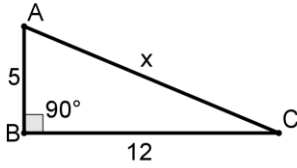


Day 1: Trigonometric Functions

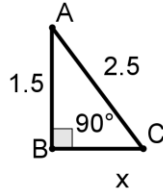
Warm-Up:

Given the following triangles, find x.

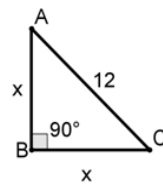
1.



2.



3.



Solve for the missing variables.

4. $x^2 - 12x = 45$

5. $y = \frac{1}{2}x - 5$
 $3x + 8y = 2$

6. Simplify $(-5\sqrt{3})^2$

Notes 9.1 and 9.2 - Trigonometric Functions

The trigonometric (trig) functions are _____, _____, and _____.

These functions can be used to find _____ measures, knowing the ratio of the sides

OR length of a _____, knowing one side and an angle measure.

They are used only for _____ triangles!

Sin θ =

Cos θ =

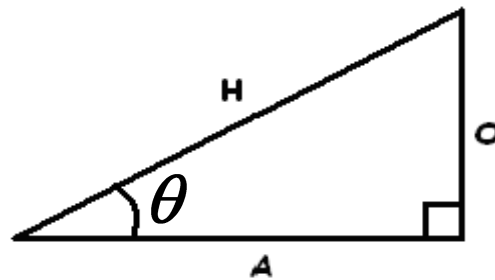
Tan θ =

O=

H=

A=

θ =



SOH CAH TOA

Example 1: $\tan(\angle B)$

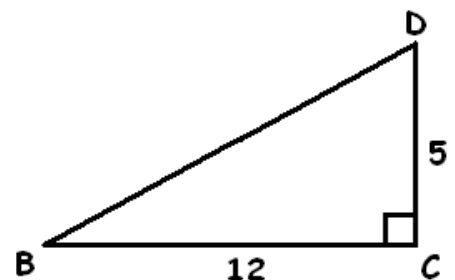
Example 2: $\tan(D)$

Example 3: $\sin(D)$

Example 4: $\cos(D)$

Example 5: $\cos(B)$

Puzzle - Practice Ratios

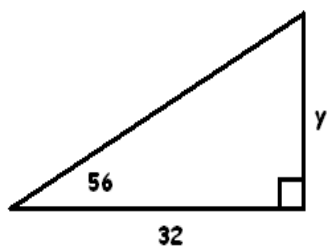


Finding missing side lengths using Trigonometric Ratios

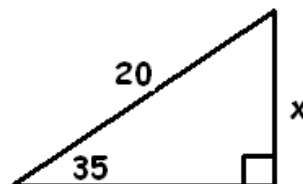
To solve for missing side lengths, set up the _____, and put the trig function _____, then cross-multiply to solve.

Use the trig ratios to find the length of the side labeled with a variable. All angle measures for these examples are in degrees. (Remember SOH CAH TOA)

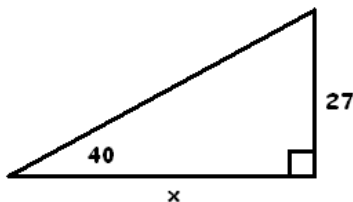
Example 1: Solve for y .



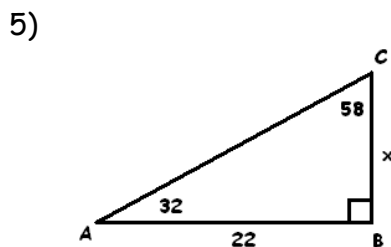
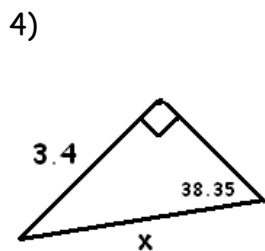
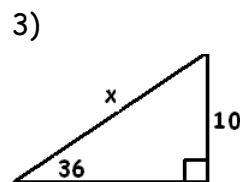
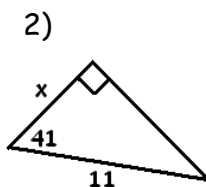
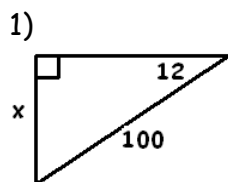
Example 2: Solve for x .



Example 3: Solve for x .



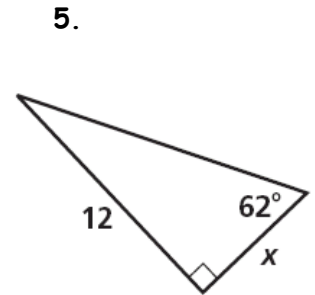
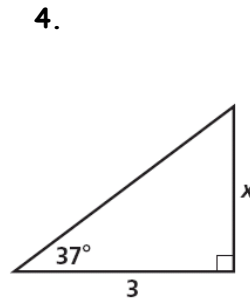
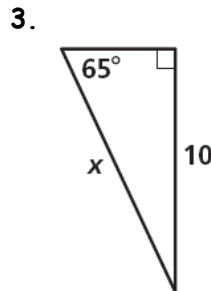
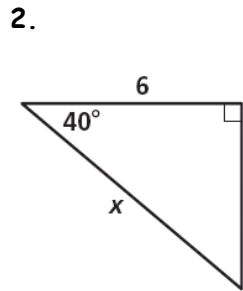
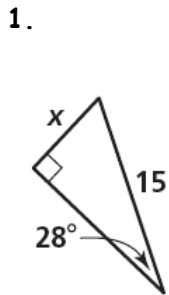
You try!!



Puzzle - Practice Finding Side Lengths

Day 2: Finding Angles using Right Triangle Trigonometry

Warm-Up: Find the value of x .



Notes Day 2: Finding Angles using Right Triangle Trigonometry

Finding missing angles with the Trigonometric Ratios

To find missing angle measures, set up the _____.

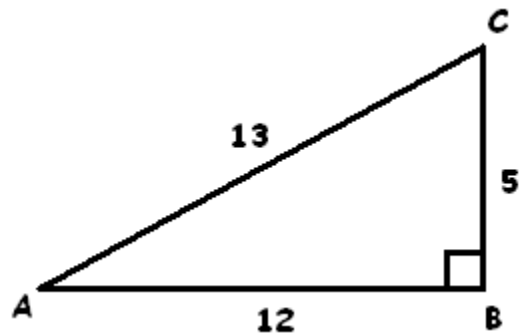
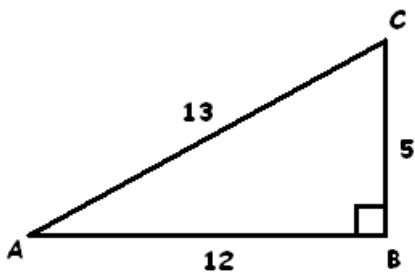
Then, you'll have to do the _____ of the trig function _____.

NOTE: the inverse of the trig function and the trig function itself cancel out!

TIP: The inverse _____ like the trig function with a _____.

Example 4: Find $\tan A$ and $\tan C$.

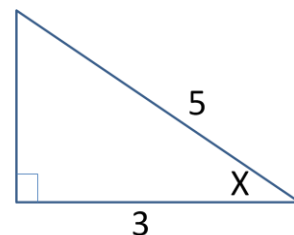
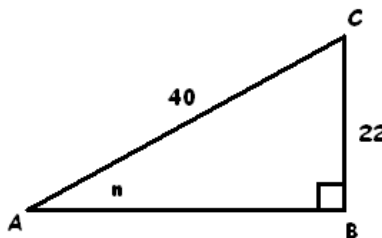
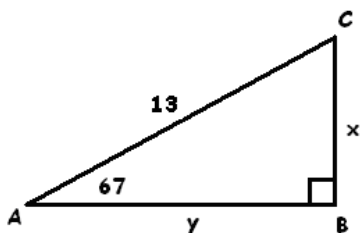
Example 5: Find A and C .



Example 6: Find x and y .

Example 7: Find n .

Ex 8: Find x .

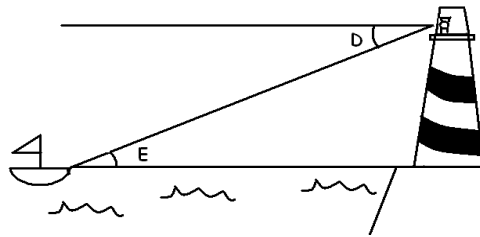


Notes Day 2: Angle of Elevation and Angle of Depression

The top of a lighthouse is 50 feet above sea level. Suppose a lighthouse operator sees a sailboat at an angle of 22° with a horizontal line straight out from his line of vision.

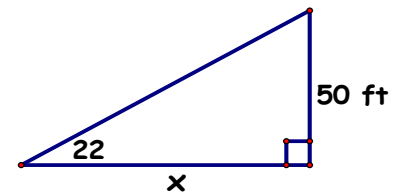
The angle between the horizontal line and the line of sight is called the _____.

At the same time, a person in the boat looks up at an angle of _____ with the horizon and sees the operator in the lighthouse. This angle is called the _____.



NOTE: The measure of the angle of depression _____ the measure of the angle of elevation.

Example 1: The distance to the lighthouse from the sailboat can be found by



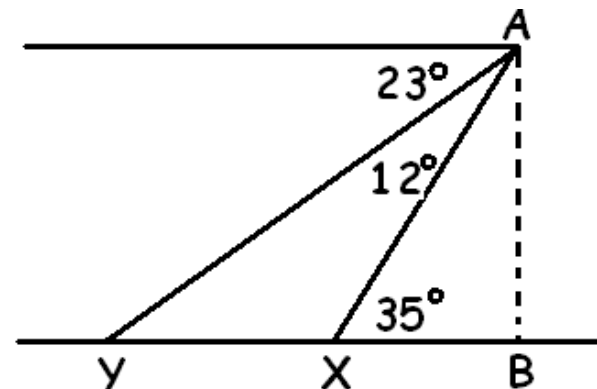
People at points X and Y see an airplane at A

The angle of elevation from X to A is _____.

The angle of depression from A to X is _____.

The angle of depression from A to Y is _____.

The angle of elevation from Y to A is _____.



Example 3 Karen drives 25 km up a hill that is a grade of 14. What horizontal distance has she covered?

Day 2: Angle of Elevation and Angle of Depression Practice
--

For each problem: 1) Sketch a diagram.

2) Set up the equation.

3) Solve.

1) The leg opposite the 50 degree angle in a right triangle measures 8 meters. Find the length of the hypotenuse.

2) A cliff is 90 feet above the sea. From the cliff, the angle of depression to a boat measures 46 degrees. How far is the boat from the base of the cliff?

3) A ramp is 60 feet long. It rises a vertical distance of 8 feet. Find the angle of elevation.

4) A tree casts a 50-foot shadow while the angle of elevation of the sun is 48. How tall is the tree?

Day 3: Applications of Trigonometric Functions

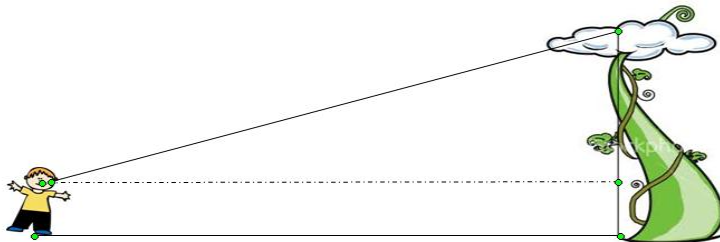
Warm-Up:

- | | | |
|---|---|--|
| <p>1. A tree casts a shadow 21m long while the sun's angle of elevation is 51°. What is the height of the tree?</p> | <p>2. A guy wire reaches from the top of a 120m TV tower to the ground making an angle of 63° with the ground. Find the length of the wire.</p> | <p>3. A 40 foot escalator rises to a height of 21 feet. What is the angle of inclination (elevation) of the escalator?</p> |
|---|---|--|

Notes Day 3 - More with Applications of Trigonometric Functions
--

Preparation for Clinometer Lab**Example:**

Jack was bragging about climbing a beanstalk. One of his friends, tired of hearing the story for the umpteenth time asked, "Jack, how tall was the beanstalk?" Knowing that his friends would pester him forever, Jack decided to find out...

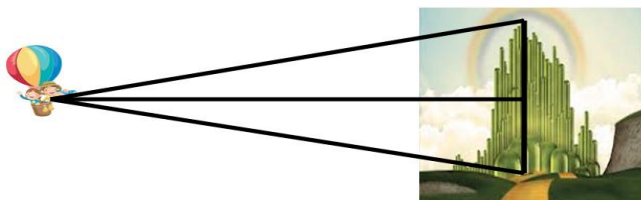


Jack stood 100 yards away from the point directly under where the beanstalk meets the clouds and used his clinometer to look at the top of the stalk (where it met the clouds). He measured the angle of elevation to be 27.5° . Using this information, what is the distance from the top of the bean stalk to Jack's line of sight?

Jack then measured from his eyes to the ground (it was 48 inches). He then concluded that the stalk was _____ feet tall.

Example

While flying in a hot air balloon, Dorothy and the Wizard looked back at the Emerald City. Dorothy wondered, "How high was that lovely green castle?" Using her clinometer, she decided to find out! She knew (using her range finder) that the horizontal distance to the city was 150 yards.



Dorothy measured the angle of depression from the balloon to the base of the emerald castle to be 15° and the angle of elevation to the top of the castle to be 25° . Based on these measurements, how tall is the castle?

Day 4: Law of Sines, Area of Triangles with Sine

Warm-Up:

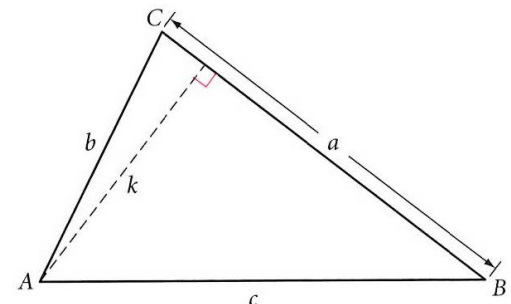
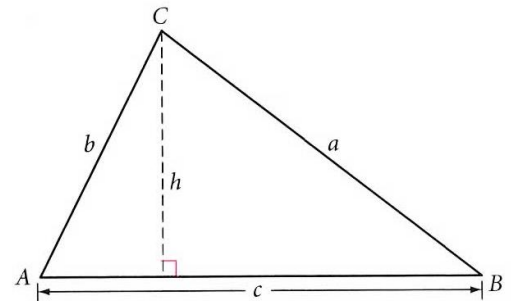
1. A tree 10 meters high cast a 17.3 meter shadow.
Find the angle of elevation of the sun.
2. A car is traveling up a slight grade with an angle of elevation of 2° .
After traveling 1 mile, what is the vertical change in feet?
(1 mile = 5280 ft)
3. A person is standing 50 meters from a traffic light. If the angle of elevation from the person's feet to the top of the traffic light is 25° , find the height of the traffic light.

Notes Day 4 Part 1 - Solving Oblique Triangles with Law of Sines

In trigonometry, the _____ can be used to find missing parts of triangles that are _____ triangles.

Discovery for Solving Oblique Triangles!

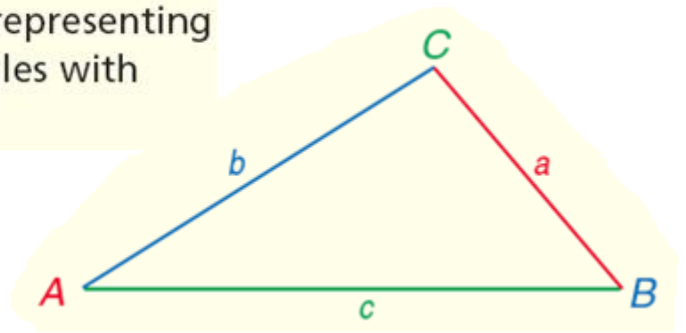
- 1) Set up the ratios for $\sin A$ and $\sin B$.
- 2) Find h in terms of a and the sine of an angle.
- 3) Find h in terms of b and the sine of an angle.
- 4) Using Algebra, show that $\frac{\sin A}{a} = \frac{\sin B}{b}$
- 5) Find k in terms of c and the sine of an angle.
- 6) Find k in terms of b and the sine of an angle.
- 7) Using Algebra, show that $\frac{\sin B}{b} = \frac{\sin C}{c}$
- 8) Combine steps 4 and 7 to complete the blanks in the following Law of Sines box.



Law of Sines

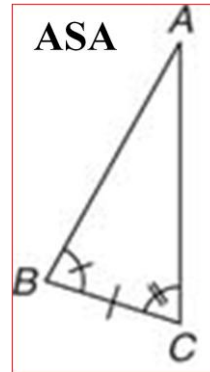
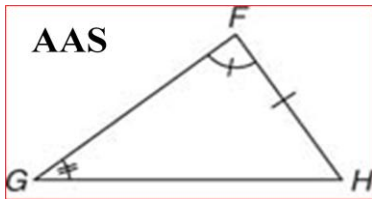
Let $\triangle ABC$ be any triangle with a , b , and c representing the measures of the sides opposite the angles with measures A , B , and C , respectively. Then

$$\sin A = \frac{\quad}{b} = \frac{\quad}{\quad}$$

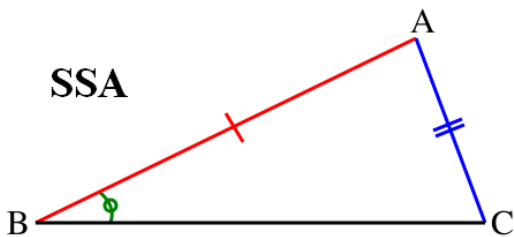


NOTE: to use the Law of Sines, we need an angle and a side _____ from each other!!

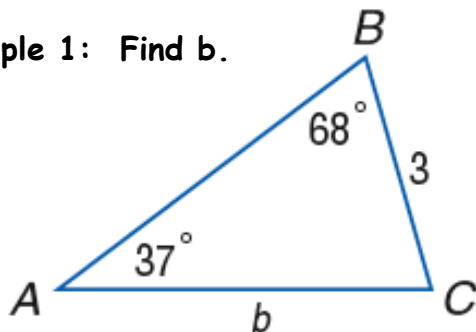
Law of Sines is useful in these cases.



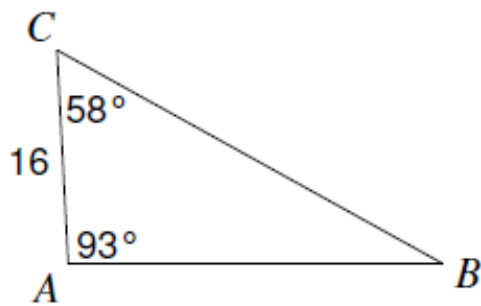
Law of Sines can also be used in this case, but it is ambiguous. We'll discuss this more later!



Example 1: Find b .



Example 2: Find B , a , and c .



The Law of Sines can be used to solve a triangle. Solving a Triangle means finding the measures of _____ and _____ of a triangle.

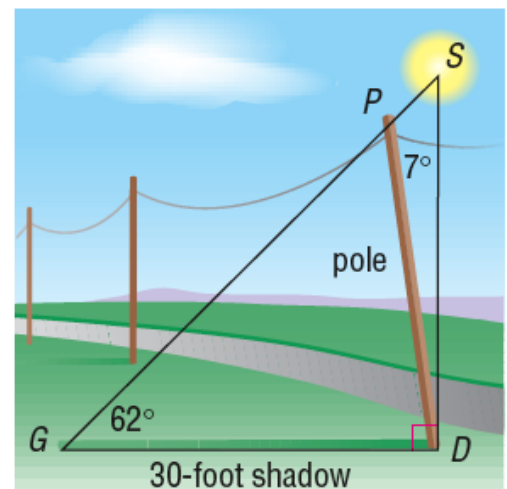
Example 3: Solve the Triangle.

Solve $\triangle ABC$ if $m\angle A = 33$, $m\angle B = 47$, and $b = 14$. Round angle measures to the nearest degree and side measures to the nearest tenth.

Ex. 4 Word Problem

Indirect Measurement

When the angle of elevation to the sun is 62° , a telephone pole tilted at an angle of 7° from the vertical casts a shadow of 30 feet long on the ground. Find the length of the telephone pole to the nearest tenth of a foot.



Concept Summary**Law of Sines**

The Law of Sines can be used to solve a triangle in the following cases.

Case 1 You know the measures of two angles and any side of a triangle. (AAS or ASA)

Case 2 You know the measures of two sides and an angle opposite one of these sides of the triangle. (SSA)

Solve each $\triangle PQR$ described below. Round angle measures to the nearest degree and side measures to the nearest tenth.

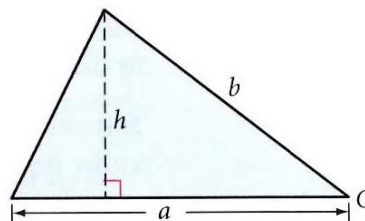
- | | | |
|--|--|------------|
| 8. $m\angle R = 66, m\angle Q = 59, p = 72$ | 9. $p = 32, r = 11, m\angle P = 105$ | SSA |
| 10. $m\angle P = 33, m\angle R = 58, q = 22$ | 11. $p = 28, q = 22, m\angle P = 120$ | SSA |
| 12. $m\angle P = 50, m\angle Q = 65, p = 12$ | 13. $q = 17.2, r = 9.8, m\angle Q = 110.7$ | SSA |

Notes Day 4 Part 2 - Finding Area of Oblique Triangles with Sine

Discovery for Area of Oblique Triangles!

1) What is the formula for the area of a triangle that you remember from other courses?

2) What is formula for the area of this triangle?



3) Find the ratio for $\sin C$.

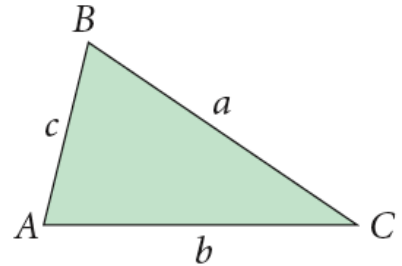
4) Suppose we only know the measures of a , b , and angle C - and that we do not know the measure of h . How could we find the area of the triangle? (Hint: use steps 2 & 3)

Theorem 9-1

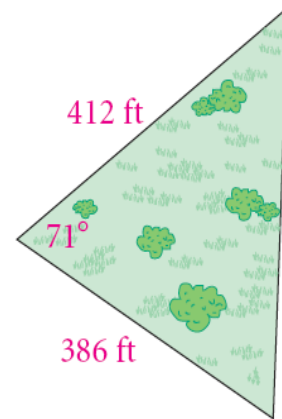
Area of a Triangle Given SAS

The area of a triangle is one half the product of the lengths of two sides and the sine of the included angle.

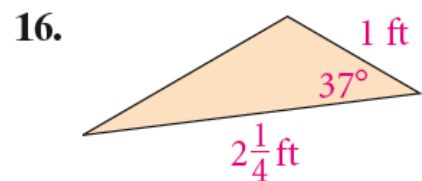
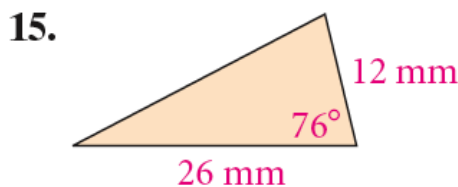
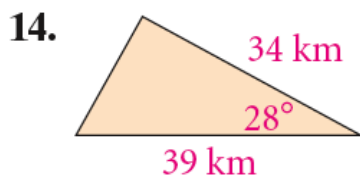
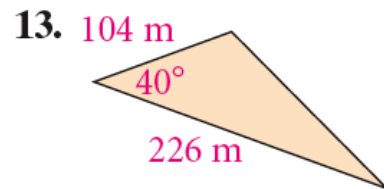
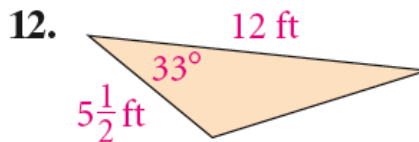
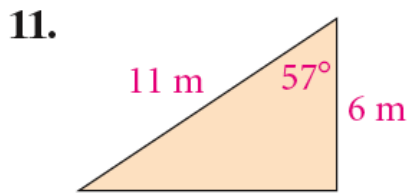
Area of $\triangle ABC =$ _____



Surveying The surveyed lengths of two adjacent sides of a triangular plot of land are 412 ft and 386 ft. The angle between the sides is 71° . Find the area of the plot.



Find the area of each triangle. Give answers to the nearest tenth.



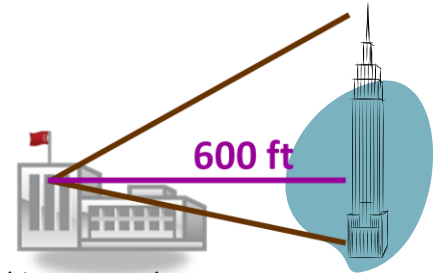
17. **Surveying** A surveyor marks off a triangular parcel of land. One side of the triangle extends 80 yd. A second side of 150 yd forms an angle of 67° with the first side. Determine the area of the parcel of land to the nearest square yard.

Day 5: Law of Sines and the Ambiguous Case

Warm-Up:

1. The straight line horizontal distance between a window in a school building and a skyscraper is 600ft. From a window in the school, the angle of elevation to the top of the skyscraper is 38 degrees and the angle of depression to the bottom of the tower is 24 degrees.

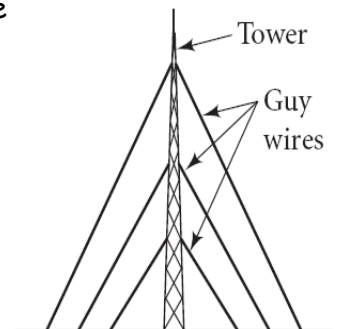
Approximately how tall is the skyscraper?



2. A communications tower is located on a plot of flat land. It is supported by several guy wires.

You measure that the longest guy wire is anchored to the ground 112 feet from the base of the tower and that it makes an 76° angle with the ground.

How long is the longest guy wire and at what height is it connected to the tower?



Notes Day 5: Law of Sines and the Ambiguous Case

With SSA situations, many *interesting* cases are possible. We will look at the 3 cases that occur given an acute angle.

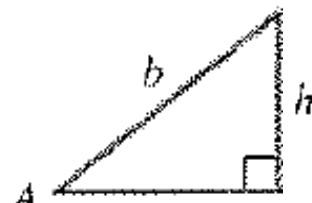
If two sides and an angle opposite one of them is given (SSA), three possibilities can occur.

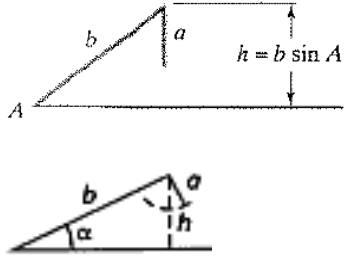
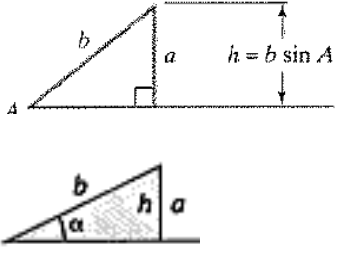
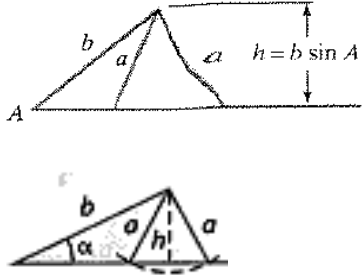
- (1) _____
- (2) _____
- (3) _____

Before we look at the cases, let's use what we know about right triangles to set up the ratio for the following triangle.

$\sin A =$

When we solve for h , we get $h =$ _____



<p>Figures</p>			
<p>Number of Triangles Possible</p>			
<p>Occurs when....</p>			
<p>Why it occurs....</p>	<p>Side across from the angle is</p>	<p>Just gives us</p> <p>_____</p> <p>_____</p> <p>Triangle</p>	<p>Ambiguous Case...</p> <p>The side across from the angle can "swing" to form</p> <p>an _____ triangle</p> <p>AND</p> <p>an _____ triangle</p>

Summary:

If the side across from the given angle is _____ than the other side, then check for the ambiguous case!

Ex. 1: SSA Ambiguous Case

Solve $\triangle ABC$ if $m\angle A = 25^\circ$, $a = 125$, and $b = 150$.

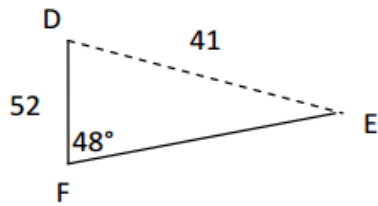
Ex. 2: Solve a triangle when one side is 27 meters, another side is 40 meters and a non-included angle is 33° .

Ex. 3: Solve for all of the missing sides and angles given $m\angle C = 48$, $c = 93$, and $b = 125$.
(Draw the triangle!)

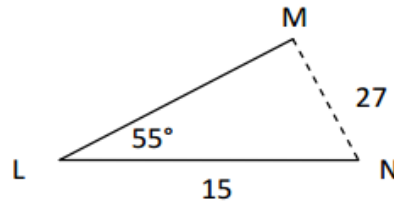
Ex. 4: Solve for all of the missing sides and angles given $m\angle A = 24$, $a = 9.8$, and $b = 17$.
(Draw the triangle!)

Law of Sines Practice:

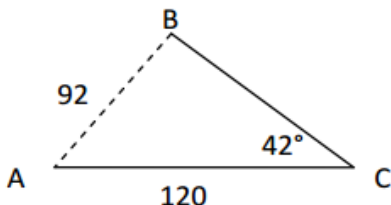
2. For $\triangle DEF$,
 $e = 52, f = 41$, and $m\angle F = 48^\circ$. Find all possible $m\angle E$ to the nearest degree.



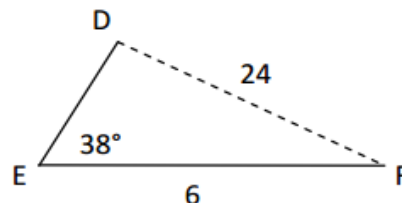
1. For $\triangle LMN$,
 $l = 27, m = 15$, and $m\angle L = 55^\circ$. Find all possible $m\angle M$ to the nearest degree.



4. For $\triangle ABC$,
 $b = 120, c = 92$, and $m\angle C = 42^\circ$. How many triangles can be formed? Find $m\angle B$.

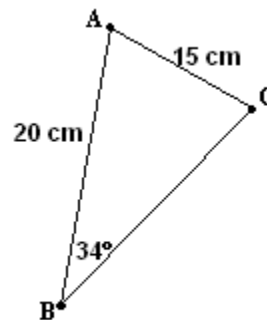


3. For $\triangle DEF$,
 $d = 6, e = 24$, and $m\angle E = 38^\circ$. How many Triangles can be formed? Find $m\angle D$.



5. For triangle DEF, $d = 25$, $e = 30$, and $m\angle E = 40^\circ$. Find all possible measurements of f to the nearest whole number.

6. Given $\triangle ABC$ with $\angle B = 34^\circ$, $b = 15\text{cm}$, and $c = 20\text{cm}$, solve the triangle.



7. Given triangle ABC, $a = 8$, $b = 10$, and $m\angle A = 34$, solve the triangle.

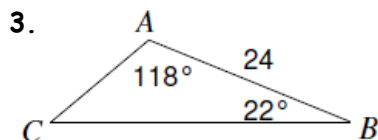
Day 6: Law of Cosines

Warm-Up: Solve each proportion:

$$1) \frac{2x - 3}{3} = \frac{10 - 4x}{2}$$

$$2) \frac{x + 3}{x + 2} = \frac{x - 1}{x - 4}$$

Solve each triangle using Law of Sines.



4. $m\angle C = 53^\circ$, $m\angle B = 44^\circ$, $b = 7$

Notes Day 6: Law of Cosines

THE LAW OF COSINES Suppose you know the lengths of the sides of the triangular building and want to solve the triangle. The **Law of Cosines** allows us to solve a triangle when the Law of Sines cannot be used.

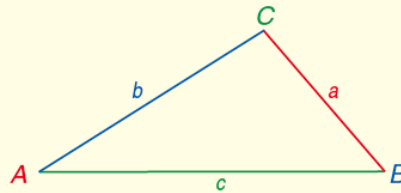
Key Concept*Law of Cosines*

Let $\triangle ABC$ be any triangle with a , b , and c representing the measures of sides opposite angles with measures A , B , and C , respectively. Then the following equations are true.

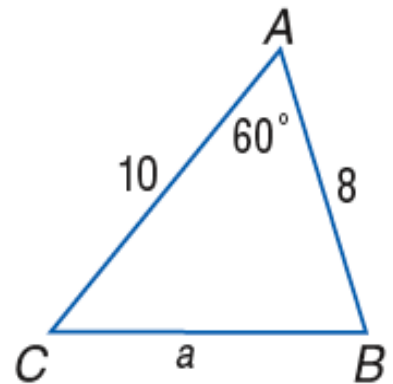
$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

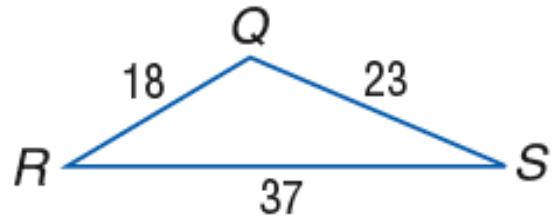
$$c^2 = a^2 + b^2 - 2ab \cos C$$



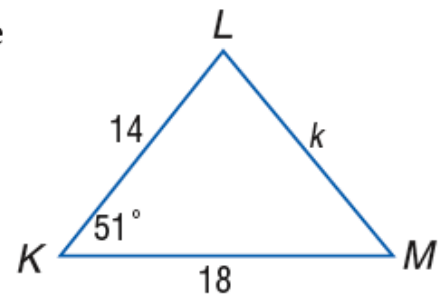
Example 1: Use Law of Cosines to find a .



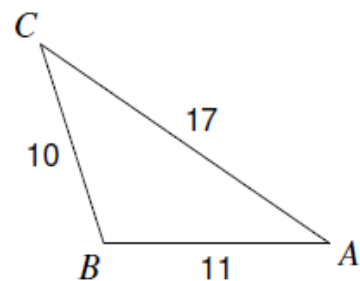
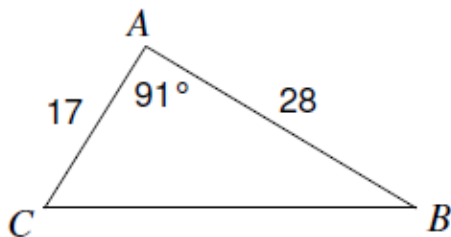
Example 2: Use Law of Cosines to find $m\angle R$.



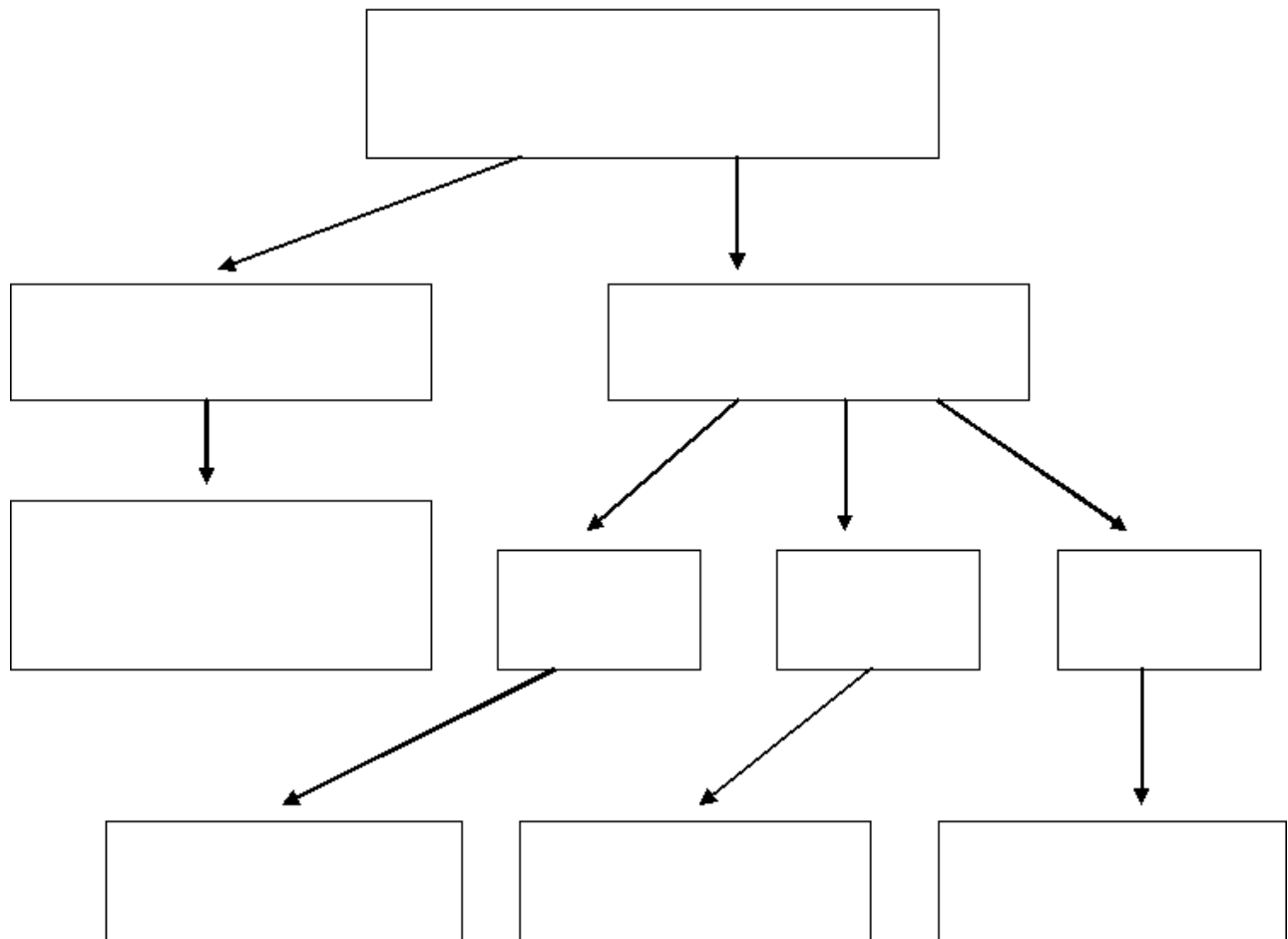
Example 3: Solve $\triangle KLM$. Round angle measure to the nearest degree and side measure to the nearest tenth.



Examples 4, 5 YOU TRY - Solve each Triangle.

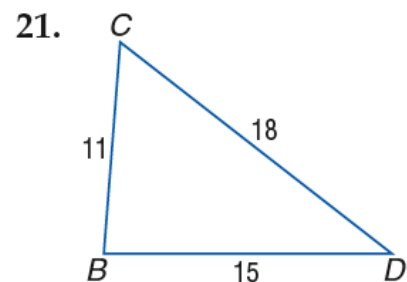
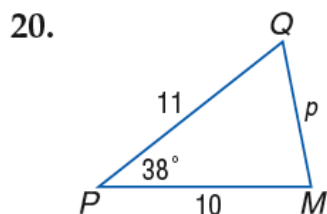
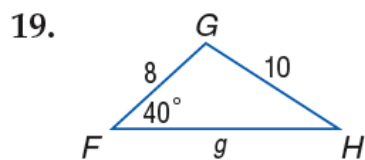


Which Formula Do I Use?



Practice

Solve each triangle using the given information. Round angle measures to the nearest degree and side measures to the nearest tenth.



22. $\triangle ABC: m\angle A = 42, m\angle C = 77, c = 6$

23. $\triangle ABC: a = 10.3, b = 9.5, m\angle C = 37$

24. $\triangle ABC: a = 15, b = 19, c = 28$

25. $\triangle ABC: m\angle A = 53, m\angle C = 28, c = 14.9$

Day 7: Classifying Triangles and their parts AND Quiz

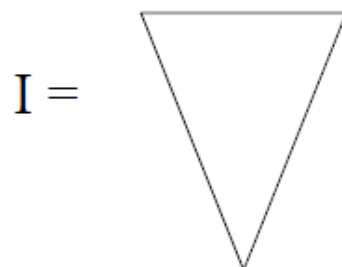
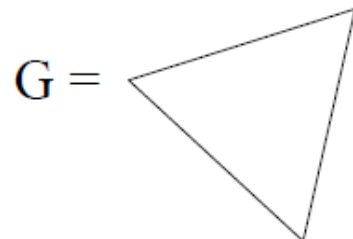
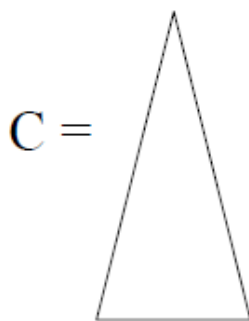
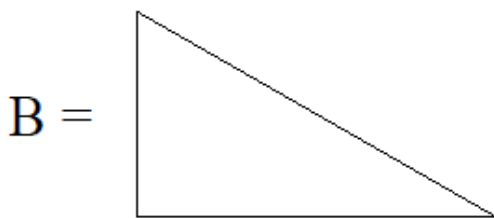
Warm-Up:

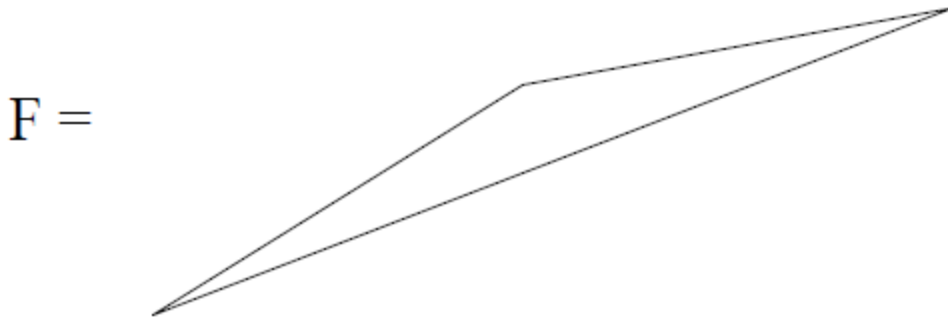
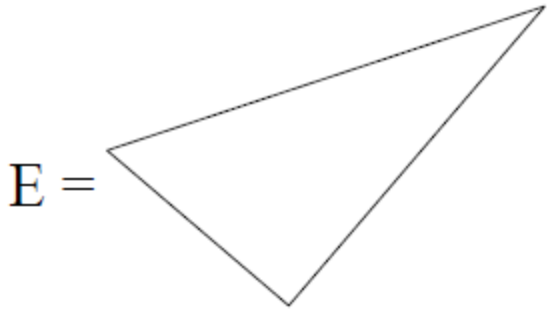
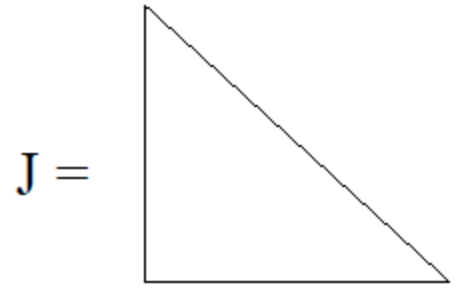
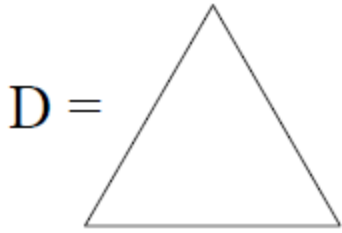
1. After flying at an altitude of 9 km, an airplane starts to descend when its ground distance from the landing field is 175 km. What is the angle of depression for this portion of the flight?
2. A ski slope is 550 yd long with an angle of depression of 14 degrees. Find the vertical drop of the slope.
3. The San Jacinto Column near Houston Texas is 570 feet tall. If the angle of elevation for Derrick's line of sight is 75 degrees and his eyes are 6 feet from the ground, how far is he from the base of the column?
4. Jimmy is 24 feet up a tree. His mom is 7 feet from the tree, telling him to come down. How far is Jimmy from his mom?

Day 7 Lesson Introduction, Classifying Triangles

Measure the sides and angles of each triangle. Write the letter of the triangle in the recording table (on the next page) by classifying the triangle.

Triangle Sort





Triangle Sort Recording Table

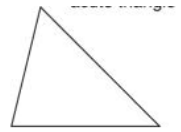
Acute	Obtuse	Right
Scalene	Isosceles	Equilateral

Day 7 Main Lesson, Notes

I. Classifying Triangles by their angles

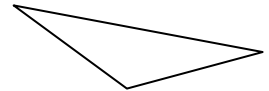
a. Acute Triangle

i. An acute triangle is a triangle that has _____



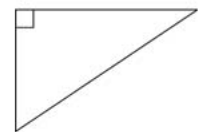
b. Obtuse Triangle

i. An obtuse triangle is a triangle that has _____



c. Right Triangle

i. A right triangle is a triangle that has _____

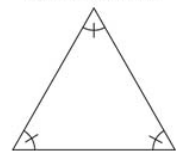


d. Oblique Triangle

i. An oblique triangle is a _____
ii. These can be _____ triangles or _____ triangles

e. Equiangular Triangle

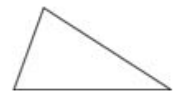
i. An equiangular triangle is a triangle that has _____



II. Classifying Triangles by their sides

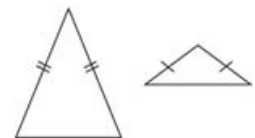
a. Scalene Triangle

i. A scalene triangle is a triangle that _____



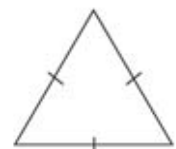
b. Isosceles Triangle

i. An isosceles triangle is a triangle that has _____



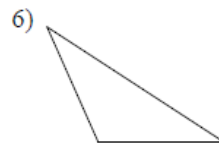
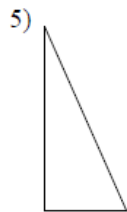
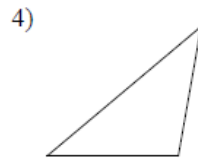
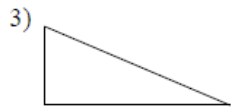
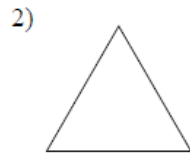
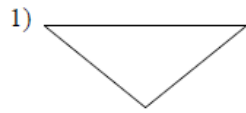
c. Equilateral Triangle

i. An equilateral triangle is a triangle that has _____



Day 7 Practice

Classify each triangle by its sides. Base your decision on the actual lengths of the sides and the measures of the angles.



Classify each triangle by its angles and sides.

