Smarter Balanced Assessments: Mathematics Overview Gail Pagano Connecticut State Department of Education



CSDE Assessment Literacy Workshops January 13 and 14, 2014





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Presentation Overview

- Common Core State Standards
 - Standards for Mathematical Practice
 - Standards for Mathematical Content
- The Three Shifts in Mathematics
- The Claims for the Mathematics Summative Assessment
- Item Response Types
- Computer Adaptive Testing
- Performance Tasks



The CCSS Mathematics Standards Not a New Way of Doing Old Business

The Structure is the Standards

Phil Daro, Bill McCallum, Jason Zimba



A Grecian urn



The Standard for Mathematical Practice

Make sense of problems and persevere in solving them

6. Attend to precision

2. Reason abstractly and quantitatively

3. Construct viable arguments and critique the reasoning of others

4. Model with mathematics

5. Use appropriate tools strategically

7. Look for and make use of structure.

8. Look for and express regularity in repeated reasoning.



The Standards for Mathematical Content

COMMON CORE STATE STANDARDS

Operations and Algebraic Thinking

3.OA

Represent and solve problems involving multiplication and division.

- Interpret products of whole numbers, e.g., interpret 5 x 7 as the total number of objects in 5 groups of 7 objects each. For example, describe a context in which a total number of objects can be expressed as 5 x 7.
- Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.
- 3. Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.¹
- 4. Determine the unknown whole number in a multiplication or division equation relating three whole numbers. For example, determine the unknown number that makes the equation true in each of the equations 8 x? = 48, 5 = [] ÷ 3, 6 x 6 = ?.



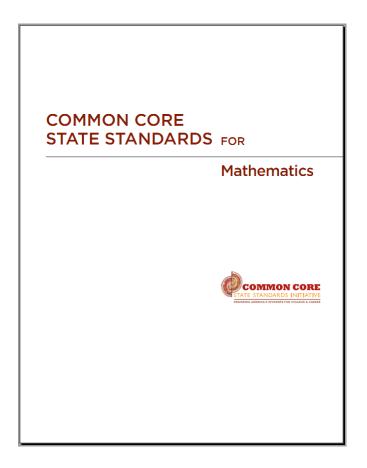
The Mathematics Assessment The Three Shifts in Mathematics





The CCSS Require Three Shifts in Mathematics

- Focus strongly where the standards focus
- Coherence: Think across grades and link to major topics within grades
- Rigor: In major topics, pursue conceptual understanding, procedural skill and fluency, and application with equal intensity





Shift #1: Focus Strongly where the Standards Focus The shape of math in A+ countries

Mathematics topics intended at each grade by at least twothirds of A+ countries

$\mathbb{E}^{\mathbb{N}}_{+}$	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	Mathematic
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¹ Schmidt, Houang, & Cogan, "A Coherent Curriculum: The Case of Mathematics." (2002).

Shift #1: Focus

Common Core State Standards – Mathematics

Learning Progressions

Kindergarten	1	2	3	4	5	6	7	8	HS
<u>Counting and</u> <u>Cardinality</u>					1				
		Number and Quantity							
		-	Numbe		o <u>nships</u> e Number Syste				
	Operatio	nc and Algol	braic Thinki	Expressions and Equations			Algebra		
	operado	iis and Aiger				<u>Functions</u>	Functions		
		<u>Geomet</u>	τ χ	<u>Geometry</u>			Geometry		
	Me	asurement a	and Data	<u>Statis</u>	tics and Probat	oility	Statistics and Probability		



Shift #1: Focus *Key Areas of Focus in Mathematics*

Grade	Focus Areas in Support of Rich Instruction and Expectations of Fluency and Conceptual Understanding					
K–2	Addition and subtraction - concepts, skills, and problem solving and place value					
3–5	Multiplication and division of whole numbers and fractions – concepts, skills, and problem solving					
6	Ratios and proportional reasoning; early expressions and equations					
7	Ratios and proportional reasoning; arithmetic of rational numbers					
8	Linear algebra and linear functions					
	Slide 11					

Shift #1: Focus

Content Emphases by Cluster

The Smarter Balanced Content Specifications help support focus by identifying the content emphasis by cluster. The notation [m] indicates content that is major and [a/s] indicates content that is additional or supporting.

Grade 4 Cluster-Level Emphases m = major clusters; a/s = additional and supporting clusters **Operations and Algebraic Thinking** [m] Use the four operations with whole numbers to solve problems. [a/s] Gain familiarity with factors and multiples. [a/s] Generate and analyze patterns. Number and Operations in Base Ten [m] Generalize place value understanding for multi-digit whole numbers. [m] Use place value understanding and properties of operations to perform multi-digit arithmetic. Number and Operations—Fractions [m] Extend understanding of fraction equivalence and ordering. [m] Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers. [m] Understand decimal notation for fractions, and compare decimal fractions.



Shift #2: Coherence

Think Across Grades, and Link to Major Topics Within Grades

- Carefully connect the learning within and across grades so that students can build new understanding on foundations built in previous years.
- Begin to count on solid conceptual understanding of core content and build on it.
 Each standard is not a new event, but an extension of previous learning.



Shift #2: Coherence Alignment in Context

Neighboring Grades and Progressions

One of several staircases to algebra designed into in the OA domain.

Expressions and Equations 6.EE

3. Apply the properties of operations to generate equivalent expressions. For example, apply the distributive property to the expression 3 (2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6 (4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y.

Operations and Algebraic Thinking 5.0A

 Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. For example, express the calculation "add 8 and 7, then multiply by 2" as 2 x (8 + 7). Recognize that 3 x (18932 + 921) is three times as large as 18932 + 921, without having to calculate the indicated sum or product.

Operations and Algebraic Thinking 3.OA

5. Apply properties of operations as strategies to multiply and divide.² Examples: If 6 x 4 = 24 is known, then 4 x 6 = 24 is also known. (Commutative property of multiplication.) 3 x 5 x 2 can be found by 3 x 5 = 15, then 15 x 2 = 30, or by 5 x 2 = 10, then 3 x 10 = 30. (Associative property of multiplication.) Knowing that 8 x 5 = 40 and 8 x 2 = 16, one can find 8 x 7 as 8 x (5 + 2) = (8 x 5) + (8 x 2) = 40 + 16 = 56. (Distributive property.)

Operations and Algebraic Thinking 1.0A

3. Apply properties of operations as strategies to add and subtract.³ Examples: If 8 + 3 = 11 is known, then 3 + 8 = 11 is also known. (Commutative property of addition.) To add 2 + 6 + 4, the second two numbers can be added to make a ten, so 2 + 6 + 4 = 2 + 10 = 12. (Associative property of addition.)



Assessments Must Attend to the Coherence in the Standards

1.0A.7 Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. For example, which of the following equations are true and which are false? 6 = 6, 7 = 8 - 1, 5 + 2 = 2 + 5, 4 + 1 = 5 + 2.

Some standards from early grades are critical through Grade 12



Shift #3: Rigor

In Major Topics, Pursue Conceptual Understanding, Procedural Skill and Fluency, and Application

- The CCSSM require a balance of:
 - Solid conceptual understanding
 - Procedural skill and fluency
 - Application of skills in problem solving situations
- Pursuit of all three requires equal intensity in time, activities, and resources.

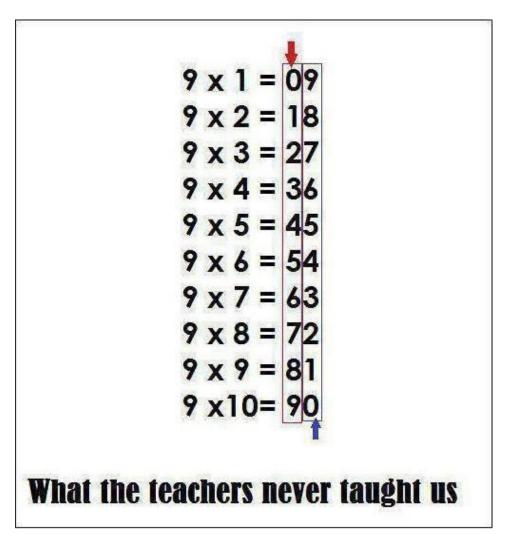


Shift #3: Rigor Solid Conceptual Understanding

- Teach more than "how to get the answer" and instead support students' ability to access concepts from a number of perspectives
- Students are able to see math as more than a set of mnemonics or discrete procedures
- Conceptual understanding supports the other aspects of rigor (fluency and application)

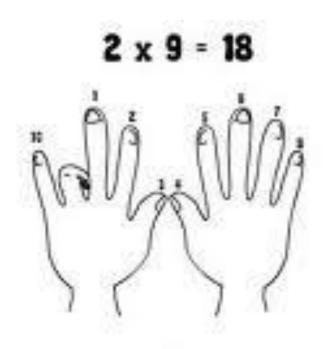


Is This Fostering Understanding of Multiplication?





And This?



- Each finger to the left of the curled finger represents 10.
 Say 10,
- Each finger to the right of the curled finger represents one.
- Count 1, 2, 3, 4, 5, 6, 7, 8. (Or 11, 12, 13, 14, 15, 16, 17, 18)
- $-2 \times 9 = 18$



MP 8: Look for and express regularity in repeated reasoning

Progressions for the Common Core State Standards in Mathematics – K, Counting and Cardinality; K - 5 Operations and Algebraic Thinking (page 26)

 $\begin{array}{l} \mbox{Patterns in multiples of 9} \\ 1\times9=9 \\ 2\times9=2\times(10-1)=(2\times10)-(2\times1)=20-2=18 \\ 3\times9=3\times(10-1)=(3\times10)-(3\times1)=30-3=27, \ \ \, \mbox{etc} \end{array}$

Source:

http://commoncoretools.files.wordpress.com/2011/05/ccss_progression_cc_oa_ k5_2011_05_302.pdf



Shift #3: Rigor

Required Fluencies for Grades K-6

Grade	Standard	Required Fluency			
K	K.OA.5	Add/subtract within 5			
1	1.OA.6	Add/subtract within 10			
2	2.OA.2 2.NBT.5	Add/subtract within 20 (know single-digit sums from memory) Add/subtract within 100			
3	3.OA.7 3.NBT.2	Multiply/divide within 100 (know single-digit products from memory) Add/subtract within 1000			
4	4.NBT.4	Add/subtract within 1,000,000			
5	5.NBT.5	Multi-digit multiplication			
6	6.NS.2,3	Multi-digit division Multi-digit decimal operations			

Slide 21

Assessment Consortium

How Can Assessments Deliver on the Promise of Focus, Coherence and Rigor?

• FOCUS: Assessments focus where the standards focus.

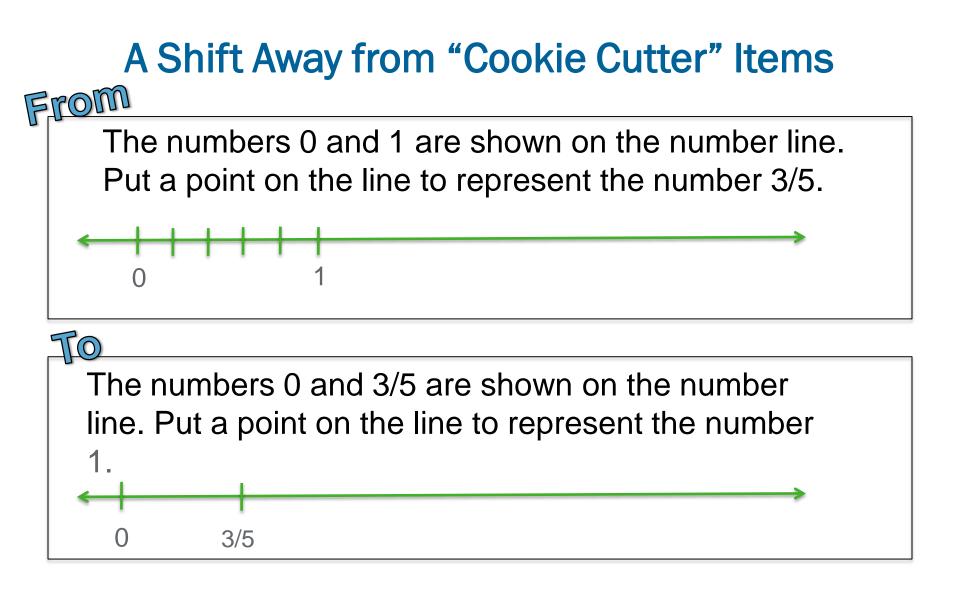
Major content represents the majority of points and problems on assessments.

COHERENCE: Assessments honor the coherence in the standards.

Balance of tasks assessing individual standards and related standards within the context of the grade and, as relevant, the progressions.

RIGOR: Assessments reflect the rigor of the standards.

Balance of tasks assessing conceptual understanding, procedural skill and fluency, and application of mathematics to solve problems.

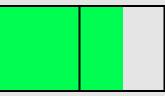




From the Progressions

^{3.NF.1} Understand a fraction 1/b as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size 1/b.

The importance of specifying the whole



Without specifying the whole it is not reasonable to ask what fraction is represented by the shaded area. If the left square is the whole, the shaded area represents the fraction $\frac{3}{2}$; if the entire rectangle is the whole, the shaded area represents $\frac{3}{4}$.

Source:

commoncoretools.files.wordpress.com/2011/08/ccss_progressi on_nf_35_2011_08_12.pdf



How do we "Shift" Item Writers' Understanding of the Mathematics?

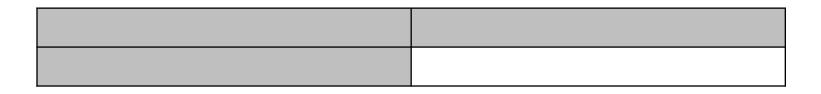
What fraction is represented by the shaded area?

Disclaimer: This is not a Smarter Balanced item.



How do we "Shift" Item Writers' Understanding of the Mathematics?

What fraction is represented by the shaded area?



Four students give responses. Explain what must be true for each student to be correct. Student A: ³/₄ Student B: 3/2 Student C: 3/1 Student D: 30/1



The Mathematics Assessment

The Claims for the Mathematics Summative Assessment





Claims for the Mathematics Summative Assessment

Overall Claim for Grades 3-8	"Students can demonstrate progress toward college and career readiness in mathematics."
Overall Claim for Grade 11	"Students can demonstrate college and career readiness in mathematics."
Claim #1 - Concepts & Procedures	"Students can explain and apply mathematical concepts and interpret and carry out mathematical procedures with precision and fluency."
Claim #2 - Problem Solving	"Students can solve a range of complex well-posed problems in pure and applied mathematics, making productive use of knowledge and problem solving strategies."
Claim #3 - Communicating Reasoning	"Students can clearly and precisely construct viable arguments to support their own reasoning and to critique the reasoning of others."
Claim #4 - Modeling and Data Analysis	"Students can analyze complex, real-world scenarios and can construct and use mathematical models to interpret and solve problems."

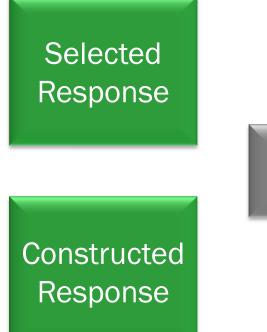


The Mathematics Assessment Response Types





Response Types



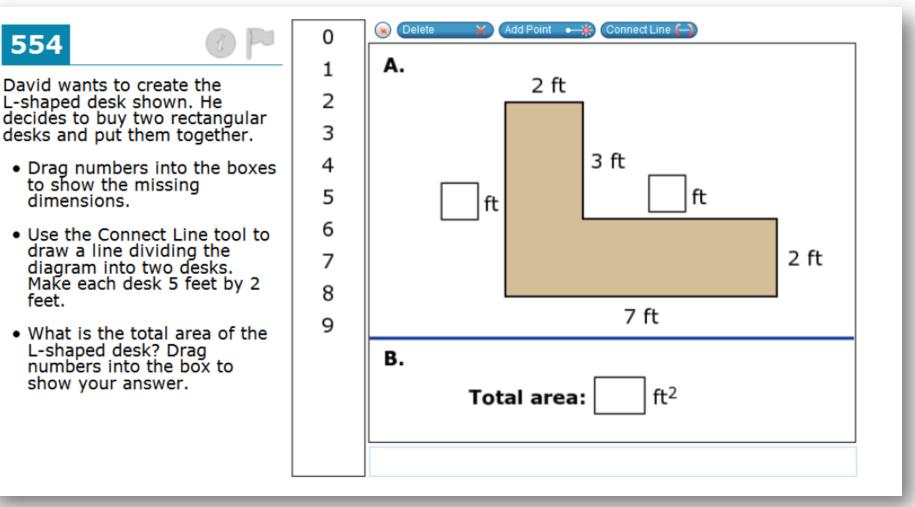


Smarter Balanced Response Types

MC with one correct response MC with multiple correct responses Two Part multiple-choice Matching Tables Yes/No or True/False Tables **Fill-in Tables** Select or order text or graphics Complex drag and drop Graphing Equation or numeric response Short Text Long Essay



Drag & Drop / Line Tool



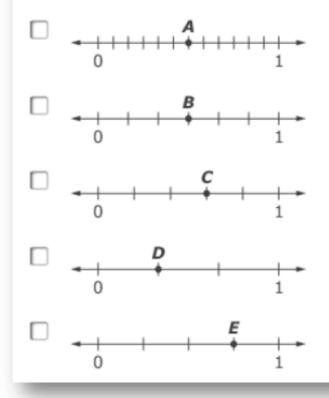


Multiple Correct Response

Use this number line to answer the question that follows.



Choose all the number lines that show a fraction equal to the fraction shown by point P.

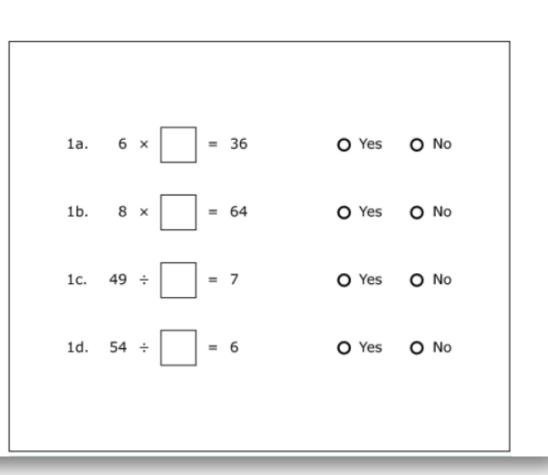




Matching Table



For questions 1a-1d, choose Yes or No to show if the number 7 will make each equation true.







In the table shown, enter the two missing values for *y* to create a relation that is **not** a linear function.

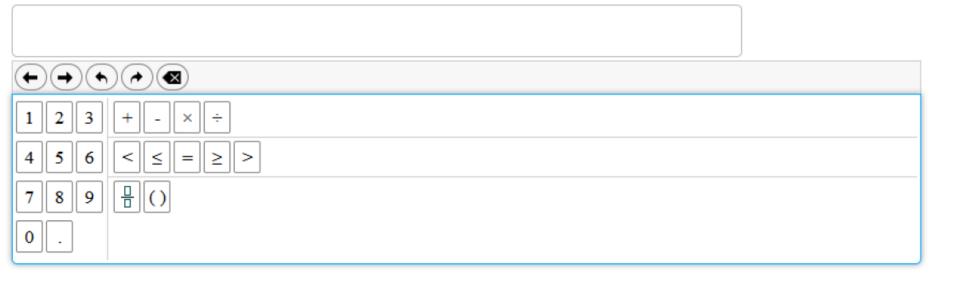
X	У
1	5
2	
3	10
4	



Equation/Numeric Grade 3

Lisa had 3 pizzas. Each pizza was cut into 8 pieces. Lisa ate 2 pieces. How many pieces were left?

Write an equation to show how many pieces were left.

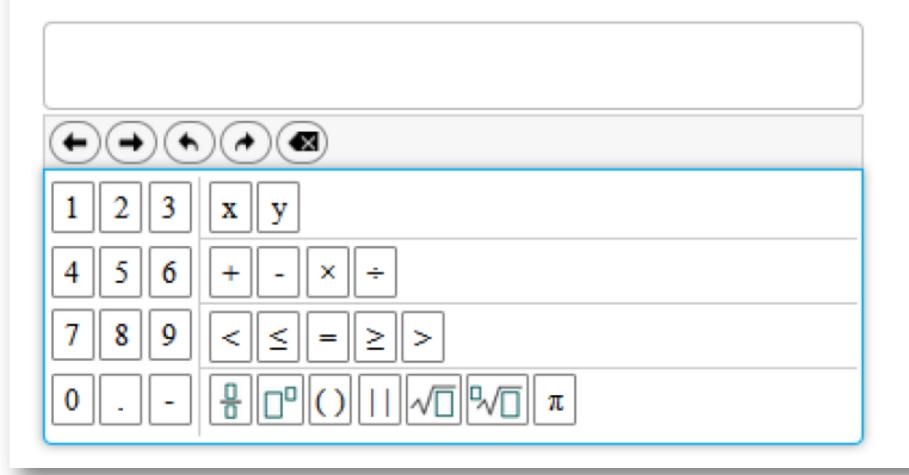




Equation/Numeric

Grade 8

Write an equation for the line in slope-intercept form.



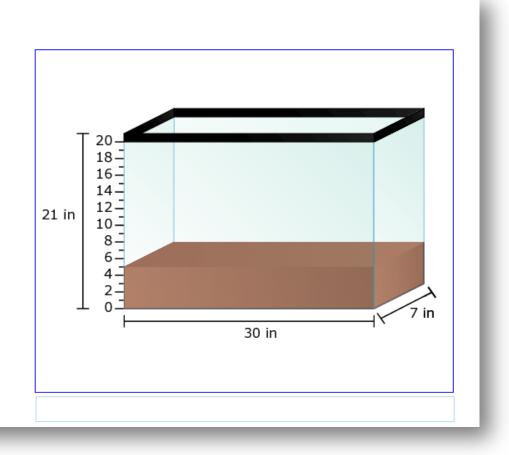
Hot Spot

18



Walter puts 1050 cubic inches of dirt into the tank shown.

Click the number line to show the height of the dirt in the tank.





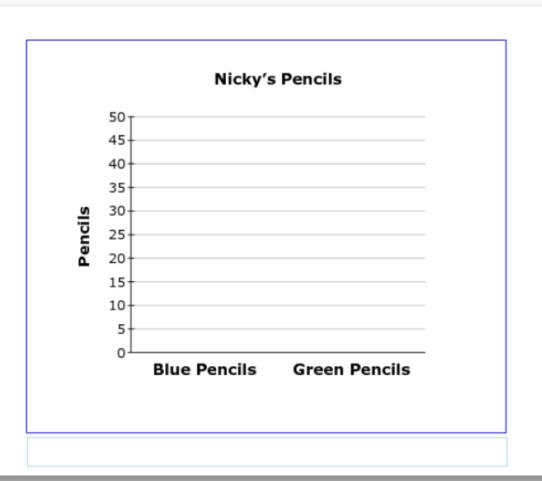
Hot Spot

557

Nicky has 4 packs of pencils. Each pack contains 15 pencils. In each pack, 5 pencils are blue and the rest green.

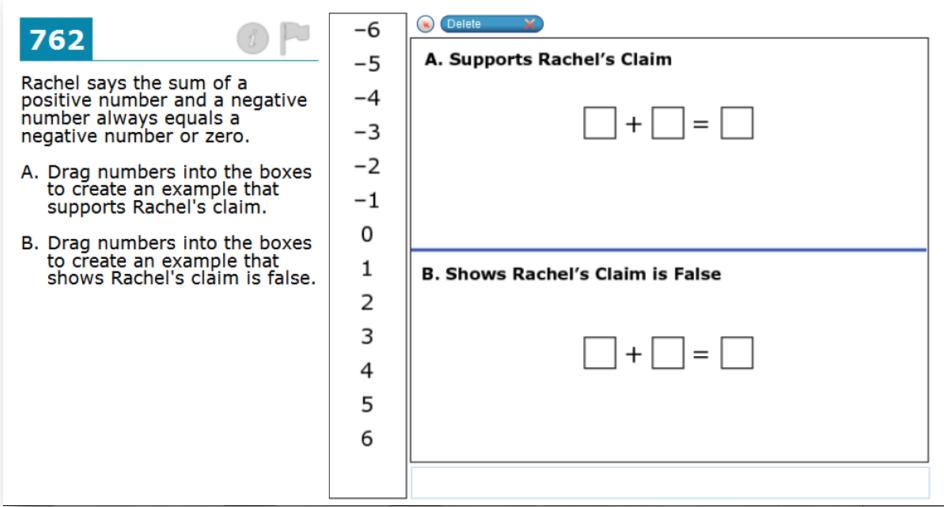
Create a bar graph to show how many of each color pencil Nicky has.

Click the graph to show where the top of the bar should go.





Multiple Parts with Drag and Drop





Short Text

Six friends are going to buy pizza. Their choices are to buy 2 medium 10-inch diameter pizzas for \$7.00 each, or 1 large 14-inch diameter pizza for \$15.00. Both prices include tax and tip.

The friends agree that their best choice is the one that gives them the most pizza for their money.

764

Which is the best choice? Explain your answer.



How to We Capture and Score Responses Like This?

Question 2: Glacier Speeds

Score 1

The honging glacien because it is movery at the greatest speed because it moves 6 feed is a day. you would take the 3 inches and multiply then by 24 because for each have, the glacin moud 3 inches and I wanted to see now much it mored in a day (24 hours). Next & tear 72 (3x24) and durding by 12 because there one 12 mones in a fool. I thogot 6 which is hiw many pert the placies moved I more fet than the shiding glacies and almost a feet more than the arquing placies. 4 ft. I day 3×24=72+12=6Ft. 10em 6×10-5 = .00006 X24 - .0144 .0012 1 day

Challenge: Distinguishing between a computation error and conceptual error.



Mathematics Reasoning Project

- Panel of combined mathematics content and technology experts met in early October to define the evidence gap between what we want to be able to measure and what we are currently able to measure.
- Two phase development plan
 - Phase 1 development is focused on items that we can already capture, but want to be able to score using technology.
 - Phase 2 will focus on newer technologies as defined and prioritized during the panel meeting.



The Mathematics Assessment Computer Adaptive Testing





Smarter Balanced: Complexity vs. Difficulty

In the Smarter Balanced adaptive test, ALL students will see items across the full range of *complexity*; appropriate items will be selected for examinees based on their calibrated *difficulty* levels.

Appendix C – <u>Cognitive Rigor Matrix/Depth of</u> <u>Knowledge (DOK)</u> – Mathematics Content Specifications.



Increasing Complexity

• Understand:

What is the value of 2(14) - 3?

- Apply:
 - Susan has \$14. Jack has \$3 less than twice the amount of money Susan has. How much money does Jack have?
 - Jack has \$3 less than twice the amount of money Susan has. Write an expression for the amount of money, (s), Susan has.



Increasing Difficulty

- What is the value of this expression?
 2(14) 3
- What is the value of this expression?
 2(9 + 8) (17 4)
- What is the value of this expression? $7(8-5) - 2^3 + (6)(3)$



Computer Adaptive Testing

Adapting Items – Adjusting the Difficulty of Grade 3 Multiplication Items

 $8 \times 5 = \Box$ Fluently multiply and divide within 100.

 $8 \times \Box = 40$ Represent and solve problems involving multiplication and division.

 $9 \times 4 = \Box \times 9$ Understand properties of multiplication...

 $4 \times \Box = 40 - 8$

9 x 4 = 2 x □

8 x □ = 4 x □

Give two different pairs of numbers that could fill the boxes to make a true equation.



The Mathematics Assessment Performance Tasks





Performance Tasks

- Integrate knowledge and skills across multiple Claims and Targets.
- Measure capacities such as depth of understanding, research skills, and/or complex analysis with relevant evidence.
- Require student-initiated planning, management of information/data and ideas, and/or interaction with other materials.
- Reflect a real-world task and/or scenario-based problem.
- Allow for multiple approaches.



Mathematics Claims for Performance Tasks

Claim #1 - Concepts & Procedures	"Students can explain and apply mathematical concepts and interpret and carry out mathematical procedures with precision and fluency."
Claim #2 - Problem Solving	"Students can solve a range of complex well-posed problems in pure and applied mathematics, making productive use of knowledge and problem solving strategies."
Claim #3 - Communicating Reasoning	"Students can clearly and precisely construct viable arguments to support their own reasoning and to critique the reasoning of others."
Claim #4 - Modeling and Data Analysis	"Students can analyze complex, real-world scenarios and can construct and use mathematical models to interpret and solve problems."
Overall Claim for Grades 3-8	"Students can demonstrate progress toward college and career readiness in mathematics."
Overall Claim for Grade 11	"Students can demonstrate college and career readiness in mathematics."





Heartbeats Performance Task Classroom Interaction

Resources needed:

- Each student should have access to a piece of paper and writing tool
- Projector or some manner to display images
- A timing device for measuring a 20 second interval

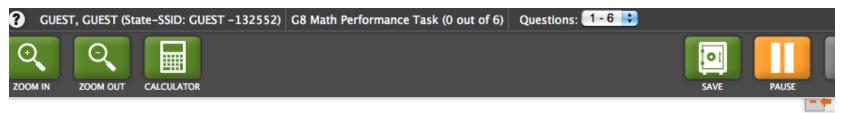
Setting the Context

Facilitator says: "The performance task you will complete allows you to explore the body weights and pulse rates of different animals."

Facilitator says: "Let's start by talking about the body weights of different animals. Imagine a chicken, a dog, a horse, and a rat. On your paper, write the animals in order from lightest to heaviest according to their body weights. [Display the animals' names for students in the order listed: chicken, dog, horse, rat.]

Facilitator asks: "Which two animals do you think are closest in weight?" [Wait for responses. Responses may include the rat and the chicken, or some students may think the dog and the horse depending on which breeds they are considering or because they have never seen the live animals. Students may explain why they made the choice they did.]





HEARTBEATS

In this task, you will use data to create a model that shows the relationship between animal body weight and pulse rate measures. Then you will examine additional data to evaluate your model.

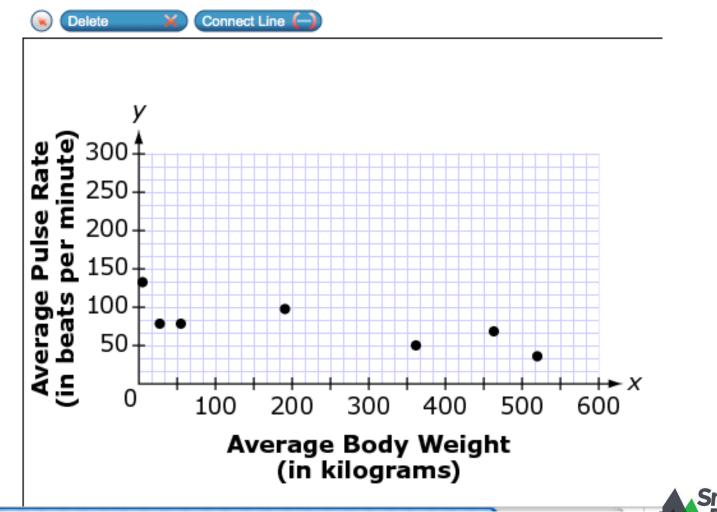
A study states that the relationship between an animal's pulse rate and body weight is approximately linear. The study data are below.

Table 1. Average Body Weight and Average Pulse Rate of Seven Animals

Animal	Average Body Weight (in kilograms)	Average Pulse Rate (in beats per minute)
Cat	3	130
Goat	28	75
Sheep	56	75
Pig	192	95
Ox	362	48
Cow	465	66
Horse	521	34



The data from Table 1 are plotted below. Use the Connect Line tool to create a linear model of these data.





The body weight and pulse rate of a guinea pig and rabbit are given in the table below.

Animal	Average Body Weight (in kg)	Average Pulse Rate (per minute)
Guinea Pig	1	250
Rabbit	2.5	265

If the study had included these data, would this change the model relating average body weight and average pulse rate? How do you know?



Calculators

- Grades 3 5: No calculator
- Grade 6: Basic Calculator
- Grades 7 and 8: Scientific Calculator
- Grade 11: Scientific/Graphing, Regression Calculator

(Available for student use on the Smarter Balanced Practice Test)



Smarter Balanced Resources





Find Out More www.SmarterBalanced.org



An Early Look at Smarter Balanced Assessments

OS Version

Available nearly two years before the assessment system is implemented in the 2014-15 school year, the Smarter Balanced Practice Tests allow teachers, students, parents, and other interested parties to experience the features of online testing and gain insight into how Smarter Balanced will assess students' mastery of the Common Core.

The following browsers are compatible with the Practice Test.

Operating System

Supported Browsers

处 TWITTER

Montana looks to move forward with the 2014 Field Test. Learn more via @cgewertz @educationweek: http://t.co/GGii3e7E6b

With AB 484, CA is moving forward with #assessments aligned to the #CCSS. Read more via @LkEIsinorePatch: http://t.co/qPatZFDWTN

ore Tweets 🕨

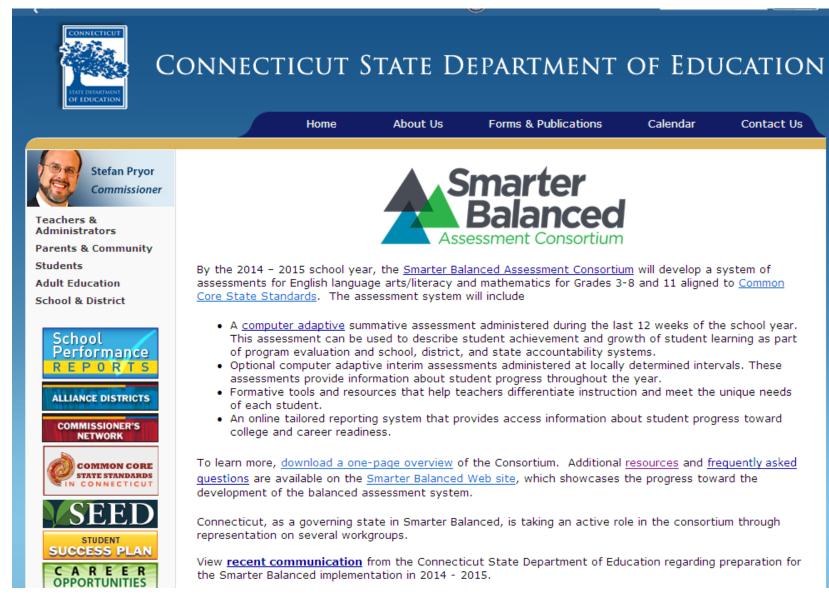


Student Interface Practice Test



Find Out More

www.sde.ct.gov/sde/SmarterBalanced









Reflect on Guiding Questions

- What do educators need to do to support student learning?
- What do educators need to do to ensure that students are prepared for the Smarter Balanced assessments?

