Soil Compressibility & Settlement

Faculty of Engineering – Cairo University
Third Year Civil
Soil Mechanics
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Soil Settlement

- Settlements are vertical deformations of a soil mass under the effect of an applied stress causing vertical movement of the supported structure
- Settlements are one of the safety and performance criteria in geotechnical assessment of structures
- Excessive settlements can result in structural damage to a building frame, or loss of functionality.

Soil Settlement

Soil Settlement

- **Uniform settlements:** equal settlements across the area of the structure
- **Differential settlements:** unequally foundation settlements in different areas of the structure.
- Differential settlement can result in severe structural damage while uniform settlements are of less consequences

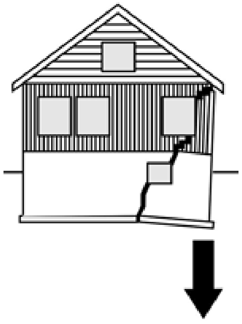
No Settlement   Total Settlement   Differential Settlement

Soil Settlement

- Differential settlements

Soil Settlement

- Differential settlements



Define: Compressibility

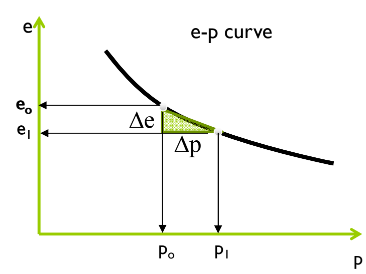
- **Compressibility:** is the property through which particles of soil are brought closer to each other, due to escapage of air and/or water from voids under the effect of an applied pressure.

Settlement of Cohesive Soils

Coefficient of compressibility (a_v):
is the rate of change of void ratio (*e*) with respect to the applied effective pressure (*p*) during compression.

$$a_v = \frac{\Delta e}{\Delta p}$$

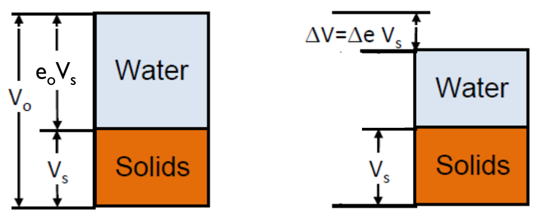
e_o = initial void ratio
p_o = initial ***effective*** stress
e_1 = final void ratio
p_1 = final ***effective*** stress
$\Delta e = e_o - e_1$
$\Delta p = p_1 - p_o$



Settlement of Cohesive Soils

Coefficient of volume compressibility (m_v):
is the volume decrease of a unit volume of soil per unit increase of effective pressure during compression.

$$m_v = \frac{\frac{\Delta V}{V_o}}{\Delta p} = \frac{\frac{\Delta e V_s}{(1+e_o)V_s}}{\Delta p} = \frac{1}{(1+e_o)}\frac{\Delta e}{\Delta p} = \frac{a_v}{(1+e_o)}$$



Initial Condition Final Condition

Settlement of Cohesive Soils

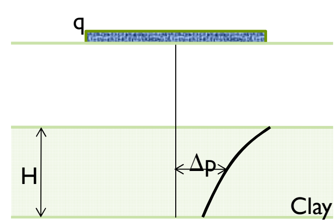
For a thin layer, Δp at mid depth is considered as an average stress within the layer.

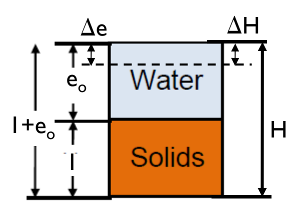
$$\frac{\Delta H}{H} = \frac{\Delta V}{V}$$

$$\Delta H = S = \frac{\Delta e}{(1+e_o)}H$$

$$S = \frac{a_v}{(1+e_o)}\Delta p H$$

$$S = m_v \Delta p H$$





where:
S = settlement
m_v = coefficient of volume compressibility
Δp = stresses at mid depth of layer due to added loads
H = total thickness of layer

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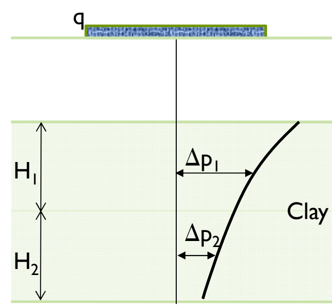
Settlement of Cohesive Soils

For a thick layer, the layer may be divided to number of sub-layers, each of thickness H_i. The stress at mid-depth of each sub-layer is Δp_i.

$$S = \sum m_{vi} \Delta p_i H_i$$



Settlement of Cohesionless Soils

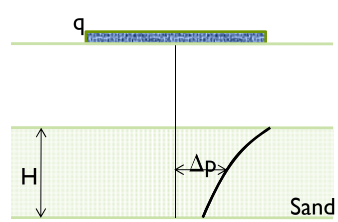
- Sand may be considered as an elastic material with Young's modulus (E)
- Young's *Modulus (E):* is the slope for the linear portion of the stress-strain curve.

$$\frac{\text{strain}}{\text{stress}} = \frac{1}{E}$$

$$\frac{\Delta H / H}{\Delta p} = \frac{1}{E}$$

$$\frac{S}{H} = \frac{\Delta p}{E}$$

$$S = \frac{1}{E} \Delta p H$$



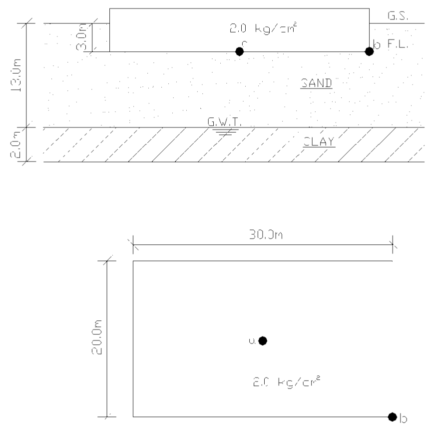
$$E = 100\,(\text{v. loose}) - 800\,(\text{v. dense}) \quad \text{kg/cm}^2$$

Example

Compute the settlement due to compressibility of the sand and clay layers at points (a) and (b) of the building shown in the figure. The building has a basement and is founded on raft foundation. The stress from the building at the foundation level is as shown in the figure.

Sand: γ = 1.70 t/m^3 E = 500 kg/cm^2

Clay: m_v = 0.03 cm^2/kg



Example

$q_{net} = q - \gamma h = 20 - 1.70\text{x}3.0 = 14.9$ t/m^2

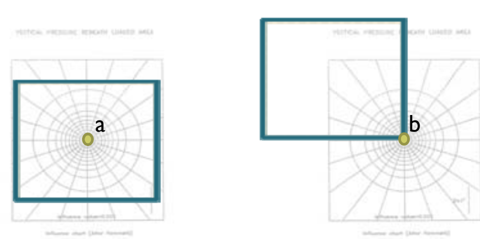
For sand layer:

z = 5.0 m = 1" = 2.5 cm

L = 30 m → 30x2.5/5.0 = 15.0 cm, B = 20 m → 20x2.5/5.0 = 10.0 cm,

n_a = 191 $\Delta\sigma_{a,sand} = n_a/200\ (q_{net})$ = 191/200 x 14.9 = 14.2 t/m^2

n_b = 50 $\Delta\sigma_{b,sand} = n_b/200\ (q_{net})$ = 50/200 x 14.9 = 3.7 t/m^2



Example

$q_{net} = q - \gamma h = 20 - 1.70x3.0 = 14.9\ t/m^2$

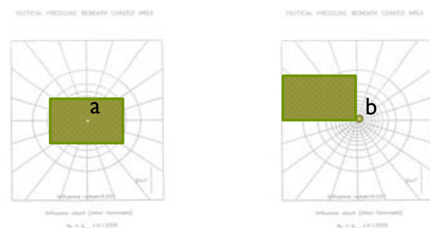
For clay layer:

z = 11.0 m = 1" = 2.5 cm

L = 30 m → 30x2.5/11 = 6.8 cm, B = 20 m → 20x2.5/11 = 4.5 cm

$n_a = 144$ $\Delta\sigma_{a,clay} = n/200\ (q_{net}) = 144/200 \times 14.9 = 10.7\ t/m^2$

$n_b = 47.3$ $\Delta\sigma_{b,clay} = n/200\ (q_{net}) = 47.3/200 \times 14.9 = 3.5\ t/m^2$



Example

$S_a = S_{sand} + S_{clay}$

$S_{sand} = 1/E\ \Delta\sigma_{a,sand} H$

H = 10.0 m E = 500 kg/cm^2 $\Delta\sigma_{a,sand} = 14.2\ t/m^2$

$S_{sand} = 1/500 \times 14.2/10 \times 1000 = 2.84$ cm

$S_{clay} = m_v\ \Delta\sigma_{a,clay} H$

H = 2.0 m $m_v = 0.03\ cm^2/kg$ $\Delta\sigma_{a,clay} = 10.7\ t/m^2$

$S_{clay} = 0.03 \times 10.7/10 \times 200 = 6.42$ cm

$S_a = S_{sand} + S_{clay} = 2.84 + 6.42 = 9.26$ cm

Example

$S_b = S_{sand} + S_{clay}$

$S_{sand} = 1/E\ \Delta\sigma_{b,sand}H$

H = 10.0 m E = 500 kg/cm^2 $\Delta\sigma_{b,sand} = 3.7$ t/m^2

$S_{sand} = 1/500 \times 3.7/10 \times 1000 = 0.74$ cm

$S_{clay} = m_v\ \Delta\sigma_{b,clay}H$

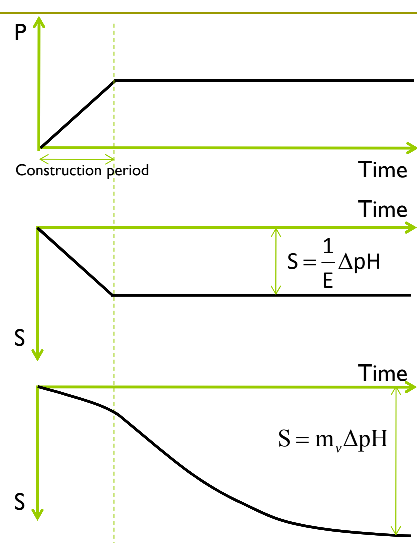
H = 2.0 m $m_v = 0.03$ cm^2/kg $\Delta\sigma_{b,clay} = 3.5$ t/m^2

$S_{clay} = 0.03 \times 3.5/10 \times 200 = 2.1$ cm

$S_b = S_{sand} + S_{clay} = 0.74 + 2.1 = 2.84$ cm

Time of settlement compared to construction time

- Loading Diagram
- Settlement of Sand
- Settlement of Clay



Causes of Settlement

- Static Loads
- Dynamic Loads
- Groundwater table lowering
- Loads from adjacent buildings
- Capillary forces

Theory of Consolidation

- **Consolidation:** is the process of squeezing water out of saturated soil under the effect of loads.
- In sandy soils, high permeability, drainage of water out of the soil under the effect of loading happens immediately.
- In clay soils, low permeability, drainage of water out of the soil under the effect of loading is time dependent.
- Therefore, parameters governing consolidation process include: soil properties, stress (p), time (t), drainage conditions.

Mechanism of Consolidation Process

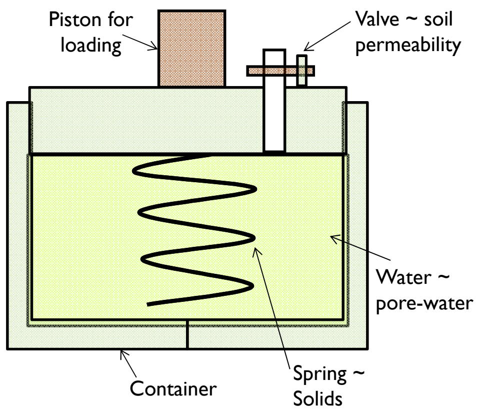
Analogy:

Confined saturated soil (solids and voids filled with water)

Versus

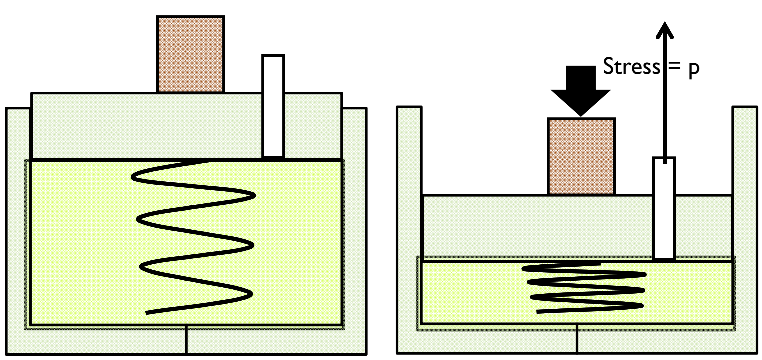
Container filled with water that has a spring and a valve

Remember: water is incompressible



Mechanism of Consolidation Process

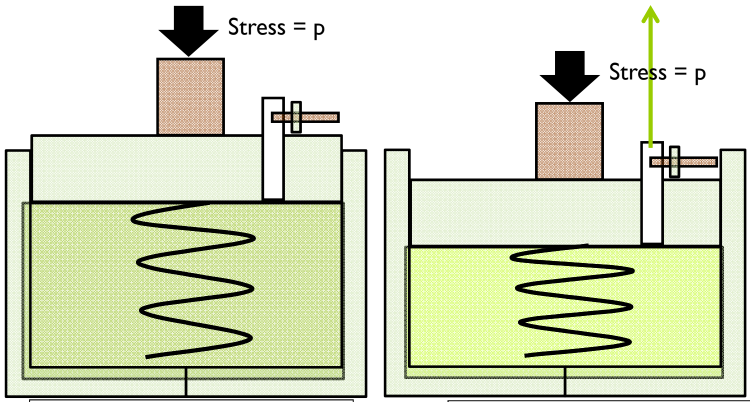
For Sands: high permeability ~ valve is open, water gets out immediately (t = 0), excess pore water pressure Δu = 0



At t = 0 (immediately), Δu = 0
Load carried by spring

Mechanism of Consolidation Process

For Clays: low permeability ~ valve is partially open, water gets out slowly (time-dependent), Δu decreases with t

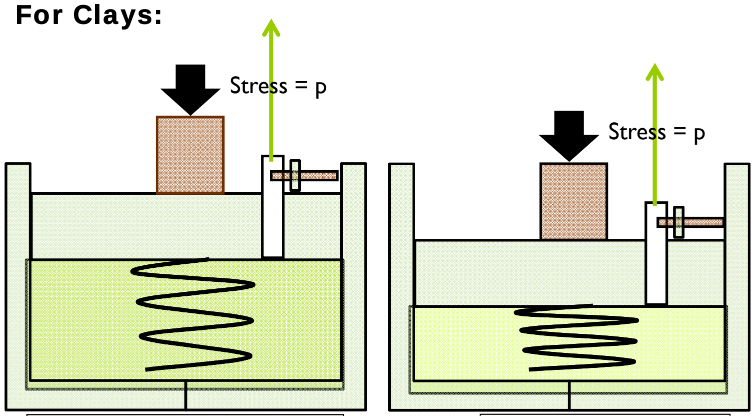


At t = 0 (immediately), $\Delta u = p$
Spring carry no load

At $t = t_I$, $0 < \Delta u < p$
Load carried by water and spring

Mechanism of Consolidation Process

For Clays:



At $t = t_I$, $0 < \Delta u < p$
Load carried by water and spring

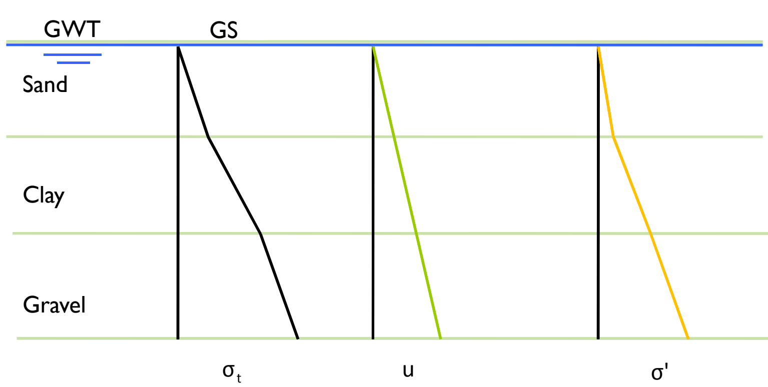
At t = *long time*, $\Delta u = 0$
Load carried by spring

Mechanism of Consolidation Process

- If we apply a certain load, and the valve is closed, then water (incompressible material) carries the entire added load.
- If the valve is opened, water flows out, and thus its volume is reduced.
- As volume is reduced, the spring starts to be compressed and carries part of the load. Meanwhile, excess pore water pressure decreases.
- The process continues until sufficient amount of water escapes from the valve, resulting in compression of the spring to carry the entire load.
- At this point, the excess pore water pressure decreases to zero, the spring carries the entire load, the system is in equilibrium.

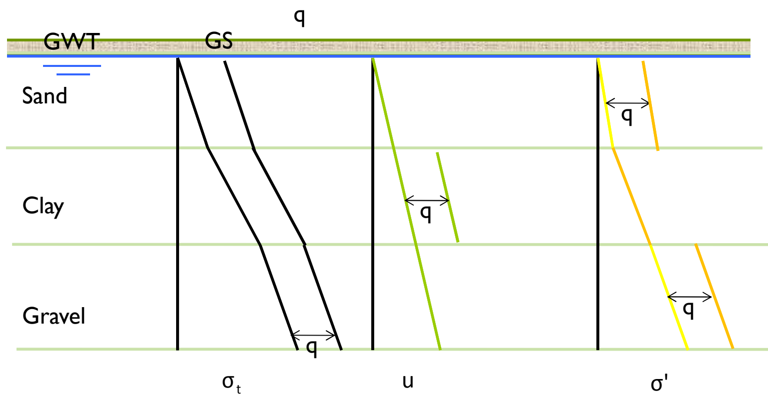
Application on Soil

- Stress distribution before application of load



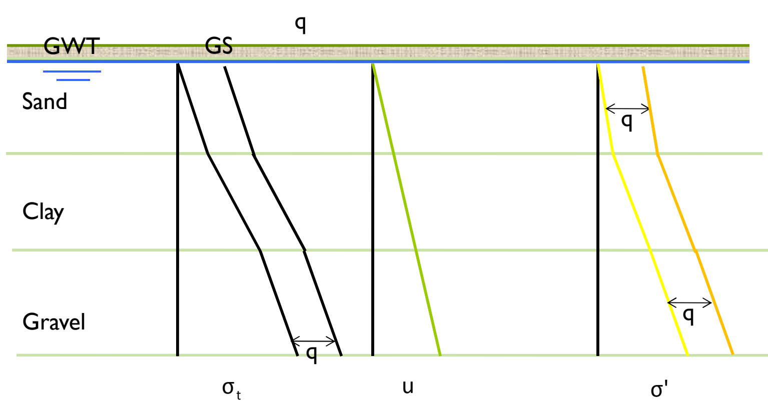
Application on Soil

- Short time (immediately) after application of q (t = 0)



Application on Soil

- Long time after application of q (t = ∞)

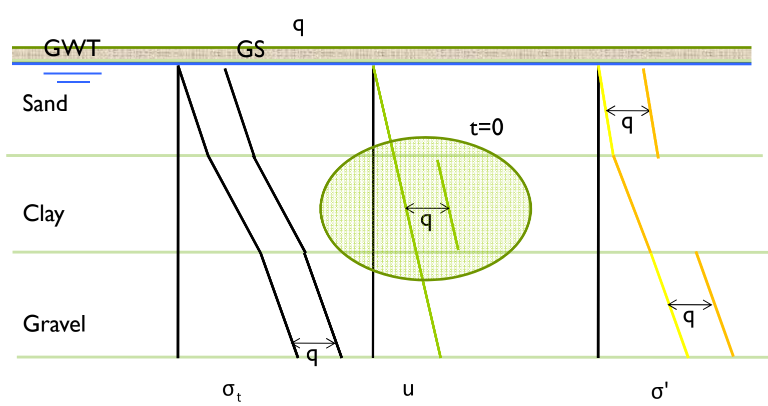


Application on Soil

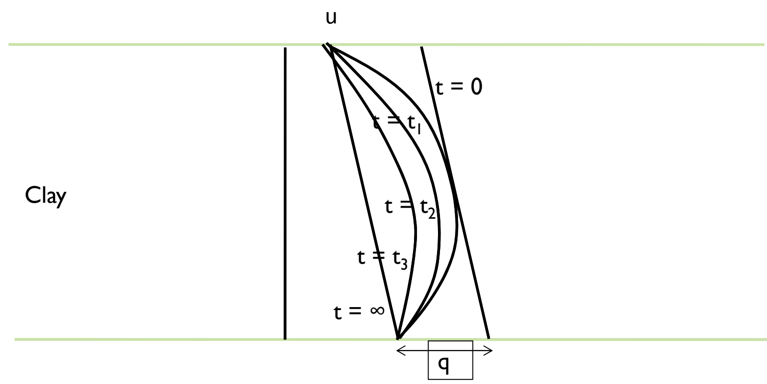
- Total stress: $\sigma_t = \Sigma\gamma h + q$
- Pore water pressure:
  - t = 0 ➔ $u = \gamma_w h_w + q$ (clay)
    ➔ $u = \gamma_w h_w$ (sand)
  - t = ∞ ➔ $u = \gamma_w h_w$ (clay)
    ➔ $u = \gamma_w h_w$ (sand)
- Effective stress:
  - t = 0 ➔ $\sigma' = \Sigma\gamma' h$ (clay)
    ➔ $\sigma' = \Sigma\gamma' h + q$ (sand)
  - t = ∞ ➔ $\sigma' = \Sigma\gamma' h + q$ (clay)
    ➔ $\sigma' = \Sigma\gamma' h + q$ (sand)

At time t ???

Rate of Consolidation



Rate of Consolidation



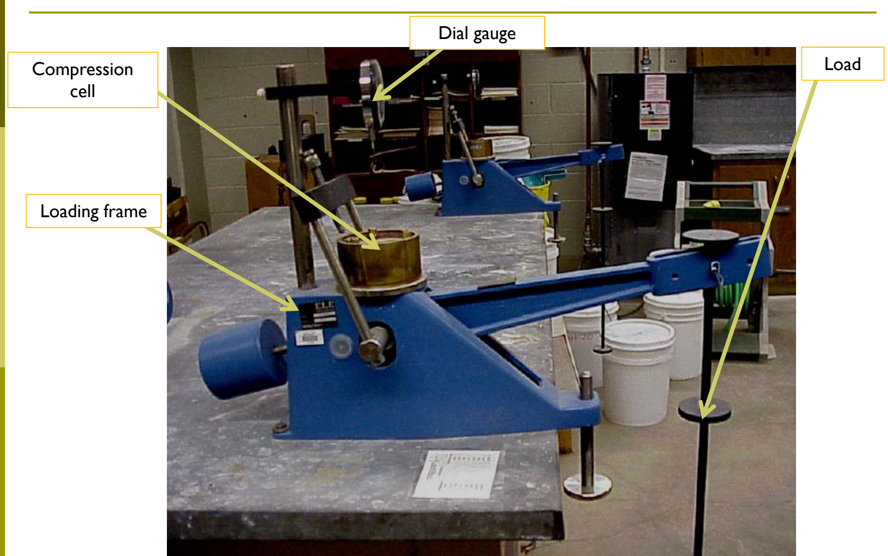
Assumptions of Theory of Consolidation

1. Clay is homogeneous, isotropic, and saturated
2. Water and clay particles are incompressible
3. Darcy's law is valid
4. The clay layer is laterally confined
5. One-dimensional compression, and one-dimensional flow
6. Consolidation parameters from test applies to clay layer from which sample for test was taken
7. Soil properties are constant with time

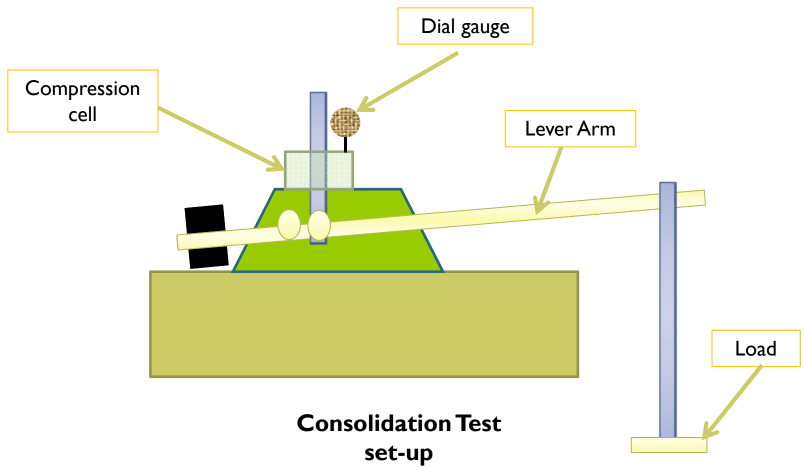
Consolidation Test

- Objectives:
  - Volume change-effective pressure relationship
  - Stress history of soil
  - Volume change – time – pore water dissipation relationship

Consolidation Test

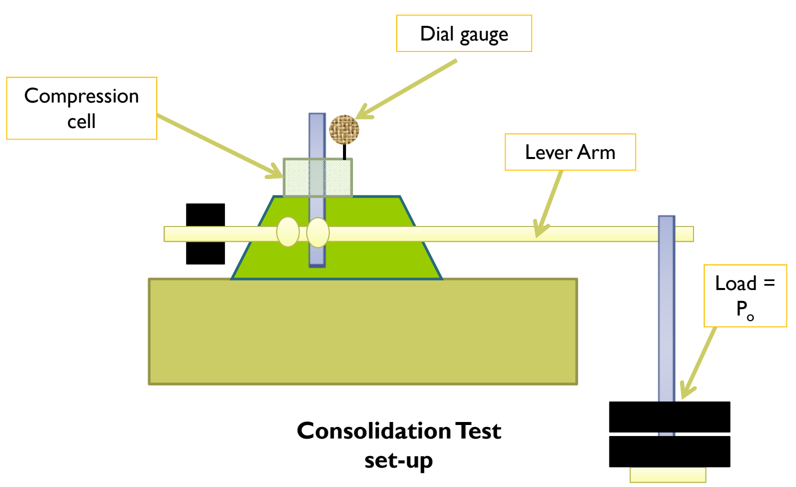


Consolidation Test



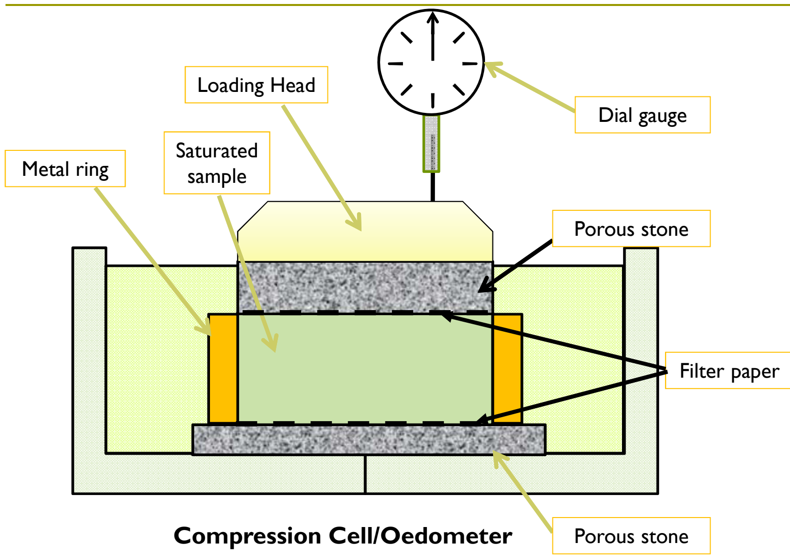
Consolidation Test set-up

Consolidation Test

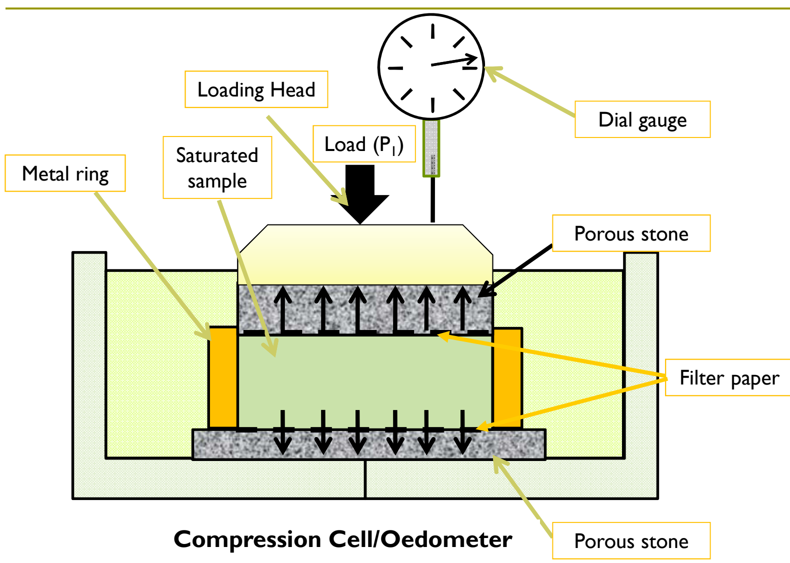


Consolidation Test set-up

Consolidation Test



Consolidation Test



Consolidation Test

- Equipment:
  1. Compression cell / Oedometer
  2. Loading frame to apply weights (P_o)
  3. Dial gage to measure soil compression

  Note:

  Load acting on the cell (P_1) is magnified by the lever arm ratio of the loading frame (*LAR*), where: $P_1 = LAR \times P_o$

Consolidation Test

- Procedure:
  1. Trim the sample into the metal ring and place filter paper on both sides of sample.
  2. Measure initial conditions of sample: h_o, e_o, w_o, G_s
  3. Place the ring into the compression cell between the two porous stones.
  4. A metal cap (loading head) is placed over the top porous stone, on which the load (P_1) is applied.
  5. Set-up the dial gage to zero reading. Fill the compression cell with water until the top porous stone is covered with water.

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Consolidation Test

- Procedure:
  6. Place the hanging load (P_o) such that the pressure on the clay sample is equal to the first loading step.
  7. Record readings of dial gage (compression of sample) with time until compression stops (usually within 24 hours).
  8. Repeat steps 6 and 7 for subsequent loading and unloading increments.

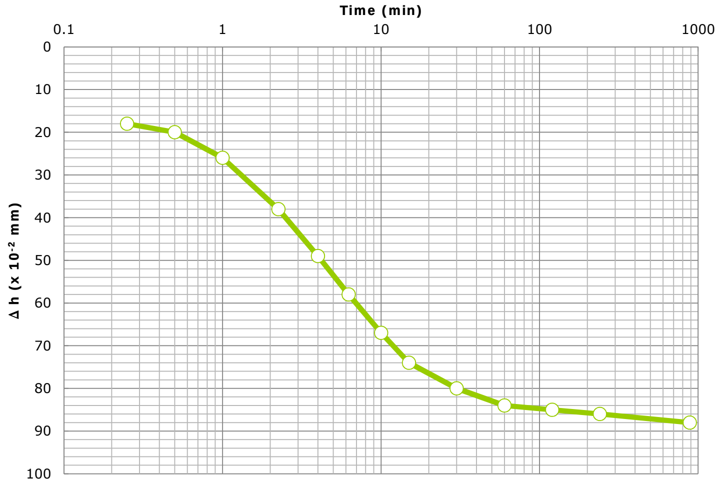
Consolidation Test

- Loading scheme, for example:
  - 0.25, 0.5, 1.0, 2.0, 4.0, 8.0, 16.0 (kg/cm^2) Loading
  - 16.0, 4.0, 1.0, 0.25 (kg/cm^2) Unloading

- Results: for *each* loading/unloading increment, we record: dial gage (Δh) that measures sample compression versus elapsed time (t).

Consolidation Test Data Reduction

- For each loading increment, plot Δh-log t:

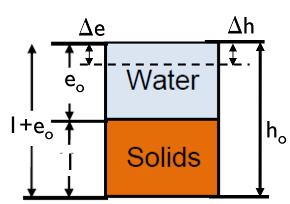


Consolidation Test Data Reduction

- At the end of each loading increment:
  1. Calculate effective pressure (p) = Load x LAR/sample area
  2. Measure Δh = sample compression during this loading increment
  3. Calculate void ratio (e):

$$\frac{\Delta e}{1+e_o} = \frac{\Delta h}{h_o}$$

$$e = e_o - \Delta e$$



- Plot e-p curve

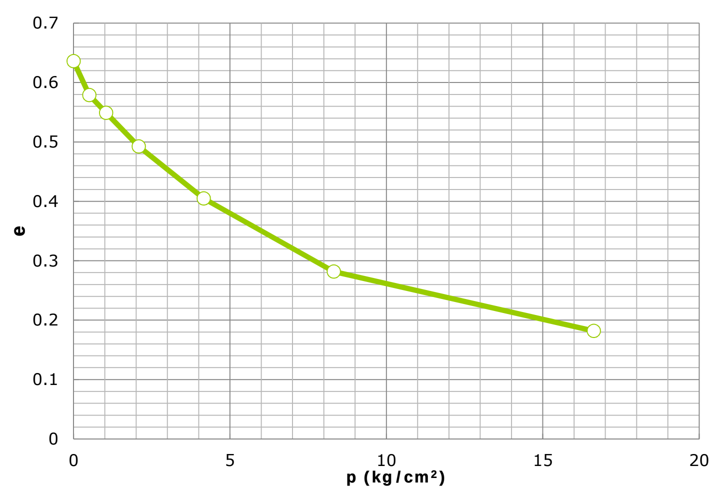
Consolidation Test Data Reduction

Example: LAR = 3, sample area = 41.85 cm², h_o = 2.54 cm, e_o = 0.636

Load (kg)	0	7	14.5	29	58	116	232
Dial Reading x 10^{-2} (mm) = Δh	0	89	135	223	359	550	705
Stress (p) = Load x lever arm ratio/area (kg/cm²)	0	7 x 3/41.85 = 0.50	1.04	2.08	4.16	8.32	16.63
$\Delta e = \Delta h/h_o\ (1+e_o)$	0	0.89/25.4 (1+0.636) = 0.0573	0.087	0.1436	0.2312	0.3543	0.4541
$e = e_o - \Delta e$	0.636	0.636-0.0573 = 0.5787	0.5490	0.4924	0.4048	0.2817	0.1819

Consolidation Test Data Reduction

Plot e-p curve:



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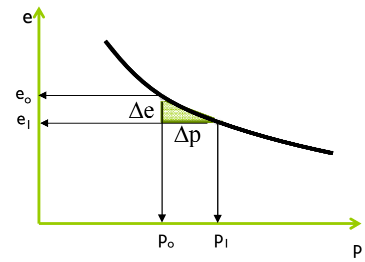
Consolidation Test Data Reduction

- From *e*-*p* curve, for a certain stress range, calculate a_v and m_v, where:

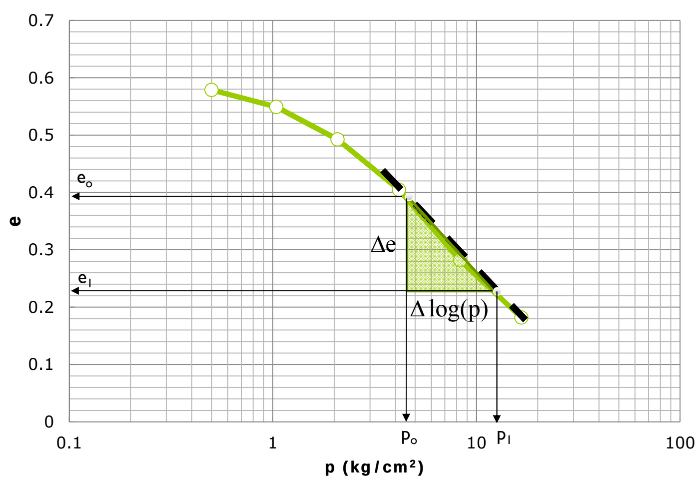
$$a_v = \frac{\Delta e}{\Delta p}$$

$$m_v = \frac{a_v}{(1+e_o)}$$



Consolidation Test Data Reduction

Plot e-log(p) curve:



Consolidation Test Data Reduction

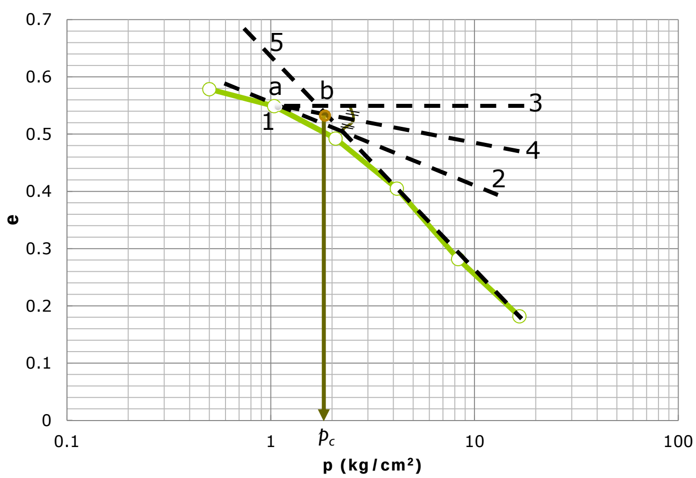
- From *e-log(p)* curve, the slope of the linear portion is the Compression Index (C_c):

$$C_c = \frac{\Delta e}{\Delta \log(p)}$$

- Determine preconsolidation pressure (p_c): is the largest effective pressure that has been applied on the soil in its geological history.

Consolidation Test Data Reduction

Determination of preconsolidation pressure:



Determination of Preconsolidation Pressure

1. Determine point of maximum curvature (a).
2. At (a), draw tangent to *e-log(p)* curve.
3. At (a), draw horizontal line.
4. Draw a bisector to the angle enclosed between the tangent and horizontal lines.
5. Extend the linear portion of the curve until it intersects the bisector at point (b).
6. The preconsolidation pressure (p_c) is the x-coordinate of the point (b).

Types of clays w.r.t. preconsolidation

1. **Normally consolidated clay:**
   Clay that has never been loaded in the past by more than the existing **effective** overburden pressure (p_o) $\quad p_c \simeq p_o$
2. **Overconsolidated (preconsolidated) clay:**
   Clay that has been loaded in the past by more than the existing **effective** overburden pressure (p_o) $\quad p_c > p_o$
   Causes: pre-existing structures, erosion of overburden
3. **Underconsolidated clay:**
   Clay that has been loaded partially by the existing **effective** overburden pressure (p_o) $\quad p_c < p_o$

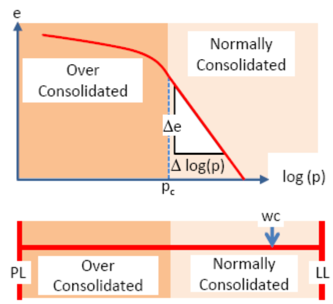
Define: Overconsolidation Ratio (OCR):

$$OCR = \frac{p_c}{p_o}$$

Determination of Compression Index

- For normally consolidated clays, C_c can be calculated as follows:
  - From *e-log(p)* curve:

$$C_c = \frac{\Delta e}{\Delta \log(p)}$$

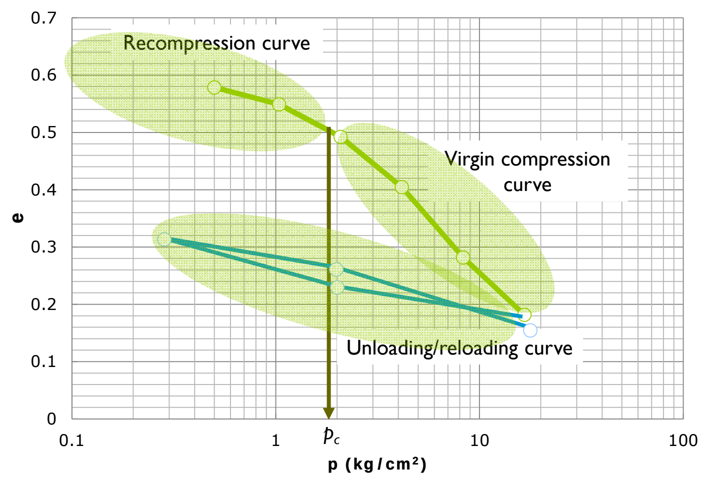


  - From liquid limit (empirical):

$$C_c \approx 0.009(w_L - 10)$$

where w_L is in (%)

Definitions



Definitions

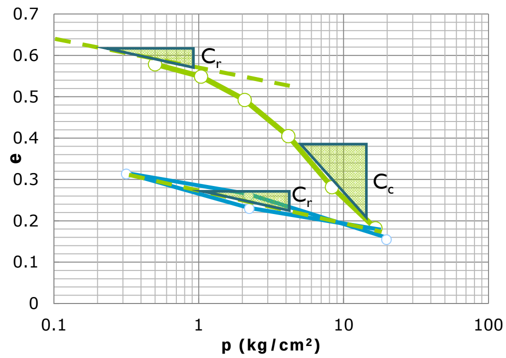
- From *e-log(p)* curve, the recompression index (C_r) is the slope of the equivalent linear portion of the unloading/reloading curve.

$$C_r = \frac{\Delta e}{\Delta \log(p)}$$

Empirical relationship for C_r:

$$C_r \approx (1/10 - 1/5) C_c$$

$$C_r \approx (1/8) C_c$$



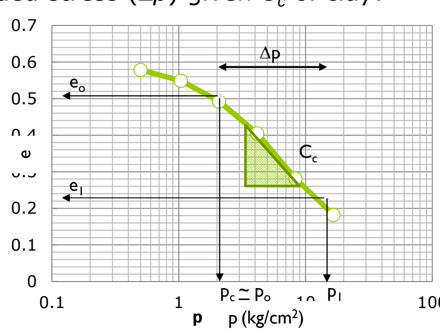
Settlement

- Calculate settlement (*S*) of normally consolidated clay layer of thickness (*H*) due to added stress (*Δp*) given C_c of clay:

$$C_c = \frac{\Delta e}{\Delta \log(p)}$$

$$C_c = \frac{e_0 - e_1}{\log(p_1) - \log(p_0)}$$

$$C_c = \frac{\Delta e}{\log(\frac{p_1}{p_0})}$$



$$\Delta e = C_c \log(\frac{p_1}{p_0}) \text{.........................} 1$$

$$\frac{\Delta e}{1+e_o} = \frac{\Delta h}{h_o} = \frac{S}{H} \text{..................................} 2$$

Settlement

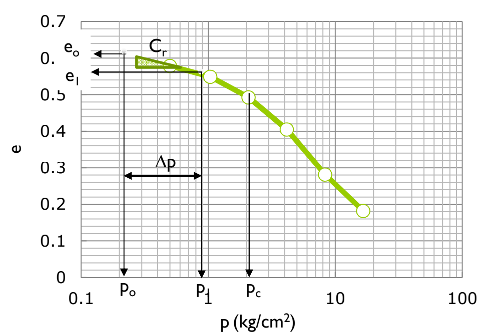
- From 1 and 2:

$$C_c \log(\frac{p_1}{p_o}) = \frac{S}{H}(1+e_o)$$

$$S = \frac{C_c}{1+e_o} H \log(\frac{p_1}{p_o})$$

Settlement

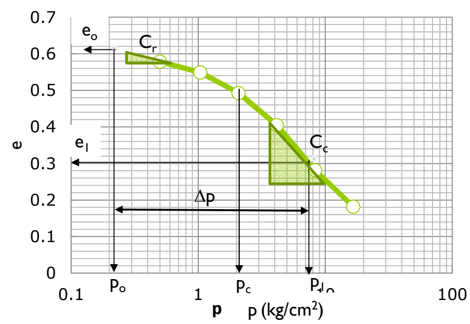
- For overconsolidated clay, (p_o and p_1) < p_c:



$$S = \frac{C_r}{1+e_o} H \log(\frac{p_1}{p_o})$$

Settlement

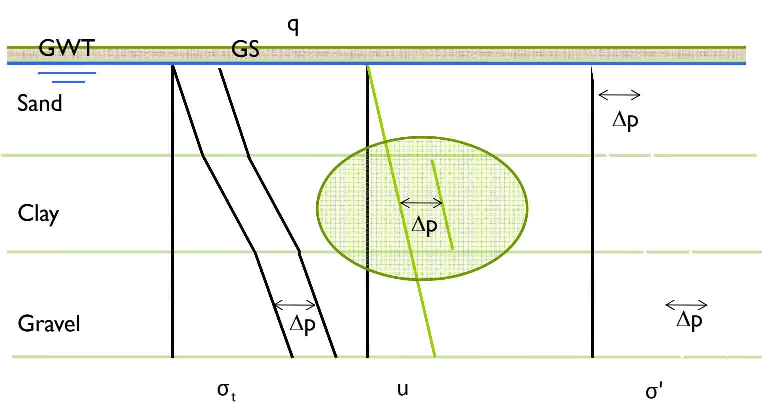
- For overconsolidated clay, $p_o < p_c$ and and $p_1 > p_c$



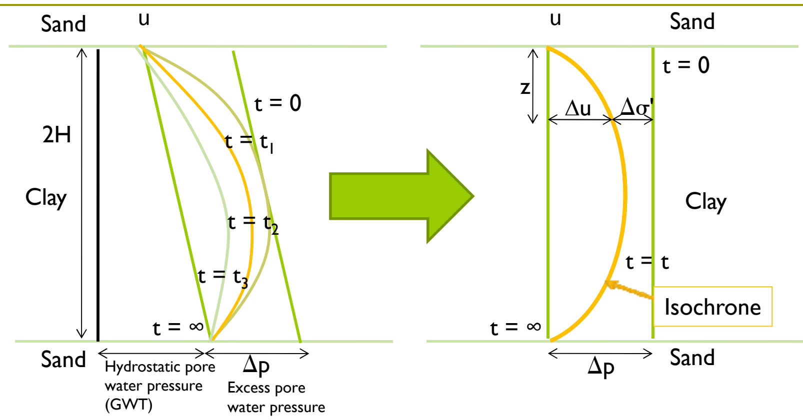
$$S = \frac{C_r}{1+e_o} H \log(\frac{p_c}{p_o}) + \frac{C_c}{1+e_o} H \log(\frac{p_1}{p_c})$$

Rate of Consolidation

Short time after application of load "q"



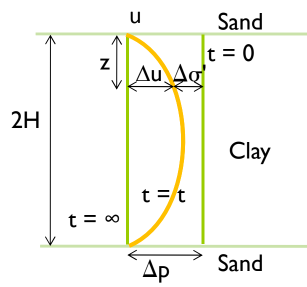
Rate of Consolidation



At time "t" and depth "z":

$\Delta p = \Delta u + \Delta\sigma'$ = increase in total stress due to added load
Δu = excess pore water pressure due to added load
$\Delta\sigma'$ = increase in effective stress due to added load

Rate of Consolidation



Degree of Consolidation (U_{tz}) at time "t" and depth "z":

$$U_{tz} = \frac{\Delta\sigma'}{\Delta p} = \frac{\Delta p - \Delta u}{\Delta p} = 1 - \frac{\Delta u}{\Delta p}$$

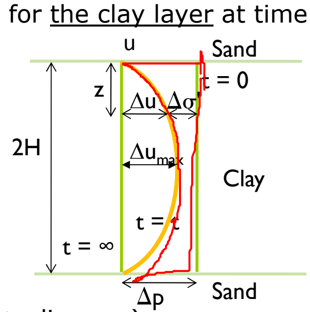
Rate of Consolidation

Average degree of consolidation (U_t) for the clay layer at time "t":

$$U_t = 2\int_0^H U_{tz} dz$$

where:

$$U_{tz} = \frac{\Delta\sigma'}{\Delta p} = \frac{\Delta p - \Delta u}{\Delta p}$$



U_t = (area of $\Delta\sigma'$ diagram)/(area of Δp diagram)

U_t = (area of Δp diagram – area of Δu diagram)/(area of Δp diagram)

where:

$$\text{area of } \Delta p \text{ diagram} = \Delta p \ 2H$$

$$\text{area of } \Delta u \text{ diagram} = \frac{2}{3}\Delta u_{max} 2H$$

Rate of Consolidation

- Settlement of clay layer of thickness 2H at time t=∞:

  $S_{ultimate} = m_v \Delta p(2H)$

- Settlement of clay layer of thickness 2H at time t=t:

  $S_t = m_v \Delta\sigma'(2H)$

  $S_t = m_v(\Delta p\ U_t)(2H) = U_t\ S_{ultimate}$

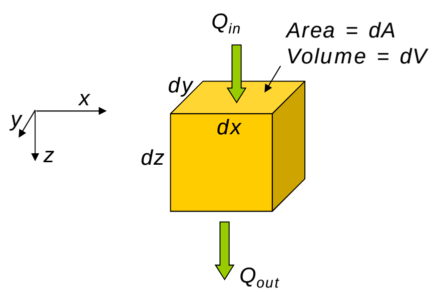
- Average degree of consolidation (U_t) at time "t":

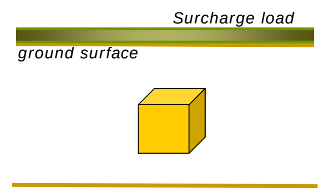
  $U_t = S_t/S_{ultimate}$

  The degree of consolidation at any time (t) is the percentage of settlement that took place at this time w.r.t. to ultimate consolidation settlement.

Differential equation of consolidation

- Equation governing rate of consolidation
- Consider a differential element of soil





Differential equation of consolidation

$$\frac{\partial(dV)}{\partial t} = Q_{out} - Q_{in} = \left(\frac{\partial Q}{\partial z}\right)dz$$

compression and flow 1-D

$$Q = kidA$$

Darcy's law is valid

$$= k\frac{\partial h}{\partial z}dA$$

$$= k\frac{\partial}{\partial z}\left(\frac{u}{\gamma_w}\right)dA$$

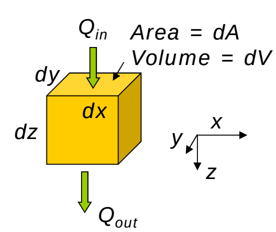
u = excess p.w.p

$$= \frac{k}{\gamma_w}\frac{\partial u}{\partial z}dA$$

$$\frac{\partial(dV)}{\partial t} = \frac{\partial}{\partial z}\left(\frac{k}{\gamma_w}\frac{\partial u}{\partial z}dA\right)dz$$

$$\boxed{\frac{\partial(dV)}{\partial t} = \frac{k}{\gamma_w}\frac{\partial^2 u}{\partial z^2}dV}$$

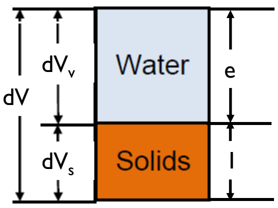
Equation (1) **soil is homogeneous**



Differential equation of consolidation

$$\frac{dV_v}{dV} = \frac{e}{1+e} \Rightarrow dV_v = \frac{e}{1+e} dV$$

$$\frac{dV_s}{dV} = \frac{1}{1+e} \Rightarrow dV_s = \frac{1}{1+e} dV$$



$$\frac{\partial(dV)}{\partial t} = \frac{\partial(dV_v)}{\partial t} = \frac{\partial(\frac{e}{1+e}dV)}{\partial t} = \frac{\partial(edV_s)}{\partial t} = \frac{\partial e}{\partial t} dV_s = \frac{\partial e}{\partial t}\frac{dV}{1+e}$$

$$\boxed{\frac{\partial(dV)}{\partial t} = \frac{\partial e}{\partial t}\frac{dV}{1+e}}$$ Equation (2)

From equations 1 and 2,

$$\frac{k}{\gamma_w}\frac{\partial^2 u}{\partial z^2}\cancel{dV} = \frac{\partial e}{\partial t}\frac{\cancel{dV}}{1+e}$$

Differential equation of consolidation

$$\frac{k(1+e)}{\gamma_w}\frac{\partial^2 u}{\partial z^2} = \frac{\partial e}{\partial t}$$

$$\frac{\partial e}{\partial t} = \frac{\partial e}{\partial \sigma'}\frac{\partial \sigma'}{\partial t}$$ (chain rule)

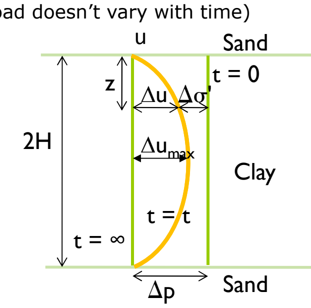
$$\frac{\partial e}{\partial t} = -a_v \frac{\partial \sigma'}{\partial t} \qquad : a_v = \frac{\partial e}{\partial \sigma'}$$

$$\frac{\partial e}{\partial t} = -a_v\left(\frac{\partial \sigma}{\partial t} - \frac{\partial u}{\partial t}\right) = a_v \frac{\partial u}{\partial t} \qquad : \frac{\partial \sigma}{\partial t} = 0$$ (load doesn't vary with time)

$$\frac{k(1+e)}{\gamma_w}\frac{\partial^2 u}{\partial z^2} = a_v \frac{\partial u}{\partial t}$$

$$\frac{\partial u}{\partial t} = \frac{k(1+e)}{a_v \gamma_w}\frac{\partial^2 u}{\partial z^2}$$

$$\boxed{\frac{\partial u}{\partial t} = c_v \frac{\partial^2 u}{\partial z^2}} \qquad : c_v = \frac{k(1+e)}{a_v \gamma_w} = \frac{k}{m_v \gamma_w}$$



Differential equation of consolidation

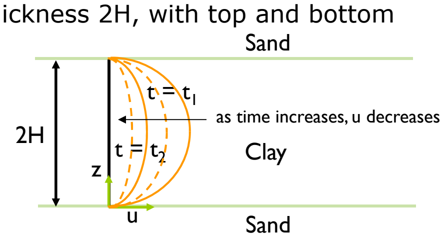
- *Coefficient of Consolidation: c_v*

$$c_v = \frac{k}{m_v \gamma_w}$$ cm²/sec

where:
k = soil permeability
m_v = coefficient of volume compressibility

Solution of differential equation of consolidation

- Boundary conditions:
  Example: a clay layer, of thickness 2H, with top and bottom boundaries freely draining
  - u(z=0,t>0) = 0
  - u(z=2H,t>0) = 0



- Initial condition:
  For wide fill, applied instantly to saturated soil, the initial excess pore-water pressure u(z,t=0) is independent of depth and equals the applied stress (Δp)
  - u(z,t=0) = u_i = Δp

Solution of differential equation of consolidation

- Solution:

$$u(z,t) = \sum_{m=0}^{\infty} \frac{2u_i}{M} (\sin MZ) \exp(-M^2 T_v) \quad \text{................. Equation 3}$$

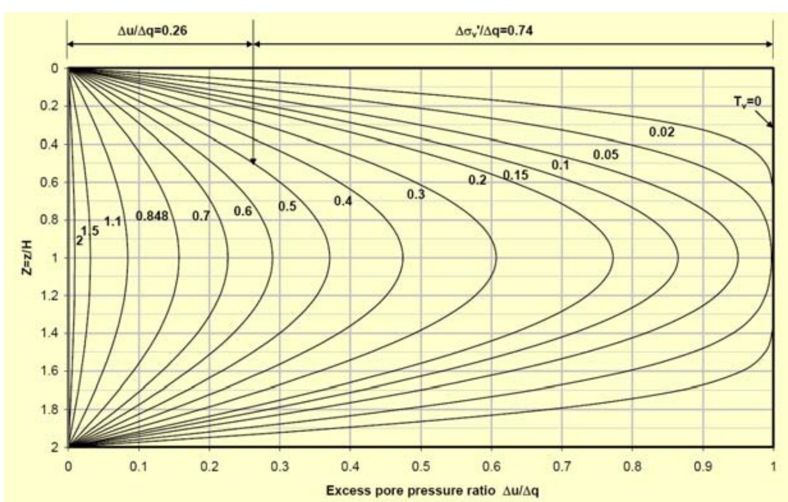
where:

$$M = \frac{\pi}{2}(2m+1)$$

$$T_v = \frac{c_v t}{H^2}$$ T_v = Time factor

Solution of differential equation of consolidation

Equation (3) was solved ➡ chart: $\Delta u/\Delta p = f(T_v, z)$



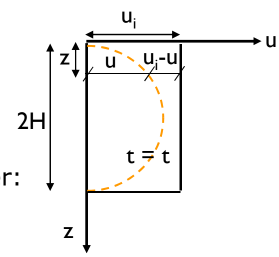
Solution of differential equation of consolidation

$$u(z,t) = \sum_{m=0}^{\infty} \frac{2u_i}{M}(\sin MZ)\exp(-M^2 T_v)$$

- Solution in terms of degree of consolidation (U):

$$U(z,t) = \frac{u_i - u}{u_i} = 1 - \frac{u}{u_i}$$

$$U(z,t) = 1 - \sum_{m=0}^{\infty} \frac{2}{M}(\sin MZ)\exp(-M^2 T_v)$$



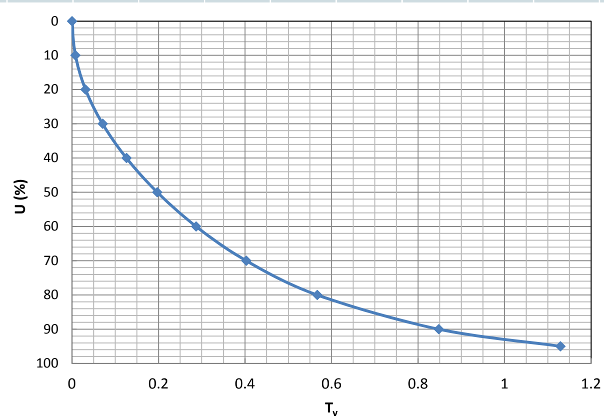
- Average degree of consolidation for entire layer:
  U = area / area
  By integration:

$$U_t = 1 - \frac{\int_0^{2H} u\,dz}{u_i} \qquad U_t = 1 - \sum_{m=0}^{\infty} \frac{2}{M^2} exp(-M^2 T_v)$$

Solution of differential equation of consolidation

Equation (4) was solved ➡ Table and chart: T_v = f(*U*)

U (%)	0	10	20	30	40	50	60	70	80	90	95	99	100
T_v	0.0	0.008	0.031	0.071	0.126	0.197	0.287	0.403	0.567	0.848	1.129	1.781	∞



Solution of differential equation of consolidation

- Time Factor: T_v

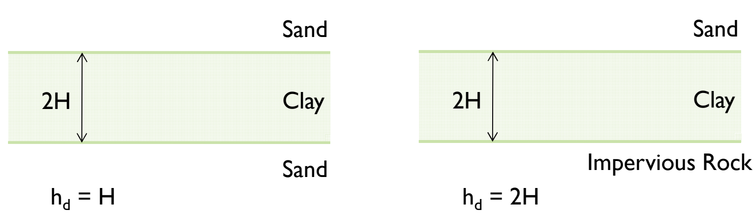
$$T_v = \frac{c_v t}{h_d^2}$$

where:
T_v = time factor corresponding to degree of consolidation occurring at time t
c_v = coefficient of consolidation (determined from consolidation test)
h_d = longest drainage path within clay layer

The above equation holds for consolidation of sample in the lab, and for clay layer consolidating in the field

Solution of differential equation of consolidation

- *Drainage Path: h_d*



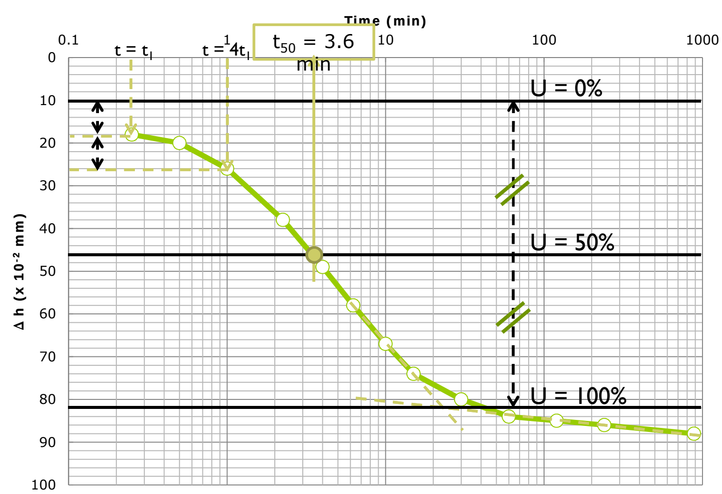
Soil Mechanics – Third Year Civil Eng.

Determination of c_v - logarithm-of-time method

- c_v is determined from the consolidation test data
- There is a c_v value that corresponds to each loading increment, i.e. c_v is stress dependent.
- Two methods to calculate c_v:
  - Logarithm-of-time method
  - Square–root-of-time method
- For the following test data corresponding to a certain loading increment, find c_v.

Time (min)	0	0.25	0.5	1	2.25	4	6.25	10	15	30	60	120	240	885
Dial Reading x 10^{-2} (mm)	135	153	155	161	173	184	193	202	209	215	219	220	221	223
= Δh x 10^{-2} (mm)	135-135 = 0	153-135 =18	20	26	38	49	58	67	74	80	84	85	86	88

Determination of c_v - logarithm-of-time method



Determination of c_v - logarithm-of-time method

1. Plot the (Δh) or (e) on the vertical axis (arithmetic scale), and the time (t) on horizontal axis (logarithmic scale) → S-shape
2. Determine (Δh) or (e) corresponding to 100% consolidation:
   - Extrapolate the linear portion at the middle of the consolidation curve
   - Extrapolate the linear portion at the end of the consolidation curve
   - The two lines intersect at a point corresponding to (Δh) or (e) at 100% consolidation
3. Determine (Δh) or (e) corresponding to 0% consolidation:
   - Consider (Δh_1) corresponding to a small time (t_1)
   - Consider (Δh_2) corresponding to ($4t_1$)
   - Determine the difference between (Δh_1) and (Δh_2) → y
   - Determine (Δh) corresponding to 0% consolidation at a distance equivalent to y above (Δh_1)

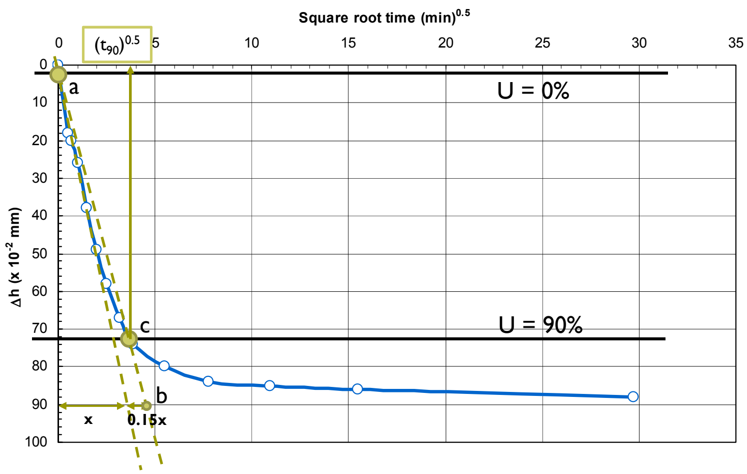
Determination of c_v - logarithm-of-time method

4. Determine (Δh) or (e) corresponding to 50% consolidation at mid distance between 0% consolidation and 100% consolidation.
5. Determine t_{50}
6. Calculate c_v:

$$T_v = \frac{c_v t}{h_d^2}$$

where: T_{50} = 0.197, h_d = ½ sample height (sample drains from both sides), and t = t_{50} from consolidation curve

Determination of c_v – square root-of-time method



Determination of c_v – square root-of-time method

1. Plot the (Δh) or (e) on the vertical axis (arithmetic scale), and the square root of time on horizontal axis (arithmetic scale)
2. Determine (Δh) or (e) corresponding to 90% consolidation:
   - Draw a straight line through the initial linear portion of the curve. This line will intersect the vertical axis at a point (a) that corresponds to U = 0%
   - Measure distance (x) between the vertical axis and the drawn line
   - Measure additional distance (0.15x) ➡ point (b)
   - Draw a line connecting points (a) and (b) and extend it until it intersects the original curve at point (c) ➡ corresponds to U = 90%
3. Determine $(t_{90})^{0.5}$

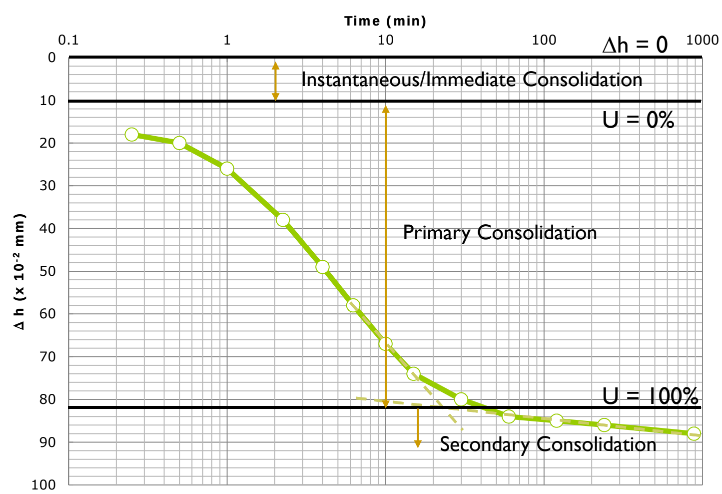
Determination of c_v – square root-of-time method

4. Calculate c_v:

$$T_{90} = \frac{c_v t_{90}}{h_d^2}$$

where: T_{90} = 0.848, h_d = ½ sample height (sample drains from both sides), and t = t_{90} from consolidation curve

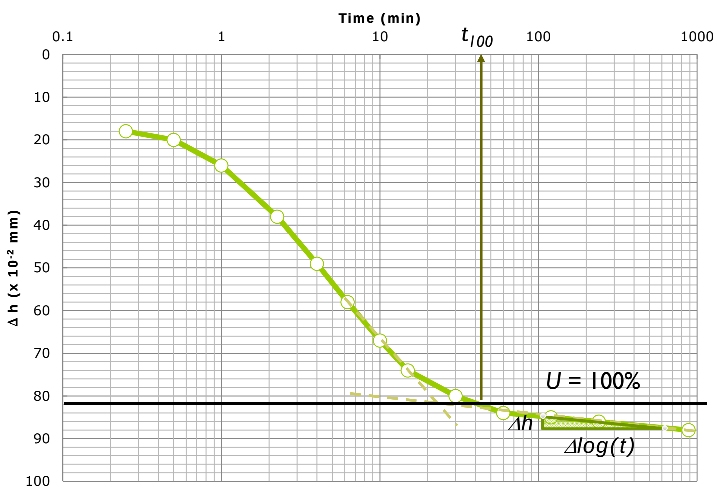
Types of Compression



Types of Compression

1. Instantaneous/Immediate Compression: due to compression of entrapped air and machine parts
2. Primary Compression: due to consolidation by escapage of water (relief of excess pore-water pressure) under the effect of pressure (loading)
3. Secondary compression: compression at constant effective stress (no excess pore-water pressure)

Secondary Compression Index



Secondary Consolidation Settlement (S_s)

- Secondary compression index (C_α):

$$C_\alpha = \frac{\Delta e}{\Delta \log(t)}$$

where:

$$\Delta e = \frac{\Delta h}{H}(1 + e_o)$$

- Secondary settlement (S_s):

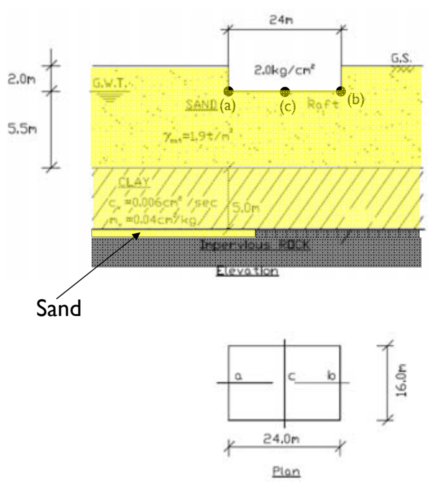
$$S_s = \frac{C_\alpha}{1 + e_o} H \log(\frac{t_1}{t_o})$$

Example

- A rectangular building 24m × 16m is founded on a raft and has its foundation level at 2.0 m below ground surface. If the building exerts a net stress of 2.0 kg/cm^2 on soil at foundation level, determine:

i. The ultimate settlement of points a, b and c.

ii. The time required for these points to settle 5.0 cm, and the time required to undergo 90% consolidation.



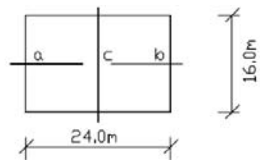
Example

i. q_{net} = 20 t/m^2

z = 8.0 m

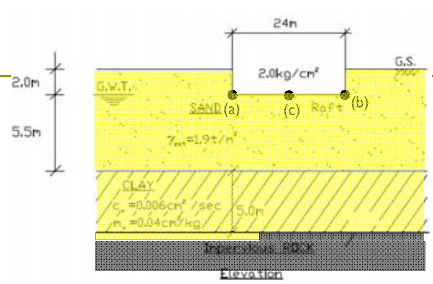
H = 5.0 m

Using Influence Chart:



Points	a & b	c
L (m)	24	12
B (m)	8	8
m	3	1.5
n	1	1
I_z	0.193	0.203
$\Delta\sigma$ (t/m^2)	0.193x20x2 = 7.72	0.203x20x4 = 16.24
$S = m_v \Delta\sigma H$	0.004x7.72x5 = 0.154 m	0.004x16.24x5 = 0.325 m

Example



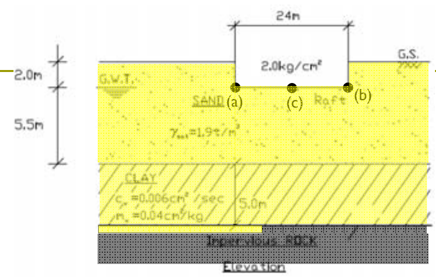
ii.

c_v = 0.006 cm^2/sec

Points	a	c	b
$U = S_t/S_f$	=5/15.4 = 0.325	5/32.5 = 0.154	=5/15.4 = 0.325
From Chart (T_v)	0.083	0.019	0.083
$T_v = \frac{c_v t}{h_D^2}$	$0.083 = \frac{0.006\,t}{(250)^2}$	$0.019 = \frac{0.006\,t}{(500)^2}$	$0.083 = \frac{0.006\,t}{(500)^2}$
t (days)	10.0	9.2	40.0

Soil Mechanics – Third Year Civil Eng.

Example



Points	a	c	b
$U = S_t/S_f$	0.90	0.90	0.90
From Chart (T_v)	0.848	0.848	0.848
$T_v = \frac{c_v t}{h_D^2}$	$0.848 = \frac{0.006\,t}{(250)^2}$	$0.848 = \frac{0.006\,t}{(500)^2}$	$0.848 = \frac{0.006\,t}{(500)^2}$
t (days)	102.2	409.0	409.0
S_t (cm)	0.90 x 15.4 = 13.9 cm	0.90 x 32.5 = 29.3 cm	0.90 x 15.4 = 13.9 cm

Potential Sources of Error

- Sample disturbance: due to improper handling and/or trimming. Disturbance affects e-logp curve, and thus preconsolidation pressure. Larger samples help reduce effect of trimming on sample disturbance.

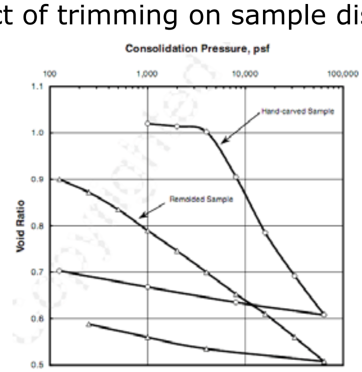


Figure 3.23 Consolidation curves for undisturbed and remolded clay.

Kontopoulos, Nikolaos S., 2012

Potential Sources of Error

- Specimen not completely filling the ring: erroneous volume change due to compression of voids.
- Clogged porous stones: porous stones should be cleaned after every test to remove embedded soil particles.
- Friction between specimen and consolidation ring. Part of load applied to specimen can be lost by side friction. This effect can be reduced by lining the consolidation ring with Teflon.