SL Trig Identities 2008-14 with MS

4a. *[2 marks]*

Let $f(x) = \sin^3 x + \cos^3 x \tan x,\ \frac{\pi}{2} < x < \pi$.

Show that $f(x) = \sin x$.

4b. *[5 marks]*

Let $\sin x = \frac{2}{3}$. Show that $f(2x) = -\frac{4\sqrt{5}}{9}$.

5a. *[2 marks]*

Let $p = \sin 40^\circ$ and $q = \cos 110^\circ$. Give your answers to the following in terms of *p* and/or *q* .

Write down an expression for

(i) $\sin 140^\circ$;

(ii) $\cos 70^\circ$.

5b. *[3 marks]*

Find an expression for $\cos 140^\circ$.

5c. *[1 mark]*

Find an expression for $\tan 140^\circ$.

6a. *[3 marks]*

Given that $\cos A = \frac{1}{3}$ and $0 \le A \le \frac{\pi}{2}$, find $\cos 2A$.

6b. *[3 marks]*

Given that $\sin B = \frac{2}{3}$ and $\frac{\pi}{2} \le B \le \pi$, find $\cos B$.

7a. *[3 marks]*

The expression $6 \sin x \cos x$ can be expressed in the form $a \sin bx$.

Find the value of *a* and of *b* .

7b. *[4 marks]*

Hence or otherwise, solve the equation $6 \sin x \cos x = \frac{3}{2}$, for $\frac{\pi}{4} \le x \le \frac{\pi}{2}$.

8a. *[3 marks]*

Let $\sin 100^\circ = m$. Find an expression for $\cos 100^\circ$ in terms of *m*.

8b. *[1 mark]*

Let $\sin 100^\circ = m$. Find an expression for $\tan 100^\circ$ in terms of *m*.

8c. *[2 marks]*

Let $\sin 100^\circ = m$. Find an expression for $\sin 200^\circ$ in terms of *m*.

9a. *[2 marks]*

Let $f(x) = (\sin x + \cos x)^2$.

Show that $f(x)$ can be expressed as $1 + \sin 2x$.

10a. *[2 marks]*

Show that $4 - \cos 2\theta + 5 \sin\theta = 2\sin^2\theta + 5\sin\theta + 3$.

10b. *[5 marks]*

Hence, solve the equation $4 - \cos 2\theta + 5\sin\theta = 0$ for $0 \le \theta \le 2\pi$.

11a. *[1 mark]*

The straight line with equation $y = \frac{3}{4}x$ makes an acute angle θ with the *x*-axis.

Write down the value of $\tan\theta$.

11b. *[6 marks]*

Find the value of

(i) $\sin 2\theta$;

(ii) $\cos 2\theta$.

12a. *[2 marks]*

Let $f(x) = \cos 2x$ and $g(x) = 2x^2 - 1$.

Find $f\left(\frac{\pi}{2}\right)$.

12b. *[2 marks]*

Find $(g \circ f)\left(\frac{\pi}{2}\right)$.

12c. *[3 marks]*

Given that $(g \circ f)(x)$ can be written as $\cos(kx)$, find the value of *k*, $k \in \mathbb{Z}$.

13. *[7 marks]*

Solve $\cos 2x - 3\cos x - 3 - \cos^2 x = \sin^2 x$, for $0 \le x \le 2\pi$.

Let $f(x) = \sqrt{3}e^{2x} \sin x + e^{2x} \cos x$, for $0 \le x \le \pi$. Solve the equation $f(x) = 0$.

16a. *[3 marks]*

Let $\sin\theta = \frac{2}{\sqrt{13}}$, where $\frac{\pi}{2} < \theta < \pi$.

Find $\cos\theta$.

16b. *[5 marks]*

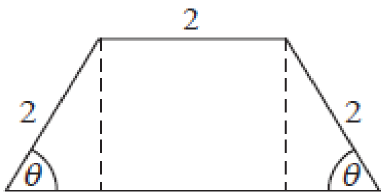
Find $\tan 2\theta$.

17. *[6 marks]*

Let $h(x) = \frac{6x}{\cos x}$. Find $h'(0)$.

18a. *[5 marks]*

The diagram below shows a plan for a window in the shape of a trapezium.



Three sides of the window are 2m long. The angle between the sloping sides of the window and the base is θ, where $0 < \theta < \frac{\pi}{2}$.

Show that the area of the window is given by $y = 4\sin\theta + 2\sin 2\theta$.

18b. *[4 marks]*

Zoe wants a window to have an area of 5 m^2. Find the two possible values of θ.

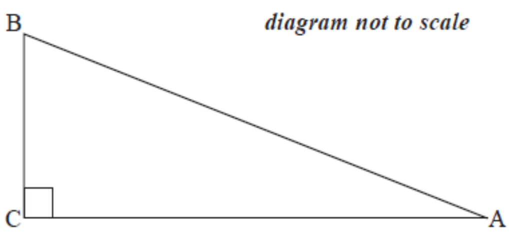
18c. *[7 marks]*

John wants two windows which have the same area *A* but different values of θ.

Find all possible values for *A*.

19a. *[2 marks]*

The following diagram shows a right-angled triangle, ABC, where $\sin \text{A} = \frac{5}{13}$.



Show that $\cos A = \frac{12}{13}$.

19b. *[3 marks]*

Find $\cos 2A$.

SL Trig Identities 2008-14 MS

1a. *[5 marks]*

Markscheme

(i) range of *f* is $[-1, 1]$, $(-1 \le f(x) \le 1)$ ***A2 N2***

(ii) $\sin^3 x \Rightarrow 1 \Rightarrow \sin x = 1$ ***A1***

justification for one solution on $[0, 2\pi]$ ***R1***

e.g. $x = \frac{\pi}{2}$, unit circle, sketch of $\sin x$

1 solution (seen anywhere) ***A1 N1***

[5 marks]

Examiners report

This question was not done well by most candidates. No more than one-third of them could correctly give the range of $f(x) = \sin^3 x$ and few could provide adequate justification for there being exactly one solution to $f(x) = 1$ in the interval $[0, 2\pi]$.

1b. *[2 marks]*

Markscheme

$f'(x) = 3\sin^2 x \cos x$ ***A2 N2***

[2 marks]

Examiners report

This question was not done well by most candidates.

1c. *[7 marks]*

Markscheme

using $V = \int_a^b \pi y^2 \mathrm{d}x$ ***(M1)***

$V = \int_0^{\frac{\pi}{2}} \pi(\sqrt{3} \sin x \cos^{\frac{1}{2}} x)^2 \mathrm{d}x$ ***(A1)***

$= \pi \int_0^{\frac{\pi}{2}} 3\sin^2 x \cos x \mathrm{d}x$ ***A1***

$V = \pi[\sin^3 x]_0^{\frac{\pi}{2}} \left(= \pi\left(\sin^3\left(\frac{\pi}{2}\right) - \sin^3 0\right)\right)$ ***A2***

evidence of using $\sin\frac{\pi}{2} = 1$ and $\sin 0 = 0$ ***(A1)***

e.g. $\pi(1 - 0)$

$V = \pi$ ***A1 N1***

[7 marks]

Examiners report

This question was not done well by most candidates. No more than one-third of them could correctly give the range of $f(x) = \sin^3 x$ and few could provide adequate justification for there being exactly one solution to $f(x) = 1$ in the interval $[0, 2\pi]$. Finding the derivative of this function also presented major problems, thus making part (c) of the question much more difficult. In spite of the formula for volume of revolution being given in the Information Booklet, fewer than half of the candidates could correctly put the necessary function and limits into $\pi \int_a^b y^2 \mathrm{d}x$ and fewer still could square $\sqrt{3} \sin x \cos^{\frac{1}{2}} x$ correctly. From those who did square correctly, the correct antiderivative was not often recognized. All manner of antiderivatives were suggested instead.

2a. *[3 marks]*

Markscheme

attempt to use substitution or inspection ***M1***

e.g. $u = 1 + \mathrm{e}^x$ so $\frac{\mathrm{d}u}{\mathrm{d}x} = \mathrm{e}^x$

correct working ***A1***

e.g. $\int \frac{\mathrm{d}u}{u} = \ln u$

$\ln(1 + \mathrm{e}^x) + C$ ***A1 N3***

[3 marks]

Examiners report

[N/A]

2b. *[4 marks]*

Markscheme

METHOD 1

attempt to use substitution or inspection ***M1***
e.g. let $u = \sin 3x$
$\frac{du}{dx} = 3\cos 3x$ ***A1***
$\frac{1}{3}\int u du = \frac{1}{3} \times \frac{u^2}{2} + C$ ***A1***
$\int \sin 3x \cos 3x dx = \frac{\sin^2 3x}{6} + C$ ***A1 N2***
METHOD 2
attempt to use substitution or inspection ***M1***
e.g. let $u = \cos 3x$
$\frac{du}{dx} = -3\sin 3x$ ***A1***
$-\frac{1}{3}\int u du = -\frac{1}{3} \times \frac{u^2}{2} + C$ ***A1***
$\int \sin 3x \cos 3x dx = \frac{\cos^2 3x}{6} + C$ ***A1 N2***
METHOD 3
recognizing double angle ***M1***
correct working ***A1***
e.g. $\frac{1}{2}\sin 6x$
$\int \sin 6x dx = \frac{-\cos 6x}{6} + C$ ***A1***
$\int \frac{1}{2}\sin 6x dx = -\frac{\cos 6x}{12} + C$ ***A1 N2***
[4 marks]
Examiners report
[N/A]
3a. *[4 marks]*
Markscheme
derivative of $2x$ is 2 (must be seen in quotient rule) ***(A1)***
derivative of $x^2 + 5$ is $2x$ (must be seen in quotient rule) ***(A1)***
correct substitution into quotient rule ***A1***
eg $\frac{(x^2+5)(2)-(2x)(2x)}{(x^2+5)^2}$, $\frac{2(x^2+5)-4x^2}{(x^2+5)^2}$
correct working which clearly leads to given answer ***A1***
eg $\frac{2x^2+10-4x^2}{(x^2+5)^2}$, $\frac{2x^2+10-4x^2}{x^4+10x^2+25}$
$f'(x) = \frac{10-2x^2}{(x^2+5)^2}$ ***AG N0***
[4 marks]
Examiners report
[N/A]
3b. *[4 marks]*
Markscheme
valid approach using substitution or inspection ***(M1)***
eg $u = x^2 + 5, \mathrm{d}\,u = 2x\mathrm{d}x,\ \frac{1}{2}\ln(x^2+5)$
$\int \frac{2x}{x^2+5} dx = \int \frac{1}{u} du$ ***(A1)***
$\int \frac{1}{u} du = \ln u + c$ ***(A1)***
$\ln(x^2+5) + c$ ***A1 N4***
[4 marks]
Examiners report
[N/A]
3c. *[7 marks]*
Markscheme
correct expression for area ***(A1)***
eg $\left[\ln\left(x^2+5\right)\right]_{\sqrt{5}}^{q}$, $\int_{\sqrt{5}}^{q} \frac{2x}{x^2+5} dx$
substituting limits into **their** integrated function and subtracting (in either order) ***(M1)***

eg $\ln(q^2+5) - \ln\left(\sqrt{5}^2+5\right)$
correct working ***(A1)***
eg $\ln\left(q^2+5\right) - \ln 10,\ \ln\frac{q^2+5}{10}$
equating **their** expression to $\ln 7$ (seen anywhere) ***(M1)***
eg $\ln\left(q^2+5\right) - \ln 10 = \ln 7,\ \ln\frac{q^2+5}{10} = \ln 7,\ \ln(q^2+5) = \ln 7 + \ln 10$
correct equation without logs ***(A1)***
eg $\frac{q^2+5}{10} = 7,\ q^2+5 = 70$
$q^2 = 65$ ***(A1)***
$q = \sqrt{65}$ ***A1 N3***
Note: Award ***A0*** for $q = \pm\sqrt{65}$.
[7 marks]
Examiners report
[N/A]
4a. *[2 marks]*
Markscheme
changing $\tan x$ into $\frac{\sin x}{\cos x}$ ***A1***
e.g. $\sin^3 x + \cos^3 x\frac{\sin x}{\cos x}$
simplifying ***A1***
e.g $\sin x(\sin^2 x + \cos^2 x)$, $\sin^3 x + \sin x - \sin^3 x$
$f(x) = \sin x$ ***AG N0***
[2 marks]
Examiners report
Not surprisingly, this question provided the greatest challenge in section A. In part (a), candidates were able to use the identity $\tan x = \frac{\sin x}{\cos x}$, but many could not proceed any further.
4b. *[5 marks]*
Markscheme
recognizing $f(2x) = \sin 2x$, seen anywhere ***(A1)***
evidence of using double angle identity $\sin(2x) = 2\sin x\cos x$, seen anywhere ***(M1)***
evidence of using Pythagoras with $\sin x = \frac{2}{3}$ ***M1***
e.g. sketch of right triangle, $\sin^2 x + \cos^2 x = 1$
$\cos x = -\frac{\sqrt{5}}{3}$ (accept $\frac{\sqrt{5}}{3}$) ***(A1)***
$f(2x) = 2\left(\frac{2}{3}\right)\left(-\frac{\sqrt{5}}{3}\right)$ ***A1***
$f(2x) = -\frac{4\sqrt{5}}{9}$ ***AG N0***
[5 marks]
Examiners report
Part (b) was generally well done by those candidates who attempted it, the major error arising when the negative sign "magically" appeared in the answer. Many candidates could find the value of cos*x* but failed to observe that cosine is negative in the given domain.
5a. *[2 marks]*
Markscheme
(i) $\sin 140^\circ = p$ ***A1 N1***
(ii) $\cos 70^\circ = -q$ ***A1 N1***
[2 marks]
Examiners report
This was one of the most difficult problems for the candidates. Even the strongest candidates had a hard time with this one and only a few received any marks at all.
5b. *[3 marks]*
Markscheme
METHOD 1
evidence of using $\sin^2\theta + \cos^2\theta = 1$ ***(M1)***

e.g. diagram, $\sqrt{1-p^2}$ (seen anywhere)
$\cos 140^\circ = \pm\sqrt{1-p^2}$ ***(A1)***
$\cos 140^\circ = -\sqrt{1-p^2}$ ***A1 N2***
METHOD 2
evidence of using $\cos 2\theta = 2\cos^2\theta - 1$ ***(M1)***
$\cos 140^\circ = 2\cos^2 70 - 1$ ***(A1)***
$\cos 140^\circ = 2(-q)^2 - 1\ (= 2q^2 - 1)$ ***A1 N2***
[3 marks]

Examiners report

Many did not appear to know the relationships between trigonometric functions of supplementary angles and that the use of $\sin^2 x + \cos^2 x = 1$ results in a $\pm$ value. The application of a double angle formula also seemed weak.

5c. *[1 mark]*

Markscheme

METHOD 1
$\tan 140^\circ = \frac{\sin 140^\circ}{\cos 140^\circ} = -\frac{p}{\sqrt{1-p^2}}$ ***A1 N1***
METHOD 2
$\tan 140^\circ = \frac{p}{2q^2-1}$ ***A1 N1***
[1 mark]

Examiners report

This was one of the most difficult problems for the candidates. Even the strongest candidates had a hard time with this one and only a few received any marks at all. Many did not appear to know the relationships between trigonometric functions of supplementary angles and that the use of $\sin^2 x + \cos^2 x = 1$ results in a $\pm$ value. The application of a double angle formula also seemed weak.

6a. *[3 marks]*

Markscheme

evidence of choosing the formula $\cos 2A = 2\cos^2 A - 1$ ***(M1)***
Note: If they choose another correct formula, do not award the ***M1*** unless there is evidence of finding $\sin^2 A = 1 - \frac{1}{9}$
correct substitution ***A1***
e.g. $\cos 2A = \left(\frac{1}{3}\right)^2 - \frac{8}{9}$, $\cos 2A = 2 \times \left(\frac{1}{3}\right)^2 - 1$
$\cos 2A = -\frac{7}{9}$ ***A1 N2***
[3 marks]

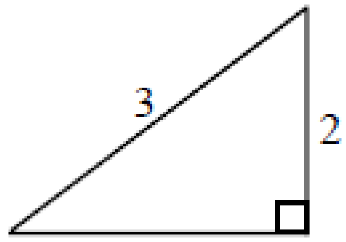
Examiners report

This question was very poorly done, and knowledge of basic trigonometric identities and values of trigonometric functions of obtuse angles seemed distinctly lacking. Candidates who recognized the need of an identity for finding $\cos 2A$ given $\cos A$ seldom chose the most appropriate of the three and even when they did often used it incorrectly with expressions such as $2\cos^2 \frac{1}{9} - 1$.

6b. *[3 marks]*

Markscheme

METHOD 1
evidence of using $\sin^2 B + \cos^2 B = 1$ ***(M1)***
e.g. $\left(\frac{2}{3}\right)^2 + \cos^2 B = 1$, $\sqrt{\frac{5}{9}}$ (seen anywhere),
$\cos B = \pm\sqrt{\frac{5}{9}}\ \left(= \pm\frac{\sqrt{5}}{3}\right)$ ***(A1)***
$\cos B = -\sqrt{\frac{5}{9}}\ \left(= -\frac{\sqrt{5}}{3}\right)$ ***A1 N2***
METHOD 2
diagram ***M1***



for finding third side equals $\sqrt{5}$ ***(A1)***

$\cos B = -\frac{\sqrt{5}}{3}$ ***A1 N2***

[3 marks]

Examiners report

This question was very poorly done, and knowledge of basic trigonometric identities and values of trigonometric functions of obtuse angles seemed distinctly lacking. Candidates who recognized the need of an identity for finding $\cos 2A$ given $\cos A$ seldom chose the most appropriate of the three and even when they did often used it incorrectly with expressions such as $2\cos^2 \frac{1}{9} - 1$.

7a. *[3 marks]*

Markscheme

recognizing double angle ***M1***

e.g. $3 \times 2\sin x \cos x$, $3\sin 2x$

$a = 3$, $b = 2$ ***A1A1 N3***

[3 marks]

Examiners report

[N/A]

7b. *[4 marks]*

Markscheme

substitution $3\sin 2x = \frac{3}{2}$ ***M1***

$\sin 2x = \frac{1}{2}$ ***A1***

finding the angle ***A1***

e.g. $\frac{\pi}{6}$, $2x = \frac{5\pi}{6}$

$x = \frac{5\pi}{12}$ ***A1 N2***

Note: Award ***A0*** if other values are included.

[4 marks]

Examiners report

[N/A]

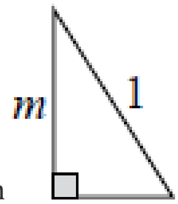
8a. *[3 marks]*

Markscheme

Note: All answers must be given in terms of *m*. If a candidate makes an error that means there is no *m* in their answer, do not award the final ***A1FT*** mark.

METHOD 1

valid approach involving Pythagoras ***(M1)***



e.g. $\sin^2 x + \cos^2 x = 1$, labelled diagram

correct working (may be on diagram) ***(A1)***

e.g. $m^2 + (\cos 100)^2 = 1$, $\sqrt{1 - m^2}$

$\cos 100 = -\sqrt{1 - m^2}$ ***A1 N2***

[3 marks]

METHOD 2

valid approach involving tan identity ***(M1)***

e.g. $\tan = \frac{\sin}{\cos}$

correct working ***(A1)***

e.g. $\cos 100 = \frac{\sin 100}{\tan 100}$
$\cos 100 = \frac{m}{\tan 100}$ ***A1 N2***

[3 marks]

Examiners report

While many candidates correctly approached the problem using Pythagoras in part (a), very few recognized that the cosine of an angle in the second quadrant is negative. Many were able to earn follow-through marks in subsequent parts of the question. A common algebraic error in part (a) was for candidates to write $\sqrt{1-m^2} = 1-m$. In part (c), many candidates failed to use the double-angle identity. Many incorrectly assumed that because $\sin 100^\circ = m$, then $\sin 200^\circ = 2m$. In addition, some candidates did not seem to understand what writing an expression "in terms of *m*" meant.

8b. *[1 mark]*

Markscheme

METHOD 1

$\tan 100 = -\frac{m}{\sqrt{1-m^2}}$ (accept $\frac{m}{-\sqrt{1-m^2}}$) ***A1 N1***

[1 mark]

METHOD 2

$\tan 100 = \frac{m}{\cos 100}$ ***A1 N1***

[1 mark]

Examiners report

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8c. *[2 marks]*

Markscheme

METHOD 1

valid approach involving double angle formula ***(M1)***

e.g. $\sin 2\theta = 2\sin\theta cos\theta$

$\sin 200 = -2m\sqrt{1-m^2}$ (accept $2m\left(-\sqrt{1-m^2}\right)$) ***A1 N2***

Note: If candidates find $\cos 100 = \sqrt{1-m^2}$, award full ***FT*** in parts (b) and (c), even though the values may not have appropriate signs for the angles.

[2 marks]

METHOD 2

valid approach involving double angle formula ***(M1)***

e.g. $\sin 2\theta = 2\sin\theta\cos\theta$, $2m \times \frac{m}{\tan 100}$

$\sin 200 = \frac{2m^2}{\tan 100}$ $(= 2m\cos 100)$ ***A1 N2***

[2 marks]

Examiners report

While many candidates correctly approached the problem using Pythagoras in part (a), very few recognized that the cosine of an angle in the second quadrant is negative. Many were able to earn follow-through marks in subsequent parts of the question. A common algebraic error in part (a) was for candidates to write $\sqrt{1-m^2} = 1-m$. In part (c), many candidates failed to use the double-angle identity. Many incorrectly assumed that because $\sin 100^\circ = m$, then $\sin 200^\circ = 2m$. In addition, some candidates did not seem to understand what writing an expression "in terms of *m*" meant.

9a. *[2 marks]*

Markscheme

attempt to expand ***(M1)***

e.g. $(\sin x + \cos x)(\sin x + \cos x)$; at least 3 terms

correct expansion ***A1***

e.g. $\sin^2 x + 2\sin x\cos x + \cos^2 x$

$f(x) = 1 + \sin 2x$ ***AG N0***

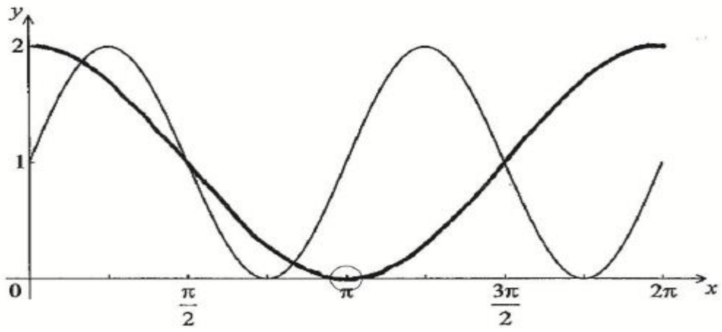
[2 marks]

Examiners report

Simplifying a trigonometric expression and applying identities was generally well answered in part (a), although some candidates were certainly helped by the fact that it was a "show that" question.

9b. *[2 marks]*

Markscheme



A1A1 N2

Note: Award ***A1*** for correct sinusoidal shape with period 2π and range $[0, 2]$, ***A1*** for minimum in circle.

Examiners report

More candidates had difficulty with part (b) with many assuming the first graph was $1 + \sin(x)$ and hence sketching a horizontal translation of $\pi/2$ for the graph of *g*; some attempts were not even sinusoidal. While some candidates found the stretch factor *p* correctly or from follow-through on their own graph, very few successfully found the value and direction for the translation.

9c. *[2 marks]*

Markscheme

$p = 2$, $k = -\frac{\pi}{2}$ ***A1A1 N2***

[2 marks]

Examiners report

Part (c) certainly served as a discriminator between the grade 6 and 7 candidates.

10a. *[2 marks]*

Markscheme

attempt to substitute $1 - 2\sin^2\theta$ for $\cos 2\theta$ ***(M1)***

correct substitution ***A1***

e.g. $4 - (1 - 2\sin^2\theta) + 5\sin\theta$

$4 - \cos 2\theta + 5\sin\theta = 2\sin^2\theta + 5\sin\theta + 3$ ***AG N0***

[2 marks]

Examiners report

In part (a), most candidates successfully substituted using the double-angle formula for cosine. There were quite a few candidates who worked backward, starting with the required answer and manipulating the equation in various ways. As this was a "show that" question, working backward from the given answer is not a valid method.

10b. *[5 marks]*

Markscheme

evidence of appropriate approach to solve ***(M1)***

e.g. factorizing, quadratic formula

correct working ***A1***

e.g. $(2\sin\theta + 3)(\sin\theta + 1)$, $(2x + 3)(x + 1) = 0$, $\sin x = \frac{-5\pm\sqrt{1}}{4}$

correct solution $\sin\theta = -1$ (do not penalise for including $\sin\theta = -\frac{3}{2}$ ***(A1)***

$\theta = \frac{3\pi}{2}$ ***A2 N3***

[5 marks]

Examiners report

In part (b), many candidates seemed to realize what was required by the word “hence”, though some had trouble factoring the quadratic-type equation. A few candidates were also successful using the quadratic formula. Some candidates got the wrong solution to the equation $\sin\theta = -1$, and there were a few who did not realize that the equation $\sin\theta = -\frac{3}{2}$ has no solution.

11a. *[1 mark]*

Markscheme

$\tan\theta = \frac{3}{4}$ (do not accept $\frac{3}{4}x$) ***A1 N1***

[1 mark]

Examiners report

Many candidates drew a diagram to correctly find $\tan\theta$, although few recognized that a line through the origin can be expressed as $y = x\tan\theta$, with gradient $\tan\theta$, which is explicit in the syllabus.

11b. *[6 marks]*

Markscheme

(i) $\sin\theta = \frac{3}{5}$, $\cos\theta = \frac{4}{5}$ ***(A1)(A1)***

correct substitution ***A1***

e.g. $\sin 2\theta = 2\left(\frac{3}{5}\right)\left(\frac{4}{5}\right)$

$\sin 2\theta = \frac{24}{25}$ ***A1 N3***

(ii) correct substitution ***A1***

e.g. $\cos 2\theta = 1 - 2\left(\frac{3}{5}\right)^2$, $\left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2$

$\cos 2\theta = \frac{7}{25}$ ***A1 N1***

[6 marks]

Examiners report

A surprising number were unable to find the ratios for $\sin\theta$ and $\cos\theta$ from $\tan\theta$. It was not uncommon for candidates to use unreasonable values, such as $\sin\theta = 3$ and $\cos\theta = 4$, or to write nonsense such as $2\sin\frac{3}{5}\cos\frac{4}{5}$.

12a. *[2 marks]*

Markscheme

$f\left(\frac{\pi}{2}\right) = \cos\pi$ ***(A1)***

$= -1$ ***A1 N2***

[2 marks]

Examiners report

In part (a), a number of candidates were not able to evaluate $\cos\pi$, either leaving it or evaluating it incorrectly.

12b. *[2 marks]*

Markscheme

$(g\circ f)\left(\frac{\pi}{2}\right) = g(-1)\ (= 2(-1)^2 - 1)$ ***(A1)***

$= 1$ ***A1 N2***

[2 marks]

Examiners report

Almost all candidates evaluated the composite function in part (b) in the given order, many earning follow-through marks for incorrect answers from part (a). On both parts (a) and (b), there were candidates who correctly used double-angle formulas to come up with correct answers; while this is a valid method, it required unnecessary additional work.

12c. *[3 marks]*

Markscheme

$(g\circ f)(x) = 2(\cos(2x))^2 - 1\ (= 2\cos^2(2x) - 1)$ ***A1***

evidence of $2\cos^2\theta - 1 = \cos 2\theta$ (seen anywhere) ***(M1)***

$(g\circ f)(x) = \cos 4x$

$k = 4$ ***A1 N2***

[3 marks]

Examiners report

Candidates were not as successful in part (c). Many tried to use double-angle formulas, but either used the formula incorrectly or used it to write the expression in terms of $\cos x$ and went no further. There were a number of cases in which the candidates "accidentally" came up with the correct answer based on errors or lucky guesses and did not earn credit for their final answer. Only a few candidates recognized the correct method of solution.

13. *[7 marks]*

Markscheme

evidence of substituting for $\cos 2x$ ***(M1)***

evidence of substituting into $\sin^2 x + \cos^2 x = 1$ ***(M1)***

correct equation in terms of $\cos x$ (seen anywhere) ***A1***

e.g. $2\cos^2 x - 1 - 3\cos x - 3 = 1$, $2\cos^2 x - 3\cos x - 5 = 0$

evidence of appropriate approach to solve ***(M1)***

e.g. factorizing, quadratic formula

appropriate working ***A1***

e.g. $(2\cos x - 5)(\cos x + 1) = 0$, $(2x - 5)(x + 1)$, $\cos x = \frac{3\pm\sqrt{49}}{4}$

correct solutions to the equation

e.g. $\cos x = \frac{5}{2}$, $\cos x = -1$, $x = \frac{5}{2}$, $x = -1$ ***(A1)***

$x = \pi$ ***A1 N4***

[7 marks]

Examiners report

This question was quite difficult for most candidates. A number of students earned some credit for manipulating the equation with identities, but many earned no further marks due to algebraic errors. Many did not substitute for $\cos 2x$; others did this substitution but then did nothing further. Few candidates were able to get a correct equation in terms of $\cos x$ and many who did get the equation didn't know what to do with it. Candidates who correctly solved the resulting quadratic usually found the one correct value of x, earning full marks.

14a. *[3 marks]*

Markscheme

(i) evidence of approach ***(M1)***

e.g. $\overrightarrow{PQ} = \overrightarrow{PO} + \overrightarrow{OQ}$, $Q - P$

$\overrightarrow{PQ} = \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}$ ***A1 N2***

(ii) $\overrightarrow{PR} = \begin{pmatrix} 2 \\ 2 \\ 4 \end{pmatrix}$ ***A1 N1***

[3 marks]

Examiners report

Combining the vectors in (a) was generally well done, although some candidates reversed the subtraction, while others calculated the magnitudes.

14b. *[7 marks]*

Markscheme

METHOD 1

choosing correct vectors $\overrightarrow{PQ}$ and $\overrightarrow{PR}$ ***(A1)(A1)***

finding $\overrightarrow{PQ} \bullet \overrightarrow{PR}$, $\left|\overrightarrow{PQ}\right|$, $\left|\overrightarrow{PR}\right|$ ***(A1) (A1)(A1)***

$\overrightarrow{PQ} \bullet \overrightarrow{PR} = -2 + 4 + 4 (= 6)$

$\left|\overrightarrow{PQ}\right| = \sqrt{(-1)^2 + 2^2 + 1^2} \left(= \sqrt{6}\right)$, $\left|\overrightarrow{PR}\right| = \sqrt{2^2 + 2^2 + 4^2} \left(= \sqrt{24}\right)$

substituting into formula for angle between two vectors ***M1***

e.g. $\cos R\hat{P}Q = \frac{6}{\sqrt{6}\times\sqrt{24}}$

simplifying to expression clearly leading to $\frac{1}{2}$ ***A1***

e.g. $\frac{6}{\sqrt{6}\times 2\sqrt{6}}$, $\frac{6}{\sqrt{144}}$, $\frac{6}{12}$

$\cos \mathrm{R\widehat{P}Q} = \frac{1}{2}$ ***AG N0***

METHOD 2

evidence of choosing cosine rule (seen anywhere) ***(M1)***

$\overrightarrow{\mathrm{QR}} = \begin{pmatrix} 3 \\ 0 \\ 3 \end{pmatrix}$ ***A1***

$\left|\overrightarrow{\mathrm{QR}}\right| = \sqrt{18}$, $\left|\overrightarrow{\mathrm{PQ}}\right| = \sqrt{6}$ and $\left|\overrightarrow{\mathrm{PR}}\right| = \sqrt{24}$ ***(A1)(A1)(A1)***

$\cos \mathrm{R\widehat{P}Q} = \frac{(\sqrt{6})^2 + (\sqrt{24})^2 - (\sqrt{18})^2}{2\sqrt{6}\times\sqrt{24}}$ ***A1***

$\cos \mathrm{R\widehat{P}Q} = \frac{6+24-18}{24} \left(= \frac{12}{24}\right)$ ***A1***

$\cos \mathrm{R\widehat{P}Q} = \frac{1}{2}$ ***AG N0***

[7 marks]

Examiners report

Many candidates successfully used scalar product and magnitude calculations to complete part (b). Alternatively, some used the cosine rule, and often achieved correct results. Some assumed the triangle was a right-angled triangle and thus did not earn full marks. Although PQR is indeed right-angled, in a "show that" question this attribute must be directly established.

14c. *[6 marks]*

Markscheme

(i) **METHOD 1**

evidence of appropriate approach ***(M1)***

e.g. using $\sin^2 \mathrm{R\widehat{P}Q} + \cos^2 \mathrm{R\widehat{P}Q} = 1$, diagram

substituting correctly ***(A1)***

e.g. $\sin \mathrm{R\widehat{P}Q} = \sqrt{1 - \left(\frac{1}{2}\right)^2}$

$\sin \mathrm{R\widehat{P}Q} = \sqrt{\frac{3}{4}} \left(= \frac{\sqrt{3}}{2}\right)$ ***A1 N3***

METHOD 2

since $\cos \widehat{\mathrm{P}} = \frac{1}{2}$, $\widehat{\mathrm{P}} = 60^\circ$ ***(A1)***

evidence of approach

e.g. drawing a right triangle, finding the missing side ***(A1)***

$\sin \widehat{\mathrm{P}} = \frac{\sqrt{3}}{2}$ ***A1 N3***

(ii) evidence of appropriate approach ***(M1)***

e.g. attempt to substitute into $\frac{1}{2} ab \sin C$

correct substitution

e.g. area $= \frac{1}{2} \sqrt{6} \times \sqrt{24} \times \frac{\sqrt{3}}{2}$ ***A1***

area $= 3\sqrt{3}$ ***A1 N2***

[6 marks]

Examiners report

Many candidates attained the value for sine in (c) with little difficulty, some using the Pythagorean identity, while others knew the side relationships in a 30-60-90 triangle. Unfortunately, a good number of candidates then used the side values of $1, 2, \sqrt{3}$ to find the area of PQR , instead of the magnitudes of the vectors found in (a). Furthermore, the "hence" command was sometimes neglected as the value of sine was expected to be used in the approach.

15. *[6 marks]*

Markscheme

$\mathrm{e}^{2x} \left(\sqrt{3} \sin x + \cos x\right) = 0$ ***(A1)***

$\mathrm{e}^{2x} = 0$ not possible (seen anywhere) ***(A1)***

simplifying

e.g. $\sqrt{3} \sin x + \cos x = 0$, $\sqrt{3} \sin x = -\cos x$, $\frac{\sin x}{-\cos x} = \frac{1}{\sqrt{3}}$ ***A1***

EITHER

$\tan x = -\frac{1}{\sqrt{3}}$ ***A1***

$x = \frac{5\pi}{6}$ ***A2 N4***

OR

sketch of $30°$, $60°$, $90°$ triangle with sides $1, 2, \sqrt{3}$ ***A1***

work leading to $x = \frac{5\pi}{6}$ ***A1***

verifying $\frac{5\pi}{6}$ satisfies equation ***A1 N4***

[6 marks]

Examiners report

Those who realized e^{2x} was a common factor usually earned the first four marks. Few could reason with the given information to solve the equation from there. There were many candidates who attempted some fruitless algebra that did not include factorization.

16a. *[3 marks]*

Markscheme

METHOD 1

evidence of choosing $\sin^2\theta + \cos^2\theta = 1$ ***(M1)***

correct working ***(A1)***

e.g. $\cos^2\theta = \frac{9}{13}$, $\cos\theta = \pm\frac{3}{\sqrt{13}}$, $\cos\theta = \sqrt{\frac{9}{13}}$

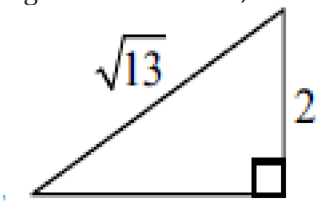
$\cos\theta = -\frac{3}{\sqrt{13}}$ ***A1 N2***

Note: If no working shown, award ***N1*** for $\frac{3}{\sqrt{13}}$.

METHOD 2

approach involving Pythagoras' theorem ***(M1)***

e.g. $2^2 + x^2 = 13$,



finding third side equals 3 ***(A1)***

$\cos\theta = -\frac{3}{\sqrt{13}}$ ***A1 N2***

Note: If no working shown, award ***N1*** for $\frac{3}{\sqrt{13}}$.

[3 marks]

Examiners report

While the majority of candidates knew to use the Pythagorean identity in part (a), very few remembered that the cosine of an angle in the second quadrant will have a negative value.

16b. *[5 marks]*

Markscheme

correct substitution into $\sin 2\theta$ (seen anywhere) ***(A1)***

e.g. $2\left(\frac{2}{\sqrt{13}}\right)\left(-\frac{3}{\sqrt{13}}\right)$

correct substitution into $\cos 2\theta$ (seen anywhere) ***(A1)***

e.g. $\left(-\frac{3}{\sqrt{13}}\right)^2 - \left(\frac{2}{\sqrt{13}}\right)^2$, $2\left(-\frac{3}{\sqrt{13}}\right)^2 - 1$, $1 - 2\left(\frac{2}{\sqrt{13}}\right)^2$

valid attempt to find $\tan 2\theta$ ***(M1)***

e.g. $\frac{2\left(\frac{2}{\sqrt{13}}\right)\left(-\frac{3}{\sqrt{13}}\right)}{\left(-\frac{3}{\sqrt{13}}\right)^2 - \left(\frac{2}{\sqrt{13}}\right)^2}$, $\frac{2\left(-\frac{2}{3}\right)}{1-\left(-\frac{2}{3}\right)^2}$

correct working ***A1***

e.g. $\frac{\frac{(2)(2)(-3)}{13}}{\frac{9}{13}-\frac{4}{13}}$, $\frac{-\frac{12}{(\sqrt{13})^2}}{\frac{18}{13}-1}$, $\frac{-\frac{12}{13}}{\frac{5}{13}}$

$\tan 2\theta = -\frac{12}{5}$ ***A1 N4***

Note: If students find answers for $\cos\theta$ which are not in the range $[-1, 1]$, award full ***FT*** in (b) for correct ***FT*** working shown.
[5 marks]

Examiners report

In part (b), many candidates incorrectly tried to calculate $\tan 2\theta$ as $2 \times \tan\theta$, rather than using the double-angle identities.

17. *[6 marks]*

Markscheme

METHOD 1 (quotient)
derivative of numerator is 6 ***(A1)***
derivative of denominator is $-\sin x$ ***(A1)***
attempt to substitute into quotient rule ***(M1)***
correct substitution ***A1***
e.g. $\frac{(\cos x)(6)-(6x)(-\sin x)}{(\cos x)^2}$
substituting $x = 0$ ***(A1)***
e.g. $\frac{(\cos 0)(6)-(6\times 0)(-\sin 0)}{(\cos 0)^2}$
$h'(0) = 6$ ***A1 N2***

METHOD 2 (product)
$h(x) = 6x \times (\cos x)^{-1}$
derivative of 6*x* is 6 ***(A1)***
derivative of $(\cos x)^{-1}$ is $(-(\cos x)^{-2}(-\sin x))$ ***(A1)***
attempt to substitute into product rule **(M1)**
correct substitution ***A1***
e.g. $(6x)(-(\cos x)^{-2}(-\sin x)) + (6)(\cos x)^{-1}$
substituting $x = 0$ ***(A1)***
e.g. $(6 \times 0)(-(\cos 0)^{-2}(-\sin 0)) + (6)(\cos 0)^{-1}$
$h'(0) = 6$ ***A1 N2***
[6 marks]

Examiners report

The majority of candidates were successful in using the quotient rule, and were able to earn most of the marks for this question. However, there were a large number of candidates who substituted correctly into the quotient rule, but then went on to make mistakes in simplifying this expression. These algebraic errors kept the candidates from earning the final mark for the correct answer. A few candidates tried to use the product rule to find the derivative, but they were generally not as successful as those who used the quotient rule. It was pleasing to note that most candidates did know the correct values for the sine and cosine of zero.

18a. *[5 marks]*

Markscheme

evidence of finding height, *h* ***(A1)***
e.g. $\sin\theta = \frac{h}{2}$, $2\sin\theta$
evidence of finding base of triangle, *b* ***(A1)***
e.g. $\cos\theta = \frac{b}{2}$, $2\cos\theta$
attempt to substitute valid values into a formula for the area of the window ***(M1)***
e.g. two triangles plus rectangle, trapezium area formula
correct expression (must be in terms of θ) ***A1***
e.g. $2\left(\frac{1}{2} \times 2\cos\theta \times 2\sin\theta\right) + 2 \times 2\sin\theta$, $\frac{1}{2}(2\sin\theta)(2+2+4\cos\theta)$
attempt to replace $2\sin\theta\cos\theta$ by $\sin 2\theta$ ***M1***
e.g. $4\sin\theta + 2(2\sin\theta\cos\theta)$
$y = 4\sin\theta + 2\sin 2\theta$ ***AG N0***
[5 marks]

Examiners report

As the final question of the paper, this question was understandably challenging for the majority of the candidates. Part (a) was generally attempted, but often with a lack of method or correct reasoning. Many candidates had difficulty presenting their ideas in a clear and organized manner. Some tried a "working backwards" approach, earning no marks.

18b. *[4 marks]*

Markscheme

correct equation ***A1***

e.g. $y = 5$, $4\sin\theta + 2\sin 2\theta = 5$

evidence of attempt to solve ***(M1)***

e.g. a sketch, $4\sin\theta + 2\sin\theta - 5 = 0$

$\theta = 0.856$ (49.0°), $\theta = 1.25$ (71.4°) ***A1A1 N3***

[4 marks]

Examiners report

In part (b), most candidates understood what was required and set up an equation, but many did not make use of the GDC and instead attempted to solve this equation algebraically which did not result in the correct solution. A common error was finding a second solution outside the domain.

18c. *[7 marks]*

Markscheme

recognition that lower area value occurs at $\theta = \frac{\pi}{2}$ ***(M1)***

finding value of area at $\theta = \frac{\pi}{2}$ ***(M1)***

e.g. $4\sin\left(\frac{\pi}{2}\right) + 2\sin\left(2 \times \frac{\pi}{2}\right)$, draw square

$A = 4$ ***(A1)***

recognition that maximum value of *y* is needed ***(M1)***

$A = 5.19615\ldots$ ***(A1)***

$4 < A < 5.20$ (accept $4 < A < 5.19$) ***A2 N5***

[7 marks]

Examiners report

A pleasing number of stronger candidates made progress on part (c), recognizing the need for the end point of the domain and/or the maximum value of the area function (found graphically, analytically, or on occasion, geometrically). However, it was evident from candidate work and teacher comments that some candidates did not understand the wording of the question. This has been taken into consideration for future paper writing.

19a. *[2 marks]*

Markscheme

METHOD 1

approach involving Pythagoras' theorem ***(M1)***

eg $5^2 + x^2 = 13^2$, labelling correct sides on triangle

finding third side is 12 (may be seen on diagram) ***A1***

$\cos A = \frac{12}{13}$ ***AG N0***

METHOD 2

approach involving $\sin^2\theta + \cos^2\theta = 1$ ***(M1)***

eg $\left(\frac{5}{13}\right)^2 + \cos^2\theta = 1,\ x^2 + \frac{25}{169} = 1$

correct working ***A1***

eg $\cos^2\theta = \frac{144}{169}$

$\cos A = \frac{12}{13}$ ***AG N0***

[2 marks]

Examiners report

[N/A]

19b. *[3 marks]*

Markscheme

correct substitution into $\cos 2\theta$ ***(A1)***

eg $1 - 2\left(\frac{5}{13}\right)^2,\ 2\left(\frac{12}{13}\right)^2 - 1,\ \left(\frac{12}{13}\right)^2 - \left(\frac{5}{13}\right)^2$

correct working ***(A1)***

eg $1 - \frac{50}{169},\ \frac{288}{169} - 1,\ \frac{144}{169} - \frac{25}{169}$

$\cos 2A = \frac{119}{169}$ ***A1 N2***

[3 marks]