Skills Practice

Name _____   Date _____

The Wizard and the King
Introduction to Exponential Functions

Vocabulary

Define each term in your own words.

1. interest

2. simple interest

3. principal

4. interest rate

5. compound interest

6. exponential function

Problem Set

Write a formula for the balance of each savings account at the end of t years, compounded annually.

   7. $P = \$3000, r = 3\%$

   8. $P = \$4000, r = 6\%$

   9. $P = \$17{,}500, r = 10\%$

  10. $P = \$25{,}000, r = 12\%$

 11. $P = \$135{,}000, r = 3.6\%$

 12. $P = \$275{,}000, r = 2.7\%$

 13. $P = \$15{,}000, r = 0.52\%$

 14. $P = \$53{,}000, r = 0.13\%$

Calculate the balance of each savings account if interest is compounded annually. Assume that no withdrawals are made from the account.

 15. $P = \$2000, r = 3\%, t = 4$

 16. $P = \$3000, r = 4\%, t = 5$

 17. $P = \$1500, r = 2.5\%, t = 6$

 18. $P = \$25{,}000, r = 4.2\%, t = 10$

 19. $P = \$1{,}500{,}000, r = 2.7\%, t = 3$

 20. $P = \$20{,}000{,}000, r = 3.5\%, t = 5$

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Use the information provided to answer each question.

21. A savings bond costs $1000 and has an interest rate of 4% compounded monthly. Calculate the value of the savings bond after 10 years.

22. You deposit $12,500 into an account with an interest rate of 6% compounded monthly. Calculate the balance after 4 years.

23. You invest $125,000 into an account with an interest rate of 3.75% compounded monthly. Calculate the balance after 15 years.

24. You invest $50,000, and have a rate of return of 6.15% compounded monthly. Calculate the balance after 12 years.

25. To buy a car, you save $15,000 in an account with an interest rate of 8.45% compounded twice a year. Calculate the balance after 5 years.

26. You deposit $500 in a certificate of deposit (CD) that earns 1.25% compounded four times a year. Calculate the balance after 8 years.

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Skills Practice

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A Review
Properties of Whole Number Exponents

Vocabulary

Identify each term using the given expression.

$$2^3$$

1. power

2. base

3. exponent

Problem Set

Use the definition of a power to simplify each expression.

4. $2 \cdot x \cdot x \cdot x$

5. $7 \cdot x \cdot x \cdot x \cdot x$

6. $14 \cdot x \cdot x \cdot x \cdot x \cdot x \cdot y \cdot y \cdot y \cdot y$

7. $13 \cdot x \cdot x \cdot x \cdot x \cdot y \cdot y \cdot y \cdot y \cdot y \cdot y \cdot y \cdot y$

8. $150 \cdot A \cdot A \cdot B \cdot B \cdot B \cdot B \cdot B \cdot C \cdot C \cdot C \cdot C$

6

9. $125 \cdot A \cdot B \cdot B \cdot B \cdot B \cdot B \cdot B \cdot C \cdot C$

10. $32 \cdot x \cdot x \cdot x \cdot y \cdot y \cdot y \cdot y \cdot y \cdot z \cdot z \cdot z$

11. $47 \cdot x \cdot x \cdot y \cdot y \cdot y \cdot z \cdot z \cdot z \cdot z \cdot z \cdot z$

Multiply the powers and simplify completely.

12. $2^3 \cdot 2^5$

13. $3^2 \cdot 3^3$

14. $4x^3 \cdot 5x^2$

15. $7x^2 \cdot 3x^6$

16. $6x^3 \cdot 8y^5$

17. $2x^5 \cdot -12y^3$

18. $-3x^3 \cdot 5y^2 \cdot 6x^4 \cdot 3y^5$

19. $4x^5 \cdot 10y^4 \cdot 2x^8 \cdot 3y^3$

Divide the powers and simplify completely.

20. $\dfrac{x^7}{x^3}$

21. $\dfrac{x^{10}}{x^4}$

22. $\dfrac{5^6}{5^4}$

23. $\dfrac{3^6}{3^8}$

24. $\dfrac{10x^8}{5x^5}$

25. $\dfrac{24x^{11}}{-6x^5}$

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26. $\dfrac{5x^7 \cdot 3x^2}{10x^3 \cdot -2x^4}$

27. $\dfrac{9x^4 \cdot 2y^2}{3x \cdot 12y^6}$

28. $\dfrac{x^5}{x^8}$

29. $\dfrac{x^8}{x^{12}}$

Raise each power to a power and simplify completely.

30. $(x^3)^4$

31. $(x^5)^3$

32. $(2^3)^3$

33. $(2^5)^2$

Raise each product to a power and simplify completely.

34. $(x^3y^4)^4$

35. $(x^2y^5)^3$

36. $(-3x^3)^2$

37. $(10x^4y)^3$

38. $(4x^2y^3)^4 \cdot (2x^3y^5)^5$

39. $(10xy)^3 \cdot (2x^3y^4)^2 \cdot (4x^2y^2)^4 \cdot (-3x^3y^4)^3$

Raise each quotient to a power and simplify completely.

40. $\left(\dfrac{3}{4}\right)^3$

41. $\left(\dfrac{10}{3}\right)^5$

42. $\left(\dfrac{x}{y}\right)^4$

43. $\left(\dfrac{2y^3}{x^5}\right)^5$

6

44. $\left(\dfrac{12x^3}{y^{10}}\right)^2$

45. $\left(\dfrac{-3x^3y^4}{4x^5y^{10}}\right)^3$

46. $\left(\dfrac{-2x^2y^5}{5xy^3}\right)^4$

47. $\left(\dfrac{10x^3y^{10}}{3x^4y^7}\right)^3$

48. $\left(\dfrac{3x^2}{y^4}\right)^3 \cdot \left(\dfrac{y^3}{x^4}\right)^2$

49. $\left(\dfrac{2x^3}{y^5}\right)^2 \cdot \left(\dfrac{y^4}{x^3}\right)^4$

Simplify each expression completely.

50. $\dfrac{x^3}{x^3}$

51. $\dfrac{y^{10}}{y^{10}}$

52. $(3x)^3$

53. $\dfrac{(4x^2)^2}{8x^4}$

54. $\dfrac{(x^2y^3)^2}{(3x^2y^2)^3}$

55. $\dfrac{(2xy^4)^3}{(x^2y^3)^4}$

56. $\left(\dfrac{-3x^2y^3}{2y^5}\right)^3 \cdot \left(\dfrac{4x^3y^3}{9x^2}\right)^2 \cdot \dfrac{-(3x^3y^2)^2}{(8xy)^3}$

57. $\left(\dfrac{5x^3y^4}{3xy^2}\right)^2 \cdot \left(\dfrac{9x^2y^4}{10y^3}\right)^3 \cdot \dfrac{-(20x^2y^3)^3}{9x^2y^3}$

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Skills Practice

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Exponents, Reciprocals, and Roots
Integral and Rational Exponents

Vocabulary

Give an example of each type of exponent.

1. negative exponent

2. rational exponent

Problem Set

Rewrite each expression without negative exponents. Then simplify completely.

3. x^{-3}

4. x^{-8}

5. $3x^{-5}y^2 \cdot 5x^{-3}y^3$

6. $11x^4y^{-3} \cdot 2x^{-5}y$

7. $\dfrac{x^{-5}y^3}{x^{-2}y^4}$

8. $\dfrac{x^3 \cdot y^{-5}}{x^{-4} \cdot y^2}$

9. $(2x^{-2}y^3)^{-2}$

10. $(3x^3y^{-4})^{-3} \cdot (-4x^2y^{-5})^2$

6

11. $\left(\dfrac{2x^5y^4}{5x^{-2}}\right)^3 \cdot \left(\dfrac{5x^{-5}}{2y^6}\right)^2 \cdot \left(\dfrac{10y^{-3}}{8x^4}\right)^2$

12. $\left(\dfrac{6x^{-5}y}{x^4y^{-2}}\right)^2 \cdot \left(\dfrac{4x^2}{9y^{-3}}\right)^3 \cdot \left(\dfrac{3x^4y^2}{8x^{-3}}\right)^2$

Solve each equation for *a*.

13. $x^a \cdot x^a \cdot x^a = x^2$

14. $x^a \cdot x^a \cdot x^a = x^4$

15. $9 = 81^a$

16. $5 = 625^a$

17. $64^a = 2$

18. $10{,}000^a = 10$

Rewrite each expression in radical form.

19. $3^{\frac{1}{2}}$

20. $7^{\frac{3}{2}}$

21. $x^{\frac{5}{4}}$

22. $x^{\frac{3}{8}}$

6

23. $x^{\frac{1}{3}} y^{\frac{1}{2}}$

24. $x^{\frac{1}{4}} y^{\frac{2}{3}}$

Rewrite each expression in radical form. Then simplify completely.

25. $125^{\frac{2}{3}}$

26. $16^{\frac{3}{4}}$

27. $81^{\frac{5}{4}}$

28. $1000^{\frac{5}{3}}$

Rewrite each expression in exponential form and simplify completely.

29. $\sqrt[3]{3^6}$

30. $\sqrt[6]{5^2}$

31. $\sqrt[3]{9^2}$

32. $\sqrt[5]{2^4}$

33. $\sqrt[9]{x^3}$

34. $\sqrt[6]{x^4}$

35. $\sqrt[2]{x^3 y^{10}}$

36. $\sqrt[3]{x^{12} y^2}$

Simplify each expression completely. Write the final answer in exponential form.

37. $\left(3^{\frac{2}{3}} y^{\frac{1}{6}}\right)^3$

38. $\left(5^{\frac{3}{4}} x^{\frac{5}{2}}\right)^4$

39. $(27 x^2 y^3)^{\frac{2}{3}} \cdot \left(x^{\frac{1}{2}} y^2\right)^4 \cdot \left(x^{-3} y^{\frac{1}{2}}\right)^{-2}$

40. $\left(2 x^3 y^{\frac{1}{3}}\right)^6 \cdot \left(256 x^8 y^{\frac{4}{3}}\right)^{\frac{1}{4}} \cdot \left(x^{\frac{1}{3}} y^3\right)^{-6}$

6

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Skills Practice

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The Hockey Stick Graph
Applications of Exponential Functions

Vocabulary

Use the words below to fill in the blanks. Each word may be used more than once.

| depreciate | exponential growth | exponential decay | half-life | decays |

1. Assets _____, or decrease in value over time.

2. A radioactive substance undergoes _____.

3. When a quantity _____, there is an amount of time after which the new sample size is half the original amount, known as its _____.

4. When a population grows by a certain percent each year, it undergoes

   _____.

5. Carbon-14 has a(n) _____ of 5730 years. After 5730 years, there will be half as much carbon-14 as in the initial amount.

Problem Set

Calculate the value of each item.

6. A new car costs $17,500 and depreciates at a rate of 7% annually. What will the value of the car be in 5 years?

7. A new computer costs $850 and depreciates at a rate of 11% annually. What will the value of the computer be in 3 years?

8. A factory buys new equipment for $125,000 and depreciates at a rate of 2.3%. What will the value of the equipment be in 12 years?

6

9. A Cessna SkyCatcher costs $111,500. If the SkyCatcher depreciates at a rate of 1.75% annually, what will the value of the airplane be in 6 years?

Calculate the value of each item.

10. A baseball card costs $30. If the card appreciates at a rate of 5% annually, what will the value of the card be in 3 years?

11. Your mother buys an antique vase for $120. If the vase appreciates at a rate of 4% annually, what will the value of the vase be in 16 years?

12. A vintage car costs $26,700. If the car appreciates at a rate of 3.7% annually, what will the value of the car be in 22 years?

13. A gold watch costs $1250. If the watch appreciates at a rate of 2.75% annually, what will the value of the watch be in 7 years?

Calculate each population.

14. Albany has a population of 15,000 people and grows at about 4% per year. What will the population be in 25 years?

15. Endwell has a population of 5650 people and grows at about 8% per year. What will the population be in 17 years?

16. A bird sanctuary contains 3250 birds. The bird population is decreasing by 3% per year. What will the population be in 23 years?

17. An aquarium contains 235 fish. The aquarium has a bacterium that kills 2% of the fish per year. How many fish will there be in 5 years?

6

18. A park contains 4652 trees and the number of trees increases by 7% per year. How many trees were in the park 8 years ago?

19. Tacoma had a population of 193,556 in the year 2000. If the population of Tacoma is decreasing by 5% per year, what was the population of Tacoma in 1994?

Calculate the amount of each material remaining.

20. Silicon-32 has a half-life of 150 years. If a computer has 3 kilograms of silicon-32 in it, how much silicon-32 will be left after 600 years?

21. Chlorine-36 has a half-life of 301,000 years. If a bucket of pool chlorine has 300 grams of chlorine-36, how much chlorine-36 will the bucket have in 100,000 years?

22. Cesium-138 has a half-life of 33 minutes. If an atomic clock has 625 grams of cesium-138, how much cesium-138 will the clock have in 4 hours?

23. Zinc-61 has a half-life of 89 seconds. If an alloy has 2 kilograms of zinc-61, how much zinc will be left after 6 minutes?

6

Complete each table of values.

24. A bacteria is being eaten by a population of zooplankton. The bacteria's population starts at 450,000 and decreases by 2%. Complete the table below to show how many bacteria there will be after 0, 40, 80, and 120 days.

Quantity Name		
Unit		
Expression		

25. An antique typewriter is bought for $250. It appreciates at a rate of 4% per year. Complete the table below to show how much the typewriter will be worth after 0, 10, 20, and 40 years.

Quantity Name		
Unit		
Expression		

26. A ring is bought for $350. It appreciates at a rate of 5.4% per year. Complete the table below to show how much the ring will be worth after 0, 20, 40, and 60 years.

| Quantity Name		
Unit		
Expression		

6

27. A road flare contains 50 grams of strontium-90, which has a half-life of about 29 years. Complete the table below to show how much strontium-90 will be in the road flare after 0, 20, 40, and 60 years.

Quantity Name		
Unit		
Expression		

Create a graph from each table.

28. A population of zebras grows 3% per year. Their growth is given by the table below.

Time	Population of Zebras
years	zebras
0	250
20	451
40	816
60	1473

6
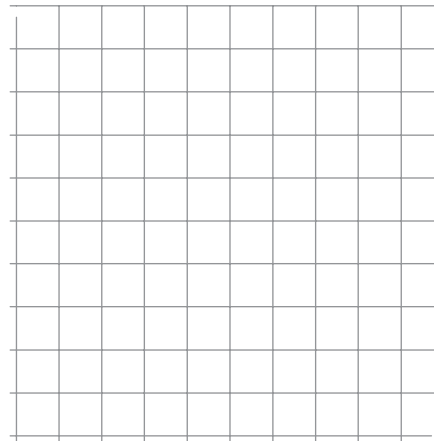
29. An airplane depreciates at a rate of 5% per year. Its value is given by the table below.

| Time	Value of Airplane
years	thousands of dollars
0	750
10	449
20	269
30	161
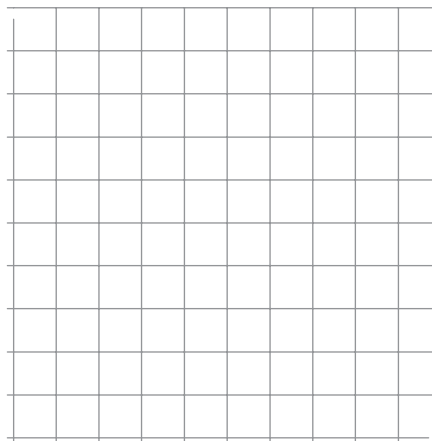
30. A bone loses half of its carbon-14 every 5730 years. The amount of carbon-14 in the bone is given below.

Time	Amount of Carbon-14
years	grams
0	140.0
1000	124.0
3000	97.4
6000	67.8

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Name _____ Date _____
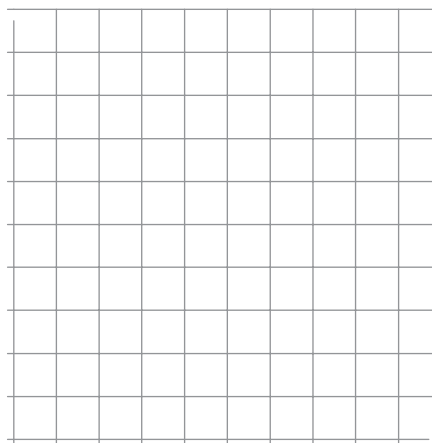
31. The population of falcons in a national park decreases by 2.5% per year. The number of falcons in the park is given in the table below.

| Time	Population of Falcons
years	falcons
0	115
5	101
10	89
15	79
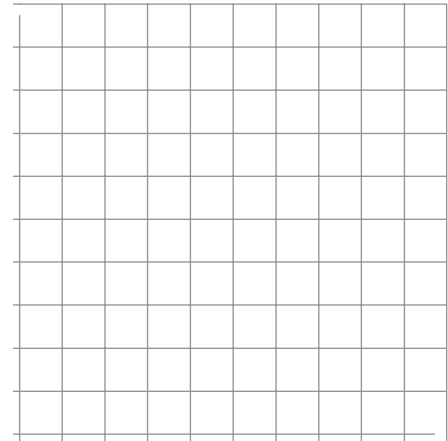
**Use each graph to approximate a solution.**

32. A car depreciates at 5.4% per year. The value of the car in thousands of dollars is given by the graph below. When will the car have a value of $15,000?


Time (years)

Value of Car (thousands of dollars)

6
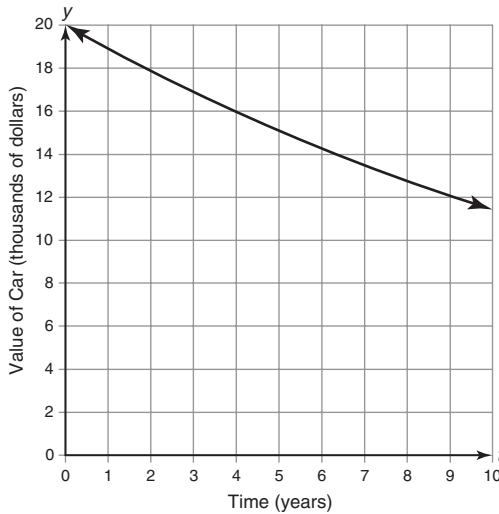
33. The population of trees in a forest grows by 1.4% per year. The tree population in the park, in thousands, is graphed below. When will the forest have 125,000 trees?
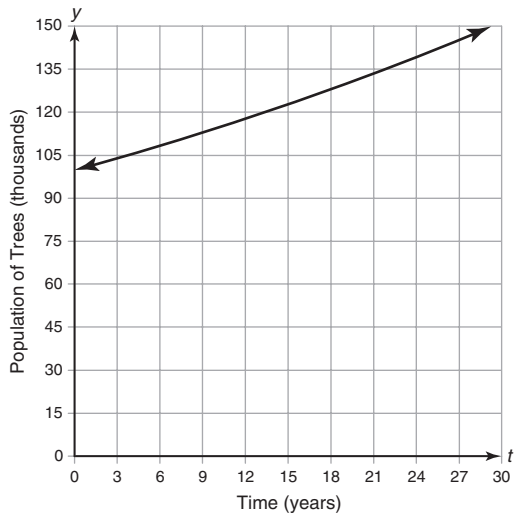
34. A population of pigeons grows by 2.3% per year. The pigeon population, in thousands, is graphed below. When will there be 3000 pigeons?
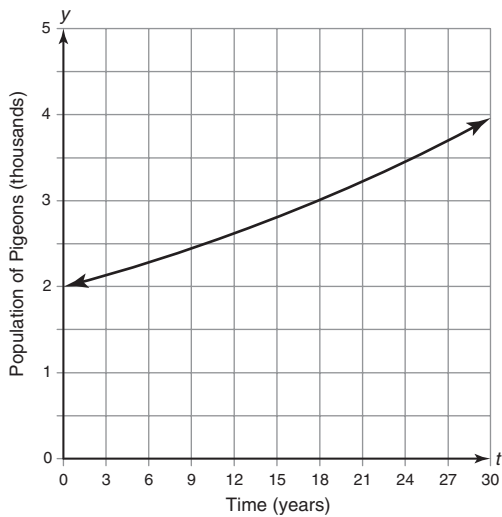
35. A prehistoric animal's bones contain an amount of calcium-41 that has a half-life of 100,000 years. The amount of calcium-41, in grams, in the animal's bones is graphed below, where the years are in thousands. When will the animal's bones contain 5 grams?

6

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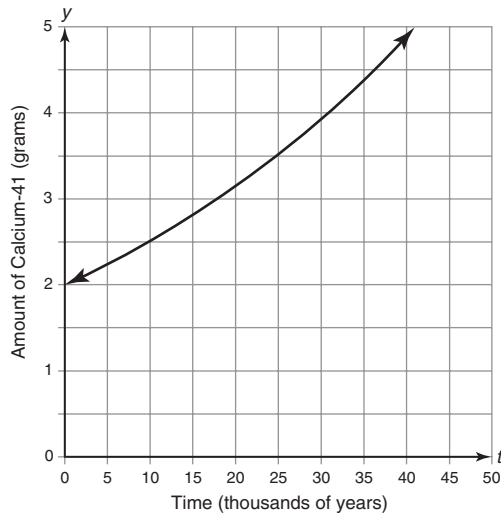
Name _____ Date _____

Log a What?
Inverses of Exponential Functions: Logarithmic Functions

Vocabulary

Define each term in your own words.

1. logarithm

2. base of a logarithm

3. common logarithm

4. natural logarithm

Problem Set

Convert each exponential equation to logarithmic form. Write a sentence to demonstrate how the logarithm equation would be spoken.

5. $7^x = 9$

6. $8^x = 12$

6

7. $3^{2x} = 4$

8. $6^{3x} = 2$

9. $e^x = 15$

10. $e^x = \dfrac{1}{2}$

11. $10^{2x-3} = 4$

12. $10^{-6x+5} = 27$

13. $7^{x-5} = 0.12$

14. $8^{-3x+2} = 0.03$

6

Evaluate each logarithm.

15. $\log_2 8$

16. $\log_5 25$

17. $\log 10,000$

18. $\log 1,000,000$

19. $\log_9 81$

20. $\log_3 81$

21. $\log_7 7^3$

22. $\log_4 4^5$

Write a sentence to demonstrate how each logarithmic equation would be spoken. Then convert each logarithmic equation to exponential form.

23. $x = \log_2 5$

24. $x = \log_4 6$

25. $4x = \log_5 2.3$

26. $0.2x = \log_8 4.2$

27. $x + 5 = \log_2 5.1$

28. $x - 11 = \log_3 0.02$

29. $2x + 4 = \log_3 6$

30. $3x - 6 = \log_7 15$

31. $x = \log_4 7 + 3$

32. $x = \log 8 - 4$

6

Solve each equation by writing each side of the equation as an exponential expression with the same base.

33. $2^x = 32$

34. $3^x = 81$

35. $4^{x+2} = 64$

36. $5^{-x-3} = 125$

Solve each equation by converting to logarithmic form.

37. $10^{-x+3} = 1{,}000{,}000$

38. $10^{4x-3} = 100{,}000$

39. $10^{2x+4} = 0.000001$

40. $10^{-2x-5} = 0.001$

41. $2^{5x} - 8 = 0$

42. $9^{4x} - 3 = 0$

6

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Write the inverse of each function.

43. $f(x) = 5^x$

44. $f(x) = 4^x$

45. $f(x) = \log x$

46. $f(x) = \log_{1.3} x$

Complete each table of values for $f(x)$ and $f^{-1}(x)$.

47.

x	$f^{-1}(x) = \log_5 x$

x	$f(x) = 5^x$
0	
1	
−1	
2	
−2	

48.

x	$f(x) = 4^x$
0	
1	
−1	
2	
−2	

x	$f^{-1}(x) = \log_4 x$

6

49.

x	$f(x) = \log_{10} x$
1	
10	
100	
$\frac{1}{10}$	
$\frac{1}{100}$	

x	$f^{-1}(x) = 10^x$

50.

x	$f^{-1}(x) = 3^x$

x	$f(x) = \log_3 x$
1	
3	
$\frac{1}{3}$	
9	
$\frac{1}{9}$	

6

51.

x	$f(x) = 9^x$
0	
1	
−1	
2	
−2	

x	$f^{-1}(x) = \log_9 x$

52.

x	$f^{-1}(x) = \log_{25} x$

x	$f(x) = 25^x$
0	
1	
−1	
2	
−2	

6

Given each graph of _f_(_x_), graph its inverse, _f_⁻¹(_x_).

53. $f(x) = 2^x$

$f(x) = 2^x$

54. $f(x) = 6^x$

$f(x) = 6^x$

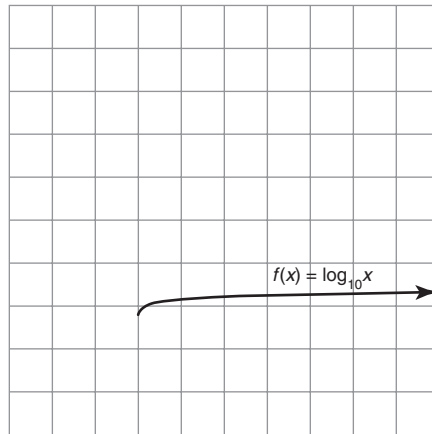**55.** $f(x) = \log_{1.5} x$

$f(x) = \log_{1.5} x$

56. $f(x) = \log_{10} x$

$f(x) = \log_{10} x$

6
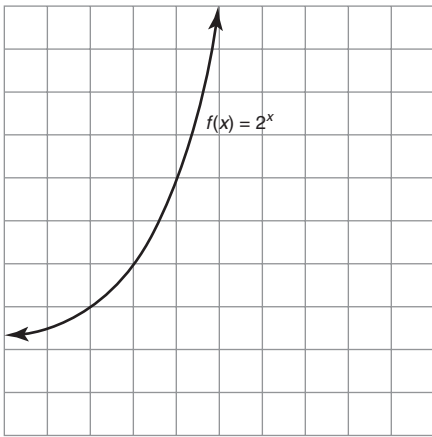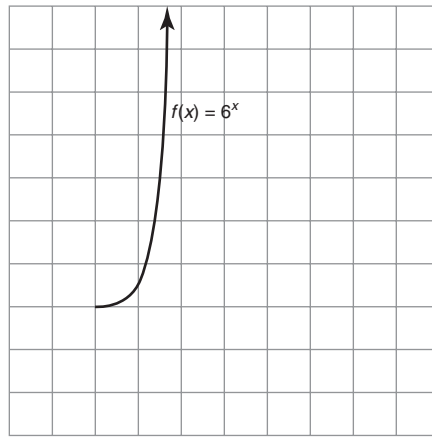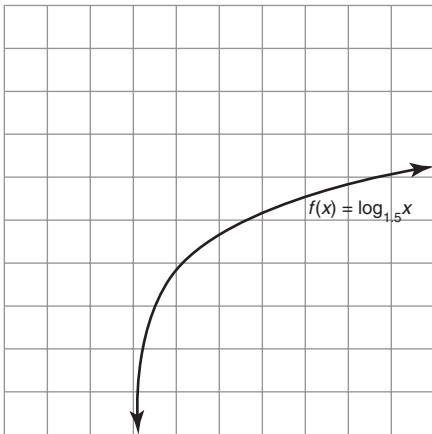
Skills Practice

Name _____ Date _____

Properties of Logarithms
Deriving the Properties of Logarithms

Vocabulary

Match each expression below with the best word that describes it.

1. x^{-3}

a. base with an exponent

2. 4^2

b. logarithm

3. $\log_2 x$

c. base with a negative exponent

Problem Set

For each property of exponents, write the corresponding property of logarithms.

4. $a^m \cdot a^n$

5. $\dfrac{a^m}{a^n}$

6. $(a^m)^2$

7. $(a^n)^{\frac{1}{2}}$

Rewrite each expression using common logs. Then evaluate the expression.

8. $\log_5 7$

9. $\log_9 11$

10. $\log_4 215$

11. $\log_{13} 144$

12. $\log_{23} 22$

13. $\log_{100} 25$

14. ln 6

15. ln 12

Write each logarithmic expression in expanded form.

16. $\log_3 5x^2$

17. $\log_2 3x^4$

18. $\log_8 6x^3y^5$

19. $\log_5 10x^9y^7$

20. $\log_4 \dfrac{1}{x^2}$

21. $\log_5 \dfrac{y}{x^4}$

22. $\log_6 \dfrac{8x^4}{y^6}$

23. $\log_3 \dfrac{2}{x^2y^{10}}$

24. $\ln 4x^3y^6$

25. $\ln \dfrac{x^3}{4y^2}$

Write each logarithmic expression using a single logarithm. Evaluate the logarithm if possible.

26. $\log_{11} 5 + 2 \log_{11} y$

27. $\log_3 6 + 5 \log_3 x$

28. $\log_4 10 + 3 \log_4 x + 6 \log_4 y$

29. $\log_7 122 + 5 \log_7 x + 8 \log_7 y$

30. $3 \log 20 - 2 \log 4$

31. $3 \log 5 + 8 \log 2$

32. $2 \log 3 - 3 \log x + 6 \log y$

33. $4 \log 5 + 5 \log x - 8 \log y$

34. $4(\ln 2 - \ln y) + 2(\ln x - \ln 5)$

35. $3(\ln 3 + \ln x - 2 \ln y) + 2(\ln 4 - 3 \ln y)$

6

Skills Practice

Name _____ Date _____

Continuous Growth, Decay and Interest
Solving Exponential and Logarithmic Equations

Vocabulary

Write the term from the box that best completes each statement.

compound interest	natural number	simple interest

1. The number e is called the _____.

2. _____ is when interest is paid only as a percent of the principal.

3. _____ is interest on both the principal and previously earned interest.

Problem Set

Answer each question about population.

4. The town of Klamath has a population of 4000 and grows 3% annually. When will the population be 5000?

6

5. The city of Brooklyn has a population of 135,000 and grows 4.2% annually. When will the population be 150,000?

6. The township of O'Hare has a population of 350 and grows 1.5% annually. When will the population double?

7. The municipality of Noren has a population of 12,000 and grows 2.9% annually. How long will it take the population to triple?

6

8. The population of Italy is 58 million in 2009 and decreases 0.047% annually. Assuming that the population will decrease at this rate, when will the population reach 50 million?

9. The population of Ukraine is about 45 million in 2009 and decreases 0.632% annually. Assuming that the population will decrease at this rate, when will the population reach 40 million?

6

Calculate the time to double each investment.

10. A bond of $100 earns 1.3% compounded monthly.

11. A money market account of $1000 earns 3.5% compounded monthly.

6

12. A mutual fund of $2000 earns 4.3% compounded annually.

13. A Roth IRA account of $5000 earns 6% compounded annually.

14. A Swiss bank account of $7000 earns 4.2% compounded continuously.

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15. An ad in the paper is asking for an investment of $10,000 and claims to offer you 7.8% compounded continuously.

Answer each question about decay.

16. Sodium-22 has a half-life of 2.6 years. Determine the age of a piece of salt, if 10% of the sodium-22 is left in the salt.

17. Copper-60 has a half-life of 5.3 years. A museum has bought what it thinks to be a prehistoric tool. Determine the age of the tool if 0.002% of the copper-60 is left in the tool. Decide if the tool is from a prehistoric period at least 12,000 years ago.

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18. Determine the decay constant for lead-210, if it has a half-life of 22.3 years.

19. Determine the decay constant for platinum-193, if it has a half-life of 50 years.

Answer each question about the spread of disease.

20. In a small town of 2000 people, 8 people have a cold. Each victim can infect 4 people a day. When will the entire town have the cold?

21. At a basketball conference of 64 athletes, 2 have a foot fungus. Each victim can infect 3 people a day. When will all the athletes have the fungus?

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22. A park of 240,000 trees experiences a blight that infects 12 trees, and each infected tree infects 6 more a day. When will all of the trees in the park be infected with the blight?

23. A puma population of 250 experiences a disease that infects 2, and each infected puma infects 2 more a day. When will all of the pumas be infected with the disease?

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