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Solvability of One Problem for Differential Equations of Shell Theory of Timoshenko Type

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Abstract

During the solution of complex problems of the theory of elasticity, the problem of the chosen model adequacy to real processes comes to the fore. The existence of existence theorems for a rigorous mathematical study of boundary value problem solvability makes it easy to prove the convergence of numerical methods to an exact real solution. Therefore, a rigorous study of boundary value problem solvability, the proof of existence theorems is a very urgent problem in the theory of elasticity. By now, the solvability of boundary value problems for nonlinear partial differential equations describing the equilibrium state of shells in the framework of the simplest Kirchhoff - Love model has been studied sufficiently. Fundamental results in this area were obtained by I.I. Vorovich, N.F. Morozov and their students. At the same time, there is an increased interest in the study of nonlinear boundary value problem solvability for partial differential equations in the framework of more complex models that are not based on the Kirchhoff - Love hypothesis. This is explained not only by the relative development of the classical theory, but also by the significant expansion of the field concerning the engineering application of the theory. The need to study boundary value problems for more complex differential equations was pointed out by the Academician I.I. Vorovich and such problems were included in the list of unsolved problems in the mathematical theory of shells. This work is devoted to the study of a nonlinear boundary value problem solvability for shallow isotropic shells by S.P. Timoshenko with hinged edges. The research method consists in reducing the original boundary value problem to a single nonlinear operator equation in Sobolev space. The method is based on integral representations for displacements, which are constructed using general solutions of the inhomogeneous Cauchy - Riemann equation. Integral representations contain arbitrary holomorphic functions, which are found using explicit representations of solutions to the Riemann - Hilbert problem in the single circle. Finding holomorphic functions is one of the main and difficult moments of the proposed research. The integral representations constructed in this way make it possible to reduce the original problem to one nonlinear equation, the solvability of which is established using the principle of contracted mappings.

Key words: the system of equilibrium equations, integral representations, contraction mapping principle, existence theorem.

I. INTRODUCTION

In this paper, they consider the following model of the theory of shallow shells by S.P. Tymoshenko:

1) the relationship of deformation - displacement [1, pp. 168 – 170, 269]:

$$\gamma_{jj}^0 = w_{j\alpha^j} - k_j w_3 + w_{3\alpha^j}^2/2,\ j=1,2,\quad \gamma_{12}^0 = w_{1\alpha^2} + w_{2\alpha^1} + w_{3\alpha^1}w_{3\alpha^2},$$

$$\gamma_{jj}^1 = \psi_{j\alpha^j},\ j=1,2,\qquad \gamma_{12}^1 = \psi_{1\alpha^2} + \psi_{2\alpha^1}, \tag{1}$$

$$\gamma_{j3}^0 = w_{3\alpha^j} + \psi_j,\ j=1,2,\quad \gamma_{33}^0 = \gamma_{k3}^1 \equiv 0,\ k=\overline{1,3},$$

where $\gamma_{ij}^k\left(i,j=\overline{1,3},k=0,1\right)$ - the components of the deformations of the shell middle surface S_0; $w_i(i=1,2)$ и w_3 - tangential and normal displacements of points S_0; $\psi_i\left(i=1,2\right)$ - the angles of normal section rotation; α^1,α^2 - Cartesian coordinates of points of a flat bounded domain Ω with the boundary Γ, homeomorphic S_0;

2) constitutive relations: $\sigma^{ij} = B^{ijkn}\gamma_{kn},\ i\le j,k\le n; i,j,k,n=\overline{1,3};$ hereinafter, the summation is carried out by the repeated Latin indices from 1 to 3, by the Greek ones - from 1 to 2, where $\gamma_{kn} = \gamma_{kn}^0 + \alpha^3\gamma_{kn}^1;$ B^{ijkn} - the elastic characteristics of the

shell: $B^{1111}=B^{2222}=E/(1-\mu^2)$, $B^{1122}=\mu E/(1-\mu^2)$, $B^{1212}=E/(2(1+\mu))$, $B^{1313}=B^{2323}=Ek^2/(2(1+\mu))$; the rest $B^{ijkn}=0$; $\mu=const$ - Poisson's ratio, $E=const$ - Young's modulus, $k_1,k_2=const$ - the main curvatures; $k^2=const$ - shear coefficient;

3) boundary conditions on Γ:

$$w_1=\psi_1=0; \tag{2}$$

4) mass $\vec{F}(\alpha^1,\alpha^2,\alpha^3)$ and surface $\vec{F}^{\pm}(\alpha^1,\alpha^2)$ forces act on the shell, and the forces $\vec{F}^0(s,\alpha^3)$ are applied at the shell boundary.

Using the variational Lagrange principle, we obtain the equilibrium equations:

$$\begin{gathered} T^{i\lambda}_{\alpha^\lambda}+R^i=0, i=1,2,\\ T^{\lambda 3}_{\alpha^\lambda}+k_\lambda T^{\lambda\lambda}+\left(T^{\lambda\mu}w_{3\alpha^\mu}\right)_{\alpha^\lambda}+R^3=0,\\ M^{i\lambda}_{\alpha^\lambda}-T^{i3}+L^i=0, i=1,2 \end{gathered} \tag{3}$$

and the static boundary conditions on Γ:

$$\begin{gathered} T^{12}\,d\alpha^2/ds-T^{22}\,d\alpha^1/ds=P^2(s),\\ T^{13}\,d\alpha^2/ds-T^{23}\,d\alpha^1/ds+T^{11}w_{3\alpha^1}\,d\alpha^2/ds-T^{22}w_{3\alpha^2}\,d\alpha^1/ds+T^{12}(w_{3\alpha^2}\,d\alpha^2/ds-w_{3\alpha^1}\,d\alpha^1/ds)=P^3(s),\\ M^{12}\,d\alpha^2/ds-M^{22}\,d\alpha^1/ds=N^2(s), \end{gathered} \tag{4}$$

where T^{ij} - the efforts, M^{ij} - the moments:

$$T^{ij}\equiv T^{ij}(a)=D^{ijkn}_0\gamma^0_{kn}, M^{ij}\equiv M^{ij}(a)=D^{ijkn}_2\gamma^1_{kn}, D^{ijkn}_m=\int_{-h/2}^{h/2}B^{ijkn}(\alpha^3)^m\,d\alpha^3; \tag{5}$$

$R^i\,(i=\overline{1,3}), L^j\,(j=1,2), P^2, P^3, N^2$ depend on external forces; $a=(w_1,w_2,w_3,\psi_1,\psi_2)$ - the vector of generalized displacements, $h=const$ - shell thickness.

Problem A. It is required to find a solution to the system (3) that satisfies the boundary conditions (2), (4).

II. METHODS

We will study the boundary value problem A in a generalized setting. Let the following conditions be satisfied: a) Ω is a simply connected domain with the boundary $\Gamma\in C^1_\beta$, *b)* $\vec{F}\in L_p(\Omega)\times L_1[-h/2,h/2]$, $\vec{F}^{\pm}\in L_p(\Omega)$, $\vec{F}^0\in C_\beta(\Gamma)\times L_1[-h/2,h/2]$; here and below, everywhere $p>2, 0<\beta<1$.

During displacements, the equilibrium equations (3) take the following form

$$\begin{gathered} w_{1\alpha^1\alpha^1}+\mu_1 w_{1\alpha^2\alpha^2}+\mu_2 w_{2\alpha^1\alpha^2}=f_1,\\ \mu_1 w_{2\alpha^1\alpha^1}+w_{2\alpha^2\alpha^2}+\mu_2 w_{1\alpha^1\alpha^2}=f_2,\\ k^2\mu_1\left(w_{3\alpha^1\alpha^1}+w_{3\alpha^2\alpha^2}+\psi_{1\alpha^1}+\psi_{2\alpha^2}\right)+k_3 w_{1\alpha^1}+k_4 w_{2\alpha^2}-k_5 w_3+\\ +k_3 w^2_{3\alpha^1}/2+k_4 w^2_{3\alpha^2}/2+\beta_2\left[\left(T^{\lambda\mu}w_{3\alpha^\lambda}\right)_{\alpha^\mu}+R^3\right]=0\\ \psi_{1\alpha^1\alpha^1}+\mu_1\psi_{1\alpha^2\alpha^2}+\mu_2\psi_{2\alpha^1\alpha^2}=\tilde{g}_1,\\ \mu_1\psi_{2\alpha^1\alpha^1}+\psi_{2\alpha^2\alpha^2}+\mu_2\psi_{1\alpha^1\alpha^2}=\tilde{g}_2, \end{gathered} \tag{6}$$

the boundary conditions on Γ are transformed to the following form

$$\mu_1\left(w_{1\alpha^2}+w_{2\alpha^1}\right)(t)\,d\alpha^2/ds-\left(\mu w_{1\alpha^1}+w_{2\alpha^2}\right)(t)\,d\alpha^1/ds=\varphi_2(w_3)(t), \tag{7}$$

$$\mu_1\left(\psi_{1\alpha^2}+\psi_{2\alpha^1}\right)(t)\,d\alpha^2/ds-\left(\mu\psi_{1\alpha^1}+\psi_{2\alpha^2}\right)(t)\,d\alpha^1/ds=\tilde{\varphi}_2(t), \tag{8}$$

$$T^{13}\,d\alpha^2/ds - T^{23}\,d\alpha^1/ds + T^{11}w_{3\alpha^1}\,d\alpha^2/ds - T^{22}w_{3\alpha^2}\,d\alpha^1/ds + \\ +T^{12}(w_{3\alpha^2}\,d\alpha^2/ds - w_{3\alpha^1}\,d\alpha^1/ds) = P^3(s). \qquad (9)$$

The following designations are adopted in (6) - (9)

$$f_j \equiv f_j\left(w_3\right) = k_{j+2}w_{3\alpha^j} - w_{3\alpha^j}w_{3\alpha^j\alpha^j} - \mu_2 w_{3\alpha^{3-j}}w_{3\alpha^1\alpha^2} - \mu_1 w_{3\alpha^j}w_{3\alpha^{3-j}\alpha^{3-j}} - \beta_2 R^j,\quad j=1,2,$$

$$\tilde{g}_j = g_j + k_0\psi_j,\quad g_j \equiv g_j\left(w_3\right) = k_0 w_{3\alpha^j} - \beta_1 L^j,\, j=1,2,\, \mu_1 = \left(1-\mu\right)/2,\, \mu_2 = \left(1+\mu\right)/2, \qquad (10)$$

$$\varphi_2\left(w_3\right)(t) = \beta_2 P^2\left(s\right) + \left[-k_4 w_3(t) + \mu w_{3\alpha^1}^2(t)/2 + w_{3\alpha^2}^2(t)/2\right]d\alpha^1/ds - \mu_1\, w_{3\alpha^1}(t)w_{3\alpha^2}(t)d\alpha^2/ds,$$

$$\tilde{\varphi}_2\left(t\right) = \beta_1 N^2\left(s\right),\quad t = t(s) = \alpha^1(s) + i\alpha^1(s) \in \Gamma,\, k_3 = k_1 + \mu k_2,\, k_4 = k_2 + \mu k_1,$$

$$k_5 = k_1^2 + k_2^2 + 2\mu k_1 k_2,\, k_0 = 6k^2\left(1-\mu\right)/h^2,\, \beta_1 = 12\left(1-\mu^2\right)/\left(h^3 E\right),\, \beta_2 = \left(1-\mu^2\right)/\left(Eh\right).$$

Definition. A generalized solution of the problem A is the vector of generalized displacements $a = \left(w_1, w_2, w_3, \psi_1, \psi_2\right) \in W_p^{(2)}(\Omega)$, $p > 2$, almost everywhere satisfying the system (6) and pointwise boundary conditions (2), (7), (8), (9).

Currently, there is a large number of works devoted to the calculation of the strength of elastic structures taking into account geometric and (or) physical nonlinearity. This is due to the widespread use of elastic structures in aviation, space technology, shipbuilding, mechanical engineering and construction. In very rare cases, nonlinear problems are solved in a closed form. For this reason, a wide range of approximate computer methods is used to solve them. In the numerical solution of problems, the problem of the numerical solution convergence to the exact (real) solution of the problem always comes to the fore. As is known, the solution to this problem is based on a rigorous mathematical study of boundary value problem solvability and the proof of existence theorems. At one time, Professor S.G. Mikhlin noted that “the proof of the solution existence is a test of the chosen model adequacy” (The Journal “St. Petersburg University", No. 8, 2008). The existence of existence theorems makes it easy to prove the convergence of numerical methods to an exact real solution and contributes to a deep understanding of the studied mechanical phenomena. Therefore, a rigorous study of boundary value problem solvability, the proof of existence theorems and the development of the methods for finding solutions are a very urgent problem in the mathematical theory of elasticity.

At present, the solvability of nonlinear boundary value problems in the theory of thin shallow elastic shells has been sufficiently studied in the framework of the simplest Kirchhoff-Love model. The issues of the existence of solutions to nonlinear problems within the framework of more general models of the theory of shells that are not based on the Kirchhoff-Love hypotheses were included in the well-known list of unsolved problems in the mathematical theory of shells by I.I. Vorovich and until recently remained open. Nowadays, there is a number of works devoted to the study of nonlinear problems in the framework of the Timoshenko shear model [2–9]. The studies in [2–9] are based on integral representations for generalized displacements containing arbitrary holomorphic functions, which are found in such a way that generalized displacements satisfy the given boundary conditions. Two approaches are used to construct them. The first approach is based on the use of explicit representations of solutions to the Riemann - Hilbert problems for holomorphic functions in the single circle. Therefore, a flat domain homeomorphic to the middle surface of the shell is either assumed from the very beginning to be the single circle [2–5], or is mapped conformally onto the single circle [6], [9]. In the second approach, the theory of one-dimensional singular integral equations [7], [8] is used to define holomorphic functions. In this paper, the method of conformal mapping is used to study a nonlinear problem for arbitrary shallow shells with other boundary conditions.

III. RESULTS AND DISCUSSION

Let's consider the system of the first two equations in (6), in which the deflection is assumed to be fixed temporarily. The general solution of the system (1) has the form [2]:

$$\omega_0(z) = w_2 + iw_1 = \Phi_2(z) + iTd[\Phi_1 + Tf](z),\quad z = \alpha^1 + i\alpha^2,\quad f = (f_1 + if_2)/2, \qquad (11)$$

where $\Phi_1(z) \in C_\alpha\left(\overline{\Omega}\right), \Phi_2(z) \in C_\alpha^1\left(\overline{\Omega}\right)$ – arbitrary holomorphic functions;

$$Tf = -\frac{1}{\pi}\iint_\Omega \frac{f\left(\zeta\right)}{\zeta - z}d\xi d\eta, \zeta = \xi + i\eta,\; d[g] = d_1 g + d_2\overline{g},\quad d_j = (\mu_1 + (-1)^j)/(4\mu_1), j = 1,2.$$

where c_0 – arbitrary real constant.

Let us find the holomorphic functions $\Phi_j(z), j=1,2$ so that the tangential displacements w_1, w_2 (11) satisfy the boundary conditions (2), (7). Following [6], for tangential displacements w_1, w_2 under the condition of solvability of the form

$$\int_\Gamma P^2(s)\,ds + \iint_\Omega R^2 d\alpha^1 d\alpha^2 = 0, \qquad (12)$$

we obtain the required representation

$$\omega_0(z) = H_0 w_3(z) + c_0, \quad z \in \Omega, \qquad (13)$$

$$H_0 w_3(z) \equiv H_0[f(w_3); l(w_3)](z) = \Phi_2[l(w_3)](\psi(z)) + iTd[\Phi_1[l(w_3)](\psi(\zeta)) + Tf(w_3)(\zeta)](z),$$

where

$$\Phi_2[l(w_3)](\psi(z)) = -\frac{1}{2\pi}\int_{\partial K}(\operatorname{Re}Td[\Phi_1[l(w_3)]](t) + \operatorname{Re}Td[Tf](t))\frac{t+\psi(z)}{t-\psi(z)}\frac{dt}{t},$$

$$\Phi_1[l(w_3)](\tilde\zeta) = 2(\mu-1)S_{\partial K}(\operatorname{Re}Td[Tf](\varphi(t)))(\tilde\zeta) + \frac{(\mu-1)}{\pi}\int_{\partial K}\frac{l(w_3)(\varphi(t))|\varphi'(t)|}{t(t-\tilde\zeta)}dt,$$

$$S_{\partial K} f(\tilde\zeta) = \frac{1}{2\pi i}\int_{\partial K}\frac{f(t)}{(t-\tilde\zeta)^2}dt, \quad \tilde\zeta \in K, t \in \partial K,$$

$$l(w_3)(\tau) = \frac{\varphi_2(w_3)(\tau)}{\mu-1} + \operatorname{Re}\{t' Sd[Tf]^+(\tau) - \mu_3\, d\alpha^1/ds \operatorname{Re}Tf(\tau)\} \equiv l[f(w_3); \varphi_2(w_3)], \quad \tau \in \Gamma;$$

via $Sd[\Phi_1]^+(\tau)$ the limit of the function $Sd[\Phi_1](z)$ is denoted at $z \to \tau \in \Gamma$ within the domain Ω; $z = \varphi(\zeta)$ – conformal mapping of a single circle $\overline{K}: |\zeta| \le 1$ on the domain $\overline{\Omega}$; $\zeta = \psi(z)$ – the function inverse to $z = \varphi(\zeta)$; $t' = dt/d\sigma, d\sigma$ – the element of a circular arc $\partial K, c_0$ – an arbitrary real constant.

We proceed to finding the functions ψ_1, ψ_2 from the last two equations in the system (6) that satisfy the conditions (2), (8) on Γ.

Let's note that the structure of the left-hand sides of the last two equations in (6) is the same as in the case of tangential displacements; they differ only in the right-hand sides. Therefore, for the angles of rotation ψ_1, ψ_2 with fixed right-hand sides, we immediately obtain a representation similar to (11):

$$\psi = \psi_2 + i\psi_1 = H_0[\tilde g(\upsilon); \tilde l(\upsilon)] + c_1, \quad z \in \Omega, \qquad (14)$$

where the following designations are accepted

$$\upsilon = \upsilon_2 + i\upsilon_1, \quad \tilde g(\upsilon) = (\tilde g_1(\upsilon) + i\tilde g_2(\upsilon))/2, \qquad (15)$$

$$\upsilon_j = w_{3\alpha^j} + \psi_j, \quad \tilde g_j(\upsilon) = k_0\upsilon_j - \beta_1 L^j, \ \tilde l(\upsilon)(t) = \tilde\varphi_2(t)/(\mu-1) + h_2\tilde g(\upsilon)(t),$$
$$h_2\tilde g(\upsilon)(t) = -\operatorname{Re}\{t' Sd[T\tilde g(\upsilon)]^+(t) - \mu_3 d\alpha^1/ds \operatorname{Re}T\tilde g(\upsilon)(t)\}, \quad t \in \Gamma,$$

c_1 – arbitrary real constants; the operator $H_0[\tilde g(\upsilon); \tilde l(\upsilon)]$ is defined by the formula in (13).

In this case, the solvability condition must be satisfied.

$$\beta_1\left(\int_\Gamma N^2(s)\,ds + \iint_\Omega L^2 d\alpha^1 d\alpha^2\right) - k_0\iint_\Omega \upsilon_2\, d\alpha^1 d\alpha^2 = 0, \qquad (16)$$

where N^2, L^2 – the components of the external load, $\upsilon_2 \in W_p^{(1)}(\Omega)$ – the function introduced in the formula (15).

Thus, under the conditions (12), (16), the problem A with fixed w_3, $\upsilon_j(j=1,2)$ is solvable with respect to tangential displacements and angles of rotation; its solutions are given by the formulas (13), (14).

For further research, we use the method of [2], [6]. Taking into account the solution of the system (6) with respect to tangential displacements and angles of rotation, the conditions (2), (7), (8), integral representations (13), (14) are developed for $w_j, \psi_j (j=1,2)$. Now let us examine the third equation in (6). Before proceeding to it, we express the deflection w_3 and its derivatives through $\upsilon_j(j=1,2)$. Then, using the functions $\upsilon = \upsilon_2 + i\upsilon_1$, $\upsilon_j = w_{3\alpha^j} + \psi_j (j=1,2)$, the generalized displacements $w_j, \psi_j (j=1,2), w_3$ are represented as

$$\omega_0 \equiv \omega_0(\upsilon) = \omega_{01}(\upsilon) + \omega_{02}(\upsilon) + \omega_{0*}, \quad \psi \equiv \psi(\upsilon) = \psi^0 + \psi^1(\upsilon) + \psi_*, \tag{17}$$

$$w_3 \equiv w_3(\upsilon) = w_{30} + w_{30}(\upsilon) + w_{3*},$$

where

$$\omega_{0j}(\upsilon) = w_{2j}(\upsilon) + i w_{1j}(\upsilon) = H_0[f^j(\upsilon); \varphi^j(\upsilon)], \quad j = 1,2, \tag{18}$$

$$\psi^n(\upsilon) = \psi_{2n}(\upsilon) + i\psi_{1n}(\upsilon) = H_0[g^n(\upsilon); \tilde{\varphi}^n], \quad n = 0,1,$$

$$w_{30} = -\int_{(0,0)}^{(\alpha^1,\alpha^2)} \psi_{10} d\alpha^1 + \psi_{20} d\alpha^2, \quad w_{31}(\upsilon) = \int_{(0,0)}^{(\alpha^1,\alpha^2)} [\upsilon_1 - \psi_{11}(\upsilon)] d\alpha^1 + [\upsilon_2 - \psi_{21}(\upsilon)] d\alpha^2,$$

$$\omega_{0*} = -c_1 k_4 (\alpha^2)^2/2 + (k_4 c_3 - c_1^2/2)\alpha^2 + c_1 k_4/4 + c_0,$$

$$\psi_*(z) = \psi_{2*}(z) + i\psi_{1*}(z) = c_1, \quad w_{3*} = -c_1\alpha^2 + c_2;$$

c_j, $j = \overline{0,3}$ – arbitrary real constants.

Further, using (17), the problem A is reduced to one nonlinear operator equation of the form

$$\upsilon - G_* \upsilon = 0, \tag{19}$$

where $G_*\upsilon$ – a nonlinear bounded operator in *and for any* $\upsilon^j (j=1,2) \in W_p^{(1)}(\Omega)$, belonging to the ball $\|\upsilon\|_{W_p^{(1)}} < r$, the estimate $\|G_*\upsilon^1 - G_*\upsilon^2\|_{W_p^{(1)}(\Omega)} \le q_* \|\upsilon^1 - \upsilon^2\|_{W_p^{(1)}(\Omega)}$. is fair.

In this case, a solvability condition of the following form appears

$$\int_\Gamma (k_1\alpha^1 T^1(a) + k_2\alpha^2 P^2 + P^3) ds + \iint_\Omega (k_1\alpha^1 R^1 + k_2\alpha^2 R^2 + R^3) d\alpha^1 d\alpha^2 = 0,$$

$$T^1(a) = T^{11} d\alpha^2/ds - T^{12} d\alpha^1/ds,$$

which is performed by choosing the constant c_2.

IV. CONCLUSIONS

Suppose that the radius r of the ball and the external forces acting on the shell are such that the following conditions are satisfied:

$$q_* < 1, \quad \|G_*(0)\|_{W_p^{(1)}(\Omega)} < (1 - q_*) r. \tag{20}$$

Then the principle of squeezed mappings can be applied to the equation (19) [10, p. 146], according to which the equation (19) in the ball $\|\upsilon\|_{W_p^{(1)}(\Omega)}<r$ has a unique solution $\upsilon\in W_p^{(1)}(\Omega)$, $2<p<2/(1-\beta)$.

Thus, the following theorem is true.

Theorem. *Let the conditions a), b) from the problem A, and the inequality (20) be satisfied. Then, for the solvability of the geometrically nonlinear equilibrium problem A for shallow elastic shells of the Timoshenko type under boundary conditions (2), it is necessary and sufficient that the condition (12) be satisfied. If it is fulfilled, the problem has a generalized solution* $a=(w_1,w_2,w_3,\psi_1,\psi_2)\in W_p^{(2)}(\Omega)$, $2<p<2/(1-\beta)$.

V. SUMMARY

All existence theorems currently known in the nonlinear theory of elastic shells are obtained within the framework of the simplest Kirchhoff-Love model. In this case, topological and (or) variational methods were used in various energy spaces. At the same time, one of the founders of the mathematical theory of shells, the Academician I.I. Vorovich pointed out the need to obtain existence theorems within the framework of more complex models that do not rely on the Kirchhoff - Love hypothesis. The present study is devoted to the solvability of a nonlinear boundary value problem for partial differential equations in the framework of a more complex model, not based on the Kirchhoff - Love hypothesis. In this paper, the existence theorem is proved within the framework of the shear model by S.P. Tymoshenko. When passing to the model by S.P. Tymoshenko, the methods previously used in the Kirchhoff - Love model cease to work. This is primarily due to the impossibility of constructing energy spaces in which the coercivity inequalities are valid. Therefore, a new analytical method is used, which consists in studying the original system of five equilibrium equations in classical Sobolev spaces under given boundary conditions by reducing it to a single nonlinear operator equation. Analytical (explicit) solutions of shell deformation problem in a geometrically nonlinear formulation are developed. The solvability of boundary value problems describing the equilibrium state of shells in the framework of the simplest Kirchhoff - Love model is quite large. Therefore, consideration of this study provides a significant expansion of engineering application of the theory. Shell structures (various building structures, domes, etc.) require more and more reliable, accurate design data and often pose completely new challenges. Thus, in this paper, an existence theorem is proved and an analytical method is used for finding solutions of geometrically nonlinear, physically linear boundary value problems for elastic shallow isotropic homogeneous shells of the type by S.P. Timoshenko with a different version of hinge-supported edges. The scientific novelty of the considered problem lies in the fact that, first, it is a new, unexplored problem of the mathematical theory of elasticity; secondly, a new method is proposed to solve it that makes it possible to study the solvability of such a class of problems within the framework of more general models for a wider class of elastic structures. The solution of the problem posed makes a significant contribution to the development of the mathematical theory of elasticity, and can be useful in creating new software packages for elastic structure calculation.

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