. Selection effects are strong and wages evolutions are not in line with price changes.
2. Preferences for working in different occupations has changed.
3. Labor market frictions are important so that the reallocation of labor cannot be attributed to the evolution of relative productivity.

In this section we explore the relative importance of these factors and conclude that accounting for frictions is key.

## Selection

Figure 3 presented in our Motivating Facts section above documents a low correlation between employment and wages evolution. Using our estimates, we first assess the extent controlling for selection improves on this dimension. Recall that Figure 3 presents the changing mean value of $w\left(j, S_{i t}\right)$ conditional on $j$. This can change for two reasons: changes in the wage function $w(j, \cdot)$ over time or changes in the distribution of $\mathcal{S}_{i t}$ (which is what we refer

to as selection). To account for this we try to control for selection by holding fixed the distribution of the human index supplied in each occupation to its initial value at the beginning of the period, 1979. We then plot the evolution over time of the intermediate output/hedonic pricing equation, $f_{j t}\left(\mathcal{S}_{j t}\right)$ produced on average by individuals from that fixed distribution. Figure 7 reports a much more strongly positive correlation between the growth in $f_{j t}(\cdot)$ and the growth in employment than for wages without the selection equation.

Therefore, expanding occupations have benefited from relatively favorable demand shocks on average. Yet, selection is not strong enough to completely solve the puzzle. For example it can not account for lack of convergence of wages between shrinking operators and expanding clerical. Furthermore, average prices mask considerable heterogeneity within occupations. The average price declines substantially for most occupations but it declines much less for the human capital index at the bottom or at the top of the distribution, as reported in Figure 4. Finally, note that even though the occupation share of managers and clerical are stable, there are strong selection effects at play due to changes in the prices series. Managerial occupations saw a large influx of high skilled individuals and as a result average wages grew faster than average prices. It is the opposite for clerical occupations.

## Preferences

Could it be that manual jobs became less attractive jobs over time for non-pecuniary reasons? That is, could the decline in operators relative to clerical workers occur because younger workers dislike operator jobs relative to previous cohorts? We use the change in the "winning" rate of occupation between the two NLSY cohorts to identify changes in preferences. That is

Table 3: Winning rate - Model Fit NLSY79 vs NLSY97

|  | Data |  | Model (baseline) | Model (constant $\left.\vartheta_{j y_{i}}\right)$ |  | $\Delta \vartheta_{j y_{i}}$ |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Occupation | NLSY79 | NLSY97 | NLSY79 | NLSY97 | NLSY79 | NLSY97 | (in \%) |
| Managers | 0.5582 | 0.5772 | 0.5539 | 0.5630 | 0.5560 | 0.5506 | 4.51 |
|  | $(0.014)$ | $(0.029)$ |  |  |  |  |  |
| Clerical | 0.5009 | 0.4838 | 0.5045 | 0.4995 | 0.5100 | 0.4685 | 1.60 |
|  | $(0.012)$ | $(0.018)$ |  |  |  |  |  |
| Services | 0.4603 | 0.4823 | 0.4679 | 0.4795 | 0.4778 | 0.4594 | 1.88 |
|  | $(0.012)$ | $(0.017)$ |  |  |  |  |  |
| Operators | 0.4905 | 0.4828 | 0.4876 | 0.4792 | 0.4782 | 0.4801 | -3.07 |
|  | $(0.010)$ | $(0.021)$ |  |  |  |  |  |
| Mechanics | 0.5100 | 0.5607 | 0.5071 | 0.5206 | 0.5259 | 0.5439 | 3.63 |
|  | $(0.014)$ | $(0.027)$ |  |  |  |  |  |
| Construction | 0.5066 | 0.5242 | 0.5170 | 0.5199 | 0.5211 | 0.5238 | 4.86 |
|  | $(0.012)$ | $(0.021)$ |  |  |  |  |  |
| Precision | 0.5190 | 0.5045 | 0.5257 | 0.5197 | 0.5204 | 0.5216 | -1.86 |
|  | $(0.014)$ | $(0.033)$ |  |  |  |  |  |
| Transport | 0.4917 | 0.4810 | 0.4923 | 0.4855 | 0.4809 | 0.5126 | -2.30 |
|  | $(0.008)$ | $(0.015)$ |  |  |  |  |  |

we look at workers that experience job to job transitions from one occupation to another as evidence of revealed preference. ${ }^{11}$ We look at this by constructing the winning rate between any two distinct occupation $j_{1}$ and $j_{2}$ as the ratio of the total number of transition from occupation $j_{1}$ to occupation $j_{2}$ compared to the total number of transitions between these two occupations. If younger cohorts preference for manual jobs is declining they should be less likely to move to them and more likely to move from them which would be reflected as a lower winning rate. Table 3 reports the winning rate in the data from two different cohorts: the NLSY79 and the NLSY97. It also shows simulations from the baseline model and from a restricted model where we force preferences to be stable over time. The last column reports the relative change over time in the static non-pecuniary payoff of working for each occupation.

First note that there is no evidence in the raw data of preferences shifting away from manual skills. Most importantly note that the Operators winning rate is very similar in the two cohorts. In fact, the baseline model can fit the data well without large changes in preferences for working in each occupation. There is small positive increase in the utility

[^9]Figure 8: Evolution of occupation share by year

for working for services and a small declining utility for working as operators. This could in principle explain the rise of services relative to operators: workers have increasingly chosen services over any other occupation and it goes in the other direction for operators. However, the magnitude of the changes are comparable to changes observed in all other occupations. It is therefore not surprising that this won't be enough to explain the large reallocation of labor across these occupations as will be shown in the next subsection.

## Fitting Occupational Composition

We now turn to explaining the evolution of occupational composition. It will become clear that while we do find some evidence of selection in Figure 7 and changes in preferences in Table 3, neither are important determinants of changes in occupational composition.

Figure 8 reports the evolution of employment share by occupation. The circle are from the CPS data. The straight line is from the baseline simulated model. The dotted lines are from a restricted model where we re-estimate the model without search frictions to see if this can fit the data.

One can see that the baseline model fits the data very well while the restricted model
does not. ${ }^{12}$ In particular it does not match the decline of operators at all nor the rise of services.

The change in occupational composition can be explained by a combination of changes in prices reported in Figure 4 above and change in capital costs over time that are reported in Table 4.

Table 4: Capital cost $c_{j t}$

| Occupation | 1979 | 2017 | Difference (in \% ) |
| :--- | :---: | :---: | :---: |
| Managers | 3.01 | 3.39 | 12.6 |
|  | $(0.01)$ | $(0.03)$ |  |
| Clerical | 2.21 | 2.24 | 1.5 |
|  | $(0.03)$ | $(0.08)$ |  |
| Services | 1.75 | 1.43 | -18.6 |
|  | $(0.04)$ | $(0.05)$ |  |
| Operators | 1.28 | 2.56 | 100.5 |
|  | $(0.05)$ | $(0.07)$ |  |
| Mechanics | 3.26 | 3.95 | 21.4 |
|  | $(0.04)$ | $(0.05)$ |  |
| Construction | 2.77 | 2.8 | 0.8 |
|  | $(0.05)$ | $(0.06)$ |  |
| Precision | 3.83 | 4.39 | 14.7 |
|  | $(0.03)$ | $(0.04)$ |  |
| Transport | 1.32 | 1.76 | 33.1 |
|  | $(0.06)$ | $(0.06)$ |  |

Operators occupations experienced the most significant increase in capital intensity. This yields to a lower supply of these jobs but does not directly affect their wages which allows the model to explain the large decline of operators despite the relatively modest decline in wages. One explanation for this increased capital intensity is that these values reflect the evolution of the manufacturing sector where many low skilled workers have been replaced by machines. Yet, some workers, the most talented one, are now in charge of operating these machines and whose skills became much more important than in the past. This is an example that one would expect to lead to a rise in wages for high skill workers, a decline for the median worker and a decline in the number of workers in that occupation.

We also find that the rise of not-working since 2000 cannot be attributed to price changes. Indeed, the evolution of the price series alone cannot match the growing share of low skilled

[^10]men not-working. Preferences did not play a significant role either. In our model the rise of not-working is due to an overall increase in capital intensity which lowers the job finding rate. It could as well be explained with a rise of the value of not working (Aguiar et al., 2021), nonphysical aspects of some jobs (Kaplan and Schulhofer-Wohl, 2018) or health considerations (Borella et al., 2019). We do not attempt to incorporate these factors into our model as it is not our main focus.

## 9 Skill Premium

We have demonstrated a decrease in wages for low skilled men. This raises the question of what policies might best address this problem. While we do not try to provide a precise answer to this question, in this section we examine which of our skills is most important and how that has changed over time.

## Preliminary Evidence

We begin with a simple exercise in which we classify the individuals from our estimated simulated model based on their endowment in each of the three skills at labor market entry. Precisely, for each skill we look at whether their initial endowment is in the top third, bottom third, or in the middle of the distribution. We then compare the mean wage of these different groups of individuals as we simulate the model. Note that if initial skills were observable in the CPS, this is something we could produce from the raw data without a model. Figure 9 reports the results.

The top-left panel reports the difference in mean wages between the top and bottom skill groups while the top-right panel reports the difference between the middle group and the bottom group. The bottom-left panel reports regression results. For each year, we regress log-wage on a measure of each skills at labor market entry, controlling for age dummies. We normalize each skill to have mean 0 and standard deviation 1. The bottom left panel reports the coefficient on each skill over time. The bottom-right panel reports analogous results from the same regression specification but where the left-hand side variable is a dummy variable for working.

At the beginning of the period, manual skills have the most predictive power on wages followed by cognitive skills. Individuals in the top third in manual skills earn more than $40 \%$ more than individuals in the bottom third (at labor market entry) in 1979. The corresponding number is around $30 \%$ for cognitive skills. By contrast, the pay gap associated with inter-

Figure 9: Skill Premium

personal skills is only $2 \%$ in 1979. The regression coefficients have the same ordering. A one standard deviation difference in manual, cognitive and inter-personal skills, is associated with higher $\log$ wages by, respectively, $20 \%, 15 \%$ and $1 \%$.

The pay premium associated with each three skills increased in the 1980s. The top mean wages compared to the bottom rises by 8 percentage points in manual skills, 5 for interpersonal skills and around 0.5 for cognitive skills. The patterns markedly differ by skills after that. Since 1990, the premium slowly fell for cognitive skills while the premium for inter-personal skills kept steadily rising. Hence, the gap between the premium for cognitive and the premium for inter-personal skills is becoming increasingly narrow. The premium for manual skills is about $50 \%$ and therefore remains the highest. It was stable in 1990s and it has been rising since 2000. The regression coefficients show the same ordering when either the wage or a dummy for working are on the left hand side. However, since 1990 individuals at the bottom in either manual or cognitive skills did relatively better that individuals in the middle due to the flattening of the price series at the bottom in some occupations. This is not the case for inter-personal skills.

## Counterfactual Skill Investment

Should we conclude that we should invest in individuals manual skills before they enter the labor market? The previous results were suggestive of this, but were not causal. We next do an exercise to help this question. Since we don't know the relative costs of investing in the three different skills nor exactly how one might do it, we consider a thought exercise: Suppose that it were equally costly to increase skill by a standard deviation, in which skill would we prefer to invest? We investigate this question by increasing the endowment of each individual at labor market entry by one-standard deviation for each skill. These are partial equilibrium experiments because the hedonic pricing function is fixed though they are equilibrium effects in that we allow the equilibrium labor market tightness to adjust. ${ }^{13}$ Figure 10 reports the average wage gain of each policy.

Figure 10: Skill improvement program


Improving manual skills of low-skilled men before labor market entry has the highest rewards. This is true throughout the period of observations. This reflects the fact that this group predominantly uses manual skills. The returns to improving cognitive skills increased during the 1980s and fell since. Perhaps surprisingly, manual skills not only were important

[^11]Table 5: Skill improvement program: probability of working

| Skill | 1979 | 2017 | $\Delta$ |
| :--- | :---: | :---: | :---: |
| Cognitive | 0.0416 | 0.0375 | -0.0041 |
| Inter-personal | 0.0033 | 0.0265 | 0.0232 |
| Manual | 0.0579 | 0.1057 | 0.0478 |

in the initial period but have substantially gained in importance over time. Interpersonal skill investment had little returns at the beginning of the period, but increased throughout the period of analysis and it is now only slightly lower than the returns to improving cognitive skills.

These policies also improve the probability of working as reported in Table 5. Increasing cognitive skills or manual skills by one standard deviation raises the probability of working by about 4 and 6 percentage points respectively in 1979. It remains fairly stable over time for cognitive skills while it increased by more than 4 percentage points for manual skills. On the other hand, improving inter-personal skills had little impact, less than 0.5 percentage points, at the beginning of the period while it boosts the probability of working by more than 2.5 percentage points at the end of the period.

## Heterogeneity

Are the returns to these policies heterogenous across individuals? To answer this question, we again classify people depending on whether their initial endowment in a particular skill is in the top third or the bottom third of the distribution. ${ }^{14}$ Table 6 reports the returns for each subgroup at the beginning of the period (column $t_{0}$ ) and at the end of the period (column $t_{1}$ ).

The analysis by group confirms that investing in manual group has the highest return for virtually all groups throughout the period of analysis. This is the policy that has the highest return for almost any endowment. This is true both at the end and the beginning of the period of analysis.

The only exception is for people that are in the top group in both cognitive and interpersonal skills but are in the bottom in manual skills. For this case cognitive skills have the highest returns. This is driven by management occupations which attracts the highest

[^12]Table 6: Counterfactual: heterogeneity and interactions

| Endowment |  |  | Policy |  |  |  |  |  | Best |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Cog | Inter | Man |  |  |  |  |  |  |  |  |
|  |  |  | $t_{0}$ | $t_{1}$ | $t_{0}$ | $t_{1}$ | $t_{0}$ | $t_{1}$ | tor | $t_{1}$ |
| w | Low | Low | 0.0804 | 0.0281 | 0.0029 | 0.0179 | 0.146 | 0.115 | Man | Man |
| Low | Low | High | 0.0937 | 0.0673 | 0.0038 | 0.0337 | 0.185 | 0.3417 | Man | Man |
| Low | High | Low | 0.0874 | 0.0622 | 0.0052 | 0.0389 | 0.1527 | 0.151 | Man | Man |
| Low | High | High | 0.1063 | 0.0912 | 0.0077 | 0.0516 | 0.1957 | 0.3506 | Man | Man |
| High | Low | Low | 0.1133 | 0.0883 | 0.0147 | 0.0433 | 0.1679 | 0.1625 | Man | Man |
| High | Low | High | 0.1179 | 0.1048 | 0.0084 | 0.0592 | 0.2037 | 0.3492 | Man | Man |
| High | High | Low | 0.122 | 0.2286 | 0.0109 | 0.1501 | 0.1595 | 0.1988 | Man | Cog |
| High | High | High | 0.1192 | 0.1691 | 0.0057 | 0.1205 | 0.1981 | 0.2889 | Man | Man |

skilled workers. This is interesting as it happens simultaneously as the premium for cognitive skills declined overall.

## 10 Conclusions

We propose and estimate a model to understand the evolution of the wage structure of low skilled men since 1979. We allow for a rich specification of change in the demand for workers which has heterogenous effects on different occupations and different parts of the skill distribution. We document the relative role of demand-side factors and supply-side factors.

We have three main findings. First, the reallocation away from manual jobs towards services and changes in the wages structure were driven by demand factors while the supply of skills and changes in preferences played almost no role. We find little evidence that preferences changed across cohorts and supply of skills changed very little.

Second, frictions play a crucial role in preventing wages in traditional blue collar occupations from falling substantially relative to other occupations. We show that our model without search frictions can not fit the data. For example, it can not explain the decline in operators who have experienced only a small decline in wages. In our model this is explained by an increase in the capital cost. A higher cost leads to less labor market entry and fewer jobs, but due to search frictions this does not directly translate to lower wages. It is important to point out that while our model without frictions can not explain the data, we have not explored a large set of alternatives models. Our base model can fit the data but there may be other mechanisms that can as well. We leave it to further work to explore this
further.
Finally, while we see an increase in the payoff to interpersonal skills, manual skills still remain the most important skill type for low educated males. This is true for almost all initial skill types.

In going forward, this paper has only shed light on a small part of the picture. If we want to increase wages of low skilled workers we should invest in their skills. The results suggest that interpersonal skills have become much more important for this group than they were before but that manual skills remain clearly the most important. Thus we conclude that it would be helpful for these workers to invest in manual skills. At the same time, we recognize that it is not clear precisely how to implement this in policy terms. Vocational education or a German apprenticeship type system are possibilities-but how effective they would be is not something we address here. Further progress on these problems would focus on how to invest in skills, incorporation of this model into a general equilibrium framework, and inclusion of other demographic groups.

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## Appendix

## A Datasets description

ORG CPS Wages are calculated using Outgoing Rotation Group data from the Current Population Survey for earnings years 1979-2017 for all male workers aged 20-60 with 12 years of education or less who are not in the military, institutionalized or self-employed. We do the same data trimming as Acemoglu and Autor (2011). Wages are weighted by CPS sample weights. Hourly wages are equal to the logarithm of reported hourly earnings for those paid by the hour and the logarithm of usual weekly earnings divided by hours worked last week for non-hourly workers. Top-coded earnings observations are multiplied by 1.5. Hourly earners of below $\$ 1.675 /$ hour in 1982 dollars are dropped, as are hourly wages exceeding $1 / 35$ th the top-coded value of weekly earnings. All earnings are deflated by the chain-weighted (implicit) price deflator for personal consumption expenditures (PCE). Allocated earnings observations are excluded in all years, except where allocation flags are unavailable. We start from the cohort that left or graduated from high school no latter than 1915 and we end with cohorts that left or graduated from high school no earlier than 2017.

NLSY79 We use the 1979-2015 survey years of the National Longitudinal Survey of Youth, 1979 (NLSY79). The NLSY79 is a representative sample of US households that was administered yearly from 1979-1994 by the Bureau of Labor Statistics, and once every two years since. We use both the core sample and the supplemental sample that over-represents economically disadvantaged respondents and minorities. We reweight observations to have a representative sample. In any given year, we only consider earnings observations for individuals who work 30 or more total hours in a week and who work full time at least 20 of the past 24 weeks. We construct measures of labor market experience using the work history file. We define work experience and occupation-specific experience as, respectively, the sum of weeks worked since labor market entry and the sum of weeks worked in a particular occupation since labor market entry. ${ }^{15}$

O*NET We use O*NET to obtain data on the skill intensity of different occupations. It is a representative survey of occupations developed by the U.S. Department of Labor.

[^13]Individuals were asked to complete a survey asking about the tasks and activities workers perform in those occupations.

NLSY97 We follow Deming (2017) and combine NLSY79 and NLSY97. We restrict the sample to ages $25-33$ to exploit the overlap in ages across surveys. This means comparing the returns to different skills for individuals of similar ages during the late 1980s and early 1990s, compared to the more recent 2004-2015 period. We use respondents' standardized scores on the Armed Forces Qualifying Test (AFQT) to proxy for cognitive skill as in Altonji et al. (2012). And following Deming (2017), we construct a measure of social skills to maximize the comparability of the two measures of social skills across NLSY waves. All test scores are normalized to have mean 0 and standard deviation 1 .

Table A1: Occupation Categories Low Educated Men

|  | Occupations | Label |
| :--- | :--- | :--- |
| 1 | Executive, Administrative, and Managerial | Managers |
|  | Professional Specialty | Clerical |
| 2 | Technicians <br>  <br>  <br>  <br> Sales |  |
| 3 | Housekeeping and Cleaning | Services |
|  | Protective Service <br> Other Services |  |
| 4 | Farming, Forestry, and Fishing <br>  <br>  <br> Machine Operators, Assemblers, Inspectors | Operators |
| 5 | Mechanics and Repairers | Mechanics |
| 6 | Construction Trades, Extractive | Construction |
| 7 | Precision Production | Production |
| 8 | Transportation and Material Moving | Transportation |

## B Occupation share in the Census/ACS

We use the Census to assess the robustness of our results on the evolution of occupation composition and the share of men not-working that we reported in the main text using CPS data.

We use data from the 1980 , 1990, 2000 Census and the 2001-2016 American Community Survey (ACS). We include all males with at most 12 years of education between the ages of 20 and 60 . We exclude individuals in the military. The number of observations range between 284,400 in 2002 and 3,816,849 in 2000.

Figure B1 reports the share not-working in the Census and in the CPS.

Figure B1: Share not-working by year: Census and CPS


The trend is about similar in both sample with a rise until 2010 and a decrease since. However, the level is about 3 percentage point higher in the Census since 2010.

Figure B2 reports the occupational composition in the CPS and in the Census for all relevant years.

Overall, the numbers are reassuringly close to each other. Between 1982 and 1983, there is an apparent discontinuity in the proportions of managers and the proportions of clerical in the CPS data. It can be attributed to the change in the occupational classification scheme. Up to 1982, occupations were coded using the 1970 Census classification scheme. In 1983 (and up to 1991), occupations were coded using the 1980 Census classification scheme.

To smooth out the discontinuity, we assume the CPS data have a constant share of manager misclassified as clerical worker for the year between 1979 and 1982. To recover the bias in the CPS data, we assume the proportion of managers in 1980 is measured without error in the Census. The line "CPS-corrected" reports the corrected occupational share.

Figure B2: Occupational composition: Census and CPS


- cps - cps with correction
census


## C Probability of going to college by cohort

We allow labor market endowments to differ across cohorts. Figure C1 reports the probability of going to college by year-of-birth in the Census and ACS. We report two different measures. The first measure simply calculates the fraction of the sample that goes to college in the census data for all men aged between 20 and 60 . To calculate the second measure, we first regress a dummy variable taking the value one for individuals that went to college and zero otherwise. The regressors are a set of dummies for each age and for each cohort. We then report the cohort fixed effects.

Figure C1: Probability of going to college by cohort


## D Identification

## Notation/Assumptions

We use $(i, t)$ to represent a given observation at a point in time.
There are a total of $L$ types and we assume this number is known. In practice we could easily relax this to assume that the maximum number is known and also suspect the results would go through if it were not known (but still finite). Since each individual $i$ is of one type, we let $\ell_{i} \in\{1, \ldots, L\}$ denote that type.

For simplicity we focus on the case in which all jobs are available for all $\mathcal{S}_{i j t}$ in all periods. This is not necessary but simplifies the exposition as we do not need to worry about all of these possibilities.

We let observed wages be

$$
\begin{equation*}
\widetilde{w}_{i t}=w\left(j, \mathcal{S}_{i t}\right)+u_{i t} \tag{8}
\end{equation*}
$$

where $u_{i t}$ is the measurement error which has distribution $G^{u}$ and characteristic function $\psi^{u}$. We assume that this characteristic function does not vanish.For notational convenience we denote $\widetilde{w}_{i t}=0$ when the individual is non-employed $\left(j_{i t}=0\right)$.

We assume that for each $j$, the tastes for occupations $\vartheta_{j}+\nu_{i j t}$ is i.i.d. across time and independent of the tastes for other jobs, but do not restrict the distribution $G_{j}^{\nu}$ to a parametric form and allow it to differ across jobs. We assume it has full support across the real line so that all jobs available have some mass of workers and normalize its mean value for non-employment to be zero. We also define $G^{\nu} \equiv\left(G_{0}^{\nu}, \ldots, G_{J}^{\nu}\right)$.

Similarly, we do not make a parametric assumption about the distribution of $\chi_{i j t}$ but as in the text continue to assume it is iid across time with distribution $G^{\chi}$. We normalize the value of not searching to zero and assume the distribution of the remaining $\chi_{i \kappa t}$ has full support.

An important part of our notation is to use $X_{i t}$ to denote $i$ 's labor market history at the end time $t$. That is if individual $i$ is a member of a cohort that enters the labor market at time $t$ then $X_{i t}=\left\{j_{i t}\right\}$ and

$$
\begin{equation*}
X_{i t+d}=\left\{j_{i t}, \ldots, j_{i t+d}\right\} \tag{9}
\end{equation*}
$$

Recall that our state vector is

$$
\begin{equation*}
\mathcal{S}_{i t}=\left(\theta_{i t}, a_{i t}, \tau_{i t}, j_{i t-1}, t\right) \tag{10}
\end{equation*}
$$

Our definition of $X_{i t}$ is broad enough that it contains all of the information in state variables
$\left(a_{i t}, \tau_{i t}, j_{i t-1}\right)$. Note as well that human capital is perfectly determined by initial human capital and $X_{i t}$ from the assumption $\theta_{i t+1}^{l}=d\left(\theta_{i t}^{l}, j_{i t}\right)$ (which is more general than our parametric model). Thus the state variables are perfectly described by type $\ell_{i}$, job history $X_{i t}$, and current time $t$. Since $\theta_{i t}$ is not observed by the econometrician, but $X_{i t}$ is, it is also useful to define the following objects (abusing notation somewhat):

$$
\begin{align*}
\omega_{t}\left(\ell_{i}, X_{i t}\right) & \equiv w\left(j, \mathcal{S}_{i t}\right)  \tag{11}\\
f_{t}\left(\ell_{i}, X_{i t}\right) & \equiv f_{j t}\left(\mathcal{S}_{i t}\right)  \tag{12}\\
\mu_{t}\left(\ell_{i}, X_{i t}\right) & \equiv \mu_{i t} \tag{13}
\end{align*}
$$

It is also useful to define

$$
\begin{align*}
V_{t}^{1}\left(\ell, x_{t-1}\right) & \equiv E_{\chi}\left(V^{1}\left(\mathcal{S}_{i t-1}, \chi_{i t}\right) \mid \ell_{i}=\ell, X_{i t-1}=x_{t-1}\right)  \tag{14}\\
V_{t}^{3}\left(\ell, x_{t-1}, \kappa\right) & \equiv E_{v}\left(V^{3}\left(\mathcal{S}_{i t}, \mathcal{B}_{i t}, v_{i t}\right) \mid \ell_{i}=\ell, X_{i t-1}=x_{t-1}, \kappa \in \mathcal{B}_{i t} \backslash j_{i t-1}\right)  \tag{15}\\
\Pi_{t}\left(\ell, x_{t-1}\right) & \equiv E\left(\Pi_{j t}\left(\mathcal{S}_{i t}\right) \mid \ell_{i}=\ell, X_{i t-1}=x_{t-1}\right) \tag{16}
\end{align*}
$$

and will also use the convention $V_{t}^{3}\left(\ell, x_{t-1}, 0\right)$ to denote the state of the world where the only job options are $j_{i t-1}$ and 0 .

We use the term generic identification in the same sense as Blume et al. (2015). We mean that they are typically over-identified in the sense of more nonlinear equations than unknowns but we are not ruling out very special cases that can be constructed in which they are not identified.

## Step 1: Identification of distribution of wages and occupational mobility by cohort and distribution of measurement error

To give the intuition of the result we first discuss a simple case and then we later generalize to a more complicated model and formalize. In the simple case we study a single cohort in which individuals live for two periods, there are two occupations $a$ and $b$. We abstract from $t$ as it will be collinear with experience for this cohort. Since non-employment is a third labor force status there are $3^{2}=9$ different labor market sequence. For each of the 8 patterns that involve work, there are $L$ different wage patterns (though most involve at least one period of non-employment).

We first identify the distribution of the measurement error by conditioning on one particular sequence, say $a$ in period 1 and $b$ in period 2 . We can observe the full two dimensional
distribution of wages in the two periods-but this depends on a single one dimensional object, the measurement error, and a finite number of parameters: the conditional probability of each of the $L$ types, each of their wage in period one, and each of their wages in period 2. As we show formally below, this is substantially over-identified and the measurement error distribution is identified.

Given that we know the measurement error we can uncover the $L$ different wage patterns for each of the 8 different patterns. We can also identify the conditional distribution of the $\ell$ types. Note that for each labor market sequence this gives us the $L$ different wage sequences. However, it does not connect them to each. That is if $L=2$ for the $a, b$ sequence we can identify the two wage sequences say

$$
\begin{equation*}
\left\{\left(\omega_{t}\left(\ell^{1}, a\right), w_{t+1}\left(\ell^{1},\{a, b\}\right)\right),\left(\omega_{t}\left(\ell^{2}, a\right), w_{t+1}\left(\ell^{2},\{a, b\}\right)\right)\right\} \tag{17}
\end{equation*}
$$

and for the $b, a$ sequence we can identify 2 more sequences

$$
\begin{equation*}
\left\{\left(\omega_{t}\left(\ell^{3}, b\right), w_{t+1}\left(\ell^{3},\{b, a\}\right)\right),\left(\omega_{t}\left(\ell^{4}, b\right), w_{t+1}\left(\ell^{4},\{b, a\}\right)\right)\right\} . \tag{18}
\end{equation*}
$$

However, we can not match them. That is, at this point, we don't know whether $\ell^{1}=\ell^{3}$ or $\ell^{1}=\ell^{4}$.

We can avoid this problem with sequences that begin with the same starting occupation. That is, suppose the third sequence we considered was the $a, a$ sequence and it gave

$$
\begin{equation*}
\left.\left.\left\{\left(\omega_{t}\left(\ell^{1}, a\right\}\right), w_{t+1}\left(\ell^{1},\{a, a\}\right)\right),\left(\omega_{t}\left(\ell^{2}, a\right\}\right), w_{t+1}\left(\ell^{2},\{a, a\}\right)\right)\right\} . \tag{19}
\end{equation*}
$$

The first wage corresponds to the wage $\omega_{t}\left(\ell^{1}, a\right)$ for both our first case (equation 17) and the third (equation 19) So as long as $\omega_{t}\left(\ell^{1}, a\right) \neq \omega_{t}\left(\ell^{2}, a\right)$ we can match these sequences since if $\ell^{3}=\ell^{1}$ then $\omega_{t}\left(\ell^{1}, a\right)=\omega_{t}\left(\ell^{3}, a\right)$ and $\omega_{t}\left(\ell^{1}, a\right)=\omega_{t}\left(\ell^{3}, a\right)$. This argument generalizes beyond two periods-what is important is that the first occupation in the first working period is the same.

Note as well that we have identified the distribution of types for each sequence. Since we can match types across sequences conditional on the first occupation worked we can use Bayes theorem to calculate the subsequent conditional choice probabilities for each type across occupations.

We now make this argument again more formally for a more general case with an arbitrary number of periods and occupations.

We begin by showing we can identify the distribution of the measurement error. We do this by considering a single cohort in the first two periods. We condition on individuals in that cohort that have any particular job sequence in the first two periods (in which neither of these correspond to non-employment). We (arbitrarily) choose the sequence $\{a, b\}$. Formally we are conditioning on a set of individuals for which $X_{i t+1}=\{a, b\}$. Under the conditions above there will be $L$ types in this sequence. The econometrician observed these with i.i.d. measurement error $u_{i t}$. We observe the full joint distribution of $\left(\widetilde{w}_{i t}, \widetilde{w}_{i t+1}\right)$ conditional on $X_{i t+1}=\{a, b\}$. From this, the joint distribution of $\left(\omega_{t}(\ell, a), \omega_{t+1}(\ell,\{a, b\})\right)$ is over identified. To see this consider the ratio of the characteristic function of the joint distribution of $\left(\widetilde{w}_{i t}, \widetilde{w}_{i t+1}\right)$ conditional on $X_{i t+1}=\{a, b\}$ divided by the characteristic function of the two conditional marginal distributions. We use the notation $E_{a b}$ and $P r_{a b}$ as short hand for the conditional expectation and probability conditioning on $X_{i t+1}=\{a, b\}$ ). We also use the notation $\iota \equiv \sqrt{-1}$.

$$
\begin{align*}
& \frac{E_{a b} \exp \left(\iota\left(s_{1} \widetilde{w}_{i t}+s_{2} \widetilde{w}_{i t+1}\right)\right)}{E_{a b} \exp \left(\iota\left(s_{1} \widetilde{w}_{i t}\right)\right) E_{a b}\left(\exp \left(\iota\left(s_{2} \widetilde{w}_{i t+1}\right)\right)\right)} \\
= & \frac{E_{a b}\left(\exp \left(\iota\left(s_{1}\left[\omega_{t}\left(\ell_{i}, a\right)+u_{i t}\right]+s_{2}\left[\omega_{t+1}\left(\ell_{i},\{a, b\}\right)+u_{i t+1}\right]\right)\right)\right)}{E_{a b} \exp \left(\iota s_{1}\left[\omega_{t}\left(\ell_{i}, a\right)+u_{i t}\right]\right) E_{a b} \exp \left(\iota s_{2}\left[\omega_{t+1}\left(\ell_{i},\{a, b\}\right)+u_{i t+1}\right]\right)} \\
= & \frac{E_{a b} \exp \left(\iota\left(s_{1} \omega_{t}\left(\ell_{i}, a\right)+s_{2} \omega_{t+1}\left(\ell_{i},\{a, b\}\right)\right)\right) \psi^{u}\left(s_{1}\right) \psi^{u}\left(s_{2}\right)}{E_{a b} \exp \left(\iota s_{1} \omega_{t}\left(\ell_{i}, a\right)\right) E_{a b}\left(\exp \left(\iota s_{2} \omega_{t+1}\left(\ell_{i},\{a, b\}\right)\right)\right) \psi^{u}\left(s_{1}\right) \psi^{u}\left(s_{2}\right)} \\
= & \frac{\sum_{\ell} \exp \left(\iota\left(s_{1} \omega_{t}(\ell, a)+s_{2} \omega_{t+1}(\ell,\{a, b\})\right)\right) P r_{a b}\left(\ell_{i}=\ell\right)}{\left[\sum_{\ell} \exp \left(\iota\left(s_{1} \omega_{t}(\ell, a)\right)\right) P r_{a b}\left(\ell_{i}=\ell\right)\right]\left[\sum_{\ell} \exp \left(\iota\left(s_{2} \omega_{t+1}(\ell,\{a, b\})\right)\right) P r_{a b}\left(\ell_{i}=\ell\right)\right]} . \tag{20}
\end{align*}
$$

In this case our model is clearly generically over-identified as we have a continuum of equations: this will be true for all $\left(s_{1}, s_{2}\right)$, but only $3 L-1$ unknowns: $L$ values of $\omega_{t}(\ell, a), L$ values of $\omega_{t+1}(\ell,\{a, b\})$, and $L$ values of $\operatorname{Pr}_{c}\left(\ell_{i}=\ell\right)$ (which must add up to one). ${ }^{16}$

After identifying this distribution we can also identify the distribution of the measurement error:

$$
\begin{equation*}
\psi^{u}\left(s_{1}\right)=\frac{E_{c} \exp \left(\iota\left(s_{1} \widetilde{w}_{i t}\right)\right)}{\sum_{\ell} \exp \left(i\left(s_{1} \omega_{t}(\ell, a)\right)\right) P r_{c}\left(\ell_{i}=\ell\right)} . \tag{21}
\end{equation*}
$$

Next consider any cohort who enters the labor market at time $t$ and any history $x_{t, t+d}$ that involves work at some point (up until retirement). We need some additional notation to refer to components of $x_{t, t+d}$. We use the notation $j\left(\tau, x_{t, t+d}\right)$ to denote the labor force status in the $\tau^{t h}$ period of this sequence and $x_{t, t+d}^{\tau}$ to denote the first $\tau$ elements (i.e. the

[^14]relevant history for period $\tau$ ). We can identify the characteristic function
\[

$$
\begin{align*}
& E\left(\exp \left(\iota \sum_{\tau=1}^{d+1} s_{\tau} 1\left[j\left(\tau+1, x_{t, t+d}\right)>0\right] \widetilde{w}_{i t+\tau-1}\right) \mid X_{i t+d}=x_{t+d}\right)  \tag{22}\\
= & E\left(\exp \left(\iota \sum_{\tau=1}^{d} s_{\tau} 1\left[j\left(\tau, x_{t, t+d}\right)>0\right] \omega_{t+\tau-1}\left(\ell_{i}, x_{t, t+d}^{\tau}\right)\right) \mid X_{i t+d}=x_{t, t+d}\right) \times \\
& \prod_{\tau=1}^{d} \psi^{u}\left(s_{\tau} 1\left[j\left(\tau, x_{t, t+d}\right)>0\right]\right) \\
= & {\left[\sum_{\ell} \exp \left(\iota \sum_{\tau=1}^{d} s_{\tau} 1\left[j\left(\tau, x_{t, t+d}\right)>0\right] \omega_{t+\tau-1}\left(\ell, x_{t, t+d}^{\tau}\right)\right) \operatorname{Pr}\left(\ell_{i}=\ell \mid X_{i t+d}=x_{t, t+d}\right)\right] \times } \\
& \prod_{\tau=1}^{d} \psi^{u}\left(s_{\tau} 1\left[j\left(\tau, x_{t, t+d}\right)>0\right]\right) . \tag{23}
\end{align*}
$$
\]

Since $\psi^{u}$ and the left hand side are identified this means that t for each $\ell=1, \ldots, L$ up to labelling we can identify $\operatorname{Pr}\left(\ell_{i}=\ell \mid X_{i t+d}=x_{t, t+d}\right)$ and the sequence of wages $\left(\omega_{t}\left(\ell, x_{t, t+d}^{1}\right), \ldots, \omega_{t+d}\left(\ell_{i}, x_{, t+d}^{d}\right)\right.$ (normalizing the wage to be zero when people don't work).

At this point we have identified the distribution of $\ell_{i}$ and wage sequence conditional on any $X_{i t+d}=x_{t, t+d}$ up to labelling, but have not shown we can connect the labels across sequences. We next show that we can connect within sets for which the initial job (excluding non-employment) within the sequence is the same. That is define

$$
\mathcal{X}_{t}(\tau, \kappa)=\left(x_{t, t+d}: j\left(s, x_{t, t+d}\right)=\left\{\begin{array}{ll}
0 & s<\tau  \tag{24}\\
\kappa & s=\tau
\end{array}\right)\right.
$$

This is the set of histories for people who enter the labor market at time $t$, take their first job in their $\tau^{t h}$ working period and that job is occupation $\kappa$. For any history in this set, the first observed wage for type $\ell$ will be the same. For example for $\tau=2$ and $\kappa=a$ it will be $\omega(\ell,\{0, a\})$. Assuming that we have $L$ distinct values, we can define the labelling by the order of this initial wage across $\ell$. Once we have done this, we can identify the wage along any sequence in this set at any point in time for type $\ell$.

We can also identify the conditional choice probabilities. That is for any $x_{t, t+d}$ (prior to the retirement period) in this set, $\left\{x_{t, t+d}, j\right\}$ for $j=0, \ldots, J$ must also be in the set. Since we can identify $\operatorname{Pr}\left(\ell_{i}=\ell \mid X_{i t+d}=x_{t, t+d}\right)$ for all of the different $\ell$, for any value of $\ell$ we can
identify the conditional choice probability

$$
\begin{aligned}
& \operatorname{Pr}\left(j_{i t+d+1}=\kappa \mid \ell_{i}=\ell, X_{i t+d}=x_{t, t+d}\right) \\
& \quad=\frac{\operatorname{Pr}\left(\ell_{i}=\ell \mid X_{i t+d}=x_{t, t+d}, j_{i t+d+1}=\kappa\right) \operatorname{Pr}\left(j_{i t+d+1}=\kappa \mid X_{i t+d}=x_{t, t+d}\right)}{\sum_{k=0}^{J} \operatorname{Pr}\left(\ell_{i}=\ell \mid X_{i t+d}=x_{t, t+d}, j_{i t+d+1}=k\right) \operatorname{Pr}\left(j_{i t+d+1}=k \mid X_{i t+d}=x_{t, t+d}\right)} .
\end{aligned}
$$

## Step 2: Generic Identification of $G^{\nu}$ and $G^{\chi}, R, c_{j t}$, and $f_{j t}$

We focus on a simple case in which workers only work for three periods and there are two jobs $a$ and $b$. We focus on the final period for identification initially, so since this period is $t$, we assume that they enter the labor market at time $t-2$. All of the arguments should carry over for more general cases, but we focus on this one for simplicity.

Typically non-parametric identification of distributions require strong support conditions. This is clear in a simple binary choice model. If $\operatorname{Pr}(D=1 \mid X)=F\left(X^{\prime} \beta\right)$, even if $\beta$ were known and we want to non-parametrically identify $F(\cdot)$, if the support of $X^{\prime} \beta$ were bounded above by $x^{u}$, the shape of $F$ would not be identified above $x^{u}$. For this reason we will proceed assuming full support conditions on the joint distribution of wages and of labor market tightness proxied by $\alpha\left(\mu_{t}\left(\ell_{i}, X_{i t}\right)\right)$.

## Identification of distributions of $\nu_{i 0 t}-\nu_{i a t}$ and $\nu_{i 0 t}-\nu_{i b t}$

We first condition on workers who worked in occupation $a$ during the first two periods (i.e. $j_{i t-2}=j_{i t-1}=a$ ). Note that this particular choice is arbitrary-the argument would work for any path that involves work during the second period. We have shown in Step 1 that we can identify the distribution of types and the time $t$ and the wages associated with each type in the last time period $\left.\omega_{t}(\ell,\{a, a, a\}), \omega_{t}(\ell,\{a, a, b\})\right)$ as well as the conditional choice probability $\operatorname{Pr}\left(j_{i t}=\kappa \mid \ell_{i}=\ell, X_{i t-1}=\{a, a\}\right)$. Intuitively the advantage of looking at this group is twofold. First, by looking at people in the last period we do not need to worry about the continuation value. Second we will focus on the decision to move from work to not working. This simplifies the analysis because we do not need to worry about the application or matching procedures-people who currently have a job always have the option of whether to keep it or move to non-employment.

Now looking at this both across cohorts and types, given the support conditions, we can condition on cohorts for which $\operatorname{Pr}\left(j_{i t}=b \mid \ell_{i}=\ell, X_{i t-1}=\{a, a\}\right)$ is arbitrarily close to zero (which can either result from low wages or low arrival rates but we do not need to worry
about why). Then using our choice model

$$
\begin{equation*}
\operatorname{Pr}\left(j_{i t}=a \mid \ell_{i}=\ell, X_{i t-1}=\{a, a\}\right) \approx \operatorname{Pr}\left(\nu_{i 0 t}-\nu_{i a t} \leq \omega_{t}(\ell,\{a, a, a\})\right) \tag{25}
\end{equation*}
$$

which is the $C D F$ of $v_{i 0 t}-v_{i 1 t}$ evaluated at $\omega_{t}(\ell,\{a, a, a\})$ (and we have assumed people keep working if indifferent). By varying $\omega_{t}(\ell,\{a, a, a\})$ we identify this cdf.

The analogous argument (i.e. conditioning on $j_{i t-1}=b$ ) gives the cdf of $v_{i 0 t}-v_{i b t}$.

Identification of $\mathbf{G}^{\chi}$ We next discuss identification of the cost of applying for jobs. We continue to focus on the final period for simplicity, but now condition on $X_{i t-1}=\{a, 0\}$. The advantage of this group is that since they worked in the first period we can condition on type from Step 1 and can look at their decision to move from non-work to work. Since they are unemployed in the second period, the preference to become re-employed conditional on an offer depends on $\nu_{i 0 t}-\nu_{i a t}$ and $\nu_{i 0 t}-\nu_{i b t}$. A complication is separating the probability of applying for a job with receiving one. We will essentially use the support condition on the probability of getting a job in $a$ and $b$ to make them sufficiently high that one can always get a job.

Now consider the problem of the worker with history $X_{i t-1}=\{a, 0\}$ who is trying to decide whether to apply for a job in occupation $\kappa$ (where $\kappa=a$ or $b$ ). In this case if the job search were successful, the value function in the second sub-period would be

$$
\begin{align*}
V_{t}^{3}(\ell,\{a, 0\}, \kappa) & =E_{v} \max \left\{\omega_{t}(\ell,\{a, 0, \kappa\})+\nu_{i \kappa t}, \nu_{i 0 t}\right\} \\
& =E_{v} \max \left\{\omega_{t}(\ell,\{a, 0, \kappa\})+\nu_{i \kappa t}-\nu_{i 0 t}, 0\right\}+E_{v}\left(\nu_{i 0 t}\right) \tag{26}
\end{align*}
$$

which is identified for $\kappa \in\{a, b\}$ since it depends only on the distribution of $\nu_{i 0 t}-\nu_{i \kappa t}$ and wages for which we have shown identification (and $E_{v}$ ( $\nu_{i 0 t}$ has been normalized to zero). Then the individual searches for a job in occupation $a$ if

$$
\begin{equation*}
\alpha\left(\mu_{i a t}\right) V_{t}^{3}\left(\ell_{i},\{a, 0\}, a\right)-\chi_{i a t}>\max \left\{\alpha\left(\mu_{i b t}\right) V_{t}^{3}(\ell,\{a, 0\}, b)-\chi_{i b t}, 0\right\} . \tag{27}
\end{equation*}
$$

Let $\widetilde{G}^{\chi}$ be the cdf of $\left(\chi_{i a t}, \chi_{i a t}-\chi_{i b t}\right)$ then the conditional probability of applying to job $a$
is

$$
\begin{align*}
& \operatorname{Pr}\left(\alpha\left(\mu_{i a t}\right) V_{t}^{3}\left(\ell_{i},\{a, 0\}, a\right)-\chi_{i a t}>\max \left\{\alpha\left(\mu_{i b t}\right) V_{t}^{3}\left(\ell_{i},\{a, 0\}, b\right)-\chi_{i b t}, 0\right\} \mid \ell_{i}=\ell, X_{i t-1}=\{a, 0\}\right) \\
= & \operatorname{Pr}\left(\chi_{i a t} \leq \alpha\left(\mu_{i a t}\right) V_{t}^{3}\left(\ell_{i},\{a, 0\}, a\right),\right. \\
& \left.\chi_{i a t}-\chi_{i b t} \leq \alpha\left(\mu_{i a t}\right) V_{t}^{3}\left(\ell_{i},\{a, 0\}, a\right)-\alpha\left(\mu_{i b t}\right) V_{t}^{3}\left(\ell_{i},\{a, 0\}, b\right) \mid \ell_{i}=\ell, X_{i t-1}=\{a, 0\}\right) \\
= & \widetilde{G}^{\chi}\left(\alpha\left(\mu_{t}(\ell,\{a, 0, a\})\right) V_{t}^{3}(\ell,\{a, 0\}, a),\right. \\
& \left.\alpha\left(\mu_{t}(\ell,\{a, 0, a\})\right) V_{t}^{3}(\ell,\{a, 0\}, a)-\alpha\left(\mu_{t}(\ell,\{a, 0, b\})\right) V_{t}^{3}(\ell,\{a, 0\}, b)\right) . \tag{28}
\end{align*}
$$

Thus

$$
\begin{align*}
& \operatorname{Pr}\left(j_{i t}=a \mid \ell_{i}=\ell, X_{i t-1}=\{a, 0\}\right) \\
= & \widetilde{G}^{\chi}\left(\alpha\left(\mu_{t}(\ell,\{a, 0, a\})\right) V_{t}^{3}(\ell,\{a, 0\}, a)\right. \\
& \left.\alpha\left(\mu_{t}(\ell,\{a, 0, a\})\right) V_{t}^{3}(\ell,\{a, 0\}, a)-\alpha\left(\mu_{t}(\ell,\{a, 0, b\})\right) V_{t}^{3}(\ell,\{a, 0\}, b)\right) \times \\
& \alpha\left(\mu_{t}(\ell,\{a, 0, a\})\right) \operatorname{Pr}\left(\nu_{i 0 t}-\nu_{i a t} \leq \omega_{t}(\ell,\{a, 0, a\})\right) . \tag{29}
\end{align*}
$$

Note further from looking at this expression that there is no tradeoff with $\alpha\left(\mu_{t}(\ell,\{a, 0, a\})\right)-$ as it increases this increases both the probability of getting a job in $a$ and the probability of applying for a job in $a$. The analogous argument works for moving to $b$. Therefore if we condition on the two wages $\omega_{t}(\ell,\{a, 0, a\})$ and $\omega_{t}(\ell,\{a, 0, b\})$ as we look at the state of the world in which the probability of staying in non-employment is minimized, this must be the state of the world in which $\alpha\left(\mu_{i a t}\right) \approx \alpha\left(\mu_{i b t}\right) \approx 1$. By conditioning on this we can identify the simplified expression

$$
\begin{equation*}
\widetilde{G}^{\chi}\left(V_{t}^{3}(\ell,\{a, 0\}, a), V_{t}^{3}(\ell,\{a, 0\}, a)-V_{t}^{3}(\ell,\{a, 0\}, b)\right) \operatorname{Pr}\left(\nu_{i 0 t}-\nu_{i a t} \leq \omega_{t}(\ell,\{a, 0, a\})\right) \tag{30}
\end{equation*}
$$

but since $\operatorname{Pr}\left(\nu_{i 0 t}-\nu_{i a t} \leq \omega_{t}(\ell,\{a, 0, a\})\right)$ is identified, we can identify $\widetilde{G}^{\chi}$ by varying $V_{t}^{3}(\ell,\{a, 0\}, a)$ and $V_{t}^{3}(\ell,\{a, 0\}, b)$ which allows us to identify $\widetilde{G}^{\chi}$. From this, $G^{\chi}$ is identified because from the joint distribution of $\left(\chi_{i a t}, \chi_{i a t}-\chi_{i b t}\right)$ one can identify the joint distribution of $\left(\chi_{i a t}, \chi_{i b t}\right)$.

## Identification of capital costs and matching parameter $B$

We continue to focus on individuals with a history of $\{a, 0\}$ but and now focus on the firms in occupation $a$ who potentially hire these workers.

First note that the values of $\alpha\left(\mu_{t}(\ell,\{a, 0, a\})\right)$ and $\alpha\left(\mu_{t}(\ell,\{a, 0, b\})\right)$ are identified from equation (29) and the analogous expression for $\operatorname{Pr}\left(j_{i t}=b \mid \ell_{i}=\ell, X_{i t-1}=\{a, 0\}\right)$ as all other
components in these expressions have been identified. We also know that

$$
\begin{equation*}
\alpha\left(\mu_{t}(\ell,\{a, 0, a\})\right)=B\left[\mu_{t}(\ell,\{a, 0, a\})\right]^{1-\eta} . \tag{31}
\end{equation*}
$$

We next consider identification of $f_{t}(\ell,\{a, 0, a\})$. From the first order condition for free entry we know $\omega_{t}(\ell,\{a, 0, a\})$ must be solve

$$
\begin{equation*}
\frac{\partial \operatorname{Pr}\left(\nu_{i 0 t}-\nu_{i a t} \leq \omega_{t}(\ell,\{a, 0, a\})\right)}{\partial \omega_{t}(\ell,\{a, 0, a\})}\left[f_{t}(\ell,\{a, 0, a\})-\omega_{t}(\ell,\{a, 0, a\})\right]=\operatorname{Pr}\left(\nu_{i 0 t}-\nu_{i a t} \leq \omega_{t}(\ell,\{a, 0, a\})\right) . \tag{32}
\end{equation*}
$$

Everything in this model is identified from this expression except for $f_{t}(\ell,\{a, 0, a\})$ so it is generically identified.

From the free entry condition of firms we know that
$\left.B\left[\mu_{t}(\ell,\{a, 0, a\})\right]^{-\eta} \operatorname{Pr}\left(\nu_{i 0 t}-\nu_{i a t} \leq \omega_{t}(\ell,\{a, 0, a\})\right)\left[f_{t}(\ell,\{a, 0, a\})-\omega_{t}(\ell,\{a, 0, a\})\right)\right]=c_{a t}$.

Now as long as we have three different values of $\ell$, we have six expressions: Equations (31) and (33) at two different values of $\ell$. We have five unknowns: $B, c_{a t}$, and three values of $\mu_{t}(\ell,\{a, 0, a\})$ (since $\eta$ is assumed known and everything else is identified). Thus generically these are all identified. An analogous argument allows us to identify $c_{b t}$. Note that since these capital costs are identical for all groups they are identified for all $c_{j t}$.

## Identification of marginal distributions of $\nu_{i j t}$.

Now consider individuals whose employment history is $\{a, b\}$. Consider the labor market of these individuals as they choose to potentially move to occupation $a$ or to non-employment.

When they make their decisions about whether to apply for job in $a$, the relevant value functions are

$$
\begin{align*}
V_{t}^{3}(\ell,\{a, b\}, 0) & =E_{v} \max \left\{\omega_{t}(\ell,\{a, b, b\})+\nu_{i b t}, \nu_{i 0 t}\right\} \\
& =E_{v} \max \left\{\omega_{t}(\ell,\{a, b, b\})+\nu_{i b t}-\nu_{i 0 t}, 0\right\} \tag{34}
\end{align*}
$$

and

$$
\begin{align*}
V_{t}^{3}(\ell,\{a, b\}, a) & =E_{v} \max \left\{\omega_{t}(\ell,\{a, b, a\})+\nu_{i a t}, \omega_{t}(\ell,\{a, b, b\})+\nu_{i b t}, \nu_{i 0 t}\right\} \\
& =E_{v} \max \left\{\omega_{t}(\ell,\{a, b, a\})+\nu_{i a t}-\nu_{i 0 t}, \omega_{t}(\ell,\{a, b, b\})+\nu_{i b t}-\nu_{i 0 t}, 0\right\} \tag{35}
\end{align*}
$$

Note that we have shown that the first object is identified since we have identified all of the pieces in (34). We have not for (35) because we have yet to show that the joint distribution of $\left(\nu_{i a t}-\nu_{i 0 t}, \nu_{i a t}-\nu_{i 0 t}\right)$ is identified.

The probability of applying to job $a$ is

$$
\begin{align*}
& \operatorname{Pr}\left(\alpha\left(\mu_{t}(\ell,\{a, b, a\})\right) V_{t}^{3}(\ell,\{a, b\}, a)+\left[1-\alpha\left(\mu_{t}(\ell,\{a, b, a\})\right)\right] V_{t}^{3}(\ell,\{a, b\}, 0)-\chi_{i a t} \geq V_{t}^{3}(\ell,\{a, b\}, 0)\right) \\
= & \operatorname{Pr}\left(\alpha\left(\mu_{t}(\ell,\{a, b, a\})\right)\left[V_{t}^{3}(\ell,\{a, b\}, a)-V_{t}^{3}(\ell,\{a, b\}, 0)\right] \geq \chi_{i a t}\right) \tag{36}
\end{align*}
$$

and thus

$$
\begin{align*}
& \operatorname{Pr}\left(j_{i t}=a \mid \ell_{i}=\ell, X_{i t-1}=\{a, b\}\right) \\
= & \operatorname{Pr}\left(\alpha\left(\mu_{t}(\ell,\{a, b, a\})\right)\left[V_{t}^{3}(\ell,\{a, b\}, a)-V_{t}^{3}(\ell,\{a, b\}, 0)\right] \geq \chi_{i a t}\right) \alpha\left(\mu_{t}(\ell,\{a, b, a\})\right) \times \\
& \operatorname{Pr}\left(\omega_{t}(\ell,\{a, b, a\})+\nu_{i a t}-\nu_{i 0 t} \geq \max \left\{\omega_{t}(\ell,\{a, b, b\})+\nu_{i b t}-\nu_{i 0 t}, 0\right\}\right) \tag{37}
\end{align*}
$$

As above note from (37) that $\operatorname{Pr}\left(j_{i t}=a \mid \ell_{i}=\ell, X_{i t-1}=\{a, 0\}\right)$ is strictly increasing in $\alpha\left(\mu_{t}(\ell,\{a, b, a\})\right)$. Thuy by conditioning on the state of the world where (37) is maximized we are conditioning on the case in which $\alpha\left(\mu_{t}(\ell,\{a, b, a\})\right) \approx 1$.

Further we economize on the notation by defining

$$
\begin{aligned}
\rho_{a}\left(w_{a}, w_{b}\right) & \equiv \operatorname{Pr}\left(w_{a}+\nu_{i a t}-\nu_{i 0 t} \geq \max \left\{w_{b}+\nu_{i b t}-\nu_{i 0 t}, 0\right\}\right) \\
\rho_{b}\left(w_{b}, w_{a}\right) & \equiv \operatorname{Pr}\left(w_{b}+\nu_{i b t}-\nu_{i 0 t} \geq \max \left\{w_{a}+\nu_{i a t}-\nu_{i 0 t}, 0\right\}\right)
\end{aligned}
$$

Thus conditioning one $\alpha\left(\mu_{t}(\ell,\{a, b, a\})\right) \approx 1$, we can identify the following three equations

$$
\begin{align*}
& \operatorname{Pr}\left(j_{i t}=a \mid \ell_{i}=\ell, X_{i t-1}=\{a, 0\}\right) \\
= & \operatorname{Pr}\left(\left[V_{t}^{3}(\ell,\{a, b\}, a)-V_{t}^{3}(\ell,\{a, b\}, 0)\right] \geq \chi_{i a t}\right) \rho_{a}\left(\omega_{t}(\ell,\{a, b, a\}), \omega_{t}(\ell,\{a, b, b\})\right)  \tag{38}\\
& \operatorname{Pr}\left(j_{i t}=b \mid \ell_{i}=\ell, X_{i t-1}=\{a, 0\}\right) \\
= & \operatorname{Pr}\left(\left[V_{t}^{3}(\ell,\{a, b\}, a)-V_{t}^{3}(\ell,\{a, b\}, 0)\right] \geq \chi_{i a t}\right) \rho_{b}\left(\omega_{t}(\ell,\{a, b, b\}), \omega_{t}(\ell,\{a, b, a\})\right) \\
& +\left[1-\operatorname{Pr}\left(\left[V_{t}^{3}(\ell,\{a, b\}, a)-V_{t}^{3}(\ell,\{a, b\}, 0)\right] \geq \chi_{i a t}\right)\right] \operatorname{Pr}\left(\omega_{t}(\ell,\{a, b, b\})+\nu_{i b t}-\nu_{i 0 t} \geq 0\right)  \tag{39}\\
& \operatorname{Pr}\left(j_{i t}=0 \mid \ell_{i}=\ell, X_{i t-1}=\{a, 0\}\right) \\
= & \operatorname{Pr}\left(\left[\gamma_{a \ell t}-\gamma_{0 \ell t}\right] \geq \chi_{i a t}\right)\left[1-\rho_{a}\left(\omega_{t}(\ell,\{a, b, a\}), \omega_{t}(\ell,\{a, b, b\})\right)-\rho_{b}\left(\omega_{t}(\ell,\{a, b, b\}), \omega_{t}(\ell,\{a, b, a\})\right)\right] \\
& +\left[1-\operatorname{Pr}\left(\left[\gamma_{a \ell t}-\gamma_{0 \ell t}\right] \geq \chi_{i a t}\right)\right] \operatorname{Pr}\left(\omega_{t}(\ell,\{a, b, b\})+\nu_{i b t}<\nu_{i 0 t}\right) \tag{40}
\end{align*}
$$

This give 3 equations in the 3 unknowns:

$$
V_{t}^{3}(\ell,\{a, b\}, a), \rho_{a}\left(\omega_{t}(\ell,\{a, b, a\}), \omega_{t}(\ell,\{a, b, b\})\right), \rho_{b}\left(\omega_{t}(\ell,\{a, b, b\}), \omega_{t}(\ell,\{a, b, a\})\right) .
$$

So these are generically identified. By varying the conditional wages we can identify the joint distribution of $\left(\nu_{i a t}-\nu_{i 0 t}, \nu_{i b t}-\nu_{i 0 t}\right)$ and thus the three marginal distributions. ${ }^{17}$

Identification of Discount Factor $R, \mu_{t}$, and $f_{t}$
We first consider identification of $f_{t}\left(\ell, x_{t}\right)$ and $\mu_{t}\left(\ell, x_{t}\right)$ in the final period and then work backwards. Note that in there are $3^{3}=27$ different labor market histories. Of these 27, 9 involve not working in the final period, so for each $\ell$, there are 18 relevant values of $f_{t}\left(\ell, x_{t}\right)$. Furthermore, $\mu_{t}\left(\ell, x_{t}\right)$ is only relevant for workers moving from one status in period $t-1$ to a different working status in period $t$. Thus of the 18 histories working in the third period, 6 involve remaining at the same occupation between periods $t-1$ and thus there are 12 relevant values of $\mu_{t}\left(\ell, x_{t}\right)$ in each $t$ for each cohort entering at $t-2$ for each $\ell$.

We start with the case in which people who are coming from non-employment in year $t-1$ which is analogous to equation (32). We generalize to any $j_{t-2} \in\{0, a, b\}$ and $j_{t} \in\{a, b\}$.We know

$$
\begin{equation*}
\frac{\partial \operatorname{Pr}\left(\nu_{i 0 t}-\nu_{i \kappa t} \leq \omega_{t}\left(\ell,\left\{j_{t-2}, 0, j_{t}\right\}\right)\right)}{\partial \omega_{t}\left(\ell,\left\{j_{t-2}, 0, j_{t}\right\}\right)}\left[f_{t}\left(\ell,\left\{j_{t-2}, 0, j_{t}\right\}\right)-\omega_{t}\left(\ell,\left\{j_{t-2}, 0, j_{t}\right\}\right)\right]=\operatorname{Pr}\left(\nu_{i 0 t}-\nu_{i \kappa t} \leq \omega_{t}\left(\ell,\left\{j_{t-2}, C\right.\right.\right. \tag{41}
\end{equation*}
$$

which identifies $f_{t}\left(\ell,\left\{\kappa_{0}, 0, \kappa\right\}\right)$ since everything else in (41) is identified.
The $\mu_{t}\left(\ell,\left\{j_{t-2}, a, \kappa\right\}\right)$ is then identified from the free entry condition:

$$
\left.B\left[\mu_{t}\left(\ell,\left\{j_{t-2}, a, j_{t}\right\}\right)\right]^{-\eta} \operatorname{Pr}\left(\nu_{i 0 t}-\nu_{i \kappa t} \leq \omega_{t}\left(\ell,\left\{j_{t-2}, 0, j_{t}\right\}\right)\right)\left[f_{t}\left(\ell,\left\{j_{t-2}, a, j_{t}\right\}\right)-\omega_{t}\left(\ell,\left\{j_{t-2}, a, j_{t}\right\}\right)\right)\right]=c_{j_{t} t}
$$

as now everything else in this expression is identified.
This has covered six different histories.
Next consider the case in which the person works in the two different occupations during the second and third period. First consider $\left(j_{t-2}, a, b\right)$ for $\kappa_{0} \in\{0, a, b\}$. The condition

[^15]determining wages is
\[

$$
\begin{align*}
& \frac{\partial \rho_{b}\left(\omega_{t}\left(\ell,\left\{j_{t-2}, a, b\right\}\right), \omega_{t}\left(\ell,\left\{j_{t-2}, a, a\right\}\right)\right)}{\partial \omega_{t}\left(\ell,\left\{j_{t-2}, a, b\right\}\right)}\left[f_{t}\left(\ell,\left\{j_{t-2}, a, b\right\}\right)-\omega_{t}\left(\ell,\left\{j_{t-2}, a, b\right\}\right)\right] \\
= & \rho_{b}\left(\omega_{t}\left(\ell,\left\{j_{t-2}, a, b\right\}\right), \omega_{t}\left(\ell,\left\{j_{t-2}, a, a\right\}\right)\right) . \tag{42}
\end{align*}
$$
\]

The only unknown here is $f_{t}\left(\ell,\left\{j_{t-2}, a, b\right\}\right)$ so it is identified. Given that, we can identify $\mu_{t}\left(\ell,\left\{j_{t-2}, a, \kappa\right\}\right)$ from the free entry condition:

$$
B\left[\mu_{t}\left(\ell,\left\{j_{t-2}, a, b\right)\right]^{-\eta} \rho_{b}\left(\omega_{t}\left(\ell,\left\{j_{t-2}, a, b\right\}\right), \omega_{t}\left(\ell,\left\{\kappa_{0}, a, a\right\}\right)\right)\left[f_{t}\left(\ell,\left\{j_{t-2}, a, b\right\}\right)-\omega_{t}\left(\ell,\left\{j_{t-2}, a, b\right\}\right)\right]=c_{b t}\right.
$$

The analogous expressions identify $f_{t}\left(\ell,\left\{j_{t-2}, b, a\right\}\right)$ and $\mu_{t}\left(\ell,\left\{j_{t-2}, b, a\right\}\right)$ for $j_{t-2} \in\{0, a, b\}$. This gives us 6 more cases so we have shown identification of 12 of the 18 wage expressions and all of the market tightness.

The final set of cases is for productivity for individuals who work in the same job the final two periods. This is substantially more complicated as the incumbent firm does not know the outside opportunity and the wage also affects the application decision. We focus on the $a$ stayers for any initial status $j_{t-2}$. Define the probability of the worker applying to a job in $b$ as a function of wages and arrival rates of jobs as
$\widetilde{\varrho}_{b}\left(w_{a}, w_{b}, \mu\right) \equiv \operatorname{Pr}\left(\alpha(\mu)\left[E_{v} \max \left\{w_{a}+\nu_{i a t}, w_{b}+\nu_{i b t}, \nu_{i 0 t}\right\}-E_{v} \max \left\{w_{a}+\nu_{i a t}, \nu_{i 0 t}\right\}\right] \geq \chi_{i a t}\right)$
and note that this function is identified. Thus the probability of the firm keeping the worker as a function of $\left(w_{a}, w_{b}, \mu\right)$ is

$$
\begin{aligned}
\varrho_{a}\left(w_{a}, w_{b}, \mu\right) \equiv & \widetilde{\varrho}_{b}\left(w_{a}, w_{b}, \mu\right) \alpha(\mu) \rho_{a}\left(w_{a}, w_{b}\right) \\
& +\left[1-\widetilde{\varrho}_{b}\left(w_{a}, w_{b}, \mu\right) \alpha(\mu)\right] \operatorname{Pr}\left(w_{a}+\nu_{i a t} \geq \nu_{i 0 t}\right) .
\end{aligned}
$$

Then the first order condition of the firm in setting wages is

$$
\begin{aligned}
& \frac{\partial \varrho_{a}\left(\omega_{t}\left(\ell,\left\{\kappa_{0}, a, a\right\}\right), \omega_{t}\left(\ell,\left\{\kappa_{0}, a, b\right\}\right), \mu_{t}\left(\ell,\left\{\kappa_{0}, a, b\right\}\right)\right)}{\partial \omega_{t}\left(\ell,\left\{\kappa_{0}, a, b\right\}\right)}\left[f_{t}\left(\ell,\left\{\kappa_{0}, a, a\right\}\right)-\omega_{t}\left(\ell,\left\{\kappa_{0}, a, a\right\}\right)\right] \\
= & \varrho_{a}\left(\omega_{t}\left(\ell,\left\{\kappa_{0}, a, a\right\}\right), \omega_{t}\left(\ell,\left\{\kappa_{0}, a, b\right\}\right), \mu_{t}\left(\ell,\left\{\kappa_{0}, a, b\right\}\right)\right)
\end{aligned}
$$

thus $f_{t}\left(\ell,\left\{\kappa_{0}, a, a\right\}\right)$ is generically identified for each $\ell$ and $\kappa_{0}$. An analogous argument gives $f_{t}\left(\ell,\left\{\kappa_{0}, b, b\right\}\right)$. There are no terms for $\mu_{t}$ since these workers are already employed in $a$.

This gives the final 6 values (for each $\ell$ ). Thus we have shown all of the relevant $f_{t}$ and
$\mu_{t}$ are identified for individuals in their last period. We next go back one period.
We next show that $R$ is identified. The key part of this expression is $V_{t}^{1}\left(\ell, x_{t-1}\right)$ which is identified (since we have shown that all its components are identified). Consider a worker of type $\ell$ in the second period who worked in a job in $a$ during the first period. Similar to above, we can condition on cohorts for which $\operatorname{Pr}\left(j_{i t-1}=b \mid \ell_{i}=\ell, X_{i t-1}=\{a\}\right) \approx 0$, then
$\operatorname{Pr}\left(j_{i t-1}=a \mid \ell_{i}=\ell, X_{i t-2}=\{a\}\right) \approx \operatorname{Pr}\left(\nu_{i 0 t-1}-\nu_{i a t-1} \leq \omega_{t-1}(\ell,\{a, a\})+\frac{1}{1+R} V_{t}^{1}(\ell,\{a\})\right)$
so since everything from this is identified other than $R$, then $R$ is identified.
Given this we can use exactly the same expressions above and identify the components for the second period $(t-1)$. In this case we have 9 histories, 6 of which involve employment in the second period and 4 of which incorporate a market tightness. We write the equations without the explanations since the explanations are identical.

First consider the cases in which workers are non-employed in the first period and employed in one of the two occupations in the second. The productivity $f_{t-1}\left(\ell,\left\{0, j_{t-1}\right\}\right)$ is identified from the wage setting equation

$$
\begin{align*}
& \frac{\partial \operatorname{Pr}\left(\nu_{i 0 t-1}-\nu_{i j_{t-1} t-1} \leq \omega_{t-1}\left(\ell,\left\{0, j_{t-1}\right\}\right)+\frac{1}{1+R} V_{t}^{1}\left(\ell,\left\{0, j_{t-1}\right\}\right)\right)}{\partial \omega_{t-1}\left(\ell,\left\{0, j_{t-1}\right\}\right)} \times \\
& {\left[f_{t-1}(\ell,\{0,\})-\omega_{t-1}\left(\ell,\left\{0, j_{t-1}\right\}\right)+\frac{1}{1+R}\left(\Pi\left(\ell,\left\{0, j_{t-1}\right\}\right)\right)\right] } \\
= & \operatorname{Pr}\left(\nu_{i 0 t-1}-\nu_{i j_{t-1} t-1} \leq \omega_{t-1}\left(\ell,\left\{0, j_{t-1}\right\}\right)+\frac{1}{1+R} V_{t}^{1}\left(\ell,\left\{0, j_{t-1}\right\}\right)\right) . \tag{43}
\end{align*}
$$

The market tightness is identified from the free entry condition

$$
\begin{aligned}
& B\left[\mu_{t-1}\left(\ell,\left\{0, j_{t-1}\right\}\right)\right]^{-\eta} \operatorname{Pr}\left(\nu_{i 0 t-1}-\nu_{i \kappa t-1} \leq \omega_{t-1}\left(\ell,\left\{0, j_{t-1}\right\}\right)+\frac{1}{1+R} V_{t}^{1}\left(\ell,\left\{0, j_{t-1}\right\}\right)\right) \times \\
& {\left[f_{t-1}\left(\ell,\left\{0, j_{t-1}\right\}\right)-\omega_{t-1}\left(\ell,\left\{0, j_{t-1}\right\}\right)+\frac{1}{1+R}\left(\Pi\left(\ell,\left\{0, j_{t-1}\right\}\right)\right)\right]=c_{j_{t-1} t-1} }
\end{aligned}
$$

The next two cases involve switching from one occupation to the other in the two periods. The productivity $f_{t-1}(\ell,\{a, b\})$ is identified from

$$
\begin{aligned}
& \frac{\partial \rho_{b}\left(\omega_{t-1}(\ell,\{a, b\})+\frac{1}{1+R} V_{t}^{1}(\ell,\{a, b\}), \omega_{t-1}\left(\ell,\{a, a\}+\frac{1}{1+R} V_{t}^{1}(\ell,\{a, a\})\right)\right)}{\partial \omega_{t-1}\left(\ell,\left\{\kappa_{0}, a, b\right\}\right)} \times \\
& {\left[f_{t-1}(\ell,\{a, b\})-\omega_{t-1}(\ell,\{a, b\})+\frac{1}{1+R}(\Pi(\ell,\{a, b\}))\right] } \\
= & \rho_{b}\left(\omega_{t-1}(\ell,\{a, b\})+\frac{1}{1+R} V_{t}^{1}(\ell,\{a, b\}), \omega_{t-1}\left(\ell,\{a, a\}+\frac{1}{1+R} V_{t}^{1}(\ell,\{a, a\})\right)\right)
\end{aligned}
$$

with $f_{t-1}(\ell,\{b, a\})$ from the analogous condition.
The $\mu_{t-1}(\ell,\{a, b\})$ is identified from the free entry condition

$$
\begin{aligned}
c_{b t}= & B\left[\mu_{t-1}(\ell,\{a, b\})\right]^{-\eta} \rho_{b}\left(\omega_{t-1}(\ell,\{a, b\})+\frac{1}{1+R} V_{t}^{1}(\ell,\{a, b\}), \omega_{t-1}\left(\ell,\{a, a\}+\frac{1}{1+R} V_{t}^{1}(\ell,\{a, a\})\right)\right) \times \\
& {\left[f_{t-1}(\ell,\{a, b\})-\omega_{t-1}(\ell,\{a, b\})+\frac{1}{1+R}(\Pi(\ell,\{a, b\}))\right] }
\end{aligned}
$$

with an analogous equation for $\mu_{t-1}(\ell,\{b, a\})$.
Finally we calculate the productivity for the occupation stayers. Productivity $f_{t}(\ell,\{a, a\})$ is identified from

$$
\begin{aligned}
& \frac{\partial \varrho_{a}\left(\omega_{t-1}(\ell,\{a, a\})+\frac{1}{1+R} V_{t}^{1}(\ell,\{a, a\}), \omega_{t-1}\left(\ell,\{a, b\}+\frac{1}{1+R} V_{t}^{1}(\ell,\{a, b\})\right), \mu_{t-1}(\ell,\{a, b\})\right)}{\partial \omega_{t-1}(\ell,\{a, b\})} \times \\
& {\left[f_{t}(\ell,\{a, a\})-\omega_{t}(\ell,\{a, a\})+\frac{1}{1+R}(\Pi(\ell,\{a, a\}))\right] } \\
= & \varrho_{a}\left(\omega_{t-1}(\ell,\{a, a\})+\frac{1}{1+R} V_{t}^{1}(\ell,\{a, a\}), \omega_{t-1}\left(\ell,\{a, b\}+\frac{1}{1+R} V_{t}^{1}(\ell,\{a, b\})\right), \mu_{t-1}(\ell,\{a, b\})\right)
\end{aligned}
$$

with an analogous expression for $f_{t}(\ell,\{b, b\})$.
Finally for the initial period things are simpler as everyone begins in non-employment, so we only have the two cases. The productivity $f_{t-2}\left(\ell, j_{t-2}\right)$ is identified from

$$
\begin{aligned}
& \frac{\left.\partial \operatorname{Pr}\left(\nu_{i 0 t-2}-\nu_{i j_{t-2} t-2} \leq \omega_{t-2}\left(\ell, j_{t-2}\right)+\frac{1}{1+R} V_{t-1}^{1}\left(\ell, j_{t-2}\right\}\right)\right)}{\partial \omega_{t-2}\left(\ell, j_{t-2}\right)} \times \\
& {\left[f_{t-2}\left(\ell, j_{t-2}\right)-\omega_{t-1}\left(\ell, j_{t-2}\right)+\frac{1}{1+R}\left(\Pi\left(\ell, j_{t-2}\right)\right)\right]} \\
& \left.=\operatorname{Pr}\left(\nu_{i 0 t-2}-\nu_{i j_{t-2} t-2} \leq \omega_{t-2}\left(\ell, j_{t-2}\right)+\frac{1}{1+R} V_{t-1}^{1}\left(\ell, j_{t-2}\right\}\right)\right) .
\end{aligned}
$$

The $\mu_{t-2}\left(\ell, j_{t-2}\right)$ is identified from

$$
\begin{aligned}
k_{\kappa t-2}= & \left.B\left[\mu_{t-2}\left(\ell, j_{t-2}\right)\right]^{-\eta} \operatorname{Pr}\left(\nu_{i 0 t-2}-\nu_{i j_{t-2} t-2} \leq \omega_{t-2}\left(\ell, j_{t-2}\right)+\frac{1}{1+R} V_{t-1}^{1}\left(\ell, j_{t-2}\right\}\right)\right) \times \\
& {\left[f_{t-2}\left(\ell, j_{t-2}\right)-\omega_{t-1}\left(\ell, j_{t-2}\right)+\frac{1}{1+R}\left(\Pi\left(\ell, j_{t-2}\right)\right)\right] . }
\end{aligned}
$$

## Step 3: Labeling types across different starting jobs

We have shown everything is identified except that the labeling of types has only been done conditional on starting jobs. In this subsection we discuss how to identify the labeling across jobs and then across cohorts.

First notice that continuing with our framework with two jobs and three periods, there are 7 different separate cases of starting occupations: each of $a$ and $b$ in each of periods $t-2, t-1$, and $t$ as well as the group that never works.

First consider the decision to work in occupation $a$ or $b$ in the first period. From step 1 we know the relative sizes of each of the groups, i.e. $\operatorname{Pr}\left(\ell_{i}=\ell \mid j_{i t-2}=a\right)$ and $\operatorname{Pr}\left(\ell_{i}=\ell \mid\right.$ $\left.j_{i t-2}=b\right)$. Given the arguments above we can also identify the value function $V_{t-2}^{3}\left(\ell, j_{t-2}\right)$ and job arrival rates $\alpha\left(\mu_{t-2}\left(\ell, j_{t-2}\right)\right)$ for each of the $L$ groups that start in $j_{t-2} \in\{a, b\}$. To see why the match is identified, suppose we take group $\ell_{a}$ from the $a$ group and group $\ell_{b}$ from the $b$ group. If $\ell_{b}=\ell_{a}$ then from our model

$$
\begin{align*}
& \operatorname{Pr}\left(j_{i t-2}=a \mid \ell_{i}=\ell_{a}=\ell_{b}\right) \\
= & \widetilde{G}^{\chi}\left(\alpha\left(\mu_{t-2}\left(\ell_{a}, a\right)\right) V_{t-2}^{3}\left(\ell_{a}, a\right), \alpha\left(\mu_{t-2}\left(\ell_{a}, a\right)\right) V_{t-2}^{3}\left(\ell_{a}, a\right)-\alpha\left(\mu_{t-2}\left(\ell_{b}, \kappa\right)\right) V_{t-2}^{3}\left(\ell_{b}, a\right)\right) \times \\
& \operatorname{Pr}\left(\nu_{i 0 t}-\nu_{\text {iat }} \leq \omega_{t}\left(\ell_{a}, a\right)\right) \tag{44}
\end{align*}
$$

which is identified since everything in the model is identified. Similarly we can identify $\operatorname{Pr}\left(j_{i t-2}=b \mid \ell_{i}=\ell_{a}=\ell_{b}\right)$. Thus we can identify their ratio. From Bayes Theorem we know

$$
\begin{equation*}
\frac{\operatorname{Pr}\left(j_{i t-2}=a \mid \ell_{i}=\ell_{a}=\ell_{b}\right)}{\operatorname{Pr}\left(j_{i t-2}=b \mid \ell_{i}=\ell_{a}=\ell_{b}\right)}=\frac{\operatorname{Pr}\left(\ell_{i}=\ell_{a}, j_{i t-2}=a\right)}{\operatorname{Pr}\left(\ell_{i}=\ell_{b}, j_{i t-2}=b\right)} \tag{45}
\end{equation*}
$$

where the left hand side comes from the expression (44) while the two objects on the the right hand side were identified in Step 1. Note that if $\ell_{a}=\ell_{b}$ this equality will hold, however if $\ell_{a} \neq \ell_{b}$, generically it will not. Thus this labelling is generically identified.

Thus we have shown that we can match the $X_{i t-2}=\{a\}$ group with the $X_{i t-2}=\{b\}$ group. We next show that we identify the distribution of these $\ell$ for the $X_{i t-2}=\{0\}$ group.

From the model and information on the conditional distribution of $\ell_{i}$, we can identify the marginal distribution of $j_{i t-2}=0$ and $\ell_{i}=\ell$ from the expression
$\operatorname{Pr}\left(j_{i t-2}=0 \mid \ell_{i}=\ell\right)=\frac{\operatorname{Pr}\left(j_{i t-2}=0, \ell_{i}=\ell\right)}{\operatorname{Pr}\left(j_{i t-2}=0, \ell_{i}=\ell\right)+\operatorname{Pr}\left(j_{i t-2}=a, \ell_{i}=\ell\right)+\operatorname{Pr}\left(j_{i t-2}=b, \ell_{i}=\ell\right)}$
where the left hand side is identified from the model and the two expressions in the denominator apart from $\operatorname{Pr}\left(j_{i t-2}=0, \ell_{i}=\ell\right)$ have been identified from stage 1. Thus $\operatorname{Pr}\left(j_{i t-2}=0, \ell_{i}=\ell\right)$ is identified. Given that from the numerator we can identify the unconditional distribution $\operatorname{Pr}\left(\ell_{i}=\ell\right)$ and the conditional distribution $\operatorname{Pr}\left(\ell_{i}=\ell \mid j_{i t-2}=1\right)$.

Note from the unconditional distribution we can now generically match groups across cohorts since these probabilities will generically be different.

We now proceed to period $t-1$ and consider people who did not work in period $t-2$ and first consider matching the $X_{i t-1}=\{0, a\}$ group with the $X_{i t-1}=\{0, a\}$ group. We have the identical problem as for $X_{i t-2}=\{a\}$ and $X_{i t-2}=\{b\}$. We can calculate the type distribution and model conditional on $X_{i t-1}=\{0, a\}$ and conditional on $X_{i t-1}=\{0, b\}$ but we have not linked them together. We can use the same approach, for any $\ell_{a}$ from $X_{i t-1}=\{0, a\}$ and $\ell_{b}$ from $X_{i t-1}=\{0, b\}$. That is from Bayes theorem we know

$$
\begin{equation*}
\frac{\operatorname{Pr}\left(j_{i t-1}=a \mid \ell_{i}=\ell_{a}=\ell_{b}, j_{i t-2}=0\right)}{\operatorname{Pr}\left(j_{i t-1}=b \mid \ell_{i}=\ell_{a}=\ell_{b}, j_{i t-2}=0\right)}=\frac{\operatorname{Pr}\left(\ell_{i}=\ell_{a}, X_{i t-1}=\{0, a\}\right)}{\operatorname{Pr}\left(\ell_{i}=\ell_{b}, X_{i t-1}=\{0, b\}\right)} \tag{46}
\end{equation*}
$$

and the left hand side is identified from the model and the right hand side is identified from Step 1. This expression will generically only hold when $\ell_{a}=\ell_{b}$.

We have linked the $\{0, a\}$ types with the $\{0, b\}$ types, but we still have not linked them to the types who work in the first period. We can use the same strategy as above using the equation
$\operatorname{Pr}\left(j_{i t-1}=0 \mid \ell_{i}=\ell, j_{i t-2}=0\right)=\frac{\operatorname{Pr}\left(X_{i t-1}=\{0,0\}, \ell_{i}=\ell\right)}{\operatorname{Pr}\left(X_{i t-1}=\{0,0\}, \ell_{i}=\ell\right)+\operatorname{Pr}\left(X_{i t-1}=\{0, a\}, \ell_{i}=\ell\right)+\operatorname{Pr}\left(X_{i t-1}=\{0\right.}$
the left hand side is identified from the model and everything apart from $\operatorname{Pr}\left(X_{i t-1}=\{0,0\}, \ell_{i}=\ell\right)$ is identified from the first stage. Thus $\operatorname{Pr}\left(X_{i t-1}=\{0,0\}, \ell_{i}=\ell\right)$ is identified. We can then also identify
$\operatorname{Pr}\left(\ell_{i}=\ell, j_{i t-2}=0\right)=\operatorname{Pr}\left(X_{i t-1}=\{0,0\}, \ell_{i}=\ell\right)+\operatorname{Pr}\left(X_{i t-1}=\{0, a\}, \ell_{i}=\ell\right)+\operatorname{Pr}\left(X_{i t-1}=\{0, b\}, \ell_{i}=\ell\right)$
but we showed that we can identify the analogue of this for the period 1 labelling. Thus generically we can connect the labelling for people who work the first period with those who first work in the second period.

The third period is analogous to the second, so using the same argument we can generically connect the labeling across all groups.

## Step 4: Identification of initial skill endowment and human capital production function

This section is looser than the others. We have shown that $f_{j t}\left(\mathcal{S}_{i t}\right)$ is identified across all potential values of $\mathcal{S}_{i t}$. We have also shown that the $c_{j t}$ is identified and from the model we know how changes in the technology $f_{j t}$ and $c_{j t}$. What we have not shown is how to identify the three dimensional initial $\theta_{i}$, the skill weights $\beta$, and the human capital production function. One could think of this as just part of a parameterization for $f_{j t}\left(\mathcal{S}_{i t}\right)$ in which case since we have shown non-parametric identification of the model. This is enough to answer many of the counterfactuals of interest.

However, this is not true for the counterfactual where we change the skill level of components of $\theta$. Understanding identification of this in our empirical approach is relatively straightforward. We obtain the level of the $\beta s$ from $\mathrm{O}^{*}$ NET and we use the contrast between the NLSY79 and NLSY97 to see how they change over time. Given that, the relative importance of the components of $\theta$ and the human capital comes from the panel data where we see the same workers move between jobs over the lifecycle. The contrast between the two NLSYs uses observable measures of $\theta$. Note that the fact that we can use observable measures would make identification of this part much easier, but since we do not use that variation for identification of anything other than changes over time in $\beta_{t}$, we consider how the model would be identified without that. Rather than formally going through this we instead provide a broader discussion.

First consider identification from a single cross section if $\beta$ were known. We continue to focus on a three period working life. Note that in this case there are $L J$ possible wages in the first period, $L(J+1) J$ in the second period and $L(J+1)^{2} J$ in the third. To give a concrete example consider a case in which $L=5$ and $J=5 .{ }^{18}$ This gives 1075 different values of $f_{t}\left(\ell_{i}, X_{i t}\right)$ that have been identified in this single period. In terms of identifying the distribution of types, we have 5 types and a three dimensional object, so this gives 15

[^16]unknowns for the initial distribution of $\theta$. Clearly we need a scale and location normalization so this brings it down to 13 . In terms of human capital for each dimension we have a depreciation parameter and human capital appreciation by occupation which is experience specific for two periods (between 1 and 2 and between 2 and 3 ). This gives $3 \times 2 \times 5+3=33$ human capital parameters. Thus far we have 1075 equations and at this point 46 unknowns. All that is left is the $f_{j t}(\cdot)$ functions. We are considering at a single time period so there are just 5 of these. The complication is that these are continuous functions, so to nonparametrically identify them from a finite set of values of human capital, $H_{j t}$ would be impossible. The best we can do is estimate this function at all of the different values of $H_{j t}$. But this clearly isn't identified, given that there are 1075 distinct values of $H_{j t}$ the model is clearly not identified. Specifically if there were no restrictions on $f_{j t}(\cdot)$, for any value of human capital and initial $\theta$, we could find values of $f_{j t}(\cdot)$ to fit the data. However, there is a very important restriction on $f_{j t}(\cdot)$ it has to be monotonically increasing in its argument (though not strictly). This will lead to inequalities-we know if the output in an occupation is higher then the human capital must be as well. Given the number of restrictions relative to the number of parameters, one might get quite tight bounds. We can then think about what we are doing in two ways. The first is to think that the true model gives bounds but we are approximating this result with a parametric model to get point estimates. The second is that with enough time periods with varying values of $\beta_{t}$, the bounds should converge to point estimates (under some additional assumptions that we have not formally worked out).

## E Auxiliary parameters based on Deming (2017)

Combining NLSY79 and NLSY97, Deming (2017) finds that social skills are a significantly more important predictor of full-time employment and wages in the NLSY97 cohort. We reproduce this result for our sample of low skilled men. We estimate the following equations with either the log hourly wage (conditional on employment) or an indicator for full-time employment as the dependent variable $y_{i t}$ :

$$
y_{i t}=\alpha+\beta_{1} \mathrm{COG}_{i}+\beta_{2} \mathrm{SS}_{i}+\beta_{3} \mathrm{COG}_{i} \times \mathrm{NLSY}_{2} 7_{i}+\beta_{4} \mathrm{SS}_{i} \times \mathrm{NLSY}_{2} 7_{i}+\zeta X_{i t}+\epsilon_{i t} .
$$

The regressors includes cognitive skills $\mathrm{COG}_{i}$ and social skills $\mathrm{SS}_{i}$, To test the hypothesis that the returns to skills have changed over time, we include the interaction between skills and an indicator for being in the NLSY97 sample NLSY97. The $X_{i t}$ vector includes age and year fixed effects and the dummy variable NLSY97 ${ }_{i}$.

The results are in Table E1.

Table E1: Reproduction of Table 4 for low skilled men in Deming (2017)

|  | Employment | Wage |
| :--- | ---: | ---: |
|  | $(1)$ | $(2)$ |
| Cognitive | $0.074^{* * *}$ | $0.126^{* * *}$ |
|  | $(0.004)$ | $(0.008)$ |
| Social | 0.005 | 0.010 |
|  | $(0.004)$ | $(0.008)$ |
| Cognitive*NLSY97 | 0.010 | $-0.057^{* * *}$ |
|  | $(0.009)$ | $(0.014)$ |
| Social*NLSY97 | $0.041^{* * *}$ | $0.030^{*}$ |
|  | $(0.008)$ | $(0.014)$ |
| age FE | Yes | Yes |
| year FE | Yes | Yes |
| NLSY97 | Yes | Yes |
| $N$ | 40,227 | 32,106 |
| $R^{2}$ | 0.065 | 0.108 |

A one standard deviation increase in cognitive skills increases the probability of employment by $7.5 \%$ and we cannot reject that the effect is the same in NLSY79 and NLSY97. It increases wages by $12.6 \%$ but the effect has decreased over time by 6 percentage points. We cannot reject that social skills have no effect on either the probability of working or log wages in NLSY79. In NLSY97, a one standard deviation increase in social skills increases the probability of employment by $4.1 \%$. And it increases wages by $3 \%$.

## F Parameter Estimates: Life Cycle

Table F1 reports heterogeneity across individuals in terms of preferences, endowment and luck.

Table F1: Heterogeneity


The actual magnitude of the skill depends on its value in different occupations, so the levels are not directly comparable. However, the levels would be directly comparable in an occupation that weighted them equally so we proceed to make these comparisons. Cognitive skills are the most unequally distributed at labor market entry, followed by manual skills and finally inter-personal skills.

Occupation-specific shocks are more predictable than search costs. There are large costs of attempting to switch occupations, though they are weighted against the variance of idiosyncratic shocks which is also large.

We find large measurement error in wages and it is on the higher side of estimates in related papers. Much of this is likely due to earnings shocks that we abstracted from. The interactions between human capital shocks and technological change is an important avenue for future research.

Each year, we estimate about $10 \%$ of individuals misreport their occupations. This number is reassuringly similar to estimates in the literature even though they are identified using different approaches. In Neal (1999) and Kambourov and Manovskii (2009), E is set using "spurious" transitions in the NLSY. These are all the within-firm occupational transitions where an individual works in occupation $j_{0}$ at both time $t$ and $t+2$, and works in $j_{1} \neq j_{0}$ at time $t$ even though he remained in these three consecutive periods with the same employer. About $10 \%$ of occupational shifts are "spurious" transitions according to this metric.

Table F2: Offer arrival rate, constant and slope of the wage function (until 1979)

| Occupation | Capital Cost |  | Constant. |  | Slope |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Managers | 3.01 | $(0.01)$ | 3.52 | $(0.03)$ | 1.69 | $(0.02)$ |
| Clerical | 2.21 | $(0.03)$ | 3.68 | $(0.01)$ | $1^{*}$ | $(0)$ |
| Services | 1.75 | $(0.04)$ | 3.7 | $(0.01)$ | 0.12 | $(0.03)$ |
| Operators | 1.28 | $(0.05)$ | 3.68 | $(0.02)$ | 0.89 | $(0.03)$ |
| Mechanics | 3.26 | $(0.04)$ | 3.82 | $(0.07)$ | 0.69 | $(0.04)$ |
| Construction | 2.77 | $(0.05)$ | 3.56 | $(0.0)$ | 1.6 | $(0.03)$ |
| Precision | 3.83 | $(0.03)$ | 3.81 | $(0.02)$ | 0.84 | $(0.04)$ |
| Transport | 1.32 | $(0.06)$ | 3.62 | $(0.01)$ | 0.98 | $(0.04)$ |

Note: The asterisk (*) indicates normalized parameters.

Table F2 reports occupation specific parameters that are identified using the NLSY79.
Table F3 reports the parameters of the human capital production function.

Table F3: Learning-by-doing parameters

| Occupation Specific |  |  |
| :---: | :---: | :---: |
| $\gamma_{01}$ | 2.0416 | $(0.6763)$ |
| $\gamma_{02}$ | 1.6846 | $(0.1958)$ |
| $\gamma_{03}$ | 0.7679 | $(0.3802)$ |
| $\gamma_{04}$ | 0.484 | $(0.732)$ |
| $\gamma_{05}$ | 0.886 | $(0.7916)$ |
| $\gamma_{06}$ | 0.6172 | $(0.7008)$ |
| $\gamma_{07}$ | 1.2914 | $(0.4881)$ |
| $\gamma_{08}$ | 1.5968 | $(0.6891)$ |
| $\gamma_{1}$ | 2.405 | $(0.2543)$ |


| General Skills |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
| $d_{01}$ | 0.8571 | $(0.0245)$ | $d_{11}$ | 0.082 | $(0.0005)$ |
| $d_{02}$ | $1^{*}$ | $(0)$ | $d_{12}$ | 0.1297 | $(0.0004)$ |
| $d_{03}$ | 0.9757 | $(0.0164)$ | $d_{13}$ | 0.1239 | $(0.0003)$ |
| $d_{04}$ | 0.8298 | $(0.0187)$ | $d_{2}$ | 0.0526 | $(0.0)$ |
| $d_{05}$ | 1.0358 | $(0.0258)$ | $d_{31}$ | 0.0121 | $(0.0)$ |
| $d_{06}$ | 0.6093 | $(0.045)$ | $d_{32}$ | 0.0323 | $(0.0003)$ |
| $d_{07}$ | 0.6449 | $(0.156)$ | $d_{33}$ | 0.0339 | $(0.0001)$ |
| $d_{08}$ | 0.9231 | $(0.0037)$ |  |  |  |

F-2

## G Auxiliary Parameters not reported in the main text

This Section presents the auxiliary parameters calculated in the NLSY79, CPS and data simulated from the model at the estimated parameters values.

Figure G1 and Figure G2 report, respectively, occupation share in the population and occupation share by age. We use the CPS data but restricts to NLSY79 cohorts.

Figure G1: Occupation Share - CPS data, NLSY cohorts


Figure G2: Occupation Share by Age - CPS data, NLSY cohorts


Clerical
Services









Figure G3 reports different quantiles of the wage distribution by occupation in the NLSY79 cohorts using CPS data.

Figure G3: Quantiles of the Wage Distribution by Occupation - CPS data, NLSY cohorts


Figure G4 plots auto-correlations of wages by age. The horizontal lines represent autocorrelations without controlling for age.

Figure G4: Auto-correlations wages levels by age - NLSY79


Figure G5: Auto-correlations wages levels by occupation - NLSY79
Stayers


Switchers


Figure G5 plots auto-correlations of wages by occupation. The upper-panel restricts the sample to stayers. The lower-panel restricts the sample to switchers.

Table G1 reports age-earnings profile by occupation in the NLSY79 cohorts using the CPS data.

Figure G6 reports different quantiles of the wage distribution by year and occupation in the CPS. These auxiliary parameters identify prices once we control for selection using the previous moments.

Table G2 and Table G3 report the percentage of stayers by age and occupation, respectively, annually and bi-annually.

Table G4 reports the mean difference in the log wages by current and lagged occupation.
Table G1: Age earnings profile by occupation - CPS data, NLSY cohorts

| A | Managers |  | Clerical |  | Services |  | Operators |  | Mechan. |  | Constr. |  | Precis. |  | Transp. |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 20 | 2.2494 | 2.4282 | 24 | 2.3656 | 2.0498 | 2.128 | 2.2676 | 2.272 | 2.3098 | 2.373 | 2.4406 | 2.557 | 2.3937 | 2.5199 | 2.2123 | 2.27 |
| 21 | 2.3 | 2.5 | 2.2769 | 2.4216 | 2.0781 | 2.1 | 2.3453 | 2.3463 | 2.4275 | 2. | 2.514 | 2.6359 | 2.5081 | 2.5818 | 2.2916 | 73 |
| 22 | 2.4402 | 2.605 | 2.3598 | 2.4728 | 2.1085 | 18 | 2.3674 | 2.3797 | 2.4309 | 2.4898 | 532 | 2.66 | 27 | 2.585 | 2.3345 | 07 |
| 23 | 475 | 2.6441 | 2.4313 | 2.5199 | 2.154 | . 203 | 2.41 | 2.4052 | . 511 | 2.5397 | 2.5961 | 2.7194 | 68 | 2.6317 | 2.345 | 2.4277 |
| 24 | 2.5727 | 2.6956 | 2.4626 | 2.5378 | 2.177 | 2.2205 | 2.395 | 2.4038 | 2.5488 | 2.5597 | 2.6261 | 2.7482 | 2.5859 | 2.6464 | 2.3607 | 2.4441 |
| 25 | 2.5 | 2.6926 | 2.493 | 2.5449 | 2.1862 | 2.2195 | 2.40 | 2.4016 | 2.5616 | 2.58 | 2.628 | 2.759 | 2.6 | 2.65 | 2.389 | 07 |
| 26 | 2.647 | 2.74 | 2.5612 | 2.5835 | 2. | 2.23 | 2.4318 | . 422 | .562 | 2.59 | .6475 | 2.77 | 2.65 | 67 | 2.4351 | 2.4663 |
| 27 | 2.73 | 2.8022 | 2.563 | 585 | . 22 | 247 | 2.490 | . 457 | . 617 | 2.61 | .7248 | 2.827 | 2.66 | 2.6939 | 2.4626 | 659 |
| 28 | 2.7 | 2.797 | 2.583 | . 5919 | 2.2 | 2.254 | 2.5046 | . 4662 | . 645 | 2.6349 | 2.7389 | 2.819 | 2.675 | 2.69 | 2.47 | 2.4784 |
| 29 | . 66 | 2.763 | 2.6069 | 6015 | 2.28 | 2.2852 | 2.493 | 2.4718 | . 65 | 2.66 | 2.713 | 2.812 | 2.73 | 2.73 | 2.44 | 04 |
| 30 | 2.7753 | 2.827 | 2.619 | . 6247 | 2.308 | 2.2994 | 2.50 | 2.47 | 2.713 | 2.693 | 2.675 | 2.80 | 2.733 | 2.768 | 2.515 | 2.5269 |
| 31 | 2.7827 | 2.83 | 2.6676 | 2.658 | 2.32 | 2.3265 | 2.552 | 2.5116 | 2.675 | 2.73 | 2.73 | 2.84 | 2.73 | 2.778 | 2.52 | . 5427 |
| 32 | 2.8472 | 2.883 | 2.6558 | 2.6649 | 2.3076 | 2.327 | 2.5158 | 507 | 2.7138 | .760 | 759 | 2.855 | 2.741 | 2.789 | 531 | 59 |
| 33 | 2.7958 | 2.8611 | 2.6 | 2.6882 | 2.3052 | 2.342 | 2.5291 | .5 | 2.7147 | 2.7 | 2.7547 | 2.85 | 2.723 | 2.80 | 2.5 | 2.5591 |
| 34 | 2.9053 | 2.9301 | 2.69 | 7597 | 2.3 | 2.38 | 2.522 | 2.53 | .71 | 2.777 | 2.782 | 2.869 | 2.7596 | 2.7977 | 2.511 | 2.565 |
| 35 | 2.870 | 2.9124 | 2.733 | 2.8227 | 2.328 | 2.3949 | 2.5725 | 2.566 | 2.795 | 2.8294 | 2.7703 | 2.8651 | 2.7872 | 2.826 | 2.572 | 2.61 |
| 36 | 2.9 | 2.94 | 2.7 | 86 | 2.354 | 2.4209 | 2.5 | 2.58 | 13 | 2.85 | 2.823 | 2.89 | 2.79 | 2.8292 | 2.5723 | . 6202 |
| 37 | 2.9876 | 2.9697 | 2.7547 | 2.8622 | 2.3704 | 2.4394 | 2.590 | 2.5957 | 2.8329 | 2.8 | 2.8234 | 2.88 | 2.7955 | .817 | 2.5723 | 2.6217 |
| 38 | 2.9616 | 2.963 | 2.7744 | 2.884 | 2.38 | 2.4401 | 2.645 | 2.6285 | 2.8426 | 2.86 | 2.863 | 2.905 | 2.8573 | 2.8213 | 2.639 | 2.6521 |
| 39 | 3.0086 | 2.992 | 2.7 | 2.8891 | 2.35 | 2.4367 | 655 | 2.6415 | 2.86 | 2.869 | 2.840 | 2.887 | 2.8766 | 2.809 | 2.650 | 2.663 |
| 40 | 3.0967 | 3.0398 | 2.8 | 90 | 2.4 | 2.46 | 2.629 | 2.637 | 2.8 | 2.88 | 2.8 | 2.90 | 2.824 | 2.811 | 2.66 | 69 |
| 41 | 3.0468 | 3.0164 | 2.832 | 2.9211 | 2.4428 | 2.4792 | 2.6596 | 2.6426 | 2.8924 | 2.8867 | 2.8813 | 2.8937 | 2.8737 | 2.8758 | 2.6505 | 46 |
| 42 | 3.0885 | , 052 | 2.8866 | 929 | 2. | 507 | 2.6517 | 6524 | 2.8847 | 2.87 | 2.8633 | 2.873 | 2.84 | 2.855 | 2.65 | 2.6556 |
| 43 | 3.137 | 3.080 | 2.823 | 897 | 2.514 | 2.4983 | 2.697 | 2.664 | 2.9709 | 2.90 | 2.8869 | 2.882 | 2.909 | 2.89 | 2.688 | 2.6586 |
| 44 | 3.1401 | 3.1003 | 2.8886 | 2.9165 | 2.4 | . 47 | 2.671 | 2.6492 | 2.9 | 2.8 | 2.971 | 2.91 | 2.9 | . 90 | 2.7162 | 85 |
| 45 | 3.1834 | 3.1584 | 2.8508 | 2.8874 | 2.4353 | 2.4497 | 2.735 | 2.6644 | 2.917 | 2.8 | 2.9473 | 2.8912 | 2.8 | 2.8599 | 2.707 | 2.657 |
| 46 | 3.1712 | 3.1796 | 2.8348 | 2.8628 | 2. | 4422 | 2.646 | .61 | 2.9443 | 2.86 | 2.9074 | 2.85 | 2.869 | 2.86 | 2.6 | 2.6469 |
| 47 | 3.1395 | 3.193 | 2.8379 | 2.852 | 2.4818 | 2.4606 | 2.680 | 2.6254 | 2.9704 | 2.8756 | 2.8916 | 2.8384 | 2.8809 | 2.8727 | 2.67 | 2.6319 |
| 48 | 3.218 | 3.226 | 2.827 | 2.832 | 2.4849 | 2.461 | 2.6917 | 2.6301 | 2.9445 | 2.8639 | 2.9445 | 2.854 | 2.8869 | 2.882 | 2.700 | 2.6371 |
| 49 | 3.1825 | 3.212 | 2.857 | 839 | 2.44 | . 43 | 2.69 | 2.6247 | 2.99 | 2.87 | 2.94 | 2.849 | 2.928 | 2.88 | 2.7 | 2.6565 |
| 50 | 3.1788 | 3.226 | 2.848 | 2.8202 | 2.505 | . 469 | 2.702 | . 623 | 2.9163 | 2.83 | 2.9081 | 2.813 | 2.866 | 2.86 | 2.69 | 2.6305 |
| 51 | 1812 | 230 | 2.8408 | 8077 | 2.4791 | . 45 | 6917 | 2.6075 | . 9955 | 2.869 | 2.9097 | 2.80 | 2.920 | 2.8957 | 2.6945 | 2.625 |
| 52 | 3.218 | 3.234 | 2.862 | 2.811 | 2.517 | 2.470 | 2.695 | 2.6161 | 2.9081 | 2.8197 | 3.0126 | 2.847 | 2.8511 | 2.8674 | 2.70 | 2.6326 |
| 53 | 3.1782 | 3.2061 | 2.8208 | 2.7853 | 2.498 | 2.4655 | 2.6806 | 2.6023 | 2.9548 | 2.8422 | 2.9756 | 2.8158 | 2.8236 | 2.8574 | 2.7427 | 2.64 |
| 54 | 3.2223 | 3.2377 | 2.8697 | 2.8097 | 2.5366 | 2.481 | 2.729 | 2.631 | 2.9239 | 2.8216 | 2.9544 | 2.8006 | 2.9062 | 2.8911 | 2.7046 | 2.6228 |
| 55 | . 166 | 3.1906 | 2.761 | 2.753 | 2.4364 | 2.438 | 2.7012 | 2.6267 | 3.0029 | 2.8627 | 2.9383 | 2.799 | 2.9355 | 2.89 | 2.725 | 2.6308 |
| 56 | 3.2414 | 3.2185 | 2.8284 | 2.8436 | 2.5171 | 2.487 | 2.6354 | 2.5925 | 2.9239 | 2.8414 | 2.9324 | 2.7917 | 2.9902 | 2.9128 | 2.7424 | 2.6288 |
| 57 | 3.238 | 3.21 | 2.828 | 2.86 | 2.50 | 2.47 | 2.68 | 2.61 | 2.953 | 2.83 | 3.042 | 2.82 | 2.953 | 2.90 | 2.7 | 2 |

Table G2: Share of Stayers by Age and Occupation (Annual)

|  | Managers |  | Clerical |  | Services |  | Operators |  | Mechan. |  | Constr. |  | Precis. |  | Transp. |  | Not-work. |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 20 | 0.35 | 0.4 | . 34 | 0.44 | 36 | 0.46 | 0. | 0.49 | 0.28 | 0.46 | 0.33 | 0.4 | 0.11 | 0. | . 37 | 0.4 | 0.48 | 0.48 |
| 2 |  | 0.49 | 43 | 0.53 |  | 0.52 | 5 | 0.54 | 0.46 | 0.54 | 0.4 | 0.51 | 0.33 | 0.54 | 0.38 | 0.54 | 0.46 | 0.53 |
| 22 | 0.36 | 0.47 | 38 | 0.5 |  | . 4 | 0.5 | 0.5 | 0. | 0.5 | 0.5 | 0.49 | 0.27 | 0.5 | 0.39 | 0.5 | 0.46 | 0.48 |
| 2 | 0.43 | 0.56 | 0.48 | 0.5 |  | 0.54 | 0.45 | 0.55 | . 6 | 0.55 | 0.46 | 0.57 | 0.36 | 0.5 | 0.47 | 0. | 0.49 | 0.52 |
| 24 | 0.38 | 0.55 | 0.41 | 0.54 | 0.52 | 0.51 | 0.57 | 0.53 | 0.5 | 0.55 | . 4 | 0.5 | . 4 | 0.55 | 0.41 | 0.5 | 0.48 | 0.51 |
| 25 | 0.56 | 62 | 0.54 | 0.62 | 0.54 | . 6 | 0. | 0.61 | 0.5 | 0.63 | . 5 | 0. | 0.42 | 0.63 | 0.4 | 0.61 | 0.4 | 0.59 |
| 26 | 0.56 | 0.65 | 0.51 | 0.63 | 0.59 | 0.6 |  | 0.61 | 0.57 | 0.62 | 0.59 | 0.62 | 0.3 | 0.63 | 0.5 | 0. | 0.5 | 0.59 |
| 27 | 0.52 | 62 | 0.51 | . 6 | . 5 | . 58 | 0.5 | 0.5 | 0.5 | 0.61 | 0.5 | 0. | 0. | 0.6 | 0. | 0.6 | 0.49 | 0.5 |
| 28 | 0.56 | 0. | 0.51 | 0.62 | . 57 | 0.61 | 0.55 | 61 | 0.57 | 0.64 | 0.5 | 0.63 | 0.4 | 0.62 | 0.56 | 0.62 | 0.54 | 0.59 |
| 29 | . 53 | 0.63 | 0.5 | 0.6 | 0.6 | 0.57 | 0.56 | 59 | 0.6 | 0.62 | 0.59 | 0.62 | 0.45 | 0.6 | 0.5 | 0.6 | 0.56 | 0.57 |
| 30 | 0.59 | 0.66 | 0.59 | 0.64 | 0.59 | 0.6 | 0.57 | 0.62 | 0.59 | 0.64 | 0.5 | 0.6 | 0.36 | 0.6 | 0.57 | 0.6 | 0.5 | 0.6 |
| 31 | 0.57 | 0.67 | 0.61 | 0.63 | 0.61 | 0.6 | 0.57 | 0.61 | 0.67 | 0.63 | 0.5 | 0.6 | 0.43 | 0.64 | 0.5 | 0.6 | 0.6 | 0.6 |
| 32 | . 56 | 0.66 | 0.56 | 0.63 | 0.67 | 0.6 | 0.62 | 0.6 |  | . 62 | 0.6 | 0.6 | 0.4 | 0.63 | 0.5 | 0.6 | 0.59 | 0.58 |
| 33 | 0.6 | 0.68 | . | 0.66 | . 74 | . 63 | 0.65 | 0.61 | 0.6 | 0.66 | 0.59 | 0.66 | 0.42 | 0.66 | 0.68 | 0.63 | 0. 6 | 0.6 |
| 34 | 0.68 | 0.72 | 0.54 | 0.7 | 0.71 | 0.66 | 0.56 | 0.68 | 0.73 | 0.69 | 0.55 | 0.69 | 0.61 | 0.7 | 0.66 | 0.67 | 0.62 | 0.64 |
| 35 | 0.73 | 0.75 | 0.73 | 0.74 | 0.71 | 0.68 | 0.59 | 0.69 | 0.72 | 0.72 | 0.67 | 0.71 | 0.5 | 0.72 | 0.64 | 0.7 | 0.71 | 0.66 |
| 36 | 0.78 | 0.79 | 0.49 | 0.76 | 0.72 | 0.7 | 0.69 | 0.7 | 0.84 | 0.75 | 0.64 | 0.75 | 0.66 | 0.73 | 0.62 | 0.72 | 0.65 | 0.68 |
| 37 | 0.72 | 0.75 | 0.75 | 0.7 | 0.8 | 0.69 | 0.96 | 0.69 | 0.79 | 0.73 | 0.72 | 0.7 | 0.66 | 0.73 | 0.84 | 0.7 | 0.78 | 0.6 |







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Figure G6: Wage quantiles by year and by occupation. CPS


Table G5 and Table G6 report the same statistics separately for individuals with, respectively, experience and tenure, above and below median. Finally, Table G7 reports the same statistics separately for above median experience individuals with tenure above or below median.

Table G8 and Table G9 report mean wages by, respectively, experience and tenure for each occupation.

Table G10 reports the regression coefficients from Deming's regression.
Table G11 and Table G12 report the occupation distribution by, respectively, cognitive skills and social skills.

| Table G4: Mean wage growth by current and lagged occupation |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Managers |  | Clerical |  | Services |  | Operators |  | Mechan. |  | Constr. |  | Precis. |  | Transp. |  |
| Managers | 0.04 | 0.06 | 0.09 | 0.28 | 0.05 | 0.29 | 0.07 | 0.3 | 0.02 | 0.22 | -0.03 | 0.13 | -0.01 | 0.14 | 0.08 | 0.29 |
| Clerical | -0.01 | 0.18 | 0.05 | 0.06 | 0.04 | 0.31 | -0.05 | 0.27 | 0.17 | 0.33 | -0.25 | 0.05 | -0.06 | 0.16 | -0.0 | 0.26 |
| Services | 0.05 | 0.27 | 0.08 | 0.29 | 0.04 | 0.04 | 0.0 | 0.25 | 0.05 | 0.24 | -0.15 | 0.08 | -0.01 | 0.16 | 0.03 | 0.25 |
| Operators | -0.02 | 0.19 | 0.13 | 0.28 | 0.11 | 0.32 | 0.05 | 0.04 | 0.04 | 0.24 | -0.02 | 0.16 | 0.06 | 0.18 | 0.06 | 0.25 |
| Mechanics | 0.07 | 0.26 | 0.09 | 0.29 | 0.14 | 0.37 | 0.09 | 0.31 | 0.05 | 0.05 | 0.05 | 0.21 | 0.01 | 0.18 | 0.01 | 0.26 |
| Construction | 0.04 | 0.26 | 0.18 | 0.37 | 0.24 | 0.42 | 0.11 | 0.33 | 0.16 | 0.33 | 0.05 | 0.03 | 0.09 | 0.21 | 0.11 | 0.31 |
| Precision | 0.04 | 0.23 | 0.07 | 0.3 | 0.1 | 0.38 | 0.08 | 0.35 | -0.0 | 0.28 | 0.07 | 0.26 | 0.06 | 0.04 | 0.13 | 0.36 |
| Transport | -0.01 | 0.25 | 0.02 | 0.26 | 0.13 | 0.34 | 0.04 | 0.28 | 0.1 | 0.28 | -0.0 | 0.17 | -0.05 | 0.15 | 0.04 | 0.05 |

Table G5: Mean wage growth by current, lagged occupation and experience

|  | Managers |  | Clerical |  | Services |  | Operators |  | Mechan. |  | Constr. |  | Precis. |  | Transp. |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| High Experience |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Managers | 0.07 | 0.13 | 0.11 | 0.36 | 0.04 | 0.35 | 0.06 | 0.37 | -0.01 | 0.24 | -0.09 | 0.17 | 0.06 | 0.21 | 0.15 | 0.37 |
| Clerical | 0.05 | 0.31 | 0.08 | 0.13 | -0.01 | 0.32 | -0.03 | 0.34 | 0.21 | 0.4 | -0.32 | 0.08 | 0.01 | 0.22 | 0.0 | 0.34 |
| Services | 0.06 | 0.41 | 0.11 | 0.41 | 0.04 | 0.08 | 0.07 | 0.35 | 0.16 | 0.34 | -0.14 | 0.17 | 0.02 | 0.22 | 0.04 | 0.33 |
| Operators | 0.01 | 0.29 | 0.16 | 0.37 | 0.18 | 0.42 | 0.07 | 0.11 | 0.12 | 0.33 | 0.01 | 0.23 | 0.08 | 0.24 | 0.08 | 0.34 |
| Mechanics | 0.07 | 0.42 | 0.19 | 0.44 | 0.13 | 0.45 | -0.01 | 0.33 | 0.08 | 0.1 | 0.09 | 0.27 | 0.11 | 0.31 | 0.03 | 0.35 |
| Construction | 0.16 | 0.4 | 0.3 | 0.53 | 0.37 | 0.52 | 0.08 | 0.37 | 0.2 | 0.41 | 0.06 | 0.08 | 0.16 | 0.26 | 0.18 | 0.4 |
| Precision | 0.08 | 0.32 | 0.08 | 0.42 | 0.15 | 0.48 | 0.07 | 0.42 | 0.03 | 0.36 | 0.07 | 0.35 | 0.09 | 0.09 | 0.14 | 0.45 |
| Transport | -0.1 | 0.32 | 0.02 | 0.35 | 0.17 | 0.43 | 0.03 | 0.35 | 0.12 | 0.34 | 0.01 | 0.22 | -0.03 | 0.21 | 0.06 | 0.12 |
| Low Experience |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Managers | 0.02 | 0.01 | 0.08 | 0.26 | 0.05 | 0.24 | 0.07 | 0.28 | 0.04 | 0.22 | 0.0 | 0.11 | -0.07 | 0.06 | 0.03 | 0.24 |
| Clerical | -0.09 | 0.1 | 0.02 | 0.01 | 0.11 | 0.29 | -0.07 | 0.21 | 0.12 | 0.29 | -0.19 | 0.04 | -0.14 | 0.07 | -0.01 | 0.2 |
| Services | 0.02 | 0.13 | -0.01 | 0.19 | 0.04 | 0.0 | -0.09 | 0.14 | -0.21 | 0.04 | -0.15 | -0.01 | -0.1 | 0.06 | 0.02 | 0.18 |
| Operators | -0.08 | 0.12 | 0.08 | 0.21 | 0.02 | 0.23 | 0.03 | -0.02 | -0.06 | 0.16 | -0.06 | 0.09 | 0.02 | 0.13 | 0.03 | 0.17 |
| Mechanics | 0.06 | 0.13 | -0.02 | 0.18 | 0.14 | 0.3 | 0.16 | 0.29 | 0.02 | 0.01 | 0.01 | 0.17 | -0.09 | 0.11 | -0.01 | 0.19 |
| Construction | -0.09 | 0.16 | 0.05 | 0.26 | 0.09 | 0.34 | 0.14 | 0.34 | 0.1 | 0.26 | 0.03 | -0.0 | -0.0 | 0.15 | 0.03 | 0.26 |
| Precision | 0.0 | 0.16 | 0.05 | 0.21 | 0.04 | 0.27 | 0.1 | 0.29 | -0.02 | 0.24 | 0.06 | 0.21 | 0.03 | 0.0 | 0.12 | 0.3 |
| Transport | 0.11 | 0.26 | 0.02 | 0.22 | 0.05 | 0.26 | 0.05 | 0.24 | 0.06 | 0.23 | -0.01 | 0.13 | -0.07 | 0.09 | 0.03 | -0.01 |

Table G6: Mean wage growth by current, lagged occupation and tenure

|  | Managers |  | Clerical |  | Services |  | Operators |  | Mechan. |  | Constr. |  | Precis. |  | Transp. |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| High Tenure |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Managers | 0.03 | 0.09 | 0.11 | 0.33 | 0.08 | 0.3 | 0.09 | 0.38 | -0.07 | 0.14 | -0.09 | 0.1 | 0.04 | 0.18 | 0.12 | 0.34 |
| Clerical | 0.04 | 0.26 | 0.08 | 0.1 | 0.03 | 0.31 | -0.05 | 0.32 | 0.22 | 0.4 | -0.25 | 0.07 | -0.02 | 0.19 | -0.01 | 0.3 |
| Services | 0.04 | 0.33 | 0.08 | 0.37 | 0.04 | 0.07 | 0.0 | 0.28 | 0.09 | 0.27 | -0.13 | 0.13 | 0.06 | 0.22 | 0.04 | 0.3 |
| Operators | -0.02 | 0.27 | 0.2 | 0.35 | 0.18 | 0.4 | 0.08 | 0.09 | 0.09 | 0.31 | -0.02 | 0.19 | 0.1 | 0.23 | 0.06 | 0.28 |
| Mechanics | 0.04 | 0.37 | 0.14 | 0.38 | 0.15 | 0.41 | 0.1 | 0.33 | 0.08 | 0.09 | 0.08 | 0.25 | -0.01 | 0.24 | 0.01 | 0.31 |
| Construction | -0.03 | 0.3 | 0.24 | 0.46 | 0.24 | 0.45 | 0.11 | 0.37 | 0.24 | 0.39 | 0.07 | 0.06 | 0.1 | 0.2 | 0.14 | 0.35 |
| Precision | 0.04 | 0.26 | 0.15 | 0.39 | 0.15 | 0.44 | 0.08 | 0.38 | 0.04 | 0.35 | 0.07 | 0.31 | 0.09 | 0.07 | 0.12 | 0.41 |
| Transport | -0.04 | 0.3 | 0.03 | 0.33 | 0.13 | 0.38 | 0.03 | 0.3 | 0.08 | 0.3 | -0.01 | 0.19 | -0.03 | 0.18 | 0.06 | 0.1 |
| Low Tenure |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Managers | 0.05 | 0.04 | 0.08 | 0.27 | -0.0 | 0.28 | 0.05 | 0.27 | 0.11 | 0.32 | 0.0 | 0.17 | -0.06 | 0.05 | 0.03 | 0.25 |
| Clerical | -0.08 | 0.14 | 0.03 | 0.03 | 0.07 | 0.32 | -0.05 | 0.24 | 0.08 | 0.28 | -0.26 | 0.05 | -0.11 | 0.1 | 0.0 | 0.25 |
| Services | 0.06 | 0.24 | 0.07 | 0.25 | 0.03 | 0.03 | 0.01 | 0.24 | -0.04 | 0.19 | -0.2 | 0.02 | -0.37 | -0.04 | 0.03 | 0.22 |
| Operators | -0.02 | 0.15 | -0.04 | 0.17 | 0.01 | 0.26 | 0.02 | 0.01 | -0.1 | 0.16 | -0.02 | 0.14 | -0.0 | 0.14 | 0.05 | 0.23 |
| Mechanics | 0.13 | 0.21 | -0.0 | 0.22 | 0.12 | 0.35 | 0.07 | 0.3 | 0.03 | 0.03 | -0.03 | 0.17 | 0.05 | 0.2 | 0.01 | 0.23 |
| Construction | 0.2 | 0.33 | 0.08 | 0.28 | 0.25 | 0.43 | 0.09 | 0.33 | -0.06 | 0.22 | 0.03 | 0.02 | 0.02 | 0.2 | 0.07 | 0.3 |
| Precision | 0.04 | 0.18 | -0.08 | 0.2 | 0.05 | 0.34 | 0.08 | 0.33 | -0.06 | 0.23 | 0.06 | 0.23 | 0.04 | 0.03 | 0.15 | 0.34 |
| Transport | 0.07 | 0.26 | -0.0 | 0.23 | 0.12 | 0.33 | 0.06 | 0.28 | 0.13 | 0.29 | 0.01 | 0.17 | -0.13 | 0.1 | 0.02 | 0.02 |

Table G7: Mean wage growth by current, lagged occupation and tenure for high experience individuals

|  | Managers |  | Clerical |  | Services |  | Operators |  | Mechan. |  | Constr. |  | Precis. |  | Transp. |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| High Tenure |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Managers | -0.05 | -0.04 | 0.14 | 0.25 | 0.04 | 0.16 | 0.09 | 0.26 | -0.05 | 0.1 | -0.25 | -0.07 | -0.02 | 0.1 | 0.08 | 0.24 |
| Clerical | -0.06 | 0.05 | 0.04 | -0.01 | 0.17 | 0.26 | -0.07 | 0.17 | 0.24 | 0.35 | -0.11 | 0.05 | -0.11 | 0.09 | 0.01 | 0.15 |
| Services | -0.24 | -0.03 | -0.07 | 0.15 | 0.04 | -0.03 | -0.29 | -0.0 | -0.23 | -0.0 | -0.09 | 0.01 | 0.2 | 0.17 | -0.01 | 0.12 |
| Operators | -0.07 | 0.14 | 0.15 | 0.23 | 0.09 | 0.23 | 0.08 | -0.04 | -0.03 | 0.16 | 0.02 | 0.11 | 0.07 | 0.14 | 0.01 | 0.1 |
| Mechanics | -0.1 | 0.1 | 0.04 | 0.19 | 0.09 | 0.21 | 0.22 | 0.25 | 0.07 | 0.01 | -0.01 | 0.13 | -0.15 | 0.06 | -0.09 | 0.12 |
| Construction | -0.15 | 0.12 | 0.11 | 0.29 | 0.09 | 0.32 | 0.17 | 0.32 | 0.25 | 0.3 | 0.05 | -0.02 | 0.01 | 0.14 | -0.02 | 0.19 |
| Precision | -0.01 | 0.14 | 0.12 | 0.19 | 0.22 | 0.3 | 0.1 | 0.24 | 0.0 | 0.26 | 0.05 | 0.2 | 0.04 | -0.01 | 0.12 | 0.25 |
| Transport | 0.12 | 0.26 | 0.04 | 0.21 | -0.01 | 0.19 | 0.08 | 0.19 | -0.01 | 0.17 | -0.03 | 0.1 | -0.07 | 0.06 | 0.01 | -0.06 |
| Low Tenure |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Managers | 0.03 | 0.03 | 0.07 | 0.28 | 0.07 | 0.32 | 0.07 | 0.28 | 0.09 | 0.31 | 0.06 | 0.19 | -0.09 | 0.04 | 0.01 | 0.25 |
| Clerical | -0.11 | 0.13 | 0.02 | 0.03 | 0.05 | 0.31 | -0.07 | 0.22 | 0.07 | 0.27 | -0.24 | 0.05 | -0.15 | 0.06 | -0.02 | 0.22 |
| Services | 0.06 | 0.19 | 0.04 | 0.23 | 0.03 | 0.02 | 0.05 | 0.23 | -0.2 | 0.08 | -0.2 | -0.02 | -0.41 | -0.08 | 0.04 | 0.2 |
| Operators | -0.09 | 0.1 | -0.01 | 0.17 | -0.05 | 0.22 | 0.02 | 0.0 | -0.08 | 0.16 | -0.12 | 0.08 | -0.0 | 0.13 | 0.03 | 0.21 |
| Mechanics | 0.09 | 0.1 | -0.06 | 0.17 | 0.21 | 0.39 | 0.1 | 0.3 | 0.01 | 0.02 | 0.04 | 0.2 | -0.04 | 0.15 | 0.02 | 0.21 |
| Construction | 0.04 | 0.23 | -0.0 | 0.23 | 0.1 | 0.36 | 0.13 | 0.35 | -0.02 | 0.22 | 0.03 | 0.02 | -0.04 | 0.15 | 0.06 | 0.3 |
| Precision | 0.04 | 0.18 | -0.02 | 0.23 | -0.09 | 0.26 | 0.1 | 0.32 | -0.04 | 0.22 | 0.07 | 0.22 | 0.02 | 0.02 | 0.12 | 0.32 |
| Transport | 0.1 | 0.27 | 0.0 | 0.24 | 0.09 | 0.32 | 0.04 | 0.27 | 0.11 | 0.26 | -0.0 | 0.14 | -0.07 | 0.11 | 0.03 | 0.02 |

Table G8: Mean wages by experience and occupation

|  | 1 |  |  | 3 |  |  |  |  |  |  |  |  | 4 |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Managers | 0.23 | -0.05 | 0.3 | 0.02 | 0.34 | 0.07 | 0.45 | 0.14 | 0.41 | 0.15 |  |  |  |  |  |  |
| Clerical | 0.09 | -0.05 | 0.13 | -0.0 | 0.24 | 0.07 | 0.31 | 0.13 | 0.29 | 0.13 |  |  |  |  |  |  |
| Services | 0.08 | 0.02 | 0.11 | 0.05 | 0.17 | 0.1 | 0.25 | 0.13 | 0.26 | 0.16 |  |  |  |  |  |  |
| Operators | 0.08 | 0.01 | 0.13 | 0.06 | 0.17 | 0.1 | 0.19 | 0.12 | 0.23 | 0.15 |  |  |  |  |  |  |
| Mechanics | 0.07 | -0.03 | 0.16 | 0.03 | 0.22 | 0.08 | 0.2 | 0.08 | 0.31 | 0.16 |  |  |  |  |  |  |
| Construction | 0.06 | -0.06 | 0.17 | 0.04 | 0.19 | 0.08 | 0.21 | 0.11 | 0.27 | 0.15 |  |  |  |  |  |  |
| Precision | 0.1 | 0.02 | 0.11 | 0.04 | 0.13 | 0.05 | 0.23 | 0.11 | 0.28 | 0.15 |  |  |  |  |  |  |
| Transport | 0.07 | -0.01 | 0.11 | 0.04 | 0.18 | 0.11 | 0.23 | 0.14 | 0.26 | 0.18 |  |  |  |  |  |  |

Table G9: Mean wages by tenure and occupation

|  | 1 |  |  | 2 |  |  | 3 |  |  | 4 |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Managers | 0.34 | 0.075 | 0.43 | 0.125 | 0.41 | 0.135 | 0.47 | 0.175 | 0.52 | 0.2 |  |  |
| Clerical | 0.14 | 0.03 | 0.28 | 0.09 | 0.3 | 0.115 | 0.32 | 0.135 | 0.36 | 0.16 |  |  |
| Services | 0.05 | 0.01 | 0.09 | 0.03 | 0.1 | 0.045 | 0.09 | 0.055 | 0.06 | 0.06 |  |  |
| Operators | 0.12 | 0.02 | 0.2 | 0.06 | 0.23 | 0.08 | 0.27 | 0.1 | 0.31 | 0.125 |  |  |
| Mechanics | 0.17 | 0.03 | 0.22 | 0.06 | 0.25 | 0.075 | 0.34 | 0.11 | 0.4 | 0.135 |  |  |
| Construction | 0.1 | 0.02 | 0.23 | 0.065 | 0.21 | 0.075 | 0.25 | 0.085 | 0.26 | 0.095 |  |  |
| Precision | 0.13 | 0.03 | 0.22 | 0.06 | 0.26 | 0.075 | 0.31 | 0.09 | 0.33 | 0.095 |  |  |
| Transport | 0.14 | 0.035 | 0.2 | 0.07 | 0.26 | 0.105 | 0.26 | 0.11 | 0.3 | 0.135 |  |  |

Table G10: Deming's regressions

|  | Employment |  | Wages |  |
| :--- | :---: | :---: | :---: | :---: |
| Cognitive | 0.0742 | 0.0582 | 0.126 | 0.1248 |
| Social | 0.0053 | 0.0059 | 0.0103 | 0.0154 |
| Cognitive*NLSY97 | 0.0097 | -0.0049 | -0.0575 | -0.047 |
| Social*NLSY97 | 0.0409 | 0.0154 | 0.03 | 0.0186 |

Table G11: Occupation distribution by cognitive skills

|  | NLSY79 |  |  |  | NLSY97 |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | High |  | Low |  | High |  | Low |  |
| Managers | 0.03 | 0.03 | 0.1 | 0.07 | 0.03 | 0.03 | 0.06 | 0.05 |
| Clerical | 0.06 | 0.09 | 0.11 | 0.13 | 0.09 | 0.09 | 0.17 | 0.14 |
| Services | 0.14 | 0.1 | 0.09 | 0.1 | 0.18 | 0.14 | 0.16 | 0.15 |
| Operators | 0.14 | 0.14 | 0.15 | 0.17 | 0.08 | 0.1 | 0.08 | 0.09 |
| Mechanics | 0.05 | 0.09 | 0.09 | 0.09 | 0.05 | 0.05 | 0.08 | 0.06 |
| Construction | 0.08 | 0.07 | 0.1 | 0.06 | 0.09 | 0.07 | 0.12 | 0.07 |
| Precision | 0.03 | 0.04 | 0.07 | 0.04 | 0.03 | 0.02 | 0.03 | 0.03 |
| Transport | 0.19 | 0.18 | 0.17 | 0.18 | 0.2 | 0.19 | 0.18 | 0.19 |
| Not-working | 0.26 | 0.25 | 0.12 | 0.16 | 0.26 | 0.31 | 0.12 | 0.21 |

Table G12: Occupation distribution by social skills

|  | NLSY79 |  |  |  | NLSY97 |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | High |  | Low |  | High |  | Low |  |
| Managers | 0.05 | 0.04 | 0.07 | 0.05 | 0.03 | 0.03 | 0.05 | 0.05 |
| Clerical | 0.07 | 0.1 | 0.09 | 0.12 | 0.11 | 0.11 | 0.15 | 0.13 |
| Services | 0.12 | 0.1 | 0.12 | 0.1 | 0.16 | 0.15 | 0.17 | 0.15 |
| Operators | 0.15 | 0.15 | 0.14 | 0.15 | 0.09 | 0.1 | 0.07 | 0.09 |
| Mechanics | 0.07 | 0.09 | 0.07 | 0.09 | 0.06 | 0.06 | 0.07 | 0.06 |
| Construction | 0.09 | 0.07 | 0.09 | 0.07 | 0.08 | 0.07 | 0.13 | 0.07 |
| Precision | 0.05 | 0.04 | 0.04 | 0.04 | 0.03 | 0.03 | 0.03 | 0.03 |
| Transport | 0.19 | 0.18 | 0.18 | 0.17 | 0.19 | 0.19 | 0.19 | 0.19 |
| Not-working | 0.2 | 0.22 | 0.21 | 0.2 | 0.24 | 0.28 | 0.14 | 0.24 |

## H Structural Parameters not reported in the main text

Table H1: Hedonic function's time trends

| Occupation | $\delta_{j t}$ : Time Trends |  |  | 2010 |
| :---: | :---: | :---: | :---: | :---: |
|  | 1980 | 1990 | 2000 |  |
| Managers | -0.012 (0.0) | 0.0056 (0.0002) | -0.0315 (0.0002) | 0.0056 (0.0038) |
| Clerical | -0.0078 (0.0001) | 0.0099 (0.0001) | -0.0055 (0.0002) | -0.0023 (0.0007) |
| Services | -0.0142 (0.0001) | 0.0076 (0.0002) | 0.0005 (0.0003) | 0.0074 (0.0005) |
| Operators | -0.0172 (0.0) | 0.0049 (0.0004) | 0.0042 (0.0003) | 0.0138 (0.0018) |
| Mechanics | -0.014 (0.0005) | -0.0005 (0.0006) | -0.0004 (0.0011) | -0.0074 (0.003) |
| Construction | -0.0 (0.0002) | 0.0001 (0.0002) | -0.0011 (0.0006) | 0.0025 (0.0021) |
| Precision | -0.0149 (0.0001) | 0.0019 (0.0008) | -0.0085 (0.0012) | 0.0008 (0.0014) |
| Transport | -0.0046 (0.0001) | 0.0087 (0.0001) | -0.0084 (0.0001) | 0.0207 (0.0015) |
| $\alpha_{1 j t}$ : Time Trends |  |  |  |  |
| Occupation | 1980 | 1990 | 2000 | 2010 |
| Managers | 0.0079 (0.0) | 0.0003 (0.0003) | 0.0239 (0.0004) | 0.0013 (0.0046) |
| Clerical | -0.0083 (0.0001) | -0.0016 (0.0001) | -0.0017 (0.0005) | 0.0044 (0.0024) |
| Services | 0.0108 (0.0006) | -0.0016 (0.0004) | -0.0119 (0.0001) | -0.0018 (0.0001) |
| Operators | 0.0036 (0.0003) | -0.0004 (0.0013) | -0.0192 (0.0015) | 0.002 (0.0013) |
| Mechanics | 0.004 (0.0012) | 0.0003 (0.0005) | 0.009 (0.0003) | 0.0077 (0.0184) |
| Construction | -0.0067 (0.0004) | -0.0081 (0.0004) | -0.0048 (0.0016) | 0.0113 (0.0018) |
| Precision | 0.0041 (0.0002) | -0.0095 (0.0005) | 0.0154 (0.0007) | 0.01 (0.0131) |
| Transport | -0.0062 (0.0002) | $0.0003 \quad(0.0003)$ | -0.0128 (0.0005) | -0.0062 (0.0011) |
| $\alpha_{2 j t}$ : Time Trends |  |  |  |  |
| Occupation | 1980 | 1990 | 2000 | 2010 |
| Managers | 0.04 (0.0003) | -0.0185 (0.0008) | 0.038 (0.0006) | -0.011 (0.0027) |
| Clerical | 0.0178 (0.0001) | -0.0012 (0.0005) | 0.0053 (0.0017) | 0.0209 (0.0032) |
| Services | 0.0077 (0.0007) | 0.0087 (0.0005) | 0.0172 (0.0003) | 0.0222 (0.0095) |
| Operators | 0.0163 (0.0001) | -0.0062 (0.0009) | 0.0076 (0.0015) | 0.0068 (0.0168) |
| Mechanics | 0.0249 (0.0022) | 0.0022 (0.0033) | 0.0087 (0.0067) | 0.0072 (0.027) |
| Construction | -0.0027 (0.0001) | 0.0156 (0.0011) | 0.0139 (0.0027) | -0.0067 (0.0112) |
| Precision | 0.0303 (0.002) | -0.0132 (0.0026) | 0.0144 (0.0016) | -0.0061 (0.0201) |
| Transport | 0.0027 (0.0003) | -0.0037 (0.0004) | 0.0254 (0.0031) | -0.0025 (0.0049) |


[^0]:    ${ }^{1}$ Specifically Table 5 of their paper shows wage decline in other sectors of similar magnitude to the wage declines in manufacturing.

[^1]:    ${ }^{2}$ Given the structural model that follows we need a sufficient number of people in an occupation in order to obtain reliable estimates of the occupation specific variables.
    ${ }^{3}$ That is for each of the three skills we use questions that ask about those skills and perform a one dimensional factor analysis for each skill. This gives us factors for each occupation. We then use the census to get a weighted average of these to the aggregated occupations listed in Table 1 and Figure 1. The weights depend on the employment for low skilled men and vary across years based on the Census.

[^2]:    ${ }^{4}$ This exact number depends substantially on how one accounts for inflation. The CPI yields a much larger decline than the PCE. However, even the PCE is not perfect as accounting for technological change and quality differences in constructing a measure is very difficult. The fact that median wages for low educated men has fallen relative to other demographic groups is very well established.

[^3]:    ${ }^{5}$ An illustrative example of the challenges ahead is the following. Consider an economy with two occupations indexed by $j$ with wage rate $w_{j}$. Individuals are identical, indexed by $i$ and derive utility from working in occupation $u_{i j}=\eta \log w_{j}+\epsilon_{i j}$, where $\epsilon_{i j}$ is an i.i.d. extreme-value distributed preference shock and $\eta>0$ is a scale parameter. Relative labor supply to occupation 1 is $\left(\frac{w_{1}}{w_{2}}\right)^{\eta}$. The aggregate production function is $\left[\left(A_{1} n_{1}\right)^{\frac{\sigma-1}{\sigma}}+\left(A_{2} n_{2}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$. Relative labor demand for occupation 1 is $\left(\frac{A_{1}}{A_{2}}\right)^{\sigma-1}\left(\frac{w_{2}}{w_{1}}\right)^{\sigma}$. Equilibrium

[^4]:    ${ }^{6}$ Formally, write a time-varying aggregate production function $G_{t}\left(H_{1 t}, \ldots, H_{J t}\right)$, where the arguments are the human capital stocks $H_{j t}$ provided by each $J$ occupation at time $t$. When workers within an occupation are perfect substitutes, we can write $H_{j t}=\int e^{h} d \Psi_{j t}(h)$ where $\Psi_{j t}$ is the distribution of human capital indexes supplied to occupation $j$ at time $t$. With competitive labor markets, the equilibrium wage function $f_{j t}$ is such that $\delta_{j t}=\log \frac{\partial}{\partial H_{j t}} G_{t}\left(H_{1 t}, \ldots, H_{J t}\right)$ and $\alpha_{j t}^{1}=\alpha_{j t}^{2}=1$.

[^5]:    ${ }^{7}$ We settle on this parametrization of the accumulation equation because more general versions lead to large standard errors. We will however present identification results for a very general accumulation process in the next section.

[^6]:    ${ }^{8}$ We have not verified that all of the arguments go through without it, so we can't state this unequivocally. In practice since we use a simulation estimator and we actually used a discrete support for the types so one could interpret our model as discrete.

[^7]:    ${ }^{9}$ See Gourieroux et al. (1993) for a general discussion of indirect inference.

[^8]:    ${ }^{10}$ Standard errors are reported in Table H1 (Appendix H). Using a Wald test, we can reject that the parameters of the function $f_{j t}$ are equal to each other for any distinct pair of decades, given any occupation $j$. We cannot reject equality of the slopes for some occupations between 1990 and 2000, a decade with little changes in the pricing equations.

[^9]:    ${ }^{11}$ Examples of other papers using similar strategies include Villanueva (2007), Sorkin (2018), Bagger and Lentz (2018), and Taber and Vejlin (2020).

[^10]:    ${ }^{12}$ One place our model misses is the rise in non-working during the great recession. Since we do not model the great recession as our focus is on the more low frequency changes this is not surprising.

[^11]:    ${ }^{13}$ This thought exercise also makes sense if we think that $f_{j t}$ is determined by the world economy and that low skilled men in the United States are small so that they are price takers in $f_{j t}$.

[^12]:    ${ }^{14}$ We discard individuals in the middle in any particular skills to save on space and because manual skills have the highest rewards for all of them throughout.

[^13]:    ${ }^{15}$ This definition of occupation-specific tenure is different from its model counterpart presented below which we simplify for computational purposes. It is also less affected by misclassification errors.

[^14]:    ${ }^{16}$ Note that the way we have done this explicitly uses the fact that we have finite types. One can extend this argument to continuous 3 dimensional heterogeneity, but it will require more than 2 periods.

[^15]:    ${ }^{17}$ Note that it seems we can relax the independence assumption and normalize $\nu_{i 0 t}$. If we went to $J=3$ this would not be true. A worker can have at most 3 choices at a time (current job, non-employment, and one other job) so we could not identify the full joint distribution of say $\left(\nu_{i a t}-\nu_{i 0 t}, \nu_{i b t}-\nu_{i 0 t}, \nu_{i c t}-\nu_{i 0 t}\right)$.

[^16]:    ${ }^{18}$ We have expanded from 2 occupations to 5 because we have a 3 dimensional vector of $\theta$ and identification will be more straight forward with $J>3$.

