GEOMETRY AND ART

Snowflake Geometry

Big Ideas

Unit of Instruction

Snowflake Geometry

Geometry Concept

- Geometric relationships found in a regular hexagon

Rationale

This project brings out artistic inventiveness while providing a great review of geometric concepts, such as lines, angles, bisectors, symmetry, and polygons, to mention a few. A real advantage of this project is that it takes almost no class time. Best of all, it is easy for all students to succeed.

NCTM 9-12 Standards

- Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships.
- Apply appropriate techniques, tools, and formulas to determine measurements.
- Use visualization, spatial reasoning, and geometric modeling to solve problems.
- Make and investigate mathematical conjectures.
- Build new mathematical knowledge through problem solving.
- Recognize and use connections among mathematical ideas.
- Understand how mathematical ideas interconnect and build on one another to produce a coherent whole.
- Use the language of mathematics to express mathematical ideas precisely.
- Use representations to model and interpret physical, social, and mathematical phenomena.

Idaho Content Standards

- G.4.1.3 Establish the validity of geometric conjectures.
- G.4.3 Apply transformations and use symmetry to analyze mathematical situations.
- G.4.3.1 Understand and represent translations, reflections, dilations, and rotations of objects in the plane.
- G.4.4.1 Draw and construct representations of two-dimensional geometric objects using a variety of tools.
- 9-12.VA.3.2.4 Select and utilize visual, spatial, and temporal concepts to enhance meaning in artwork.

Background

Vocabulary

Acute	Obtuse
Bisectors	Organic shape
Classification of angles	Parallel
Complimentary	Polygon
Congruent	Right
Geometric shape	Symmetry
Hexagon	Vertical
Kite	

Math Instruction (pre- or post-project)

This is a review of past concepts and vocabulary relating to the relationships between angles and lines.

Art Instruction
Students will need to have an understanding of positive and negative space and geometric versus organic shapes.

Driving Question

Project Objective

Students will identify, by folding paper, geometric terms and relationships.

Students will recognize that geometric relationships are not limited to the shape of the original object.

Questions to be Answered

What is this particular geometric shape (after each fold)?

What geometric characteristics define the shape?

Do geometric characteristics always hold true?

Materials

Materials Required

- 8 ½" white paper - each student may use up to six pages per day
- Colored paper- optional for second day of snowflake production
- Scissors
- School laminator
- String/thread/fishing line to hang snowflakes
- Ruler

Reference Materials

Snowflakes for All Seasons Cindy Higham (ISBN 1-58685-528-X)

Snowflake Bentley Jacqueline Briggs Martin and Mary Azarian (ISBN 0-439-13048-4)

http://britton.disted.camosun.bc.ca/jbpolyhedra.htm
http://mathforum.org/alejandre/workshops/polyhedra.html
http://snowflakebentley.com/snowflakes.htm

Lesson Outline

Description of Activity

The students will create a regular hexagon and then identify geometric relationships within the folded hexagon. Then they will use the regular hexagon to create original snowflakes and finally thematic snowflakes. In conclusion, they will write a one page essay that reflects what they learned in the experience.

Day One

Steps 1-9 How to fold a hexagon

- Students will fold a sheet of paper to create a regular hexagon. Teacher will ask questions about each form that is created (see pages 9-18).

Day Two

- In class, students will work on their projects in small groups, identifying the 50 plus relationships in the hexagonal form (see pages 20-22).

Day Three

- Teacher will demonstrate positive and negative space and geometric/organic shapes (see pages 6-8).
- Students will experiment with cutting, forming original snowflakes, perfecting their style, creating a positive and negative image snowflake.
- Snowflakes are laminated and hung in the classroom.

Note: we highly recommend visiting snowflakebentley site to demonstrate positive and negative crystals.

Optional Day Four

Students may choose a thematic snowflake design from the book *Snowflakes for All Seasons.*

Colored paper may be provided for this project.

Assessment

Rubric

Hexagon identification: 1 point given for each relationship identified.

Snowflake production: 10 points if they turn in a completed snowflake.

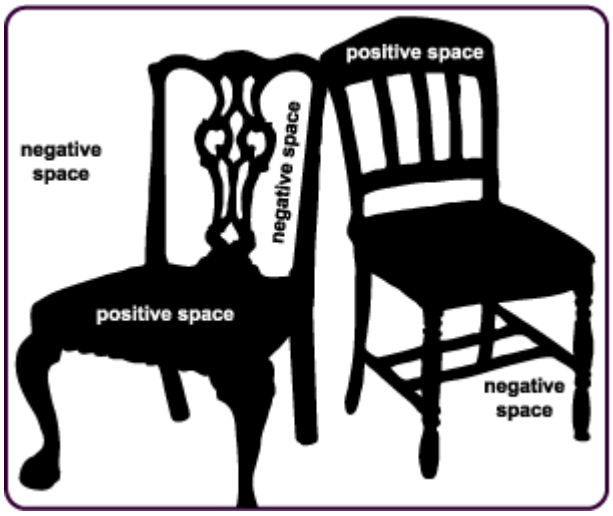
Reflection paper: 20 points

Ideas for Further Independent Student Project

Students may create three-dimensional paper sculptures.

Positive and Negative Space

Positive shapes occupy positive space. The area around positive shapes, the background, is negative space. Artists think of the entire composition, the space between the shape and the space around the shape in creating pieces of art. In the composition of the chairs, the space around the chairs, the negative space, is as clear as the positive space.



In the ceramic vase below, the negative shapes on either side of the top half of the vase nearly mirror the lower positive shape of the vase.

Geometric and Organic Shapes

Geometric Shapes / Rectilinear Shapes

Notice the shapes below and to the left of the red line. Angular shapes therefore we call them **rectilinear shapes.**

Most **rectilinear shapes** can be also called **geometric shapes** however a couple of them are not. These are circles and ovals.

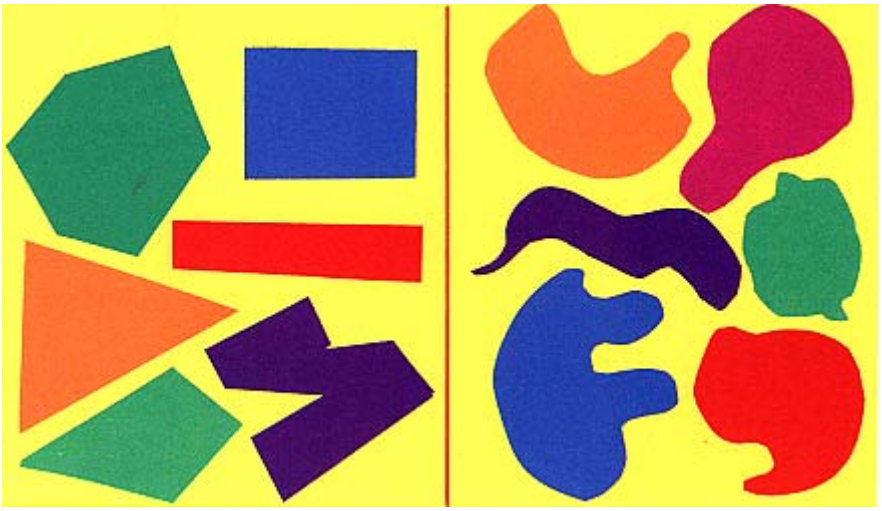
Geometric and rectilinear shapes usually are man-made. A few exceptions to the rule are crystals and honey combs.

Organic Shapes / Curvilinear Shapes

Notice the shapes below and to the right of the red line. They are all curving and flowing therefore we call them **curvilinear shapes.**

Curvilinear shapes could also be called **organic shapes.** We can include circles and ovals.

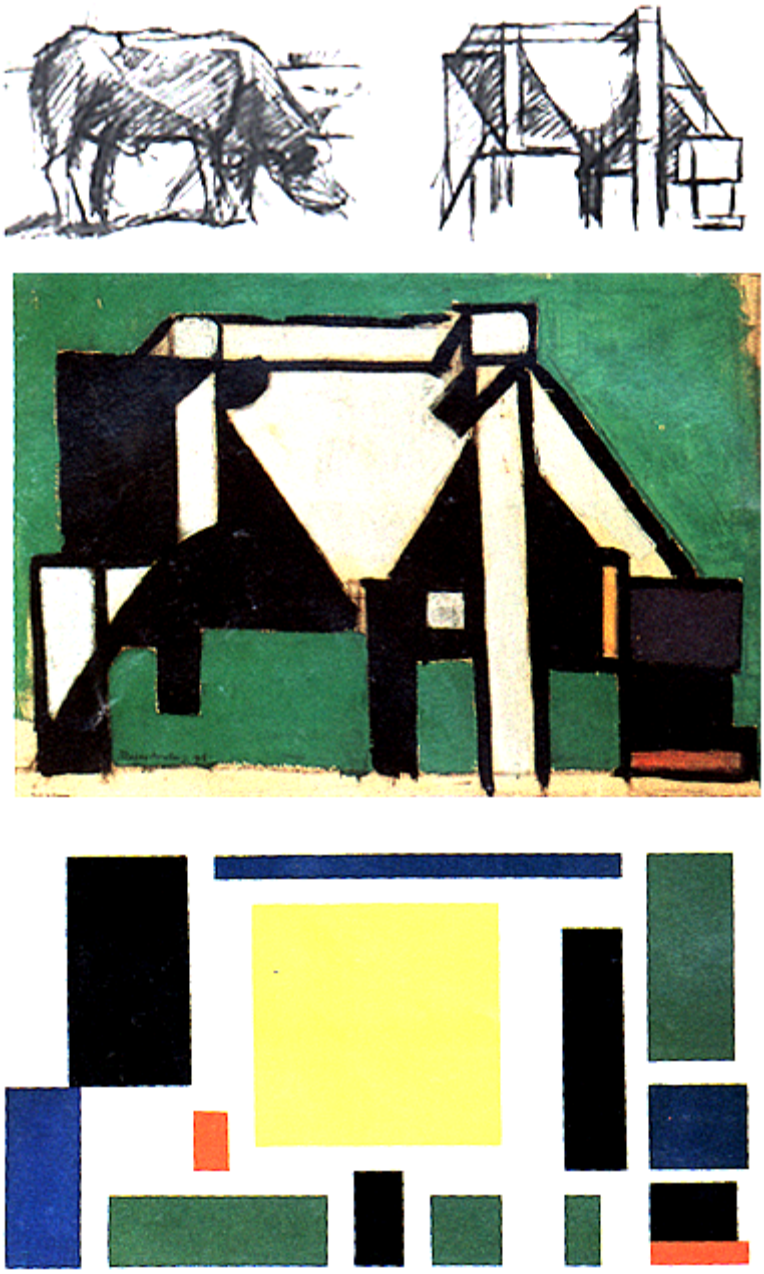
Organic and curvilinear shapes are usually natural shapes. Think of leaves, animal shapes, and plant shapes. Aren't they all organic?



So how would you answer this question?

Circles and ovals are considered to be which? There can be more than one answer.

- Geometric Shapes
- Rectilinear Shapes
- Organic Shapes
- Curvilinear Shapes



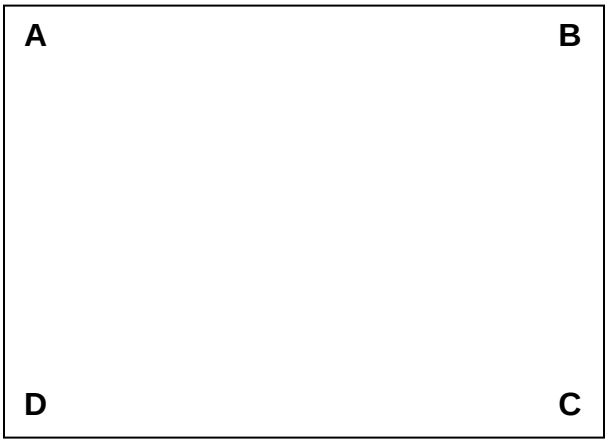
"Abstraction of a Cow, Four Stages" by Theo van Doesburg, Museum of Modern Art, NY

- Notice this 3-dimensional organic form of the cow in the first image.
- The cow becomes an abstract cow because it becomes geometric but is still 3-D looking.
- Then it becomes flat 2-dimensional shapes that one can still see as an abstract cow.
- And the fourth stage is simply geometric shapes inspired by a cow. +

How to Fold a Hexagon
Instructions & Questions

Step 1

Start with paper crosswise (face down).



Questions (and possible answers)

Q: What shape is this?
A: Rectangle

Q: What makes it a rectangle?
A: It is a parallelogram with 4 right angles

Q: What is a parallelogram?
A: A quadrilateral with opposite sides parallel

Q: What is a quadrilateral?
A: A four sided polygon

Q: What is a polygon?
A: Many sided figure in a plane having 3 or more sides

Q: What makes a figure closed?
A: A closed figure in a plane

Q: What is a plane?
A: Example: picture a flat piece of paper that goes indefinitely in all directions

Q: Why isn't it a square?
A: A square must have four equal sides

Q: Is a square a rectangle?
A: Always

Q: Is a rectangle a square?
A: Sometimes

Q: What is a right angle?
A: An angle whose measure is exactly 90 degrees

Q: Back to the rectangle, name the rectangle
A: Rectangle ABCD

Q: Can it be named another way?
A: Rectangle BCDE

Q: Can it be named by just three letters? Why not?
A: Open ended response

Q: Is the rectangle horizontal or vertical?

Q: What are the measurements of the rectangle?
A: 8.5 X 11 inches

Q: What is its length?
A: 11 inches

Q: What is its width?
A: 8.5 inches

Q: What is its height?
A: 8.5 inches

Q: Wait! I thought you said it's width was 8.5 inches, now you say the height is 8.5 inches, which is it width or height?
A: It all depends on how you look at it

Q: What is the difference between width and height? Is that true for all polygons?
A: Width: breadth, distance across from side to side Height: measurement from top to bottom

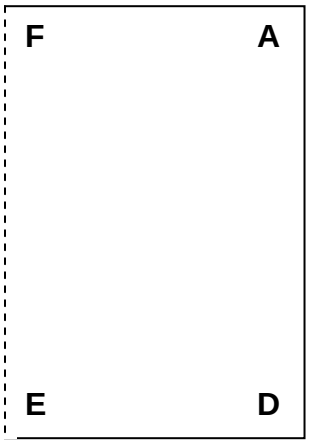
Q: How would you calculate the area of this rectangle?
A: A = bh or A = lw

Q: What is the area of this rectangle?
A: A = bh = (11) (8.5) = 93.5 square inches

Snowflake Geometry Instructions & Questions

Step 2

Fold in half, left over right.



Questions

Q: Name the new rectangle.
A: Rectangle ADEF

Q: What has happened to the area now?
A: It is half the area from before

Q: What happened to the rest of the area? Did it disappear? Where is it?
A: On the other half

Q: Why don't we count it?
A: Area is calculated in a plane

Q: Open it up. What does the fold make?
A: A line segment

Q: So if we open it up what do we have?
A: Two half planes

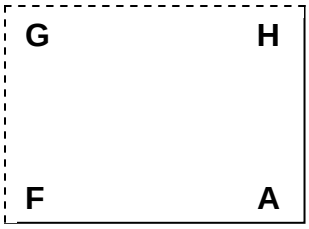
Q: Name the line segment.
A: FE

Q: What is a line?
A: A point moving through space

Snowflake Geometry Instructions & Questions

Step 3

Refold your paper. Fold in half again, folding the top down.



Questions

Q: Name the new rectangle.
A: Rectangle AFGH

Q: What have we done to the area now?
A: Halved it

Q: What relation does the area of this rectangle have to the area of the original rectangle?
A: It is one fourth the area

Q: How do we know?
A: ½ X ½ = 1/4

Q: How many folded edges are there? What makes a folded edge?
A: Three folded edges. A folded edge is made by overlapping layers of paper

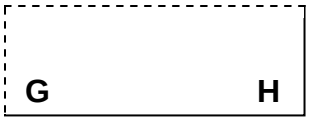
Q: How many folded edges do we see?
A: Two folded edges

Q: Name the folded edges that we see
A: FG and GH

Snowflake Geometry Instructions & Questions

Step 4

Fold in half once more, folding the top down again.



Questions

Q: What have we done to the area now?
A: Halved it

Q: What relation does the area of this rectangle have to the area of the original rectangle?
A: It is one eighth the area

Q: Can you see a pattern? What is it?
A: Every time one matches the edges it halves the area

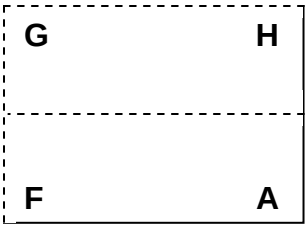
Q: How far could we continue the pattern?
A: To infinity theoretically

Q: Could we actually do it with a piece of paper?
A: No

Snowflake Geometry Instructions & Questions

Step 5

Unfold once.



Questions

Q: What does the middle folded segment create?
A: A midsegment

Q: What is a median/midsegment?
A: A segment made by connecting the midpoints of two sides

Q: Are the two rectangles similar?
A: Yes

Q: What makes geometric shapes similar?
A: Having the same shape but not necessarily the same size

Q: Are the two rectangles congruent?
A: Yes

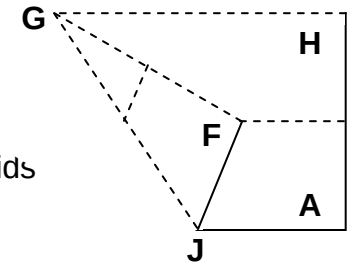
Q: How do we know?

Q: Can you prove it geometrically?
A: Answers may vary

Snowflake Geometry Instructions & Questions

Step 6

Place lower left corner **F** on center fold and crease through upper left corner **G**.



Questions

Q: What shapes do we see?
A: Two triangles, two trapezoids

Q: Name a triangle.
A: GFJ

Q: What is a triangle?
A: A polygon with 3 angles and 3 sides

Q: What is the height of the triangle?
A: Get a ruler out and measure it if you do not know (5 ½ inches)

Q: Are the two triangles similar?
A: Yes

Q: What requirements have to be met for triangles to be similar?
A: AAA similarity

Q: Name the trapezoids.
A: Can't without designating another point

Q: What is a trapezoid?
A: A quadrilateral with exactly two parallel sides

Q: How does a trapezoid differ from a rectangle?
A: A trapezoid has one pair of parallel sides; a rectangle is a parallelogram with 4 right angles

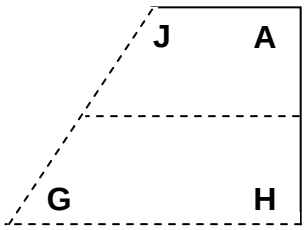
Q: What is the height of the trapezoids?
A: 4.25 inches

Q: Why isn't the height 5.5 inches?
A: Because height must be perpendicular to the base

Snowflake Geometry Instructions & Questions

Step 7

Turn over, flipping the bottom to the top.



Questions

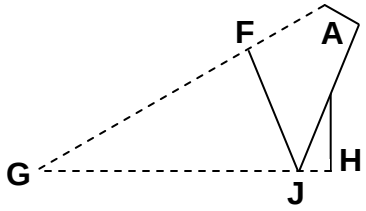
Q: What shapes do you see now?
A: Three trapezoids

Q: What can you tell me about the middle folded line?
A: It is a midsegment, corresponding angles, its length is half the length of the sum of the two bases (AJ+GH)

Snowflake Geometry Instructions & Questions

Step 8

Bisect ∠ **JGH** by matching **GJ** to **GH**. Crease on **GF**.



Questions

Q: What did we just do to angle JGH?
A: We bisected it

Q: What new angles did we form?
A: <FGD, a 30 degree angle

Q: What is bisecting an angle?
A: Dividing the measure of the angle in half

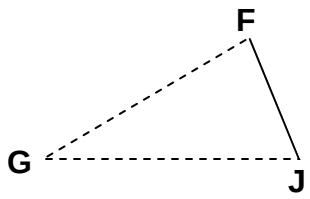
Q: What other shapes do you see?
A: Two triangles, a quadrilateral

Q: Is the quadrilateral a special type of quadrilateral?
A: No, it is not a trapezoid

Snowflake Geometry Instructions & Questions

Step 9

Cut on **JF**. Save and unfold △**GJF**.



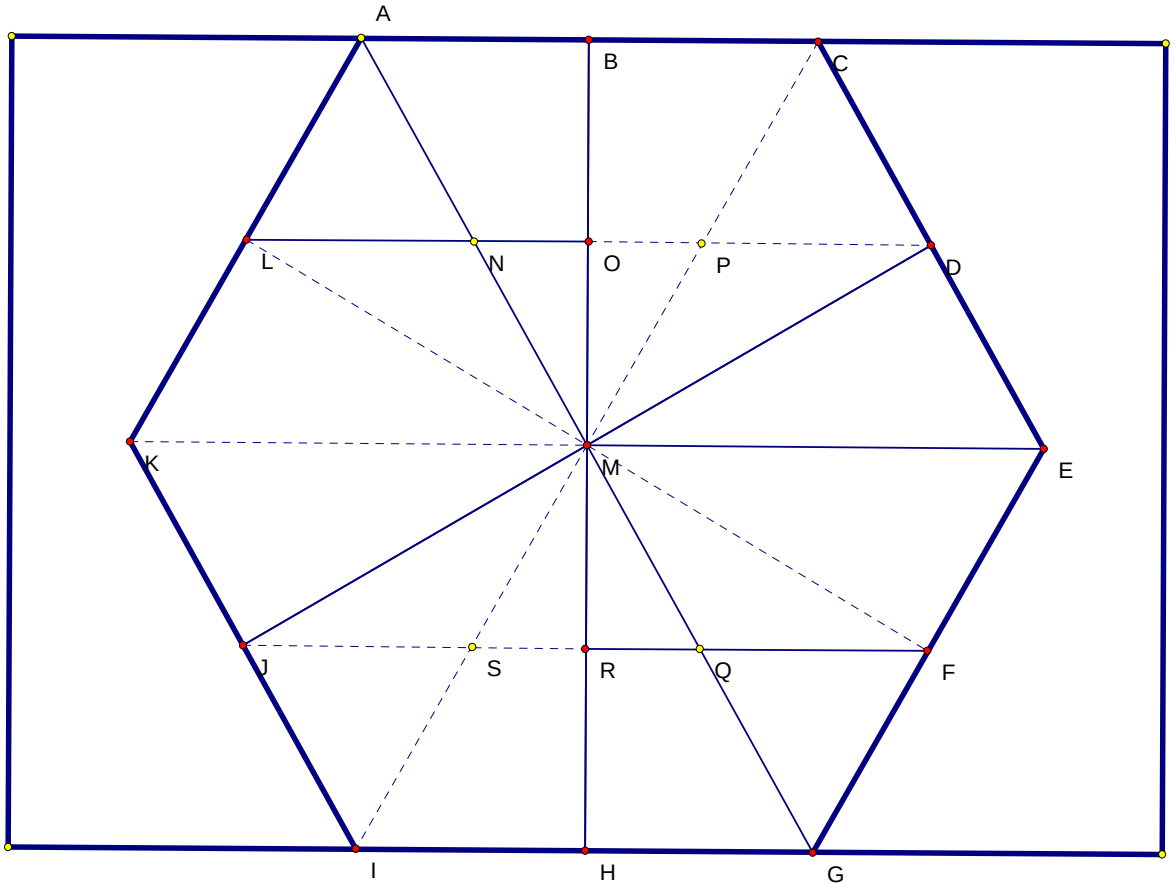
Questions

Q: What shape do you have now?
A: A triangle

Q: What kind of triangle is it?
A: Scalene, Right, 30 - 60 - 90

Q: Are there any special relationships associated with a 30 - 60 - 90 triangle?
A: Answers may vary: for example, special relationship of ratios to the sides

Points of a Snowflake



Name: _____ Period: _____

Hexagonal Geometry

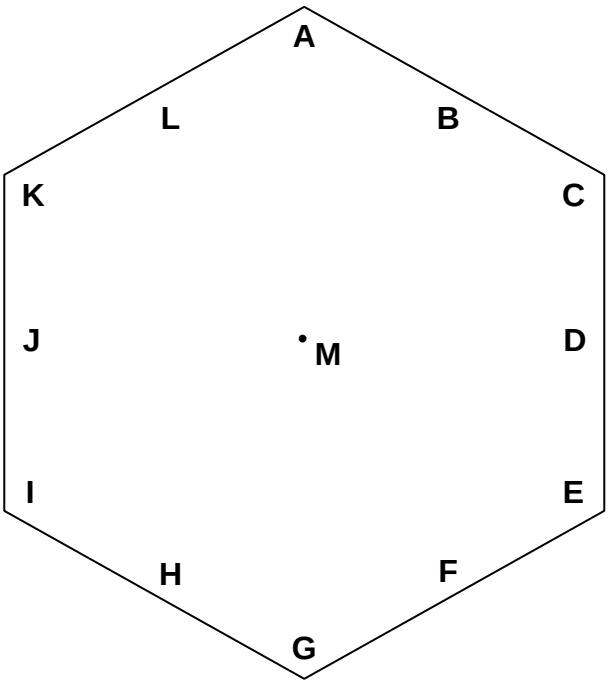
Student Assignment Sheet

1. Use a piece of paper to fold a regular hexagon, the same pattern used to make six-sided snowflakes.

2. Label all consecutive vertices and midpoints of segments in alphabetical order starting with the letter “A” and “M” as the center point as illustrated in the example at the right.

3. Identify, number, list (either on the back of the hexagon or on a separate page), and label geometry terms and/or relationships that can be formed by manipulating and folding the regular hexagon. (Hint: drawing line segments may facilitate discovery of terms and/or relationships.) One point will be awarded for each correctly identified term or relationship. Finding fifty would indicate an excellent effort, although there at least eighty.

4. Write an essay on what you learned by completing this project. Express your thoughts and feelings. Include an introduction, supporting paragraphs, and a conclusion. The essay is worth twenty points. Correct grammar and spelling are expected.



Reminder

How you present your project reflects on you. Be impressive.

Geometry & Art Snowflake Lesson: Terms and Relationships

1. **Regular Hexagon:** ⬡ ACEGHK, ⬡ MPFGHO
2. **Pentagon:** ⬠ DEGIJ
3. **Equilateral Triangle:** △ EGM, △ AEI
4. **Segment:** CE
5. **Angle:** MEG
6. **Acute Angle:** ∠ DME
7. **Obtuse Angle:** ∠ EGI
8. **Isosceles Triangle:** △KMA
9. **Parallel Segments:** BF \ \ LH
10. **Transversal:** CI
11. **Corresponding Congruent Angles:** ∠ LON = ∠ BNC
12. **Alternate Interior Congruent Angles:** ∠ LON = ∠ FNO
13. **Hexagon:** ⬡ ABFGHL
14. **Diagonals:** AE, AG, AI
15. **Rectangle:** ▭ QCER
16. **Rhombus:** ▱ ACEM
17. **Perpendicular Segments:** JD _ AG
18. **Intersection:** M
19. **Acute Triangle:** △MEG
20. **Obtuse Triangle:** △EGI
21. **Right Triangle:** △MRE
22. **Adjacent Angles:** ∠ AMD & ∠ DME
23. **Alternate Exterior Congruent Angles:** ∠ CNB = ∠ HOI
24. **Angle Bisector:** ME
25. **Apothem:** MD
26. **Center:** M
27. **Central Angle of a Polygon:** ∠ CMD
28. **Centroid of Triangle AEI:** M
29. **Circumcenter of Triangle AEI:** M
30. **Concurrency:** M
31. **Incenter of Triangle AEI:** M
32. **Orthocenter of Triangle AEI:** M
33. **Centroid of Triangle AEI:** M
34. **Collinear Points:** I, O, M, N, C
35. **Complementary Angles:** ∠ DME & ∠ EMG
36. **Supplementary Angles:** ∠ MNE & ∠ CMJ
37. **Congruent Angles:** ∠ KAC & ∠ EGI
38. **Convex Polygon:** ⬡ MNE
39. **Concave Polygon:** ⬡ IKACEM
40. **Coplanar Points:** Q, N, R
41. **Equiangular Polygon:** ⬡ AEI
42. **Hypotenuse:** AM in _AMN
43. **Isosceles Trapezoid:** ⏢ KACE
44. **Trapezoid:** ⏢ AMDC
45. **Linear Pair:** ∠ CMD & ∠ CMJ
46. **Line of Symmetry:** KE

47. **Point of Symmetry:** M
48. **Median:** NP or _MCE
49. **Midpoint:** M of IC
50. **Noncollinear Points:** K, Q, M
51. **Parallelogram:** ◇ KACM
52. **Perpendicular Bisector:** MR _ TE
53. **Point:** Q
54. **Scalene Triangle:** △ MRE
55. **Similar Figures:** ⬠ ACEGHK, ⬠MPFGHO
56. **Vertex of a Polygon:** A
57. **Triangular Pyramid:** Pyramid ERNS
58. **Vertex of an Angle:** M is the vertex of <AMD
59. **Dihedral Angle:** ∠ C-NE-I
60. **Altitude of a Triangle:** AR of △AEI
61. The median of an equilateral triangle is parallel to the base and forms a triangle that is ¼ the area of the original triangle.
62. For at least regular triangles and hexagons the regular figure will repeat within the original figure the number of sides plus one.
63. The number of diagonals of a figure is the number of sides subtract two (n-2).
64. The diagonals of a rhombus are perpendicular.
65. In a 30 - 60 - 90 triangle the hypotenuse is twice the shorter leg.
66. The sum of the measure of the interior angles of a polygon with n sides is equal to (n-2) 180.
67. The area of a regular polygon is equal to ½ the product of the apothem and the perimeter.
68. Base angles of an isosceles triangle are congruent.
69. The centroid, circumcenter, incenter, and orthocenter are the same point in an equilateral triangle.
70. Vertical angles are congruent.
71. There is exactly one perpendicular bisector to a segment.
72. Congruence of angles is reflexive, symmetric, and transitive.
73. In a triangle there can be at most one right angle or obtuse angle.
74. SSS Postulate
75. ASA Postulate
76. SAS Postulate
77. If a point lies on the perpendicular bisector of a segment then the point is equidistant from the endpoints of the segment.
78. If two points are each equidistant from the endpoints of a segment, (shaped like a kite, rhombus, or arrowhead) then the line joining the points is the perpendicular bisector of the segment.
79. An equilateral triangle is also equiangular.
80. The bisector of the vertex angle of an isosceles triangle is perpendicular to the base at the midpoint.
81. Opposite angles of a parallelogram are congruent.
82. The diagonals of a parallelogram form two congruent triangles.
83. The diagonals of a parallelogram bisect each other.
84. The diagonals of a rhombus are perpendicular.
85. The midpoint of the hypotenuse of a right triangle is the same distance from all thee vertices.
86. The diagonals of an isosceles trapezoid are congruent.
87. AA Postulate
88. The perpendicular bisectors of the sides of a triangle intersect in a point that is equidistant from the vertices of the triangle.
89. The bisectors of the angles of a triangle intersect in a point that is equidistant from the tree sides of a triangle.
90. The medians of any triangle intersect in a point that is 2/3 of the distance from each vertex.