Chapter 2

Solutions to Exercises

Exercise 2.1. Create a food web from the predator- prey table (Table 2.1 is related to the food web shown in Figure 2.1). There are various tools online that are used to construct food webs from data of this nature, but this time, try it by hand.

Question. Are there any species that are only predators and not prey, or any that are prey that are not predators? What are they? How can they be identified by looking at the food web?

Solution. See Figure S2.1.

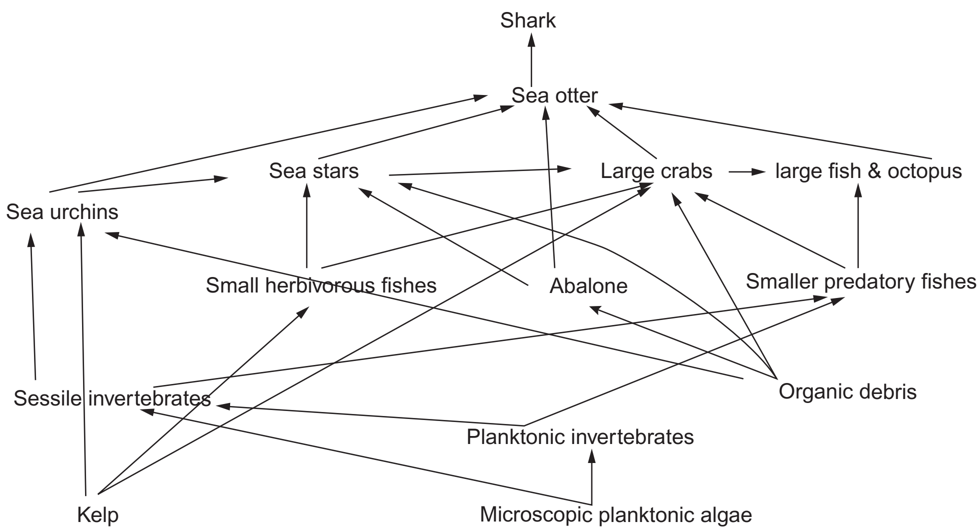

FIGURE S2.1 Marine food web.

Shark and sea otters are only predators, identified because there are no outgoing arcs. Kelp, microscopic planktonic algae, and organic debris are only prey, identified because there are no incoming arcs. □

Exercise 2.2. Expand Table 2.2 to include the trophic level of all species given in Table 2.1. Answer the following questions before moving on:

1. Do you see any challenges to using the shortest path for computing trophic level?
2. Large crabs are direct prey of sea otters, so would you think they would have different trophic levels?
3. Can you think of an alternative definition of trophic level?

Solution. See Table S2.1. □

Exercise 2.3.

1. Complete your table of trophic levels from the complete food web shown in Table 2.1 to include the trophic level using the longest path definition.

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TABLE S2.1 Solution to Exercise 2.2

Species	Trophic Level (Shortest Path)
Shark	3
Sea otter	2
Sea stars	2
Sea urchins	1
Abalone	1
Large crabs	2
Smaller predatory fishes	2
Small (herbivorous) fishes and invertebrates	1
Kelp	0
Large fishes and octopus	2
Sessile invertebrates	1
Organic debris	0
Planktonic invertebrates	1
Microscopic planktonic algae	0

2. An ecological rule of thumb is that about 10% of the energy passes from one trophic level to another. Compare these two definitions of trophic level with regard to energy loss. For example, if kelp starts with 1 million units of energy, how much energy is left for sharks using each definition?

Solution. See Table S2.2. □

Exercise 2.4. Find the trophic status of each of the species in Table 2.1.

Solution. See Table S2.3. □

Exercise 2.5. Determine the dominant species in the food web corresponding to Table 2.1 using the definition of arc removal.

Solution. Only shark is dominant using arc removal. □

Exercise 2.6.

a. Determine which species are dominant in the food web corresponding to Table 2.1 using the trophic status definition.
b. Compare your results using trophic status to the results you obtain using the definition involving arc removal.

Solution.

a. Shark, sea otter, large crabs, and large fishes and octopus.
b. More species are dominant using trophic status. □

Exercise 2.7.

a. Finish the calculations for flow-based trophic level for the food web given in Figure 2.3.
b. What happens to the flow-based trophic level for various species if an arc is removed? Try it by removing the arc from snake to hawk.

TABLE S2.2 Solution to Exercise 2.4

Species	Trophic Level Using Longest Path
Sharks	8
Sea otters	7
Sea stars	4
Sea urchins	3
Abalone	1
Large crabs	5
Smaller predatory fishes	3
Small (herbivorous) fishes and invertebrates	1
Kelp	0
Large fishes and octopus	6
Sessile invertebrates	2
Organic debris	0
Planktonic invertebrates	1
Microscopic planktonic algae	0

TABLE S2.3 Solution to Exercise 2.4

Species	Trophic Level Using Longest Path	Trophic Status
Sharks	8	62
Sea otters	7	54
Sea stars	4	13
Sea urchins	3	1
Abalone	1	1
Large crabs	5	32
Smaller predatory fishes	3	7
Small (herbivorous) fishes and invertebrates	1	1
Kelp	0	0
Large fishes and octopus	6	27
Sessile invertebrates	2	3
Organic debris	0	0
Planktonic invertebrates	1	1
Microscopic planktonic algae	0	0

Solution.

a. TL(hawk) = 2.93, TL(fox) = 3, TL(snake) = 1.9, TL(lizard) = 2, TL(rodents) = TL(other lizards) = TL(insects) = 1

b. It changes; in this case TL(hawk) = 3. □

Exercise 2.8. Answer the following questions for the weighted food web shown in Figure 2.4:

a. If you ignore the weights on the arcs, describe the effect on the food web from the removal of prairie dogs.

b. Assume that a species can survive on 50% of its normal diet. Use the weights to describe the effect on the food web from the removal of jackrabbits and small rodents.

c. Assume that a species can survive on 75% of its normal diet. Use the weights to describe the effect on the food web from the removal of black-footed ferrets.

d. Compute the flow-based trophic level of the species in the food web.

Solution.

a. The black-footed ferret has no food source and so dies off.

b. If jackrabbits and small rodents are removed, and species can survive on 50% of their normal diet, the coyote will be okay, but the golden eagle will have to increase it consumption of antelopes, prairie dogs, and ferrets, a challenge given their limited numbers.

c. If black-footed ferrets are removed, and species can survive on 75% of their normal diet, all will be okay.

d. TL(grasses) = 1, TL(jackrabbit and rodents) = TL(prairie dogs) = TL(pronghorn antelope) = 2, TL(black-footed ferrets) = 3, TL(coyote) = 3.05, TL(golden eagle) = 3.05. □

Exercise 2.9. Draw the competition graph for the food web in Figure 2.6 and *describe* the isolated vertices in the competition graph.

Solution. The competition graph is the same as the PP graph in Figure 2.14 with the addition of another isolated vertex for phytoplankton. C, HZ, and P are the isolated vertices. □

Exercise 2.10. Is the competition graph from Exercise 2.9 an interval graph? If so, show the interval representation; if not say why not.

Solution. Yes, there are many representations. □

Exercise 2.11. Give some examples of graphs that are not interval graphs.

Solution. Many examples, including any graph with a 4-cycle (square) in it. Also, the graph H in Figure 2.8 is not an interval graph. □

Exercise 2.12. Can you find a real community food web that has a competition graph that is not an interval graph?

Solution. Solution is up to the student. Real means actual ecological food web. □

Exercise 2.13. Some graphs, like H shown in Figure 2.8, cannot be represented by intervals on the real line. Can H be represented by intersecting rectangles in the plane (two-dimensional space)?

Solution. Yes, try it. □

Exercise 2.14. Find the boxicity of graph H in Figure 2.8.

Solution. The boxicity of H, box(H) = 2, is illustrated by the solution to Exercise 2.13. □

Exercise 2.15. Can you find a competition graph with boxicity 3? In other words find a graph where there is no rectangle representation of the vertices, but there is a three-dimensional box representation of the vertices so that there is an edge between two vertices if and only if the boxes overlap in three-dimensional space.

Solution. The graphs $K(n_1, n_2, \ldots n_k)$ are formed with n_1 independent vertices, n_2 independent vertices, $\ldots n_k$ independent vertices, and each of the n_i vertices is connected to each of the n_j vertices for $i \neq j$. The classic complete bipartite graph $K(2,2)$ is an example. You will note that $K(2,2)$ contains a square thus has boxicity at least 2. Roberts (2) proved that the boxicity of $K(n_1, n_2, \ldots n_k)$, denoted $\text{box}(K(n_1, n_2, \ldots n_k)) = p$, where p is the number of n_i greater than 1. Thus, $K(2,2,2)$ is an example of a graph with boxicity 3. □

Exercise 2.16. Find an example of a food web that is an interval graph, but where there is a source food web contained in it that is not an interval graph. □

Exercise 2.17. Find the connectance of the food web shown in Figure 2.11 (12, 13). Using the connectance parameter does the Shallow Water Hudson River food web appear to be stable? Why or why not?

Solution. $C = L/S^2 = 87/22^2 = 87/484 = 0.1797$

$SC = 22(.1797) = 3.95 > 2$ so likely the food web is not stable. □

Exercise 2.18. Using the partial food web in Figure 2.11 of the Hudson River, what vertices will be isolated in the competition graph? What type of species corresponds to an isolated vertex in the competition graph?

Solution. Phytoplankton, algae-attached detrius, bacteria, and water celery are isolated vertices in the competition graph. These isolated vertices are all basal species. □

Exercise 2.19. Draw the competition graph for the food web in Figure 2.11. How many isolated vertices are in the competition graph?

Solution. There are five isolated vertices, as noted in the solution to Exercise 2.18. See Figure S2.2. □

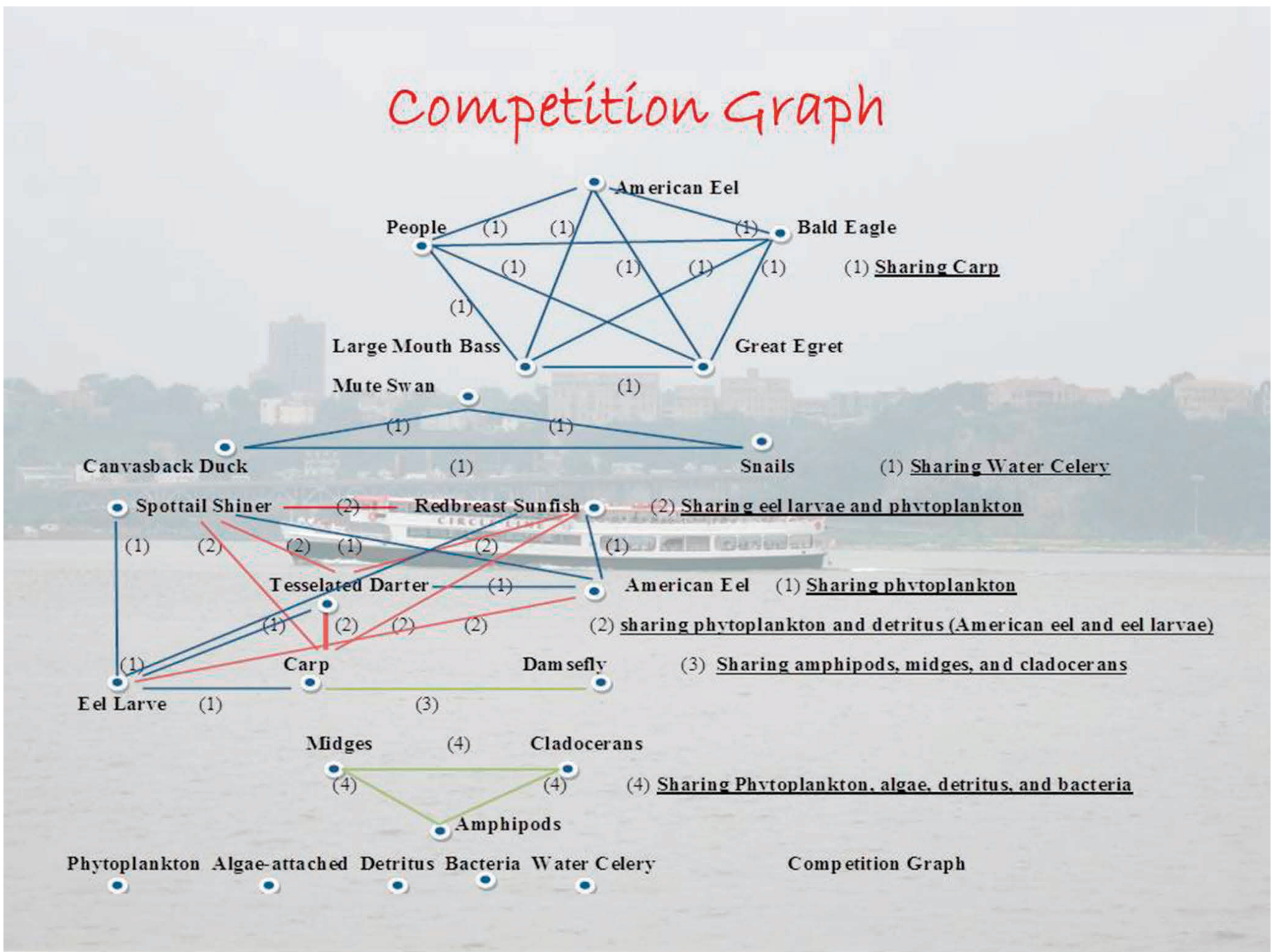

FIGURE S2.2 Competition graph for Shallow Hudson River food web.

Exercise 2.20. In this example, three isolated vertices were added to make the graph into a competition graph. Is this the least number of isolated vertices that are needed, or could we add only one, or only two isolated vertices? Experiment!

Solution. Only two are needed, as your experiment could have shown. □

Exercise 2.21. Provide a proof by construction of the statement: Any graph can be the competition graph for some acyclic directed graph by adding sufficiently many isolated vertices to it.

Proof. Given a graph G, for each pair of vertices (a,b)

- If there is an edge between a and b, build a directed graph D that includes a, b, and a vertex c along with arcs from c to a and c to b. c becomes a new isolated vertex in G.
- If there is no edge between a and b then a and b do not compete, so a and b are vertices of the directed graph, possibly connected to other vertices.

G together with isolated vertices for all such c is the competition graph of digraph D.

Example: G is the graph with vertices x, y and z and edges $\{x,y\}$ and $\{y,z\}$. D is the diagraph with vertices x, y, z, c, and d such that (c,y), (d,y), and (d,z) are arcs in D. G together with isolated vertices c and d is then the competition graph of D. □

Exercise 2.22. Compute PP and EP graphs for the food web in Figure 2.1.

Solution. See the competition graph in Figure 2.5. The PP graph is the competition graph with the isolated vertices removed; thus it is the edge {sea urchins, small fishes)

with weight = 1/6(5) = 1/30.

The EP graph is a single edge {sea urchins, large crabs} with weight = 1/6(5) = 1/30. □

Exercise 2.23. The PP and EP graphs were calculated for the food web in Figure 2.6. Can you suggest some hypotheses as a result of the network structure of these two graphs? □

Exercise 2.24. If the competition graph is and interval graph does it mean the PP and EP graphs are interval graphs, or vice versa? Experiment.

Partial Solution. The PP graph is an interval graph if and only if the competition graph is an interval graph, since basal vertices are the only differences between the two and the weights make no difference. What do you think about the EP graphs? Find a counterexample if you think it is false. □