SOLUTION TO CUBIC EQUATION USING JAVA PROGRAMMING

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ABSTRACT

Solving cubic equation is one of the non-linear Algebraic equation in mathematics. Many Cubic equations can be solved algebraically, however many cannot be solved, because of the complexity. The discriminant approach for solving cubic equation is adopted in this study to generate the solution. This study is used to design a programming solution to solve cubic equation using java programming language. This solution was developed by using the Eclipse as integrated Development Environment (IDE) to compile, run and for finding the bugs in the program. Usually large errors in using mathematical algorithms are treated by using the cubic equations. Using this application, we can outline the way of solving this cubic equation. In the view of the implication of findings, it was recommended that there is a need for design and development of computational solution and utilization into classroom study. Finally, this programme can give solution to two problems such that we can able to generate the root values for the given cubic equation. And also we can able to generate the cubic equation by giving the three real root values. Many examples have been worked out, and in most cases, we found out the exact solution.

Keywords: Cubic Equation, nonlinear Algebraic equation, Discriminant approach, Eclipse, Integrated Development Environment.

INTRODUCTION

In the computational world, Computer programming touches almost every aspect of our lives. Solving non-linear algebraic equations such as cubic equation and solving solution to nonlinear system of equation is the foundation of many scientific programming. Cubic equation is a special type of polynomial equation that is used in many fields of study. For example, a cubic equation is used to predict surface tension and spinodal limits [1]. In modern technologies to get the accurate value and to get quick answer, mathematics takes the form of computer applications. These kind of software applications support to the cognitive process reducing the memory load of students and creating awareness to the problem-solving process.

A cubic equation can be written in the form of $ax^3 + bx^2 + cx + d = 0$. It must have the term in x^3 or it would not be cubic (and so a≠0), but any or all of b, c and d can be zero. Cubic equation can be solved using synthetic division, Cardano's formula [6] or identifying discriminants by Cubic formula [3].

However, this paper proposed computer programs of solving cubic equation based on discriminant method and generate cubic equation based on given real solutions. Discriminant (Δ) of quadratic equation and cubic equation is a popular numerical method of evolutionary computing. Quadratic equations can easily be solved, by using the quadratic formula. In particular, we have $ax^2 + bx + c = 0$ if and only if $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$ [2]. The expression $b^2 - 4ac$ is known as the discriminant (Δ) of the quadratic.

According to discriminant (Δ) three cases considered for the solution. Thus, the quadratic equation has two real solutions if Δ > 0, only one real solution if Δ = 0, and no real solutions if Δ < 0. The corresponding formula for solving cubic and quartic equations are significantly more complicated. In this study, the computer program with numerical formula using discriminant was used to get approximate solutions for cubic equations. Moreover, an algorithm was proposed for a numerical method to determine the cubic equation from given real solutions.

Java

Java is a popular general-purpose programming language and computing platform. It is fast, reliable, and secure. One of the reason why java widely used is because of the availability of huge standard library. The Java environment has hundreds of classes and methods under different packages to help software developers. Java.util is a package, while Scanner is a class of this package and used to get user input. Java.lang package provides classes for performing basic numeric operations.

Eclipse

In computer programming, Eclipse is an integrated development environment (IDE) for developing applications. The main use of Eclipse is for developing Java applications, but it may also be used to develop applications in other programming languages such as C/C++, Python, PERL, Ruby etc. Eclipse contains a base workspace and an extensible plug-in system for customizing the environment. The Eclipse software development kit (SDK), which includes the Java development tools, is free and open-source software. Every year, since 2006, the Eclipse foundation releases the Eclipse Platform and a number of other plug-ins in June [7].

Cubic Formula

Let A, B, C and D be real constants such that A≠0, then a cubic equation in x is given by,

$$Ax^3 + Bx^2 + Cx + D = 0 \qquad \ldots\ldots\ldots\ldots\ldots \qquad (1.1)$$

Rewrite the equation (1.1) as,

$$x^3 + ax^2 + bx + c = 0 \qquad \ldots\ldots\ldots\ldots\ldots \qquad (1.2)$$

Where, a $= \frac{B}{A}$, b $= \frac{C}{A}$ and c $= \frac{D}{A}$.

It is important to mention that a formula called the cubic formula for finding the roots of (1.2).
The cubic formula for finding roots of (1.2) as contained is given by,

Let $p = b - \frac{a^2}{3}$ and $q = \frac{2a^3}{27} - \frac{ab}{3} + c$ Then,

Discriminant (Δ) $= \frac{q^2}{4} + \frac{p^3}{27}$

As noted earlier, the nature of the roots of a cubic equation depends on whether the associated discriminant is positive, negative or zero. The three cases are discussed in this section.

Case 1: Roots of a cubic equation when Δ > 0 there is only one real solution.

$$x1 = (\frac{-q}{2} + \sqrt{\Delta})^{1/3} + (\frac{-q}{2} - \sqrt{\Delta})^{1/3} - \frac{a}{3}$$

Case 2: Roots of a cubic equation when Δ=0 there are repeated roots.

$$x1 = -2\left(\frac{q}{2}\right)^{1/3} - \frac{a}{3} \quad \text{and} \quad x2 = x3 = \left(\frac{q}{2}\right)^{1/3} - \frac{a}{3}$$

Case 3: Roots of a cubic equation when Δ<0 there are three real solutions.

$$x1 = \frac{2}{\sqrt{3}}\sqrt{-p}\sin\left(\frac{1}{3}\sin^{-1}\left(\frac{3\sqrt{3}q}{2\left(\sqrt{-p}\right)^3}\right)\right) - \frac{a}{3}$$

$$x2 = -\frac{2}{\sqrt{3}}\sqrt{-p}\sin\left(\frac{1}{3}\sin^{-1}\left(\frac{3\sqrt{3}q}{2(\sqrt{-p})^3}\right) + \frac{\pi}{3}\right) - \frac{a}{3}$$

$$x3 = \frac{2}{\sqrt{3}}\sqrt{-p}\cos\left(\frac{1}{3}\sin^{-1}\left(\frac{3\sqrt{3}q}{2\left(\sqrt{-p}\right)^3}\right) + \frac{\pi}{6}\right) - \frac{a}{3}$$

Generate Cubic Formula from the given Solution

Let a, b, c and d be real constants such that a≠0, then a cubic equation in x is given by,

$$ax^3 + bx^2 + cx + d = 0 \qquad \ldots\ldots\ldots\ldots\ldots \qquad (2.1)$$

Assume the cubic formula (2.1) has roots α, β, and γ.

Then the cubic formula can represent by,

$$(x-\alpha)(x-\beta)(x-\gamma) = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots (2.2)$$

Extract the equation (2.2) as,

$$(x-\alpha)(x-\beta)(x-\gamma) = 0$$
$$(x-\alpha)(x^2 - \gamma x - \beta x + \beta\gamma) = 0$$
$$x^3 - \gamma x^2 - \beta x^2 + \beta\gamma x - \alpha x^2 + \alpha\gamma x + \alpha\beta x - \alpha\beta\gamma = 0$$
$$x^3 - (\propto + \beta + \gamma)x^2 + (\alpha\beta + \beta\gamma + \alpha\gamma)x - \alpha\beta\gamma = 0 \ldots\ldots\ldots\ldots (2.3)$$

Divide the equation (2.1) by a, then

$$x^3 + \frac{b}{a}x^2 + \frac{c}{a}x + \frac{d}{a} = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2.4)$$

Compare the coefficients in the equations (2.3) and (2.4),

$$\frac{b}{a} = -(\propto + \beta + \gamma),$$

$\frac{c}{a} = \alpha\beta + \beta\gamma + \alpha\gamma,$
$\frac{d}{a} = -(\alpha\beta\gamma)$

METHODOLOGY

A discriminant approach is being followed for development of this solution. The algorithm based on discriminant approach is implemented using 'Java' programming language. The steps followed are described in below.

Algorithm for Find Roots

Start
input the co-efficient values as A, B, C and D
find a $= \frac{B}{A}$, b $= \frac{C}{A}$ and c $= \frac{D}{A}$
find p as b - $\frac{a^2}{3}$, q as $\frac{2a^3}{27} - \frac{ab}{3} + c$ and Δ as $\frac{q^2}{4} + \frac{p^3}{27}$
if $\Delta > 0$ then,
find a real root x1
else if $\Delta = 0$ then,
find real roots x1 and x2
else
find real roots x1, x2 and x3
end if
print root values
end

Algorithm for Find Equation

Start
input the real root values as $\propto$, β, and γ
find the value $-(\propto + \beta + \gamma)$
find the value $\alpha\beta + \beta\gamma + \alpha\gamma$
find the value $-(\alpha\beta\gamma)$
determine the cubic equation
print the cubic equation
end

Flow- Chart

Flowchart to calculate the roots of cubic equation and construction of the cubic equation using the root values is shown in Figure 1.

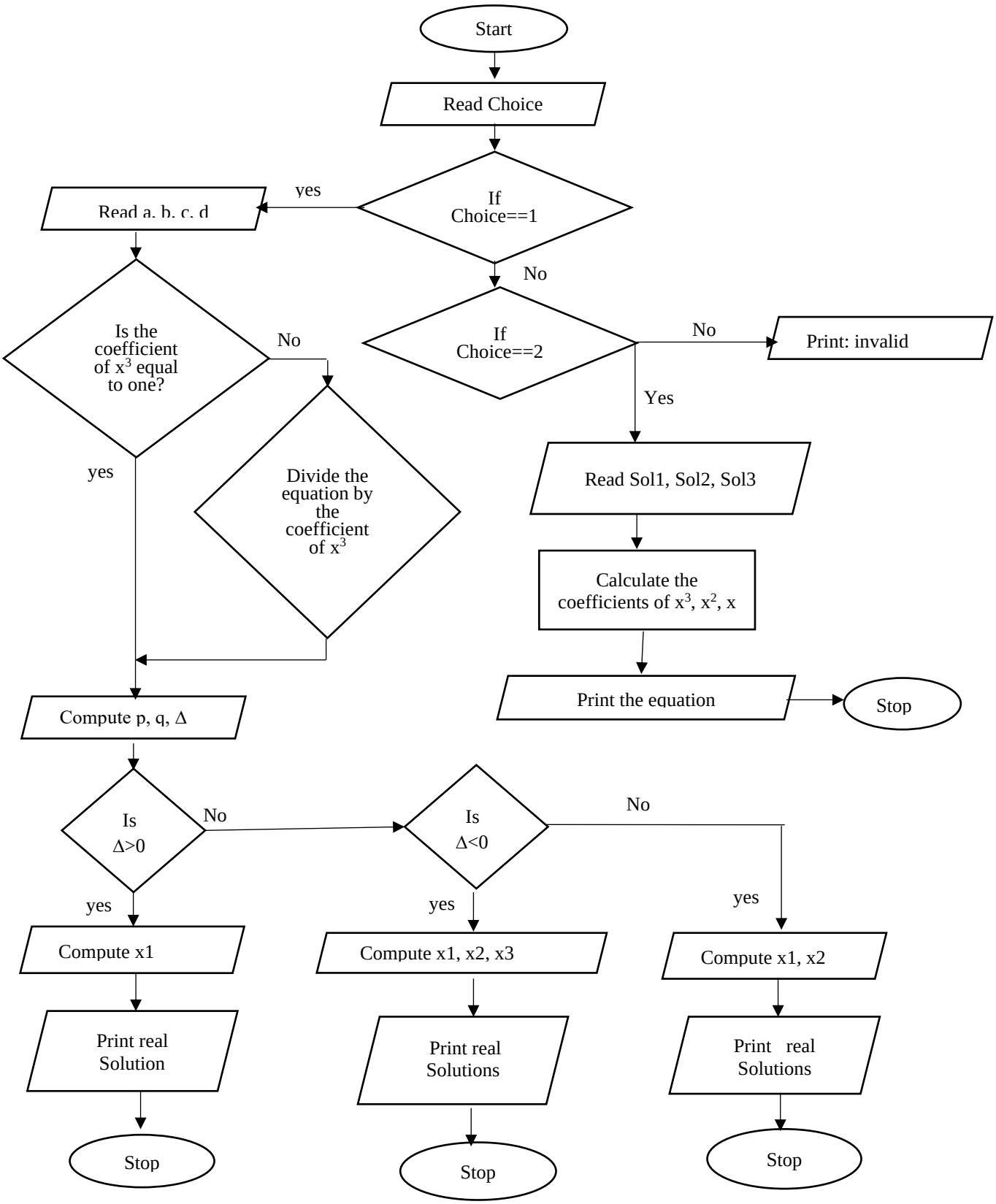

Figure 1: Flow chart for the solution

RESULTS AND DISCUSSION

Test Algorithm for Find Roots

The application for cubic solution tested using different cubic equations to find the real roots. Also the same equations used to solve manually. For the Experiment the cubic equations used to test the results are given in table 1:

Table 1: cubic equations used to test results

$x^3 -6x^2 +11x-6=0$ -------(1)	$x^3 -5x^2 +8x-4=0$ -------(2)
$x^3 -3x^2 +3x-1=0$ -------(3)	$x^3 -9x^2 +36x-80=0$ -------(4)
$3x^3 +5x^2 +4.5x+5.6=0$ -------(5)	$5x^3 -98x^2 +96x-7=0$ -------(6)
$2x^3 -7x^2 +9x+8=0$ -------(7)	$x^3 -19x^2 +118.25x-242=0$ -------(8)
$x^3 -55x^2 +65x+8=0$ -------(9)	$6x^3 -12x^2 -64x+32=0$ -------(10)

The application developed using Java programming language was tested by the given cubic equations and the sample output interface illustrates by figure 2.

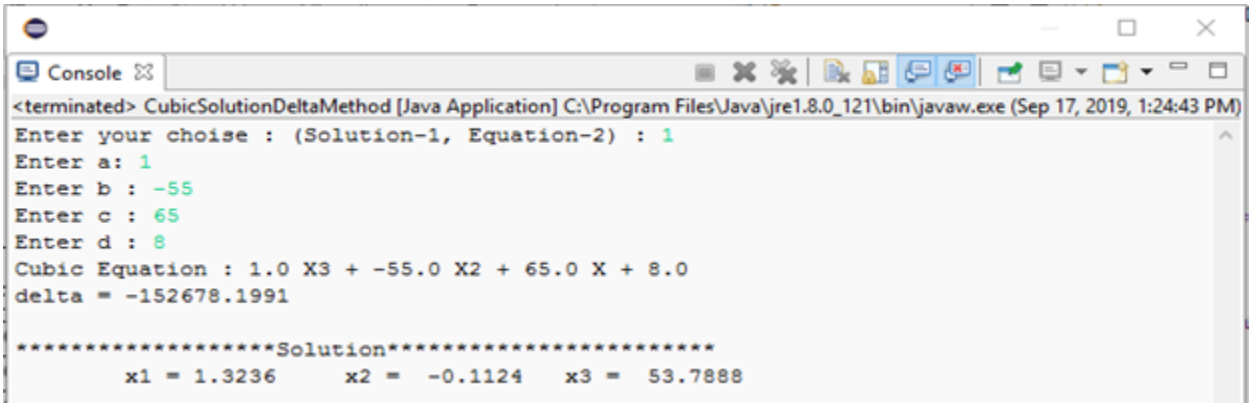

Figure 2: Sample result for find roots using Eclipse IDE

According to the comparison between computational solution and manual solution given in table 2, the difference may not be considerable.

Table 2: Computational and manual results for tested cubic equations

Equation No	Computational value			Manually calculated value		
	X_1	X_2	X_3	X_1	X_2	X_3
1	2.0	1.0	3.0	2	1	3
2	1.0	2.0	2.0	1	2	2
3	1.0	1.0	1.0	1	1	1
4	5.0	-	-	5	-	-
5	-1.4971	-	-	-1.4974	-	-
6	0.9505	0.0793	18.5701	0.9505	0.0793	18.5701
7	-0.5814	-	-	-0.5818	-	-
8	8.0	5.5	5.5	8	5.5	5.5
9	1.3236	-0.1124	53.7888	1.3236	-0.1124	53.7803
10	0.4685	-2.6941	4.2256	0.4685	-2.6941	4.2256

Test Algorithm for Find Equation

The tested algorithm has two parts, such as the application part for finding cubic equation was tested for given three real root values given in table 3.

Table 3: real root values used to test results

Solution No.	Root 1	Root 2	Root 3
1	1	1	1
2	1.1	2.1	3.3
3	0.45	-3.20	2.74
4	-1	-1	-1
5	-0.57	-5.12	0.69

The application developed using Java programming language was tested by the given real root values and the sample output interface illustrates by figure 3.

Figure 3: Sample result for find cubic equation using Eclipse IDE

Comparing the Computational Cubic Equation with actual equation given in Table 4, there is no significant difference.

Table 4: Computational and Actual cubic equation for given root values

Equation No	Computational Cubic Equation	Actual Equation
1	$x^3 -3x^2 +3x-1=0$	$x^3 -3x^2 +3x-1=0$
2	$x^3 -6.5x^2 +12.87x-7.62=0$	$x^3 -6.5x^2 +12.87x-7.623=0$
3	$x^3 -0.01x^2 -8.98x + 3.95=0$	$x^3 - 9x+ 4=0$
4	$x^3 + 3x^2 +3x +1=0$	$x^3 + 3x^2 +3x +1=0$
5	$x^3 +5x^2 -1.01x-2.01=0$	$2x^3 +10x^2 -2x-4=0$

In conclusion, the programming application for solving the cubic equation results and finding the cubic equation for given real solution in excellent performance level.

CONCLUSIONS

Nowadays, although the technologies have been developed rapidly, its operations have to be increased in the classroom studies. The use of a computer application to solve the mathematical calculation makes the future generation more independent [4,5]. As this cubic equation is commonly used in mathematics, its actual use be expected to be one of the best understood of computer algorithms. The fundamental contribution of this paper is the establishment of computer application of finding the solution of a cubic equation using the concept of discriminant method. The general algorithm for the cubic equation using discriminant method is established. The discriminant of a cubic equation, Δ, which comprises p and q, such that Δ is shown to depend on the nature of the roots of a given equation. In mathematics, a cubic equation has three distinct real roots if $\Delta < 0$, if $\Delta > 0$ one real root and if $\Delta = 0$ either two equal real roots or three equal real roots. Moreover, the programming solution of a cubic equation is implemented using the algorithm that depends on the corresponding discriminant method. Finally, numerical examples are used to substantiate the established application.

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