SOLUTION SET

Chapter 5

ENERGY LEVELS AND RADIATIVE PROPERTIES OF MOLECULES, LIQUIDS, AND SOLIDS

“LASER FUNDAMENTALS”

Second Edition

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CH5

1. Derive all of the allowed frequencies of radiation that might occur among the first four rotational energy levels of the hydrogen molecule H_2. Assume that H_2 has a permanent electric dipole moment (which it does not) so that it can radiate. Draw an energy-level diagram of those first four levels, and indicate the possible transitions and their frequencies.

According to the selection rules, Transitions only occur at $\Delta J = 0, \pm 1$. But This problem only involves $\Delta J = -1$ since Transitions only occur to lower energies.

From (5.5) $E^J_{rot} = \frac{J(J+1)\hbar^2}{2I} = J(J+1)Bhc$

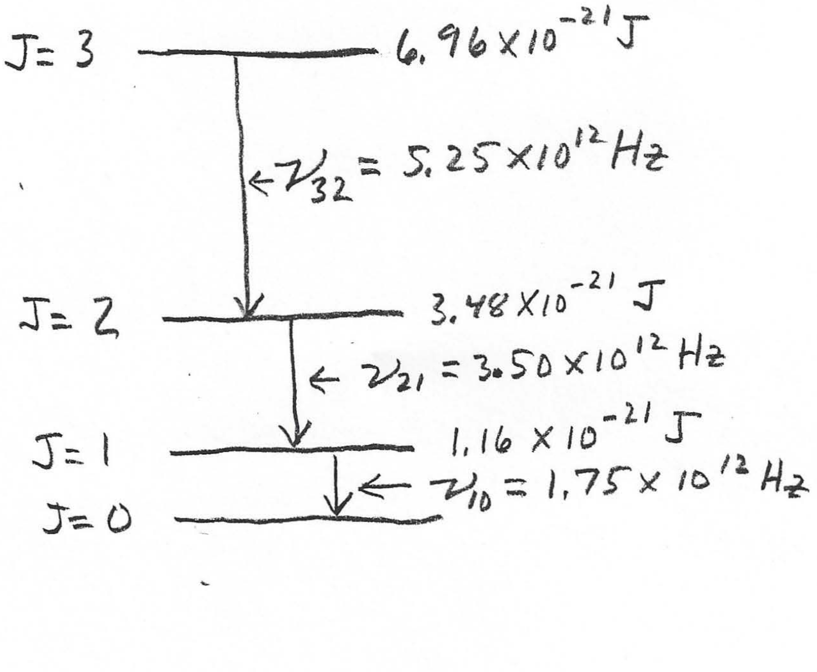
$B = \frac{h}{8\pi^2 c I}$ $I = mb^2$ 2b = separation of atom

M is combined mass

From examples on pages 140 and 141

$Bhc = (5.80 \times 10^{-22}$ Joules)

$\therefore E^J_{rot} = (5.80 \times 10^{-22})\, J(J+1)$ Joules



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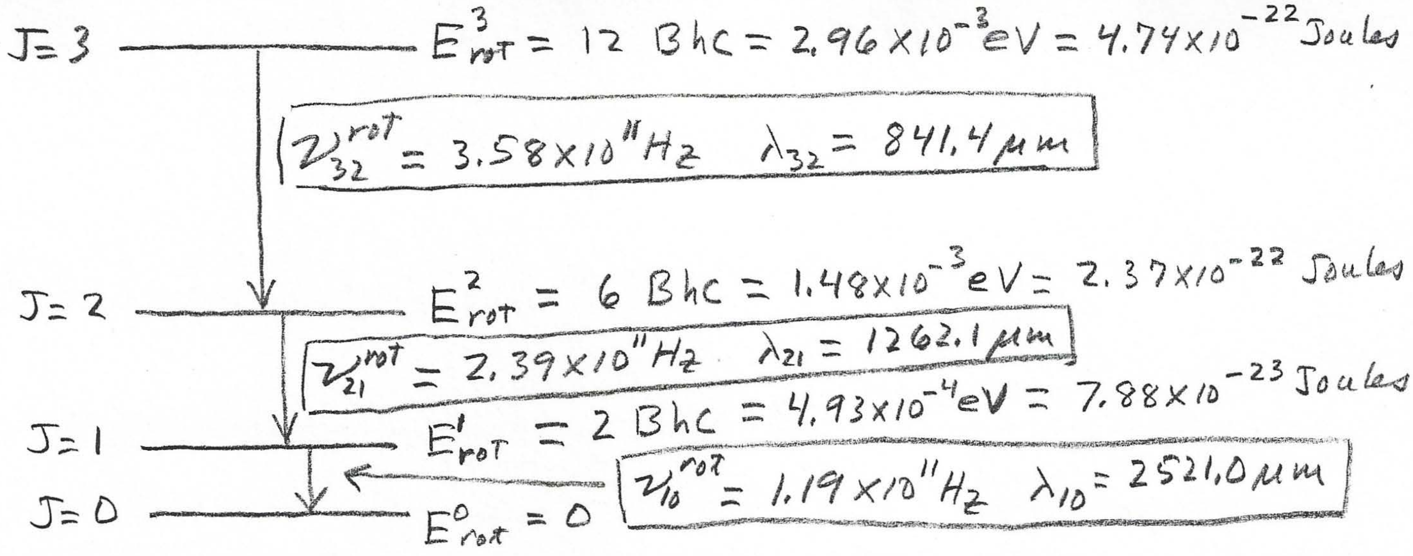


$E^{3}_{rot} = 12\ Bhc = 2.96\times10^{-3}\ eV = 4.74\times10^{-22}$ Joules

$\boxed{\nu^{rot}_{32} = 3.58\times10^{11}\ Hz \quad \lambda_{32} = 841.4\ \mu m}$

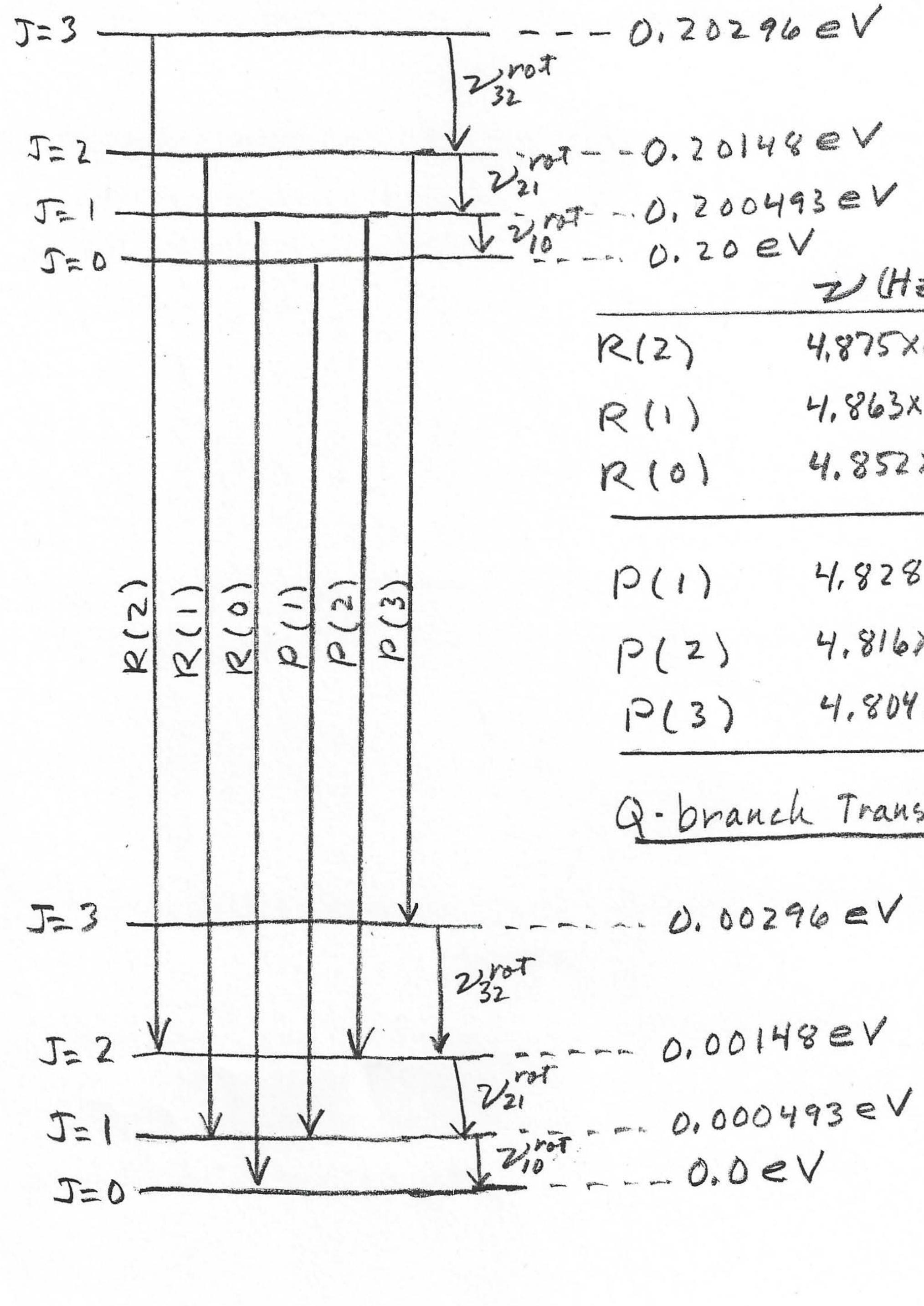
$E^{2}_{rot} = 6\ Bhc = 1.48\times10^{-3}\ eV = 2.37\times10^{-22}$ Joules

$\boxed{\nu^{rot}_{21} = 2.39\times10^{11}\ Hz \quad \lambda_{21} = 1262.1\ \mu m}$

$E^{1}_{rot} = 2\ Bhc = 4.93\times10^{-4}\ eV = 7.88\times10^{-23}$ Joules

$\boxed{\nu^{rot}_{10} = 1.19\times10^{11}\ Hz \quad \lambda_{10} = 2521.0\ \mu m}$

$E^{0}_{rot} = 0$



	ν (Hz)	λ (μm)
R(2)	4.875×10^{13}	6.154
R(1)	4.863×10^{13}	6.169
R(0)	4.852×10^{13}	6.183
P(1)	4.828×10^{13}	6.214
P(2)	4.816×10^{13}	6.230
P(3)	4.804×10^{13}	6.245

Q-branch Transitions not allowed

2. Consider the first two vibrational energy levels of the CO molecule, which we will assume have an energy separation of 0.2 eV. Consider also the first four rotational levels of each of those vibrational levels. Derive all of the frequencies of radiation that can occur among all of these levels, including pure rotational transitions and rotational–vibrational transitions. Include an energy-level diagram for those levels, and indicate the allowed transitions and wavelengths. Assume that the separation between C and O is 0.111 nm.

First consider pure rotational Transitions; $E_{rot}^{J} = J(J+1)Bhc$

$$\Delta E_{rot}(J \to J-1) = [J(J+1) - (J-1)J]\,Bhc = 2J\,Bhc$$

$$B = \frac{h}{8\pi^2 c I} \qquad I = \mu(2b)^2 \qquad \mu = \text{reduced mass } \frac{M_1 M_2}{M_1+M_2}$$

$$2b = 0.111\,nm$$

For C & O

$$I = \mu(2b)^2 = \left(\frac{12\cdot16}{12+16}\right)1.67\times10^{-27}\,kg\,(0.111\times10^{-9})^2\,m^2$$

$$= 1.41\times10^{-46}\,kg\,m^2$$

$$Bhc = \frac{(6.62\times10^{-34}\,J\text{-}s)^2}{8\pi^2\,1.41\times10^{-46}\,kg\,m^2} = 3.94\times10^{-23}\,J = 2.46\times10^{-4}\,eV$$

Thus $\Delta E_{rot}(J \to J-1) = 2J\,Bhc = 7.88\times10^{-23}\,J$ (Joules)

Both of The first Two vibrational levels have identical sets of rotational levels since rotational levels only depend upon J where J=0 is The lowest energy level of each vibrational level. Hence The first four rotational levels for both V=1 and V=2 will be spaced according To The above formula according To J

(continued next page)

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3. Label the molecular electronic energy levels listed in the following table.

S	L	
1	2	— $^3\Delta$
1	1	— $^3\Pi$
0	1	— $^1\Pi$
0	0	— $^1\Sigma$
1	2	— $^3\Delta$
1/2	1	— $^2\Pi$
3/2	2	— $^4\Delta$
1/2	3	— $^2\Phi$
1	0	— $^3\Sigma$
3/2	1	— $^4\Pi$
3/2	0	— $^4\Sigma$
1/2	2	— $^2\Delta$

selection rules

$\Delta\Lambda = 0, \pm 1$

$\Delta S = 0$

Identify which levels can have allowed transitions between them, indicating those transitions in the following form: $^2\Sigma \rightarrow {}^2\Pi$.

$^3\Delta \rightarrow {}^3\Delta$
$^3\Pi \rightarrow {}^3\Pi$
$^1\Pi \rightarrow {}^1\Pi$
$^1\Sigma \rightarrow {}^1\Sigma$
$^2\Pi \rightarrow {}^2\Pi$
$^4\Delta \rightarrow {}^4\Delta$
$^2\Phi \rightarrow {}^2\Phi$
$^3\Sigma \rightarrow {}^3\Sigma$
$^4\Pi \rightarrow {}^4\Pi$
$^4\Sigma \rightarrow {}^4\Sigma$
$^2\Delta \rightarrow {}^2\Delta$

$^3\Delta \rightarrow {}^3\Pi$
$^3\Pi \rightarrow {}^3\Sigma$
$^1\Pi \rightarrow {}^1\Sigma$
$^4\Delta \rightarrow {}^4\Pi$
$^2\Phi \rightarrow {}^2\Delta$
$^4\Pi \rightarrow {}^4\Sigma$
$^2\Delta \rightarrow {}^2\Pi$

$^3\Pi \rightarrow {}^3\Delta$
$^1\Sigma \rightarrow {}^1\Pi$
$^2\Pi \rightarrow {}^2\Delta$
$^3\Sigma \rightarrow {}^3\Pi$
$^4\Pi \rightarrow {}^4\Delta$
$^4\Sigma \rightarrow {}^4\Pi$
$^2\Delta \rightarrow {}^2\Phi$

CH 5

4. Which dyes of Figure 5-11 would make the best lasers at the following wavelengths:
   (a) the green helium–neon laser wavelength;
   (b) the yellow copper vapor laser wavelength;
   (c) the strong atomic sodium emission at 589 nm.

From dye tables

He - Ne laser	543.5 nm	Na-fluorescein dye
Yellow copper vapor laser	578,2 nm	Rh 6-G dye
strong atomic Na emission	589 nm	Rh 6-G dye

CH5

5. For the dyes given in Figure 5-11, assume that the given range of emission wavelengths corresponds to the available laser wavelengths. Assume that the dyes radiate with a quantum yield of 100%. Compute the width of the quasi-molecular energy levels of S_0 that are involved in the laser transition.

$\lambda_0 = 615\text{ nm}$

Acridine Red $\quad \Delta\lambda = (630 - 600)\text{nm} = 30\text{ nm}$

$$\Delta E = h\Delta\nu = \frac{hc}{\lambda^2}\Delta\lambda$$

$$= \frac{4.14\times10^{-15}\text{eVs}\ \ 3\times10^{8}\text{m/s}}{(615\times10^{-9})^2\text{m}^2} = \underline{0.0985\text{eV}}$$

Rh6G $\quad \Delta\lambda = (610 - 570)\text{nm} = 40\text{ nm} \quad \lambda_0 = 590\text{nm}$

$$\Delta E = \frac{4.14\times10^{-15}\ 3\times10^{8}\ 40\times10^{-9}}{(590\times10^{-9})^2} = \underline{0.143\text{eV}}$$

Rh B $\quad \Delta\lambda = (635 - 605)\text{nm} = 30\text{ nm} \quad \lambda_0 = 620\text{ nm}$

$$\Delta E = \frac{4.14\times10^{-15}\ 3\times10^{8}\ 30\times10^{-9}}{(620\times10^{-9})^2} = \underline{0.0969\text{ eV}}$$

Na-fluorescein $\quad \Delta\lambda = (560-530)\text{nm} = 30\text{nm} \quad \lambda_0 = 545\text{nm}$

$$\Delta E = \frac{4.14\times10^{-15}\ 3\times10^{8}\ 30\times10^{-9}}{(545\times10^{-9})^2} = \underline{0.125\text{eV}}$$

7 Hydroxycoumarin $\quad \Delta\lambda = (470-450)\text{nm} = 20\text{nm} \quad \lambda_0 = 460\text{n}$

$$\Delta E = \frac{4.14\times10^{-15}\ 3\times10^{8}\ 20\times10^{-9}}{(460\times10^{-9})^2} = \underline{0.117\text{eV}}$$

If dominated by collisional decay of lower level l, Then

$$\Delta\nu_{ul}^{H} \cong \frac{1}{2\pi}\frac{1}{T_1^l} \Rightarrow T_1^l = \frac{1}{2\pi\Delta\nu_{ul}^{H}}$$

Species	T_1^l
Nd: YAG	1.33×10^{-12} s
Nd: Glass	2.12×10^{-14} s $- 3.31 \times 10^{-14}$ s

If dominated by phase interruption broadening, Then

$$\Delta\nu_{ul}^{H} \cong \frac{1}{2\pi}\left(\frac{2}{T_2}\right) = \frac{1}{\pi T_2} \Rightarrow T_2 = \frac{1}{\pi\Delta\nu_{ul}^{H}}$$

Species	T_2
Nd: YAG	2.66×10^{-12} sec
Nd: Glass	4.24×10^{-14} s $- 6.62 \times 10^{-14}$ s

CH5

6. Compare the emission bandwidths of Nd:YAG and Nd:glass laser materials. Determine the collisional interaction time (either T_1 or T_2; see Section 4.3) that would be required to produce such broadening. Indicate whether this broadening is due to either T_1 or T_2.

Emission Linewidths

Species	Linewidth	Upper level lifetime
Nd : YAG	1.2×10^{11} Hz	230 μs
Nd : Glass	$4.8 - 7.5 \times 10^{12}$ Hz	290-340 μs

At room temperature the upper level lifetime is dominated by radiative decay in both materials. The lower level has no radiative decay (not allowed).

From (4.44)

$$\Delta\nu_{ul}^{H} = \frac{1}{2\pi}\left[\left(\sum_i A_{ui} + \sum_j A_{lj}\right) + \frac{1}{T_1^u} + \frac{1}{T_1^l} + \frac{2}{T_2}\right]$$

$\sum_j A_{lj} = 0$ for both YAG + Glass

For YAG, $\sum A_{ui} = A_{ul} = \frac{1}{230\,\mu\text{sec}} = 4.35 \times 10^3/\text{s}$

For Glass, $\sum A_{ui} = A_{ul} = \frac{1}{290-340\,\mu\text{s}} = 2.94 - 3.45 \times 10^3/\text{s}$

For YAG + Glass $\frac{1}{T_1^u} = 0$

$\therefore$ If the broadening is determined by the above expression, then it would have to be a result of either T_1^l or T_2. Since we do not have enough information, we can calculate for either one.

$$\therefore \quad \Delta\nu_{ul}^{H} \cong \frac{1}{2\pi}\left[\frac{1}{T_1^l} + \frac{2}{T_2}\right]$$

(continued next page)

7. Assume that both Nd:YAG and Nd:glass laser materials are doped to the same Nd concentration of $10^{26}/\text{m}^3$. Assume they are both pumped identically, so that they have approximately 0.1% of the Nd^{3+} ions in the upper laser level. Compare the radiated power of a cubic volume 10 mm on a side from each of these materials. Assume that both materials radiate at approximately the same wavelength of 1.06 μm. Compute the radiated power per unit frequency by assuming that all of the power radiated is averaged over the FWHM emission linewidth of each of the transitions.

Nd concentration in host	$10^{26}/m^3$
Percent in upper level	0.1%
Wavelength	~ 1.06 μm
Volume	$(10\ mm)^3 = (.01 m)^3$

upper level lifetime:	Nd: YAG	230 μs
	Nd: Glass	~ 300 μs

Energy radiated / photon

$$E = h\nu = \frac{hc}{\lambda} = \frac{6.62\times 10^{-34}\ Js\ \ 3\times 10^{8}\ m/s}{1.06\times 10^{-6}\ m}$$

$$= 1.87\times 10^{-19} J = 1.17 eV$$

Total energy radiated from $(0.01 m)^3 = (1cm)^3$ is

$$E_R = 0.001 \times 10^{26}/m^3 \times 10^{-6} m^3 \times \frac{hc}{\lambda} = 1.87\times 10^{-2} J$$

Power radiated is $E_R/\tau_u = P_R$

Nd: YAG $\underline{P_R = 81.3\ W}$

Nd: Glass $\underline{P_R = 62.3\ W}$

Power radiated per unit frequency is: $P_R/\Delta\nu_{FWHM}$

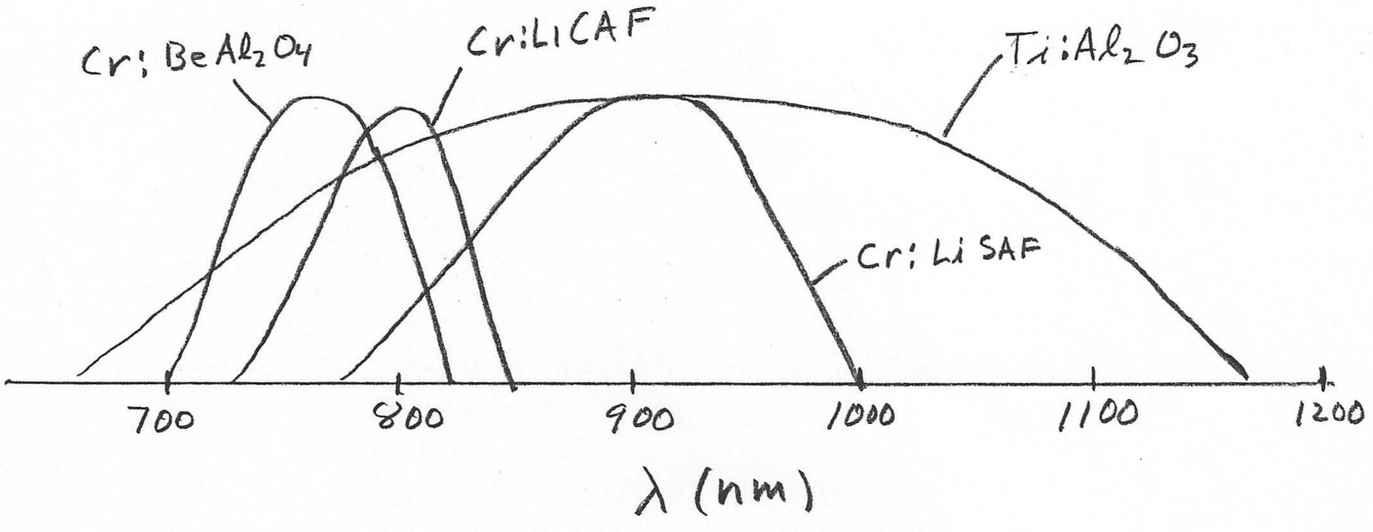
Nd: YAG $= 81.3\ W / 1.2\times 10^{11} Hz = \underline{6.78\times 10^{-10}\ W/Hz}$

Nd: Glass $= 62.3\ W / 7.5\times 10^{12} Hz = \underline{8.31\times 10^{-12}\ W/Hz}$

8. Make a sketch of the emission spectrum for Cr:BeAl$_2$O$_4$, Ti:Al$_2$O$_3$, Cr:LiSAF, and Cr:LiCaF broadband laser materials. Compute the collision interruption time T$_2$ associated with phonon broadening for each of these materials by using the experimental emission linewidths. Compare these times to the upper laser level lifetimes.

From Table 5-2 and page 570 (Table 15-12)

Material	λ_{max}	$\Delta\lambda$ (nm)	τ_u	$\Delta\nu_{FWHM}$	T_2 (sec)
Cr: BeAl$_2$O$_4$	750	120	260 μs	2.6×10^{13} Hz	1.22×10^{-14}
Ti: Al$_2$O$_3$	730	180	3.8 μs	1×10^{14} Hz	3.2×10^{-15}
Cr: Li SAF	895	200	67 μs	8.3×10^{13} Hz	3.8×10^{-15}
Cr: Li CAF	900	130	170 μs	6.4×10^{13} Hz	5.0×10^{-15}



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9. Look up and compare the electrical conductivity of glass (insulator), silicon (semiconductor), and copper (conductor).

Electrical conductivity:

glass (insulator)	$< 0.2 \times 10^{-11} (\Omega \, cm)^{-1}$
silicon (semiconductor)	$10^{0} - 10^{6} (\Omega \, cm)^{-1}$ depending upon doping concentration
copper (conductor)	$6 \times 10^{5} (\Omega \, cm)^{-1}$

CH5

10. From the chart in Figure 5-31(a), determine the laser wavelength for each of the six different compounds listed in the chart: AlP, GaP, AlAs, GaAs, InP, and InAs.

Species	Band gap	Laser wavelength
Al P	2.5 eV	~ 0.5 μm
Ga P	2.25 eV	~ 0.55 μm
Al As	2.15 eV	~ 0.58 μm
Ga As	1.4 eV	~ 0.89 μm
In P	1.3 eV	~ 0.95 μm
In As	0.4 eV	~ 3.1 μm

CH 5

11. Using the chart of Figure 5-32, determine which semiconductor laser materials could replace the He–Ne laser operating at 632.8 nm.

CdS_xSe_{1-x}

$Al_xGa_{1-x}In_{1-y}P$

$In_xGa_{1-x}As$

$GaAs_{1-x}P_x$

CH5

12. Assume that an electron is subjected to a potential energy function V of the form

$$V = \begin{cases} V_0 = 0 & \text{for } x = 0 \text{ to } x = L, \\ \infty & \text{for } x = -\infty \text{ to } x = 0, \\ \infty & \text{for } x = L \text{ to } x = +\infty. \end{cases}$$

Solve the Schrödinger equation to determine the energy levels for this system, which is called an *infinite square–well potential*. Assuming this is a quantum-well laser, how high would the first energy level occur above the bandgap?

Boundary conditions

$V = 0$ for $0 \leq x \leq L$ $V = \infty$ for $-\infty < x < 0$

for $L < x < +\infty$

Schroedinger equation

$$\nabla^2 \psi + \frac{8\pi^2 m_e}{h^2}(E - V)\psi = 0$$

or $\dfrac{d^2\psi(x)}{dx^2} + k^2\psi(x) = 0$ $k^2 = \dfrac{8\pi^2 m_e}{h^2}(E - V)$

General solution $\Rightarrow \psi(x) = A\sin kx + B\cos kx$

but at $x = 0$ $\psi(x) = 0 \Rightarrow B = 0$

Hence $\psi(x) = A\sin kx$

also at $x = L$, $\psi(x) = 0 \Rightarrow A\sin kL = 0$

or $kL = n\pi = \sqrt{\dfrac{8\pi^2 m_e}{h^2}(E - V)}\,L$

or $\left[\dfrac{8\pi^2 m_e}{h^2}(E - V)\right]L^2 = n^2\pi^2$

$\therefore E = \dfrac{n^2 h^2 \pi^2}{L^2\, 8\pi^2 m_e} + V$ but $V = 0$ in the well.

$\therefore E_n = \dfrac{h^2}{8 m_e L^2} n^2 = \dfrac{\hbar^2 \pi^2}{2 m_e L^2} n^2$

For $n = 1$

$$E_1 = \frac{\hbar^2 \pi^2}{2 m_e L^2}$$

CH5

13. Suppose that the XeF excimer upper laser level C (as labeled in Figure 5-9) is being populated in an electric discharge and decays in a time of 5 × 10^{-9} s to the lower level A in Figure 5-9. Assume the transition takes place at the value of r (the separation distance between the Xe and F atoms) as shown in the figure, and that the molecule subsequently exists temporarily in the A state at that distance. How long would it take for the molecules to be repulsed and thereby separated while in the A state to the point where they are no longer bound to each other? Assume that the molecules are at a temperature of 100°C in the discharge and that their velocity is the thermal velocity (it might be higher than that owing to the repulsive force). Compare that time of repulsion to the time the molecule exists in the upper laser level C.

Compute Thermal velocity of Xe & F

The atoms are initially separated by 2.6Å = 0.26nm

They are free when separated by 4.25Å = 0.425 nm

∴ They must Travel (0.425 − 0.26)nm = $\underline{0.165\text{ nm}}$

From (4.49)

$$\bar{v} = \sqrt{\frac{8kT}{M_N \pi}}$$

$T = 373\text{ K}$

$Xe \rightarrow M_N = 132$

$F \rightarrow M_N = 19$

$k = 1.38 \times 10^{-23}\text{ J/K}$

$M_p = 1.67 \times 10^{-27}\text{ kg}$

$$\bar{v}_{Xe} = 2.44 \times 10^{2}\text{ m/s}$$

$$\bar{v}_F = 6.43 \times 10^{2}\text{ m/s}$$

$$(\bar{v}_{Xe}\,\Delta T + \bar{v}_F\,\Delta T) = 0.165 \times 10^{-9}\text{ m}$$

$$\Delta T = \frac{0.165 \times 10^{-9}\text{ m}}{(244 + 643)\text{ m/s}} = \underline{1.86 \times 10^{-13}\text{ s}}$$

∴ separates in < 0.186 psec

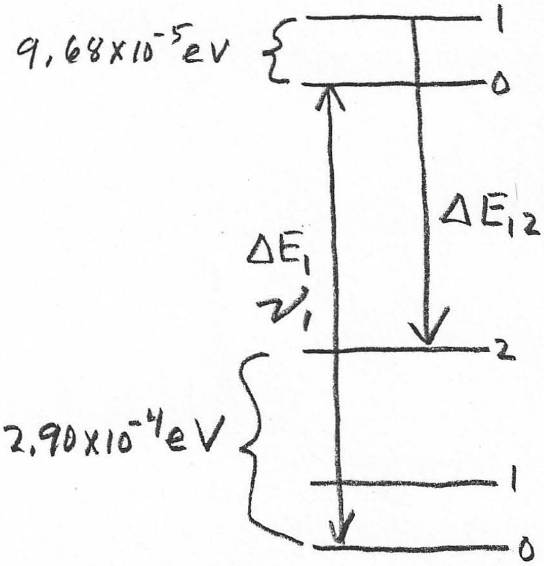
upper level decays in ~ 5 n sec

14. Using the data concerning CO_2 lasers, compute the wavelength for the P(2) transition in the V_1 symmetric stretch rotational–vibrational mode.

P(2) involves $J=1 \rightarrow J=2$

For $J=1$ $E_{rot} = 2\,(4.84\times10^{-5})\,eV = 9.68\times10^{-5}\,eV$

For $J=2$ $E_{rot} = 2\,(2+1)(4.84\times10^{-5})\,eV = 2.90\times10^{-4}\,eV$



$\nu_1 = 4.16\times10^{13}/s$

$\Delta E_1 = h\nu_1 = 4.13\times10^{-15}\,eVs\;4.16\times10^{13}/s$
$= 0.1718\,eV$

$$\nu_{12} = \frac{\Delta E}{h} = \frac{(0.1718 + 9.68\times10^{-5} - 2.90\times10^{-4})\,eV}{4.135\times10^{-15}\,eVs}$$

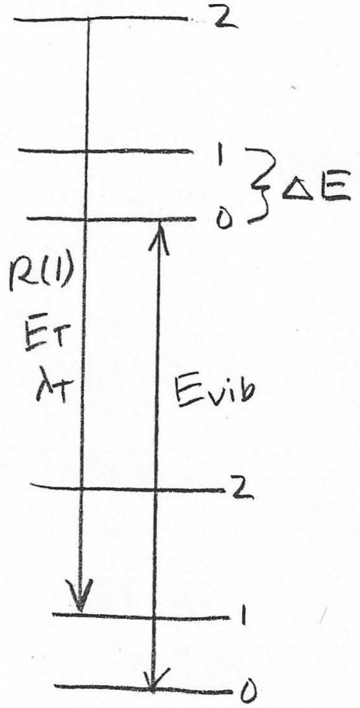
$$= \frac{1.71607\;eV}{4.135\times10^{-15}\,eVs}$$

$$= 4.150\times10^{13}\,Hz$$

$$\lambda = \frac{c}{\nu} = \frac{3\times10^{8}\,m/s}{4.150\times10^{13}/s} = \boxed{7.23\,\mu m}$$

CH 5

15. The R(1) rotational–vibrational transition in a nonhomonuclear molecule is measured to occur at a wavelength of 12.0 μm. The vibrational energy spacing for this molecule is 0.10 eV. What is the wavelength of the longest-wavelength pure rotational transition in this molecule?



$E_T = E_{vib} + 3\Delta E - \Delta E = E_{vib} + 2\Delta E$

Longest rotational wavelength is that associated with ΔE

$E_{vib} = 0.1\,eV = 1.602 \times 10^{-20}\,J$

$\Delta E = \frac{E_T - E_{vib}}{2}$

$\lambda_T = 12.0 \times 10^{-6}\,m$

$E_T = \frac{hc}{\lambda_T} = \frac{6.6256 \times 10^{-34}\,Js\; 3 \times 10^{8}\,m/s}{12.0 \times 10^{-6}\,m}$

$= 1.6564 \times 10^{-20}\,J$

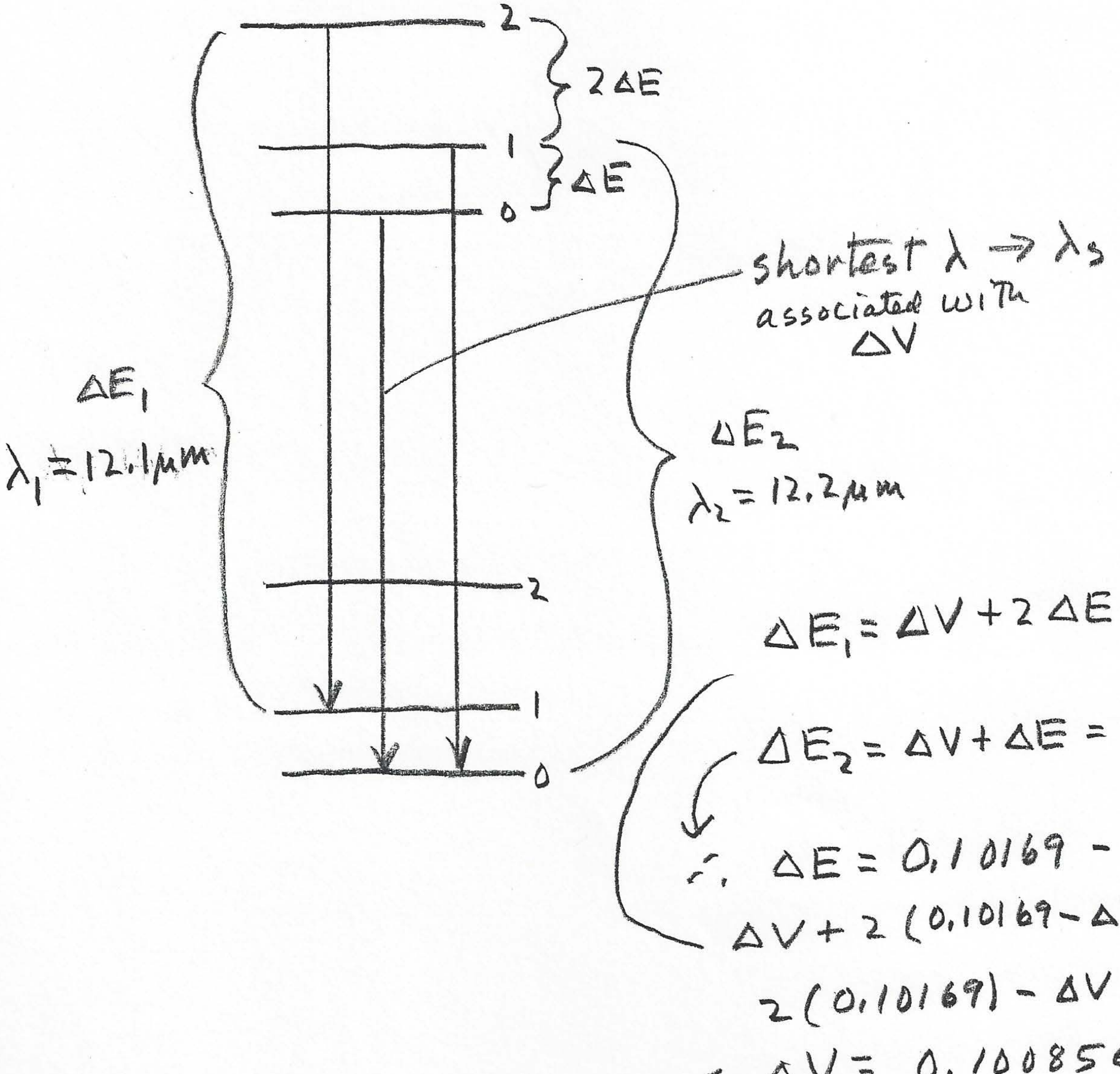
$\therefore \Delta E = \frac{1.6564 \times 10^{-20}J - 1.602 \times 10^{-20}J}{2} = 2.72 \times 10^{-22}\,J$

$\lambda = \frac{hc}{\Delta E} = \frac{6.6256 \times 10^{-34}\,Js\; 3 \times 10^{8}\,m/s}{2.72 \times 10^{-22}\,J}$

$\underline{\lambda = 7.308 \times 10^{-4}\,m = 730.8\,\mu m}$

CH 5

16. Consider a hypothetical nonhomonuclear molecule that consists of only the first two vibrational levels, each of which has only the first three rotational levels. The longest wavelength emission on a rotational–vibrational transition is measured to occur at 8.00 μm, and the next-longest wavelength transition occurs at 7.90 μm. At what wavelength would the next-shorter–wavelength rotational–vibrational transition occur, what branch would it be associated with, and what is the vibrational energy spacing (in eV) of the molecule? Assume that the Q-branch transitions are not allowed in this molecule.



$$\Delta E_1 = \Delta V + 2\Delta E = \frac{hc}{\lambda_1} = 0.10253\text{ eV}$$

$$\Delta E_2 = \Delta V + \Delta E = \frac{hc}{\lambda_2} = 0.10169\text{ eV}$$

$$\therefore \Delta E = 0.10169 - \Delta V$$

$$\Delta V + 2(0.10169 - \Delta V) = 0.10253$$

$$2(0.10169) - \Delta V = 0.10253$$

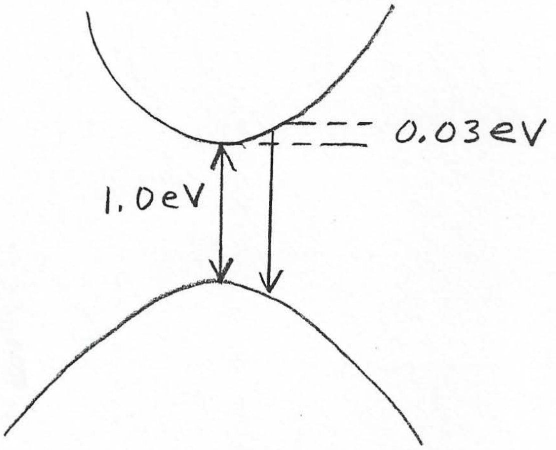
$$\therefore \underline{\Delta V = 0.10085\text{ eV}}$$

$$\lambda = \frac{hc}{\Delta V} = \frac{hc}{0.10085} = 1.23\times10^{-5}\text{ m}$$

$$= \underline{12.30\,\mu m}$$

CH5

17. A semiconductor laser material has a bandgap of 1.0 eV and an effective mass of the conduction band that is twice that of the effective mass of the valence band. If a recombination transition occurs from an energy that is 0.03 eV above the bandgap, what would be the transition wavelength?



$$E_c = \frac{\hbar^2 k^2}{2 m_c} = 0.03\,eV$$

$$E_V = \frac{\hbar^2 k^2}{2(0.5 m_c)}$$

$$\frac{\hbar^2 k^2}{m_c} = 2E_c = 0.06\,eV$$

$$\therefore h\nu = 1.0\,eV + 0.03\,eV + 0.06\,eV = 1.09\,eV$$

$$h\nu = \frac{hc}{\lambda} = 1.09\,eV$$

$$\therefore \lambda = \frac{hc}{1.09\,eV} = \frac{4.1354\times10^{-15}\,3\times10^{8}}{1.09}$$

$$= 1.138\times10^{-6}\,m$$

$$\underline{\lambda = 1.138\,\mu m}$$

CH5

18. A semiconductor laser material has a bandgap energy of 1.55 eV. This material is used to form a quantum-well laser having a 5-nm–thick active region. The effective masses of both the conduction band and the valence band are measured to be greater by a factor of 2 than the mass of an electron. What would be the longest wavelength of the laser when an electrical current is passed through the active layer?

$$E_1 = \frac{(\pi \hbar)^2}{2 m_e L^2} = \frac{\pi^2 (6.63\times10^{-34}\ \mathrm{J\cdot s})^2}{(2\pi)^2\ 2\times2\ m_e\ L^2}$$

$$= \frac{\cancel{\pi}^2 (6.63\times10^{-34})^2}{16\ \cancel{\pi}^2\ 9.1\times10^{-31}\ (5\times10^{-9})^2}$$

$$= 1.2076\times10^{-21}\ \mathrm{J} = 7.55\times10^{-3}\ \mathrm{eV}$$

$$\Delta E_T = 1.55\ \mathrm{eV} + 2\,(7.55\times10^{-3})\ \mathrm{eV}$$

$$= 1.565\ \mathrm{eV}$$

$$\Delta E_T = \frac{hc}{\lambda} \qquad \lambda = \frac{hc}{\Delta E_T} = \frac{4.1354\times10^{-15}\ \mathrm{eVs}\ 3\times10^{8}\ \mathrm{m/s}}{1.565\ \mathrm{eV}}$$

$$= 7.927\times10^{-7}\ \mathrm{m}$$

$$= \underline{0.793\ \mu\mathrm{m}}$$

CH 5

19. A gallium arsenide quantum-well semiconductor with an active region of 10 nm is excited by a pulsed laser to produce electrons in the conduction band. That excitation subsequently decays within the active region back to the valence band via recombination radiation. What would be the longest wavelength transition observed during that decay? GaAs has a bandgap energy of 1.43 eV and an index of refraction of 3.4. The effective mass for electrons in the conduction band is $0.067m_e$, and the effective mass for holes in the valence band is $0.48m_e$.

$$E_1 = \frac{(\pi \hbar)^2 n^2}{2 m_{eff} L^2} = \frac{(\pi \hbar)^2 1^2}{2 m_c L^2}$$

$m_c^* = 0.067$

$m_v^* = 0.48$

$L = 10^{-8}\ m$

$$\therefore \quad E_1 = \frac{\pi^2 \left(\frac{6.626\times10^{-34}}{2\pi}\right)^2}{(10^{-8})^2\, 2\, m^* m_e} = \frac{6.03\times10^{-22}}{m^*}$$

$$E_1^c = \frac{6.03\times10^{-22}}{0.067}\ J = 9.00\times10^{-21}\ J = 0.0562\ eV$$

$$E_1^v = \frac{6.03\times10^{-22}}{0.48}\ J = 1.26\times10^{-21}\ J = 0.00785\ eV$$

band gap = 1.43 eV

$$\therefore \quad \Delta E_{Total} = (1.43 + 0.0562 + 0.00785)\ eV$$

$$= 1.494\ eV$$

$$\Delta E = h\nu = \frac{hc}{\lambda}$$

$$\lambda = \frac{hc}{\Delta E} = \frac{4.135\times10^{-15}\ eVs\ \ 3\times10^{8}\ m/s}{1.494\ eV}$$

$$\lambda = \underline{8.30\times10^{-7} = 830\ nm}$$