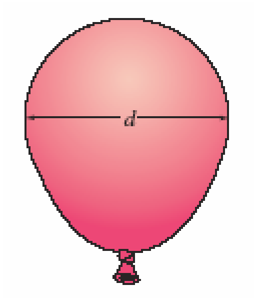
2-1 The center portion of the rubber balloon has a diameter of $d = 100$ mm. If the air pressure within it causes the balloon's diameter to become $d = 125$ mm, determine the average normal strain in the rubber.



Given: $d_0 := 100mm \qquad d := 125mm$

Solution:

$$\varepsilon := \frac{\pi d - \pi d_0}{\pi d_0}$$

$$\varepsilon = 0.2500 \frac{mm}{mm}$$ **Ans**

Ans:
$\epsilon_{CE} = 0.00250$ mm/mm, $\epsilon_{BD} = 0.00107$ mm/mm

2–2. A thin strip of rubber has an unstretched length of 375 mm. If it is stretched around a pipe having an outer diameter of 125 mm, determine the average normal strain in the strip.

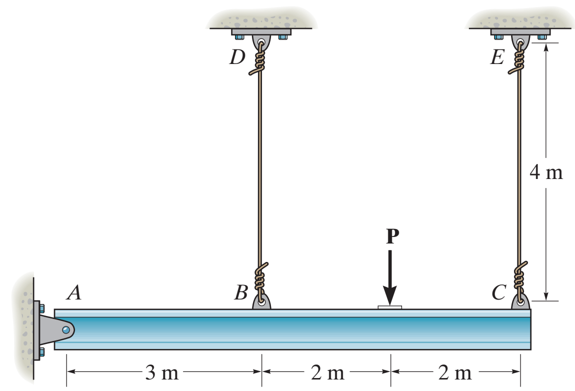
$L_0 = 375$ mm

$L = \pi(125 \text{ mm})$

$$\varepsilon = \frac{L - L_0}{L_0} = \frac{125\pi - 375}{375} = 0.0472 \text{ mm/mm}$$ **Ans.**

Ans:
$\epsilon = 0.0472$ mm/mm

2–3. The rigid beam is supported by a pin at A and wires BD and CE. If the load **P** on the beam causes the end C to be displaced 10 mm downward, determine the normal strain developed in wires CE and BD.



$$\frac{\Delta L_{BD}}{3} = \frac{\Delta L_{CE}}{7}$$

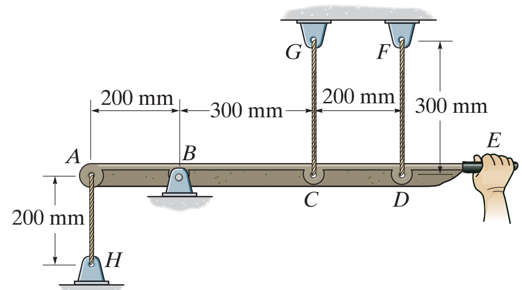
$$\Delta L_{BD} = \frac{3\,(10)}{7} = 4.286 \text{ mm}$$

$$\epsilon_{CE} = \frac{\Delta L_{CE}}{L} = \frac{10}{4000} = 0.00250 \text{ mm/mm}$$ **Ans.**

$$\epsilon_{BD} = \frac{\Delta L_{BD}}{L} = \frac{4.286}{4000} = 0.00107 \text{ mm/mm}$$ **Ans.**

Ans:
$\epsilon_{CE} = 0.00250$ mm/mm, $\epsilon_{BD} = 0.00107$ mm/mm

***2–4.** The force applied at the handle of the rigid lever causes the lever to rotate clockwise about the pin B through an angle of 2°. Determine the average normal strain developed in each wire. The wires are unstretched when the lever is in the horizontal position.



Geometry: The lever arm rotates through an angle of $\theta = \left(\frac{2°}{180}\right)\pi \text{ rad} = 0.03491 \text{ rad}$. Since θ is small, the displacements of points A, C, and D can be approximated by

$$\delta_A = 200(0.03491) = 6.9813 \text{ mm}$$

$$\delta_C = 300(0.03491) = 10.4720 \text{ mm}$$

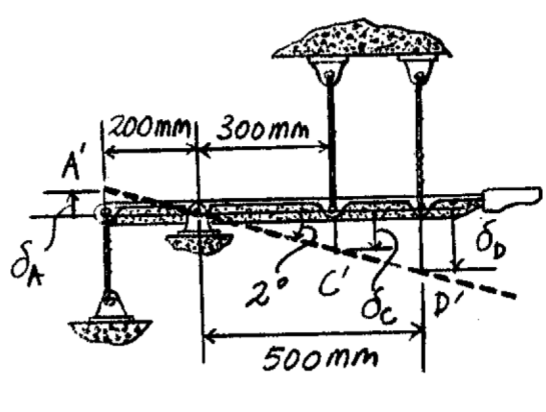
$$\delta_D = 500(0.03491) = 17.4533 \text{ mm}$$

Average Normal Strain: The unstretched length of wires AH, CG, and DF are $L_{AH} = 200$ mm, $L_{CG} = 300$ mm, and $L_{DF} = 300$ mm. We obtain

$$(\epsilon_{\text{avg}})_{AH} = \frac{\delta_A}{L_{AH}} = \frac{6.9813}{200} = 0.0349 \text{ mm/mm}$$ **Ans.**

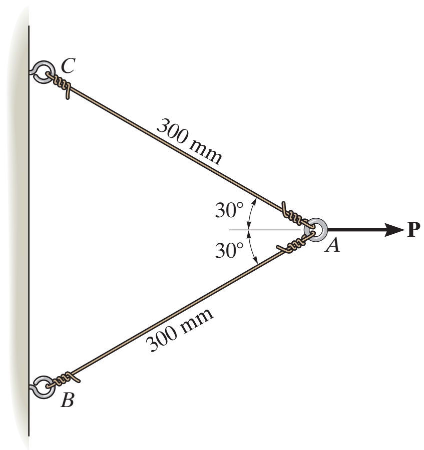
$$(\epsilon_{\text{avg}})_{CG} = \frac{\delta_C}{L_{CG}} = \frac{10.4720}{300} = 0.0349 \text{ mm/mm}$$ **Ans.**

$$(\epsilon_{\text{avg}})_{DF} = \frac{\delta_D}{L_{DF}} = \frac{17.4533}{300} = 0.0582 \text{ mm/mm}$$ **Ans.**



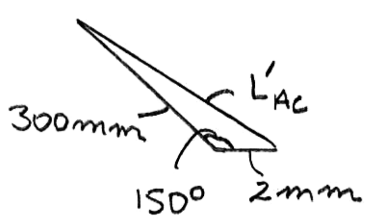
(a)

2–5. The two wires are connected together at A. If the force **P** causes point A to be displaced horizontally 2 mm, determine the normal strain developed in each wire.



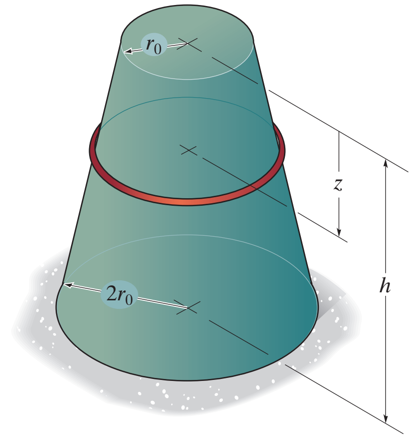
$$L'_{AC} = \sqrt{300^2 + 2^2 - 2(300)(2)\cos 150°} = 301.734 \text{ mm}$$

$$\epsilon_{AC} = \epsilon_{AB} = \frac{L'_{AC} - L_{AC}}{L_{AC}} = \frac{301.734 - 300}{300} = 0.00578 \text{ mm/mm} \qquad \textbf{Ans.}$$



Ans:
$\epsilon_{AC} = \epsilon_{AB} = 0.00578$ mm/mm

2–6. The rubber band of unstretched length $2r_0$ is forced down the frustum of the cone. Determine the average normal strain in the band as a function of z.



Geometry: Using similar triangles shown in Fig. *a*,

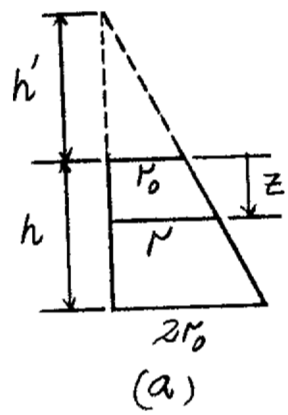
$$\frac{h'}{r_0} = \frac{h' + h}{2r_0}; \qquad h' = h$$

Subsequently, using the result of h'

$$\frac{r}{z + h} = \frac{r_0}{h}; \qquad r = \frac{r_0}{h}(z + h)$$

Average Normal Strain: The length of the rubber band as a function of z is $L = 2\pi r = \frac{2\pi r_0}{h}(z+h)$. With $L_0 = 2r_0$, we have

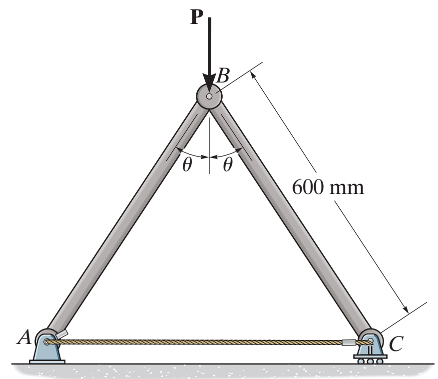
$$\epsilon_{avg} = \frac{L - L_0}{L_0} = \frac{\frac{2\pi r_0}{h}(z + h) - 2r_0}{2r_0} = \frac{\pi}{h}(z + h) - 1 \qquad \textbf{Ans.}$$



Ans:

$\epsilon_{avg} = \frac{\pi}{h}(z + h) - 1$

2–7. The pin-connected rigid rods AB and BC are inclined at $\theta = 30°$ when they are unloaded. When the force **P** is applied θ becomes 30.2°. Determine the average normal strain developed in wire AC.



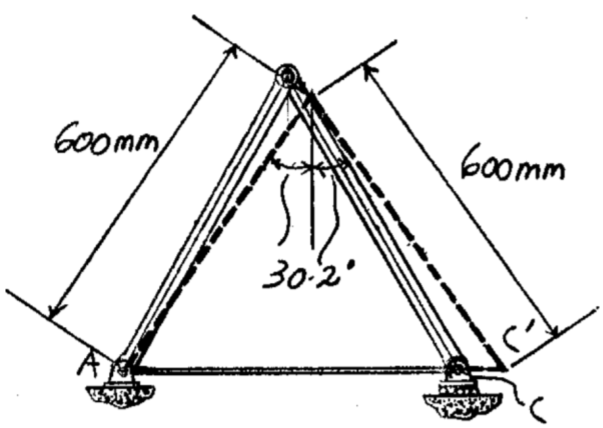
Geometry: Referring to Fig. a, the unstretched and stretched lengths of wire AD are

$$L_{AC} = 2(600 \sin 30°) = 600 \text{ mm}$$

$$L_{AC'} = 2(600 \sin 30.2°) = 603.6239 \text{ mm}$$

Average Normal Strain:

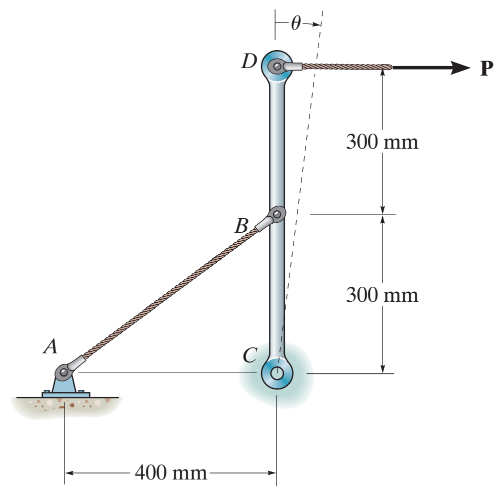
$$(\epsilon_{\text{avg}})_{AC} = \frac{L_{AC'} - L_{AC}}{L_{AC}} = \frac{603.6239 - 600}{600} = 6.04(10^{-3})\,\text{mm/mm} \qquad \textbf{Ans.}$$



(a)

Ans:
$(\epsilon_{\text{avg}})_{AC} = 6.04(10^{-3})\,\text{mm/mm}$

***2–8.** Part of a control linkage for an airplane consists of a rigid member *CBD* and a flexible cable *AB*. If a force is applied to the end *D* of the member and causes it to rotate by $\theta = 0.3^\circ$, determine the normal strain in the cable. Originally the cable is unstretched.



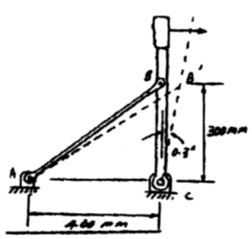
$$AB = \sqrt{400^2 + 300^2} = 500 \text{ mm}$$

$$AB' = \sqrt{400^2 + 300^2 - 2(400)(300)\cos 90.3^\circ}$$

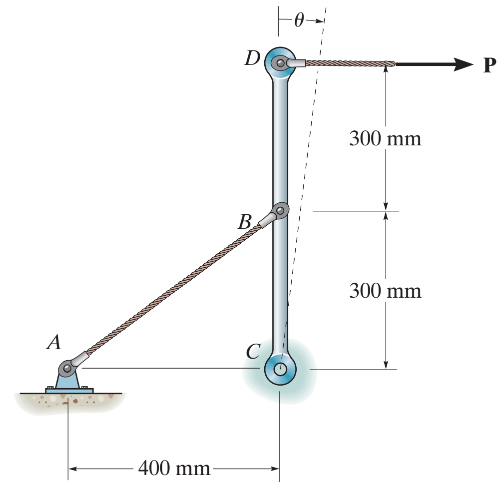
$$= 501.255 \text{ mm}$$

$$\epsilon_{AB} = \frac{AB' - AB}{AB} = \frac{501.255 - 500}{500}$$

$$= 0.00251 \text{ mm/mm}$$ **Ans.**



2–9. Part of a control linkage for an airplane consists of a rigid member *CBD* and a flexible cable *AB*. If a force is applied to the end *D* of the member and causes a normal strain in the cable of 0.0035 mm/mm, determine the displacement of point *D*. Originally the cable is unstretched.



$$AB = \sqrt{300^2 + 400^2} = 500 \text{ mm}$$

$$AB' = AB + \varepsilon_{AB}AB$$

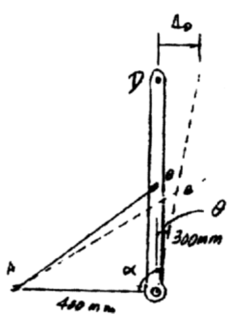
$$= 500 + 0.0035(500) = 501.75 \text{ mm}$$

$$501.75^2 = 300^2 + 400^2 - 2(300)(400)\cos\alpha$$

$$\alpha = 90.4185°$$

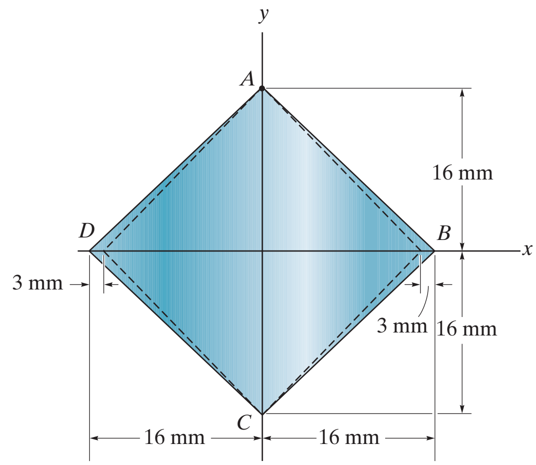
$$\theta = 90.4185° - 90° = 0.4185° = \frac{\pi}{180°}(0.4185) \text{ rad}$$

$$\Delta_D = 600(\theta) = 600\left(\frac{\pi}{180°}\right)(0.4185) = 4.38 \text{ mm}$$ **Ans.**



Ans:
$\Delta_D = 4.38$ mm

2–10. The corners of the square plate are given the displacements indicated. Determine the shear strain along the edges of the plate at *A* and *B*.



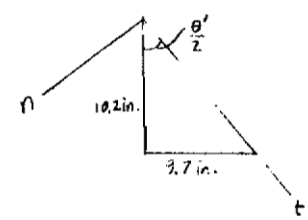
At *A*:

$$\frac{\theta'}{2} = \tan^{-1}\left(\frac{9.7}{10.2}\right) = 43.561°$$

$$\theta' = 1.52056 \text{ rad}$$

$$(\gamma_A)_{nt} = \frac{\pi}{2} - 1.52056$$

$$= 0.0502 \text{ rad}$$ **Ans.**



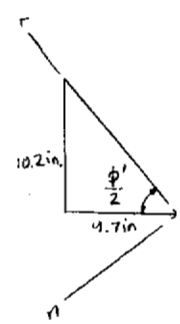
At *B*:

$$\frac{\phi'}{2} = \tan^{-1}\left(\frac{10.2}{9.7}\right) = 46.439°$$

$$\phi' = 1.62104 \text{ rad}$$

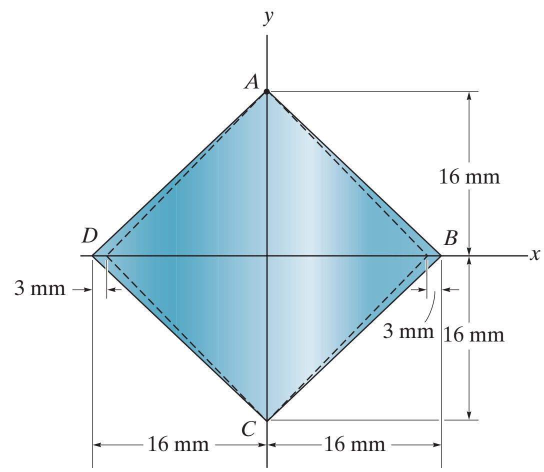
$$(\gamma_B)_{nt} = \frac{\pi}{2} - 1.62104$$

$$= -0.0502 \text{ rad}$$ **Ans.**



Ans:
$(\gamma_A)_{nt} = 0.0502$ rad, $(\gamma_B)_{nt} = -0.0502$ rad

2–11. The corners B and D of the square plate are given the displacements indicated. Determine the average normal strains along side AB and diagonal DB.



Referring to Fig. a,

$$L_{AB} = \sqrt{16^2 + 16^2} = \sqrt{512}\text{ mm}$$

$$L_{AB'} = \sqrt{16^2 + 13^2} = \sqrt{425}\text{ mm}$$

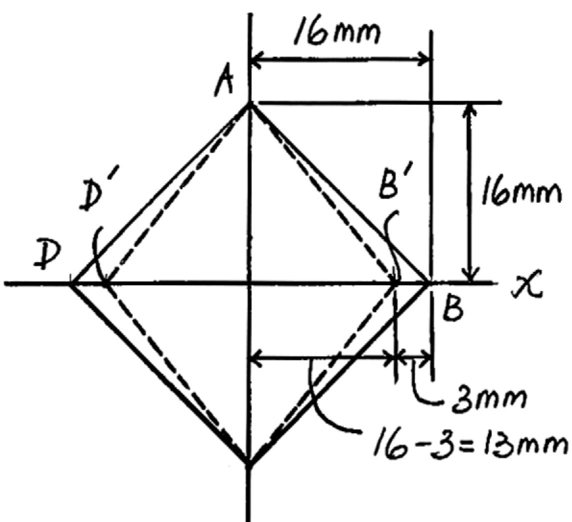
$$L_{BD} = 16 + 16 = 32\text{ mm}$$

$$L_{B'D'} = 13 + 13 = 26\text{ mm}$$

Thus,

$$\left(\varepsilon_{\text{avg}}\right)_{AB} = \frac{L_{AB'} - L_{AB}}{L_{AB}} = \frac{\sqrt{425} - \sqrt{512}}{\sqrt{512}} = -0.0889\text{ mm/mm}$$ **Ans.**

$$\left(\varepsilon_{\text{avg}}\right)_{BD} = \frac{L_{B'D'} - L_{BD}}{L_{BD}} = \frac{26 - 32}{32} = -0.1875\text{ mm/mm}$$ **Ans.**



(a)

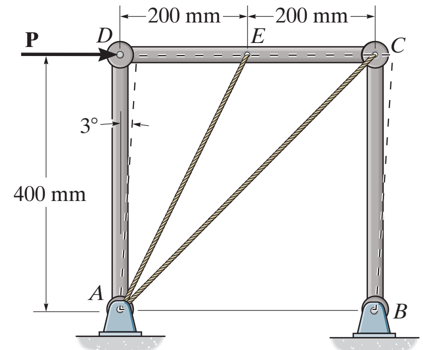
2–12

2–13

Ans:

$\epsilon_{DB} = \epsilon_{AB} \cos^2 \theta + \epsilon_{CB} \sin^2 \theta$

2–14. The force **P** applied at joint D of the square frame causes the frame to sway and form the dashed rhombus. Determine the average normal strain developed in wire AC. Assume the three rods are rigid.



Geometry: Referring to Fig. a, the stretched length of $L_{AC'}$ of wire AC' can be determined using the cosine law.

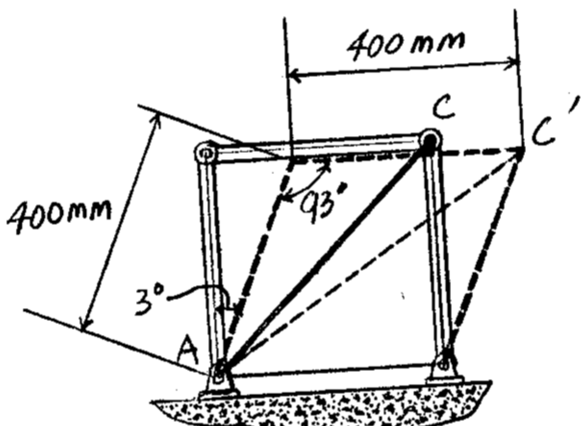
$$L_{AC'} = \sqrt{400^2 + 400^2 - 2(400)(400)\cos 93^\circ} = 580.30 \text{ mm}$$

The unstretched length of wire AC is

$$L_{AC} = \sqrt{400^2 + 400^2} = 565.69 \text{ mm}$$

Average Normal Strain:

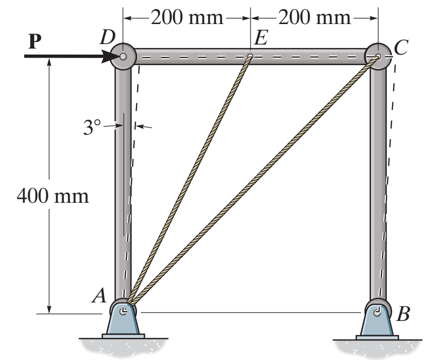
$$(\epsilon_{avg})_{AC} = \frac{L_{AC'} - L_{AC}}{L_{AC}} = \frac{580.30 - 565.69}{565.69} = 0.0258 \text{ mm/mm} \qquad \textbf{Ans.}$$



(a)

Ans:
$(\epsilon_{avg})_{AC} = 0.0258$ mm/mm

2–15. The force **P** applied at joint D of the square frame causes the frame to sway and form the dashed rhombus. Determine the average normal strain developed in wire AE. Assume the three rods are rigid.



Geometry: Referring to Fig. *a*, the stretched length of $L_{AE'}$ of wire AE can be determined using the cosine law.

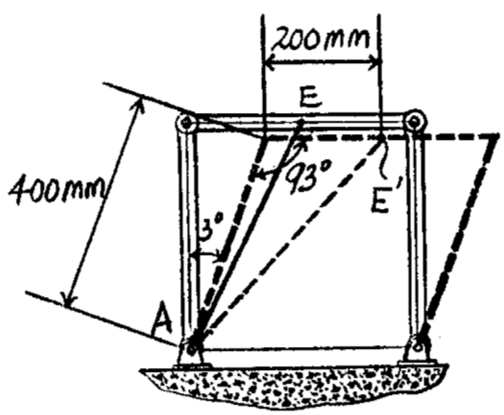
$$L_{AE'} = \sqrt{400^2 + 200^2 - 2(400)(200)\cos 93^\circ} = 456.48 \text{ mm}$$

The unstretched length of wire AE is

$$L_{AE} = \sqrt{400^2 + 200^2} = 447.21 \text{ mm}$$

Average Normal Strain:

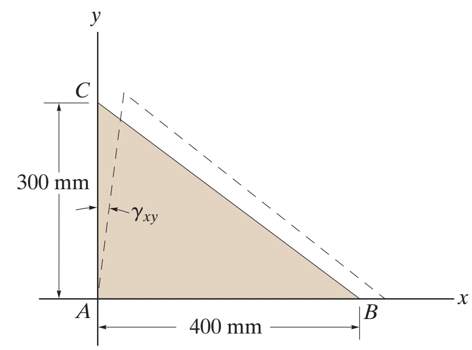
$$(\epsilon_{avg})_{AE} = \frac{L_{AE'} - L_{AE}}{L_{AE}} = \frac{456.48 - 447.21}{447.21} = 0.0207 \text{ mm/mm} \qquad \textbf{Ans.}$$



(a)

Ans:
$(\epsilon_{avg})_{AE} = 0.0207$ mm/mm

***2–16.** The triangular plate ABC is deformed into the shape shown by the dashed lines. If at A, $\varepsilon_{AB} = 0.0075$, $\epsilon_{AC} = 0.01$ and $\gamma_{xy} = 0.005$ rad, determine the average normal strain along edge BC.



Average Normal Strain: The stretched length of sides AB and AC are

$$L_{AC'} = (1 + \varepsilon_y)L_{AC} = (1 + 0.01)(300) = 303 \text{ mm}$$

$$L_{AB'} = (1 + \varepsilon_x)L_{AB} = (1 + 0.0075)(400) = 403 \text{ mm}$$

Also,

$$\theta = \frac{\pi}{2} - 0.005 = 1.5658 \text{ rad}\left(\frac{180°}{\pi \text{ rad}}\right) = 89.7135°$$

The unstretched length of edge BC is

$$L_{BC} = \sqrt{300^2 + 400^2} = 500 \text{ mm}$$

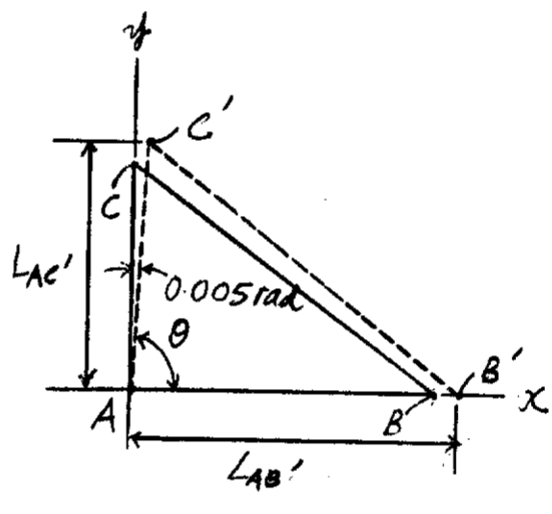
and the stretched length of this edge is

$$L_{B'C'} = \sqrt{303^2 + 403^2 - 2(303)(403)\cos 89.7135°}$$

$$= 502.9880 \text{ mm}$$

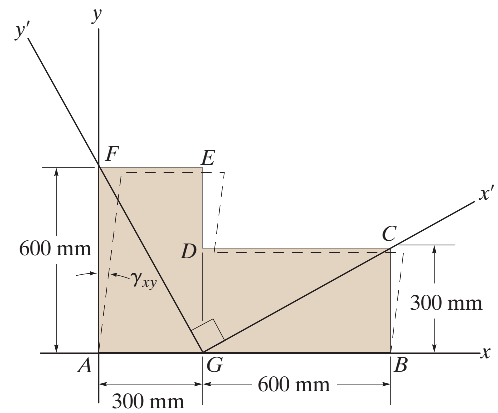
We obtain,

$$\epsilon_{BC} = \frac{L_{B'C'} - L_{BC}}{L_{BC}} = \frac{502.9880 - 500}{500} = 5.98(10^{-3}) \text{ mm/mm}$$ **Ans.**



(a)

2–17. The plate is deformed uniformly into the shape shown by the dashed lines. If at A, $\gamma_{xy} = 0.0075$ rad., while $\epsilon_{AB} = \epsilon_{AF} = 0$, determine the average shear strain at point G with respect to the x' and y' axes.



Geometry: Here, $\gamma_{xy} = 0.0075 \text{ rad}\left(\dfrac{180°}{\pi \text{ rad}}\right) = 0.4297°$. Thus,

$$\psi = 90° - 0.4297° = 89.5703° \qquad \beta = 90° + 0.4297° = 90.4297°$$

Subsequently, applying the cosine law to triangles AGF' and GBC', Fig. a,

$$L_{GF'} = \sqrt{600^2 + 300^2 - 2(600)(300)\cos 89.5703°} = 668.8049 \text{ mm}$$

$$L_{GC'} = \sqrt{600^2 + 300^2 - 2(600)(300)\cos 90.4297°} = 672.8298 \text{ mm}$$

Then, applying the sine law to the same triangles,

$$\frac{\sin\phi}{600} = \frac{\sin 89.5703°}{668.8049}; \qquad \phi = 63.7791°$$

$$\frac{\sin\alpha}{300} = \frac{\sin 90.4297°}{672.8298}; \qquad \alpha = 26.4787°$$

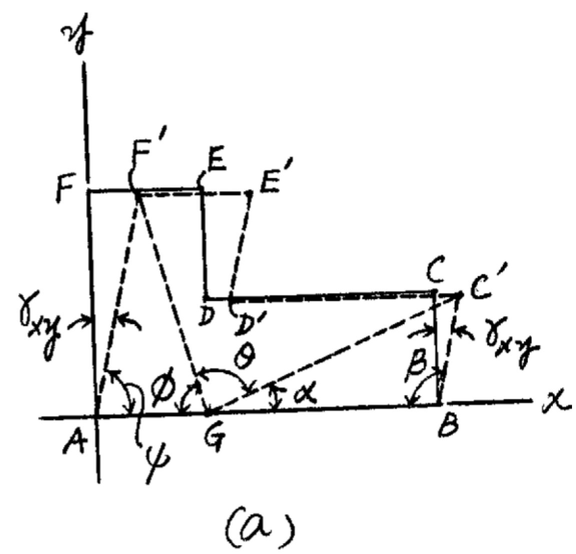
Thus,

$$\theta = 180° - \phi - \alpha = 180° - 63.7791° - 26.4787°$$

$$= 89.7422°\left(\frac{\pi \text{ rad}}{180°}\right) = 1.5663 \text{ rad}$$

Shear Strain:

$$(\gamma_G)_{x'y'} = \frac{\pi}{2} - \theta = \frac{\pi}{2} - 1.5663 = 4.50(10^{-3}) \text{ rad}$$ **Ans.**

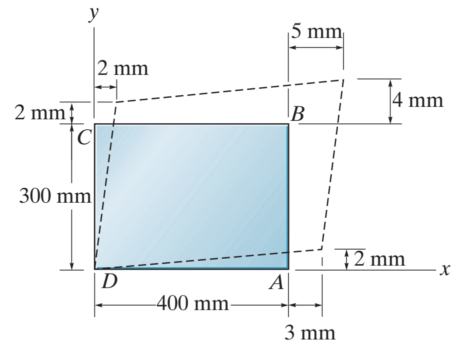


(a)

Ans:

$(\gamma_G)_{x'y'} = 4.50(10^{-3})$ rad

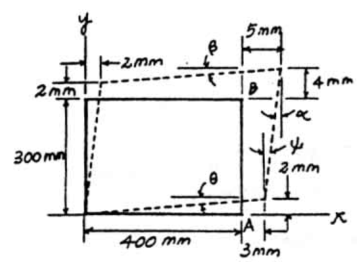
2–18. The piece of plastic is originally rectangular. Determine the shear strain γ_{xy} at corners A and B if the plastic distorts as shown by the dashed lines.



Geometry: For small angles,

$$\alpha = \psi = \frac{2}{302} = 0.00662252 \text{ rad}$$

$$\beta = \theta = \frac{2}{403} = 0.00496278 \text{ rad}$$



Shear Strain:

$$(\gamma_B)_{xy} = \alpha + \beta$$

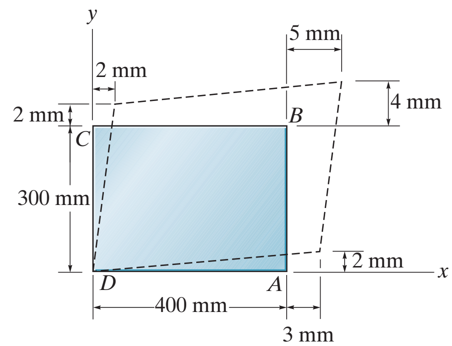
$$= 0.0116 \text{ rad} = 11.6\left(10^{-3}\right) \text{ rad} \qquad \textbf{Ans.}$$

$$(\gamma_A)_{xy} = \theta + \psi$$

$$= 0.0116 \text{ rad} = 11.6\left(10^{-3}\right) \text{ rad} \qquad \textbf{Ans.}$$

Ans:
$(\gamma_B)_{xy} = 11.6(10^{-3})$ rad,
$(\gamma_A)_{xy} = 11.6(10^{-3})$ rad

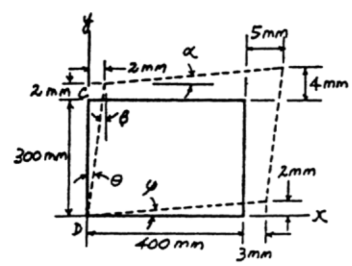
2–19. The piece of plastic is originally rectangular. Determine the shear strain γ_{xy} at corners D and C if the plastic distorts as shown by the dashed lines.



Geometry: For small angles,

$$\alpha = \psi = \frac{2}{403} = 0.00496278 \text{ rad}$$

$$\beta = \theta = \frac{2}{302} = 0.00662252 \text{ rad}$$



Shear Strain:

$$(\gamma_C)_{xy} = \alpha + \beta$$

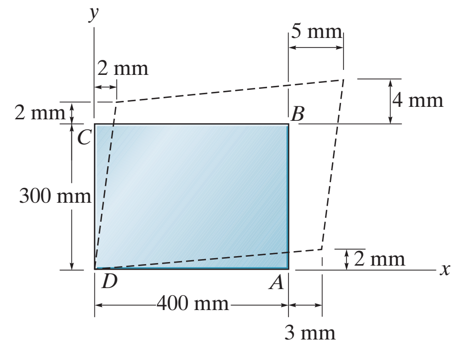
$$= 0.0116 \text{ rad} = 11.6\left(10^{-3}\right) \text{ rad}$$ **Ans.**

$$(\gamma_D)_{xy} = \theta + \psi$$

$$= 0.0116 \text{ rad} = 11.6\left(10^{-3}\right) \text{ rad}$$ **Ans.**

Ans:
$(\gamma_C)_{xy} = 11.6(10^{-3})$ rad,
$(\gamma_D)_{xy} = 11.6(10^{-3})$ rad

***2–20.** The piece of plastic is originally rectangular. Determine the average normal strain that occurs along the diagonals *AC* and *DB*.

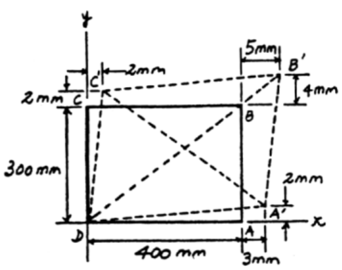


Geometry:

$$AC = DB = \sqrt{400^2 + 300^2} = 500 \text{ mm}$$

$$DB' = \sqrt{405^2 + 304^2} = 506.4 \text{ mm}$$

$$A'C' = \sqrt{401^2 + 300^2} = 500.8 \text{ mm}$$



Average Normal Strain:

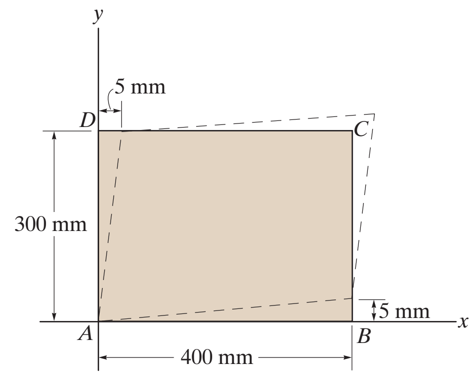
$$\epsilon_{AC} = \frac{A'C' - AC}{AC} = \frac{500.8 - 500}{500}$$

$$= 0.00160 \text{ mm/mm} = 1.60\left(10^{-3}\right) \text{ mm/mm}$$ **Ans.**

$$\epsilon_{DB} = \frac{DB' - DB}{DB} = \frac{506.4 - 500}{500}$$

$$= 0.0128 \text{ mm/mm} = 12.8\left(10^{-3}\right) \text{ mm/mm}$$ **Ans.**

2–21. The rectangular plate is deformed into the shape of a parallelogram shown by the dashed lines. Determine the average shear strain γ_{xy} at corners A and B.



Geometry: Referring to Fig. *a* and using small angle analysis,

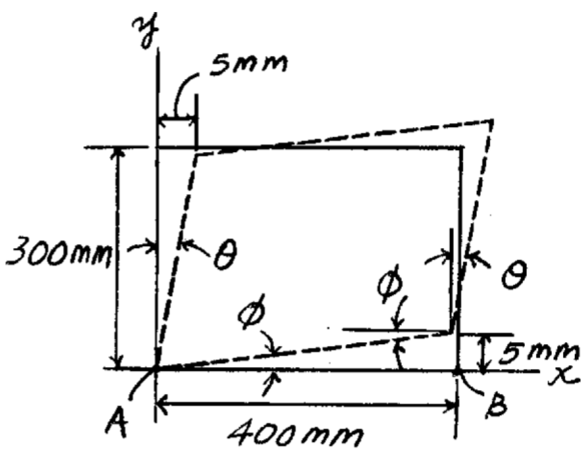
$$\theta = \frac{5}{300} = 0.01667 \text{ rad}$$

$$\phi = \frac{5}{400} = 0.0125 \text{ rad}$$

Shear Strain: Referring to Fig. *a*,

$$(\gamma_A)_{xy} = \theta + \phi = 0.01667 + 0.0125 = 0.0292 \text{ rad} \qquad \textbf{Ans.}$$

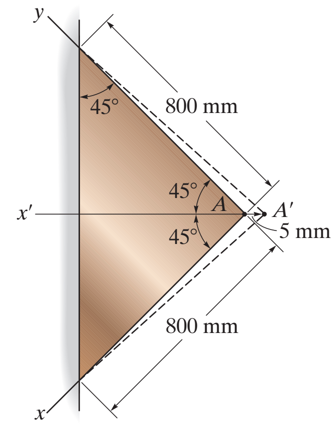
$$(\gamma_B)_{xy} = \theta + \phi = 0.01667 + 0.0125 = 0.0292 \text{ rad} \qquad \textbf{Ans.}$$



(a)

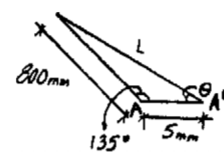
Ans:
$(\gamma_A)_{xy} = 0.0292$ rad, $(\gamma_B)_{xy} = 0.0292$ rad

2–22. The triangular plate is fixed at its base, and its apex A is given a horizontal displacement of 5 mm. Determine the shear strain, γ_{xy}, at A.



$$L = \sqrt{800^2 + 5^2 - 2(800)(5)\cos 135^\circ} = 803.54 \text{ mm}$$

$$\frac{\sin 135^\circ}{803.54} = \frac{\sin\theta}{800}; \qquad \theta = 44.75^\circ = 0.7810 \text{ rad}$$

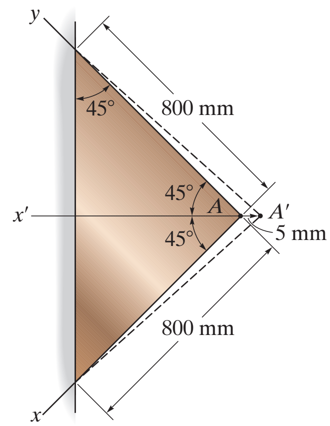


$$\gamma_{xy} = \frac{\pi}{2} - 2\theta = \frac{\pi}{2} - 2(0.7810)$$

$$= 0.00880 \text{ rad}$$ **Ans.**

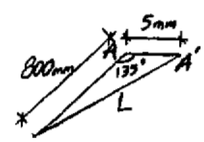
Ans:
$\gamma_{xy} = 0.00880$ rad

2–23. The triangular plate is fixed at its base, and its apex A is given a horizontal displacement of 5 mm. Determine the average normal strain ϵ_x along the x axis.



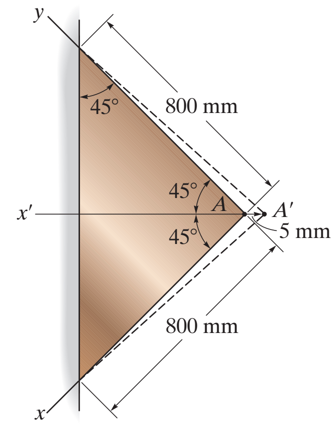
$$L = \sqrt{800^2 + 5^2 - 2(800)(5)\cos 135^\circ} = 803.54 \text{ mm}$$

$$\epsilon_x = \frac{803.54 - 800}{800} = 0.00443 \text{ mm/mm}$$ **Ans.**



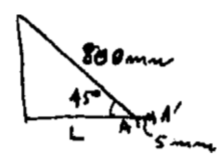
Ans:
$\epsilon_x = 0.00443$ mm/mm

***2–24.** The triangular plate is fixed at its base, and its apex A is given a horizontal displacement of 5 mm. Determine the average normal strain $\epsilon_{x'}$ along the x' axis.

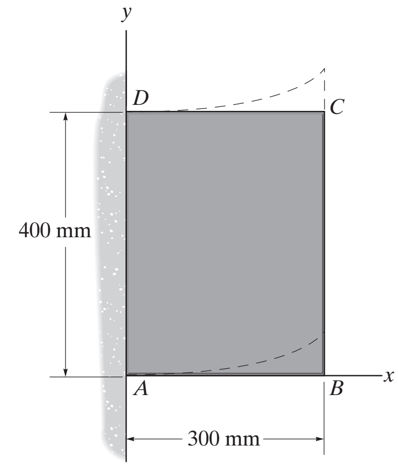


$$L = 800 \cos 45° = 565.69 \text{ mm}$$

$$\epsilon_{x'} = \frac{5}{565.69} = 0.00884 \text{ mm/mm} \qquad \textbf{Ans.}$$



2–25. The square rubber block is subjected to a shear strain of $\gamma_{xy} = 40(10^{-6})x + 20(10^{-6})y$, where x and y are in mm. This deformation is in the shape shown by the dashed lines, where all the lines parallel to the y axis remain vertical after the deformation. Determine the normal strain along edge BC.



Shear Strain: Along edge DC, $y = 400$ mm. Thus, $(\gamma_{xy})_{DC} = 40(10^{-6})x + 0.008$. Here, $\frac{dy}{dx} = \tan(\gamma_{xy})_{DC} = \tan[40(10^{-6})x + 0.008]$. Then,

$$\int_0^{\delta_c} dy = \int_0^{300\text{ mm}} \tan[40(10^{-6})x + 0.008]dx$$

$$\delta_c = -\frac{1}{40(10^{-6})}\left\{\ln\cos\left[40(10^{-6})x + 0.008\right]\right\}\Bigg|_0^{300\text{ mm}}$$

$$= 4.2003 \text{ mm}$$

Along edge AB, $y = 0$. Thus, $(\gamma_{xy})_{AB} = 40(10^{-6})x$. Here, $\frac{dy}{dx} = \tan(\gamma_{xy})_{AB} = \tan[40(10^{-6})x]$. Then,

$$\int_0^{\delta_B} dy = \int_0^{300\text{ mm}} \tan[40(10^{-6})x]dx$$

$$\delta_B = -\frac{1}{40(10^{-6})}\left\{\ln\cos\left[40(10^{-6})x\right]\right\}\Bigg|_0^{300\text{ mm}}$$

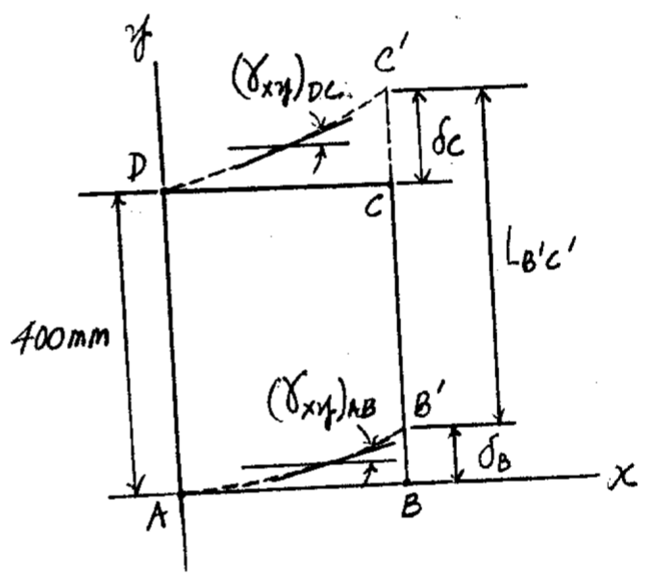
$$= 1.8000 \text{ mm}$$

Average Normal Strain: The stretched length of edge BC is

$$L_{B'C'} = 400 + 4.2003 - 1.8000 = 402.4003 \text{ mm}$$

We obtain,

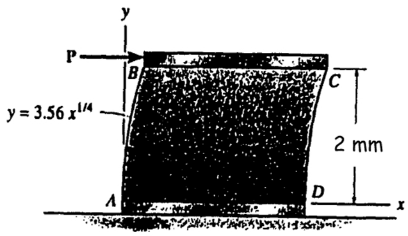
$$(\epsilon_{\text{avg}})_{BC} = \frac{L_{B'C'} - L_{BC}}{L_{BC}} = \frac{402.4003 - 400}{400} = 6.00(10^{-3}) \text{ mm/mm} \qquad \textbf{Ans.}$$



(a)

Ans:
$(\epsilon_{\text{avg}})_{BC} = 6.00(10^{-3})$ mm/mm

2–26. The Polysulfone block is glued at its top and bottom to the rigid plates. If a tangential force, applied to the top plate, causes the material to deform so that its sides are described by the equation $y = 3.56x^{1/4}$, determine the shear strain in the material at its corners A and B.



Prob. 2–33

$$y = 3.56\,x^{1/4}$$

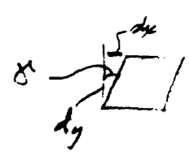
$$\frac{dy}{dx} = 0.890\,x^{-3/4}$$

$$\frac{dx}{dy} = 1.123\,x^{3/4}$$

At A, $x = 0$

$$\gamma_A = \frac{dx}{dy} = 0 \qquad \textbf{Ans}$$

At B,



$$2 = 3.56\,x^{1/4}$$

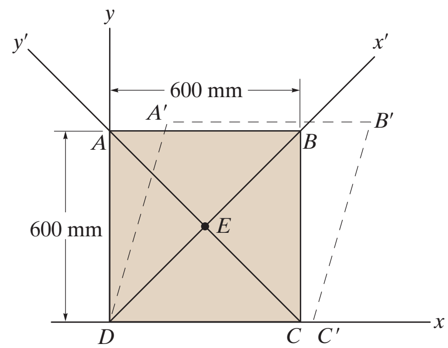
$$x = 0.0996 \text{ mm}$$

$$\gamma_B = \frac{dx}{dy} = 1.123(0.0996)^{3/4} = 0.199 \text{ rad} \qquad \textbf{Ans}$$

Ans:

$(\epsilon_{avg})_{CA} = -5.59(10^{-3})$ mm/mm

2–27. The square plate $ABCD$ is deformed into the shape shown by the dashed lines. If DC has a normal strain $\epsilon_x = 0.004$, DA has a normal strain $\epsilon_y = 0.005$ and at D, $\gamma_{xy} = 0.02$ rad, determine the shear strain at point E with respect to the x' and y' axes.



Average Normal Strain: The stretched length of sides DC and BC are

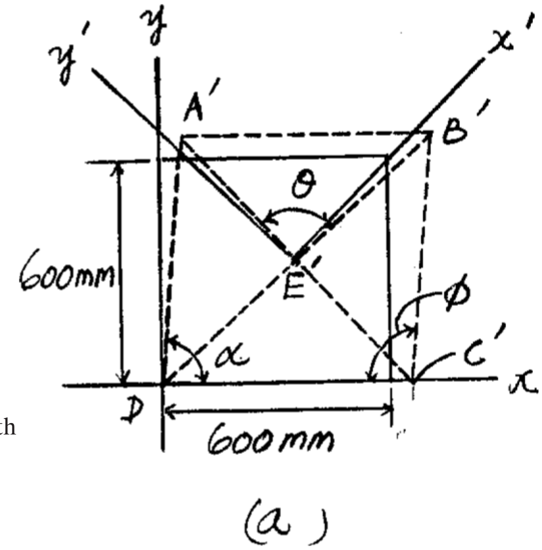
$$L_{DC'} = (1 + \epsilon_x)L_{DC} = (1 + 0.004)(600) = 602.4 \text{ mm}$$

$$L_{B'C'} = (1 + \epsilon_y)L_{BC} = (1 + 0.005)(600) = 603 \text{ mm}$$

Also,

$$\alpha = \frac{\pi}{2} - 0.02 = 1.5508 \text{ rad}\left(\frac{180°}{\pi \text{ rad}}\right) = 88.854°$$

$$\phi = \frac{\pi}{2} + 0.02 = 1.5908 \text{ rad}\left(\frac{180°}{\pi \text{ rad}}\right) = 91.146°$$



Thus, the length of $C'A'$ and DB' can be determined using the cosine law with reference to Fig. *a*.

$$L_{C'A'} = \sqrt{602.4^2 + 603^2 - 2(602.4)(603)\cos 88.854°} = 843.7807 \text{ mm}$$

$$L_{DB'} = \sqrt{602.4^2 + 603^2 - 2(602.4)(603)\cos 91.146°} = 860.8273 \text{ mm}$$

Thus,

$$L_{E'A'} = \frac{L_{C'A'}}{2} = 421.8903 \text{ mm} \qquad L_{E'B'} = \frac{L_{DB'}}{2} = 430.4137 \text{ mm}$$

Using this result and applying the cosine law to the triangle $A'E'B'$, Fig. *a*,

$$602.4^2 = 421.8903^2 + 430.4137^2 - 2(421.8903)(430.4137)\cos\theta$$

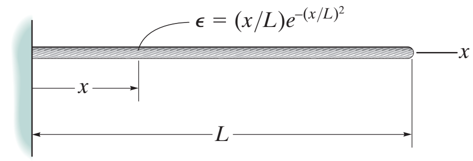
$$\theta = 89.9429°\left(\frac{\pi \text{ rad}}{180°}\right) = 1.5698 \text{ rad}$$

Shear Strain:

$$(\gamma_E)_{x'y'} = \frac{\pi}{2} - \theta = \frac{\pi}{2} - 1.5698 = 0.996(10^{-3}) \text{ rad} \qquad \textbf{Ans.}$$

Ans:
$(\gamma_E)_{x'y'} = 0.996(10^{-3})$ rad

***2–28.** The wire is subjected to a normal strain that is defined by $\epsilon = (x/L)e^{-(x/L)^2}$. If the wire has an initial length L, determine the increase in its length.

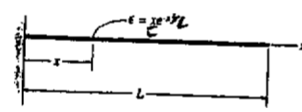


$$\Delta L = \frac{1}{L}\int_0^L x e^{-(x/L)^2} dx$$

$$= -L\left[\frac{e^{-(x/L)^2}}{2}\right]_0^L = \frac{L}{2}[1 - (1/e)]$$

$$= \frac{L}{2e}[e - 1]$$

Ans.

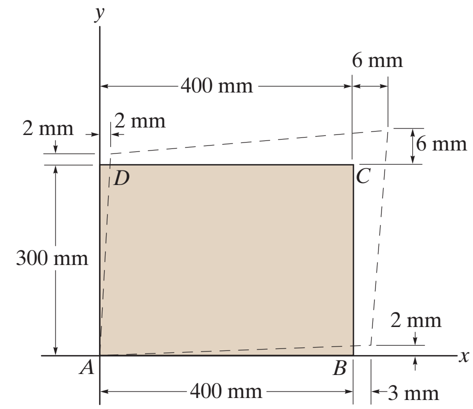


2–29.

Ans:

$(\epsilon_{avg})_{AC} = 0.0168$ mm/mm,
$(\gamma_A)_{xy} = 0.0116$ rad

2–30. The rectangular plate is deformed into the shape shown by the dashed lines. Determine the average normal strain along diagonal BD, and the average shear strain at corner B.



Geometry: The unstretched length of diagonal BD is

$$L_{BD} = \sqrt{300^2 + 400^2} = 500 \text{ mm}$$

Referring to Fig. a, the stretched length of diagonal BD is

$$L_{B'D'} = \sqrt{(300 + 2 - 2)^2 + (400 + 3 - 2)^2} = 500.8004 \text{ mm}$$

Referring to Fig. a and using small angle analysis,

$$\phi = \frac{2}{403} = 0.004963 \text{ rad}$$

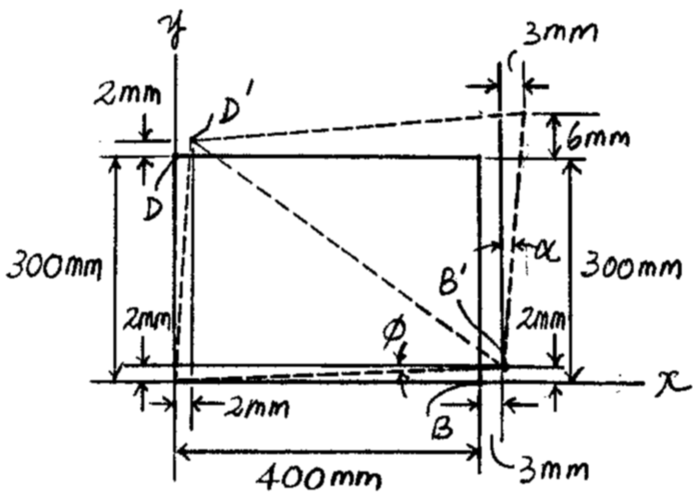
$$\alpha = \frac{3}{300 + 6 - 2} = 0.009868 \text{ rad}$$

Average Normal Strain: Applying Eq. 2,

$$(\epsilon_{avg})_{BD} = \frac{L_{B'D'} - L_{BD}}{L_{BD}} = \frac{500.8004 - 500}{500} = 1.60(10^{-3}) \text{ mm/mm} \quad \textbf{Ans.}$$

Shear Strain: Referring to Fig. a,

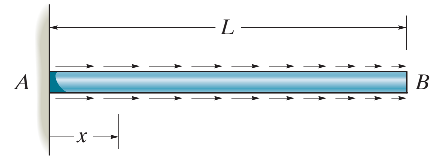
$$(\gamma_B)_{xy} = \phi + \alpha = 0.004963 + 0.009868 = 0.0148 \text{ rad} \quad \textbf{Ans.}$$



(a)

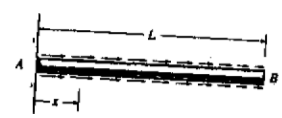
Ans:
$(\epsilon_{avg})_{BD} = 1.60(10^{-3})$ mm/mm,
$(\gamma_B)_{xy} = 0.0148$ rad

2–31. The nonuniform loading causes a normal strain in the shaft that can be expressed as $\epsilon_x = kx^2$, where k is a constant. Determine the displacement of the end B. Also, what is the average normal strain in the rod?



$$\frac{d(\Delta x)}{dx} = \epsilon_x = kx^2$$

$$(\Delta x)_B = \int_0^L kx^2 = \frac{kL^3}{3}$$ **Ans.**

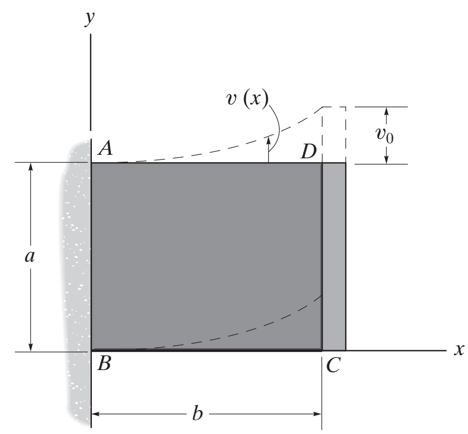


$$(\epsilon_x)_{avg} = \frac{(\Delta x)_B}{L} = \frac{\frac{kL^3}{3}}{L} = \frac{kL^2}{3}$$ **Ans.**

Ans:

$(\Delta x)_B = \frac{kL^3}{3}$, $(\epsilon_x)_{avg} = \frac{kL^2}{3}$

***2–32** The rubber block is fixed along edge AB, and edge CD is moved so that the vertical displacement of any point in the block is given by $v(x) = (v_0/b^3)x^3$. Determine the shear strain γ_{xy} at points $(b/2, a/2)$ and (b, a).



Shear Strain: From Fig. a,

$$\frac{dv}{dx} = \tan\gamma_{xy}$$

$$\frac{3v_0}{b^3}x^2 = \tan\gamma_{xy}$$

$$\gamma_{xy} = \tan^{-1}\left(\frac{3v_0}{b^3}x^2\right)$$

Thus, at point $(b/2, a/2)$,

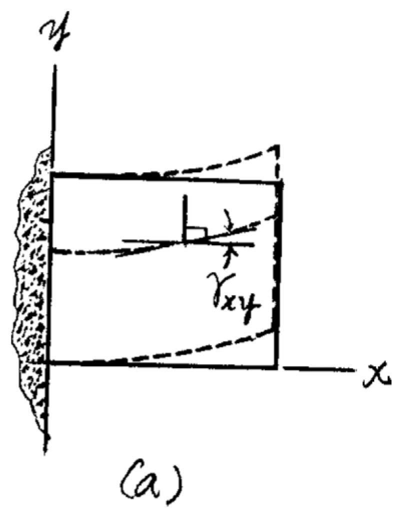
$$\gamma_{xy} = \tan^{-1}\left[\frac{3v_0}{b^3}\left(\frac{b}{2}\right)^2\right]$$

$$= \tan^{-1}\left[\frac{3}{4}\left(\frac{v_0}{b}\right)\right] \qquad \textbf{Ans.}$$

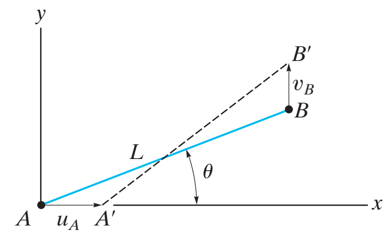
and at point (b, a),

$$\gamma_{xy} = \tan^{-1}\left[\frac{3v_0}{b^3}(b^2)\right]$$

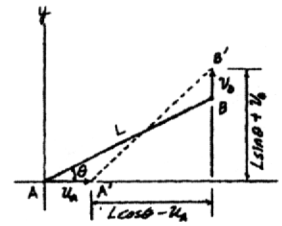
$$= \tan^{-1}\left[3\left(\frac{v_0}{b}\right)\right] \qquad \textbf{Ans.}$$



2–33. The fiber AB has a length L and orientation θ. If its ends A and B undergo very small displacements u_A and v_B, respectively, determine the normal strain in the fiber when it is in position $A'B'$.



Geometry:



$$L_{A'B'} = \sqrt{(L\cos\theta - u_A)^2 + (L\sin\theta + v_B)^2}$$

$$= \sqrt{L^2 + u_A^2 + v_B^2 + 2L(v_B\sin\theta - u_A\cos\theta)}$$

Average Normal Strain:

$$\epsilon_{AB} = \frac{L_{A'B'} - L}{L}$$

$$= \sqrt{1 + \frac{u_A^2 + v_B^2}{L^2} + \frac{2(v_B\sin\theta - u_A\cos\theta)}{L}} - 1$$

Neglecting higher terms u_A^2 and v_B^2

$$\epsilon_{AB} = \left[1 + \frac{2(v_B\sin\theta - u_A\cos\theta)}{L}\right]^{\frac{1}{2}} - 1$$

Using the binomial theorem:

$$\epsilon_{AB} = 1 + \frac{1}{2}\left(\frac{2v_B\sin\theta}{L} - \frac{2u_A\cos\theta}{L}\right) + \ldots - 1$$

$$= \frac{v_B\sin\theta}{L} - \frac{u_A\cos\theta}{L}$$ **Ans.**

Ans.
$$\epsilon_{AB} = \frac{v_B\sin\theta}{L} - \frac{u_A\cos\theta}{L}$$

2–34. If the normal strain is defined in reference to the final length, that is,

$$\epsilon'_n = \lim_{p \to p'} \left(\frac{\Delta s' - \Delta s}{\Delta s'} \right)$$

instead of in reference to the original length, Eq. 2–2 , show that the difference in these strains is represented as a second-order term, namely, $\epsilon_n - \epsilon'_n = \epsilon_n \epsilon'_n$.

$$\epsilon_B = \frac{\Delta S' - \Delta S}{\Delta S}$$

$$\epsilon_B - \epsilon'_A = \frac{\Delta S' - \Delta S}{\Delta S} - \frac{\Delta S' - \Delta S}{\Delta S'}$$

$$= \frac{\Delta S'^2 - \Delta S \Delta S' - \Delta S' \Delta S + \Delta S^2}{\Delta S \Delta S'}$$

$$= \frac{\Delta S'^2 + \Delta S^2 - 2\Delta S' \Delta S}{\Delta S \Delta S'}$$

$$= \frac{(\Delta S' - \Delta S)^2}{\Delta S \Delta S'} = \left(\frac{\Delta S' - \Delta S}{\Delta S} \right) \left(\frac{\Delta S' - \Delta S}{\Delta S'} \right)$$

$$= \epsilon_A \epsilon'_B \text{ (Q.E.D)}$$