Solutions to Probability Worksheet 3 NAME:

**Part I:** Each problem below investigates the probability that a single event occurs. For each problem, give the desired probability and either describe or list the successes. Since they involve a single event, each of the probabilities can be found be figuring

$$\text{Probability of an event} = \frac{\text{number of successes}}{\text{number of total possibilities}}$$

1. Consider a bag with ten marbles: four red, three green, and three blue. If I select one marble from the bag, what is the probability that I get a red marble?

*There are four successes (the number of red marbles) and ten possibilities (the number of marbles total). The probability is* $\frac{4}{10}$.

2. Consider a deck of poker cards. A poker deck contains four suits: diamonds, hearts, spades, and clubs. The diamonds and hearts are red and the spades and clubs are black. Each suit has thirteen cards: Ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen, and King. This makes a total of 52 cards. A face card will be defined to be a Jack, Queen, or King. If I select one card out of the deck, what is the probability I get a red Ace?

*There are two successes (the number of red Aces, one heart and one diamond) and fifty-two possibilities (the number of cards total). The probability is* $\frac{2}{52}$.

3. The following table lists the ages of the people in my knitting club, which has a total of 50 people. If I select a person from this group randomly, what is the probability that the person is 25?

| **Age** | **Number of people** |
|---|---|
| 22 | 2 |
| 25 | 8 |
| 31 | 20 |
| 39 | 15 |
| 45 | 5 |

*There are 8 successes (the number of people who are 25) and 50 possibilities (the number of people total). The probability is* $\frac{8}{50}$.

4. Margo and Juan are expecting a child. What is the probability that they have a girl?

*We assume the child is equally likely to be a boy or a girl. There is one success out of two possibilities. The probability is* $\frac{1}{2}$.

5. You roll two distinguishable dice. What is the probability that the sum is 7?

*Think about the 36 possibilities when we roll two dice. Look in your notes where I wrote out these 36 possibilities. Count the number of successes (where the sum is 7) to be 6. The probability is* $6/36$.

6. You roll a single die. What is the probability that you roll an even number?

*There are three successes (the even numbers 2, 4, and 6) and six possibilities. The probability is* $3/6$.

**Part II:** Each problem below investigates the probability that two or more events occur. For each problem, tell whether the events are mutually exclusive and/or independent. Write down the correct formula to use (from Probability Worksheet 2) and find the desired probability. By writing the formula out explicitly, it should be clear what you think the separate events are. Remember also the difference between OR and AND. Make it clear which you think applies. The first one is done for you.

1. Margo and Juan are planning on having three children. What is the probability that they have all girls? (Here, list the sample space for their three children and circle the success.)

Mutually exclusive? ___NO__

Independent? ___YES___

P(A and B and C) = P(A)*P(B)*P(C)

P(first is girl AND second is girl AND third is girl)

=P(first is girl)*P(second is girl)*P(third is girl)

$$= \frac{1}{2} * \frac{1}{2} * \frac{1}{2} = \frac{1}{8}$$

| | |
|---|---|
| GGG | BBB |
| GGB | BBG |
| GBG | BGB |
| GBB | BGG |

Three events:
1. first is girl, AND
2. second is girl, AND
3. third is girl

Notice the extension of Rule #3 from Probability Worksheet 2.

2. Consider a deck of poker cards. A poker deck contains four suits: diamonds, hearts, spades, and clubs. The diamonds and hearts are red and the spades and clubs are black. Each suit has thirteen cards: Ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen, and King. This makes a total of 52 cards. A face card will be defined to be a Jack, Queen, or King. You select one card, record its suit, replace the card in the deck, and then select another. What is the probability that you get a Spade and then a Club?
Mutually exclusive? __*NO*_____

Independent? ___*YES*____ *You replace the card, so the events are independent.*

P(A and B) = P(A)*P(B)

P($1^{st}$ is Spade AND $2^{nd}$ is Club) = P($1^{st}$ is Spade) * P($2^{nd}$ is Club)

$$= \frac{13}{52} * \frac{13}{52} = \frac{169}{2704}$$

3. Consider a deck of poker cards. A poker deck contains four suits: diamonds, hearts, spades, and clubs. The diamonds and hearts are red and the spades and clubs are black. Each suit has thirteen cards: Ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen, and King. This makes a total of 52 cards. A face card will be defined to be a Jack, Queen, or King. You select one card, record it, do **not** replace the card in the deck, and then select another. What is the probability that you get a Spade and then a Club?
Mutually exclusive? ___*NO*____

Independent? ___*NO*____ *You do* ***not*** *replace the card, so the events affect each other.*

P(A and B) = P(A) * P(B given A)

P($1^{st}$ Spade AND $2^{nd}$ Club) = P($1^{st}$ Spade)*P($2^{nd}$ Club <u>given</u> $1^{st}$ Spade)

$$= \frac{13}{52} * \frac{13}{51} = \frac{169}{2652}$$

*To find* P($2^{nd}$ is Club <u>given</u> $1^{st}$ is Spade), *we assume the first card was a Spade. So when we pick the second card, there are 51 possibilities (51 cards left in the deck) and 13 successes still (13 Clubs still left in the deck).*

4. I roll a die and toss a coin. What is the probability I get an even number and a Heads?
Mutually exclusive? ___*NO*____

Independent? ___*YES*____

P(A and B) = P(A)*P(B)

P(even on die AND heads on coin) = P(even on die)*P(heads on coin)

$$= \frac{3}{6} * \frac{1}{2} = \frac{3}{12}$$

5. I have a bag with ten marbles: four red, three green, and three blue. I will select a marble. What is the probability that I get a red or green marble?

Mutually exclusive? ___*YES*____

Independent? ___*NO*____

P(A or B) = P(A) + P(B)

P(1st red OR 1st green) = P(1st red) + P(1st green)

$$= \frac{4}{10} + \frac{3}{10} = \frac{7}{10}$$

6. I roll two distinguishable dice, a white one and a red one. What is the probability that the sum of the two dice is 6 or the white die is a 1?

Mutually exclusive? ___*NO*____

Independent? ___*NO*____

P(A or B) = P(A) + P(B) – P(A and B)

P(sum 6 OR white 1) = P(sum 6) + P(white 1) – P(sum 6 AND white 1)

= P(sum 6) + P(white 1) – P(sum 6)*P(white 1 given sum 6)

$$= \frac{5}{36} + \frac{1}{6} - \frac{5}{36} * \frac{1}{5} = \frac{5}{36} + \frac{6}{36} - \frac{1}{36} = \frac{10}{36}$$

*To figure* P(white 1 given sum 6), *think about the outcomes whose sum is 6. There are 5 such outcomes (number of possibilities). Now, out of those 5 outcomes, count the number where the white die is 1. That should be 1 of those 5 outcomes.*

7. I roll two distinguishable dice, a white one and a red one. What is the probability that the sum of the two dice is 6 or the white die is 6?

Mutually exclusive? ___*YES*____

Independent? ____*NO*___

P(A or B) = P(A) + P(B)

P(sum 6 OR white 6) = P(sum 6) + P(white 6)

$$= \frac{5}{36} + \frac{1}{6} = \frac{5}{36} + \frac{6}{36} = \frac{11}{36}$$

*Here, unlike #6, the two events are mutually exclusive because you cannot have one die be a 6 and the sum be 6 also. So we do not need to subtract off the* P(sum 6 AND white 6) *because that probability is 0.*

8. I roll two distinguishable dice, a white one and a red one. What is the probability that the sum of the two dice is 6 and the white die is 1?
Mutually exclusive? ___*NO*____

Independent? ___*NO*____ *Knowing the sum is 6 affects the probability the white is 1.*

P(A and B) = P(A)*P(B given A)

P(sum 6 AND white 1) = P(sum 6)*P(white 1 given sum 6)

$$= \frac{5}{36} * \frac{1}{5} = \frac{1}{36}$$

*To figure* P(white 1 given sum 6), *think about the outcomes whose sum is 6. There are 5 such outcomes (number of possibilities). Now, out of those 5 outcomes, count the number where the white die is 1. That should be 1 of those 5 outcomes.*

9. I roll a die and toss a coin. What is the probability I get an even number or a Heads?
Mutually exclusive? ___*NO*____

Independent? ___*YES*____

P(A or B) = P(A) + P(B) – P(A and B)

P(even OR heads) = P(even) + P(heads) – P(even AND heads)
= P(even) + P(heads) – P(even)* P(heads)

$$= \frac{1}{2} + \frac{1}{2} - \frac{1}{2} * \frac{1}{2} = \frac{1}{2} + \frac{1}{2} - \frac{1}{4} = \frac{2}{4} + \frac{2}{4} - \frac{1}{4} = \frac{3}{4}$$

10. Consider a deck of poker cards. A poker deck contains four suits: diamonds, hearts, spades, and clubs. The diamonds and hearts are red and the spades and clubs are black. Each suit has thirteen cards: Ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen, and King. This makes a total of 52 cards. A face card will be defined to be a Jack, Queen, or King. I will select one card from the deck. What is the probability that it is a red face card? (We can find this probability by finding "number of successes divided by number of possibilities" getting $6/52$. However, let's find it by considering the two events "card is red" and "card is face".)

| | |
|---|---|
| Mutually exclusive? ___*NO*__<br>Independent? ___*YES*__<br>*Technically, these two events are independent. Assuming they are not does not hurt our solution any. If you desire, you can read over the document "Exploration of independence on Number 10 on Probability Worksheet 3" for more discussion of this. You can also see this by finding P(face) and P(face given red) and seeing they are equal.* | P(A and B) = P(A) * P(B)<br>P(red AND face)<br>= P(red)*P(face)<br>$= \frac{26}{52} * \frac{12}{52} = \frac{312}{2704} = \frac{6}{52}$<br>*I reduced my answer so you can see it is equal to the answer given above.* |