Solving Equations – Quick Reference

Integer Rules

Addition:

- If the signs are the same, add the numbers and keep the sign.
- If the signs are different, subtract the numbers and keep the sign of the number with the largest absolute value.

Subtraction: Add the opposite

Keep—Change—Change

- Keep the first number the same.
- Change the subtraction sign to addition.
- Change the sign of the second number to it's opposite sign.

Multiplication and Division:

- If the signs are the same, the answer is positive.
- If the signs are different, the answer is negative.

Golden Rule for Solving Equations:

Whatever You Do To One Side of the Equation, You Must Do to the Other Side!

Combining Like Terms

Like terms are two or more terms that contain the same **variable.**

Example: 3x, 8x, 9x 2y, 9y, 10y

are like terms. are like terms.

3x, 3y ↑ ↑ are **NOT** like terms because they do **NOT** have the same variable!

Distributive Property Examples

3(x+5) = 3x + 15	Multiply the 3 times x and 5.
-2(y -5) = -2y +10	Multiply –2 times y and –5.
5(2x –6) = 10x –30	Multiply 5 times 2x and –6.

Solving Equations Study Guide

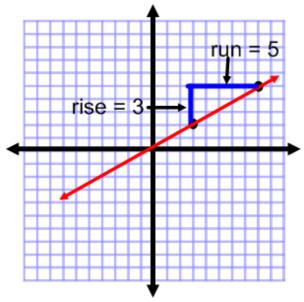
- Does your equation have fractions? Yes—Multiply every term (on both sides) by the denominator. No—Go to Step 2.
- 2. Does your equation involve the distributive property? (Do you see parenthesis?)
 - Yes—Rewrite the equation using the distributive property.
 - No-Go to Step 3.
- On either side, do you have like terms? Yes—Rewrite the equation with like terms together. Then combine like terms. (Don't forget to take the sign in front of each term!) No– Go to Step 4.
- Do you have variables on both sides of the equation? Yes—Add or subtract the terms to get all the variables on one side and all the constants on the other side. Then go to step 6. No—Go to Step 5.
- At this point, you should have a basic two-step equation. If not go back and recheck your steps above.
 - Use Addition or Subtraction to remove any constants from the variable side of the equation. (Remember the Golden Rule!)
- 6. Use multiplication or division to remove any coefficients form the variable side of the equation. (Remember the Golden Rule!)
- 7. Check your answer using substitution!

Congratulations! You are finished the problem!

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Slope= <u>rise</u> run

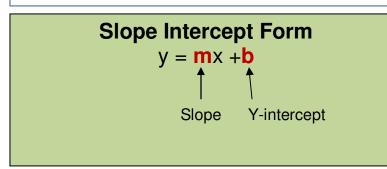
- Calculate the slope by choosing two points on the line.
- Count the rise (how far up or down to get to the next point?) This is the numerator.
- Count the run (how far left or right to get to the next point?) This is the denominator.
- •Write the slope as a fraction.



Slope = 3/5

** Read the graph from left to right. If the line is **falling**, then the slope is **negative**. If the line is **rising**, the slope is **positive**.

When counting the rise and run, if you count **down or **left**, then the number is **negative**. If you count **up** or **right**, the number is **positive**.

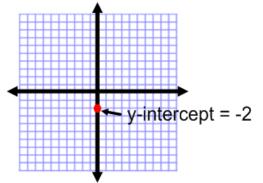




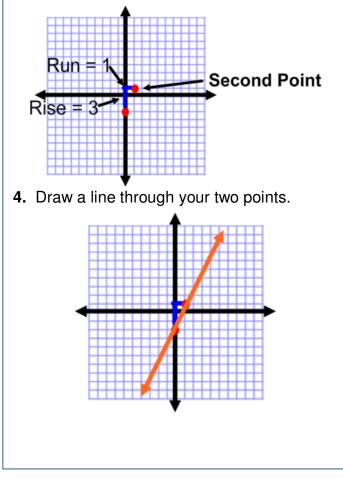
1. Identify the slope and y-intercept in the equation.

y = 3x -2 Slope Y-intercept

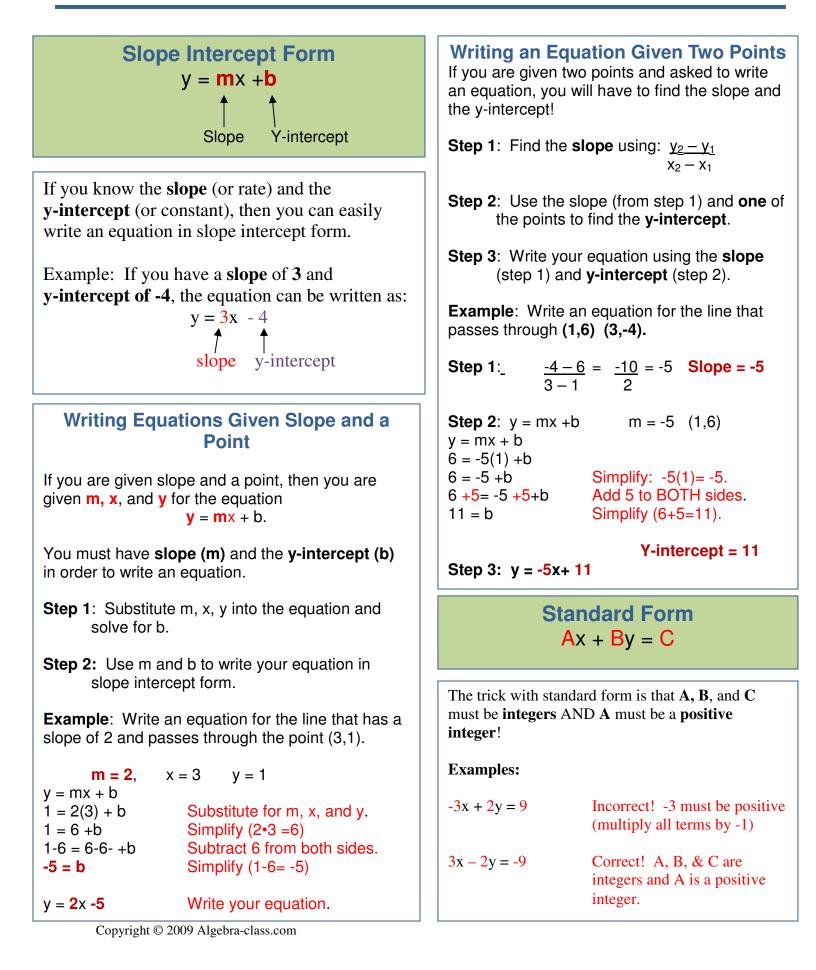
2. Plot the y-intercept on the graph.



3. From the y-intercept, count the rise and run for the slope. Plot the second point.



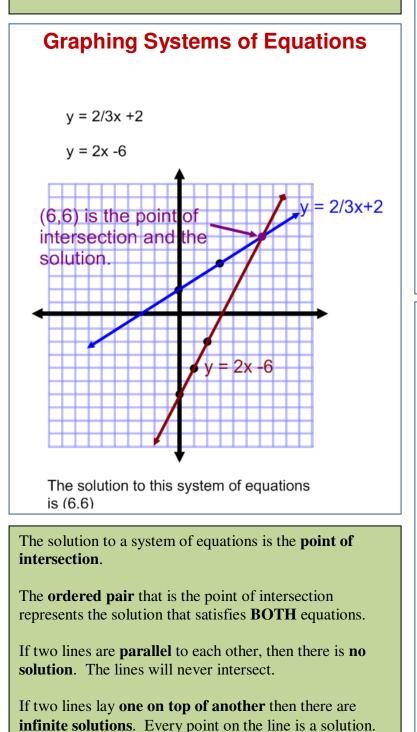
Writing Equations – Quick Reference



Systems of Equations – Quick Reference

Two linear equations form a system of equations. You can solve a system of equations using one of three methods:

- 1. Graphing
- 2. Substitution Method
- 3. Linear Combinations Method



Substitution Method			
Solve the following system of equations:			
x − 2y = -10			
y= 3x			
x - 2y = -10	Since we know $y = 3x$,		
x - 2y = -10 x - 2(3x) = -10	substitute 3x for y into		
x - 2(3x) = -10	the first equation.		
x – 6 x = -10	Simplify: Multiply		
	2(3x) = 6x.		
	(-) -		
-5x = -10	Simplify: $x - 6x = -5x$		
$\frac{-5x}{-5} = \frac{-10}{-5}$	Solve for x by dividing		
-5 -5	both sides by -5.		
x= 2	The x coordinate is 2.		
~			
y = 3x	Since we know that		
y = 3x y = 3(2) y = 6	x = 2, we can		
y = 3(2)	substitute 2 for x into		
y = 6	y = 3x.		

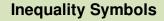
Solution: (2, 6)

Linear Combinations (Addition Method) Solve the following system of equations:

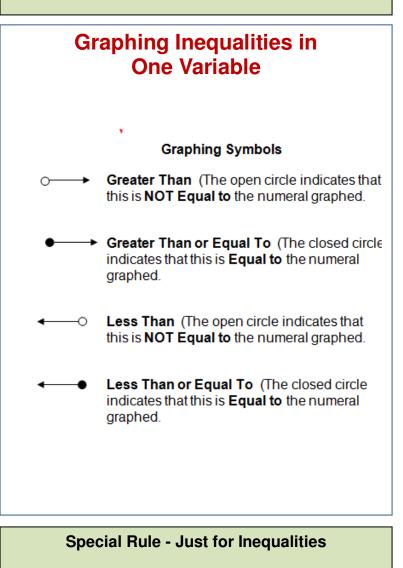
The solution!

3x+2y = 102x +5y = 3

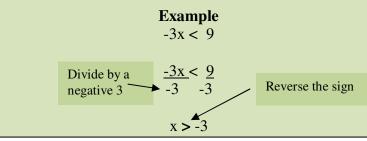
$\frac{-2(3x + 2y = 10)}{3(2x + 5y = 3)}$	Create opposite terms. I'm creating opposite x terms.
-6x - 4y = -20 $\frac{6x + 15 y = 9}{11y = -11}$	Multiply to create opposite terms. Then add the like terms.
$\frac{11y}{11} = \frac{-11}{11}$	Solve for y by dividing both sides by 11.
y = -1	The y coordinate is -1
2x + 5y = 3 2x + 5(-1) = 3	Substitute -1 for y into one of the equations.
2x - 5 = 3 2x - 5 + 5 = 3 + 5 $\frac{2x}{2} = \frac{8}{2}$	Solve for x!
x = 4	The solution (4, -1)

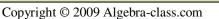


- < Less Than
- ≤ Less Than OR Equal To
- > Greater Than
- ≥ Greater Than or Equal To



Whenever you **multiply or divide** by a **negative** number, you **MUST reverse** the sign.

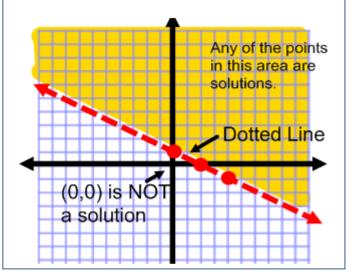




Graphing Inequalities in Two Variables

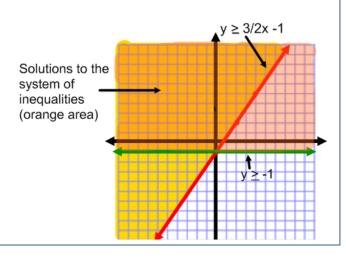
Graph for: y > -1/2x + 1

- 1. Graph y = -1/2x + 1, but dot the line since the symbol is >. The points on the line are **not** solutions.
- Pick a point such as (0,0) and substitute it into the inequality. (0,0) is **not** a solution, therefore, shade the side of the line that does not contain (0,0).



Systems of Inequalities

Graph each inequality as shown above. **ONLY** the area that is shaded by **BOTH** inequalities is the solution set (orange section)



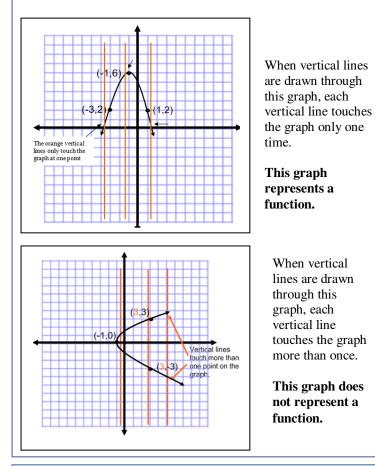
Functions – Quick Reference

Function Notation can be written as:

 $f(x) = 3x+2 \quad this translates to: "f of x" equals <math>3x+2" \\ g(x) = 3x-1 \quad this translates to: "g of x equals <math>3x-1"$

Identifying Functions using the Vertical Line Test

If a graph represents a function, that graph will only intersect with a vertical line one time.



Evaluating Functions

f(x) = 6x -1	Find f(5)	Original Problem
↓ f(x) = 6x -1		e how 5 replaces the x in function notation.
f(5) = 6(5) -1	Substitute 5 for x in the	original function.
f(5) = 29	Evaluate! This is your a	answer!

This answer means that if you substitute 5 for x, into this function, you will get an answer of 29! You "used" to write: y = 29. Now, in place of y, you will use f(5).

**(The 5 can be replaced with whatever number you substitute into the equation.)

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Linear Functions

Function notation can be confusing, but once you can identify the x and y coordinate, you can think of your typical ordered pair.

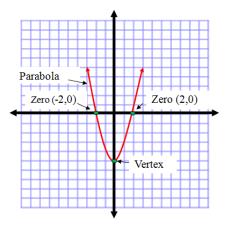
A typical ordered pair:	(2, 5)	where	(2, ↑ x	т
An ordered pair using fun	ction n	otation	f(2) =5
f(x) = y coordinate			X	т У

Quadratic Functions

Quadratic Functions will have a "squared term"

f(x) = ax ² +bx	+c	$f(x) = 2x^2 + 3x + 4$	
f(x) = ax <mark>2</mark> +bx	►	· ↑ / ↑	
coefficients	constant	coefficients con	stant

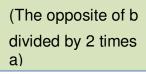
A quadratic function will result in a "parabola" when graphed.



**If the lead coefficient is positive, then the parabola will open up. Example: $3x^2 + 2x - 5$ (3 is positive) **If the lead coefficient is negative, then the parabola will open down. Example: $-2x^2 + 2x - 5$ (2 is negative)

Vertex Formula Given the function: $f(x) = ax^2 + bx + c$

Vertex Formula: $\frac{-b}{2a}$



Exponents and Monomials – Quick Reference

4 ²	This expression is read as "4 to the second power" OR "4 squared".
$4^2 = 4 \cdot 4$	It means that we multiply 4 by itself 2 times.
4 ² = 16	4 • 4 = 16

	your answer will alwa	a negative base and the exponent is odd,
	(-3) ³	This expression is read as -3 to the third power.
(-3) ³ = -3 • -3 • -3	It means that we multiply -3 by itself 3 times.
	(-3) ³ = -27	-3 • -3 • -3 = -27 9 • (-3) = -27

LAWS of EXPONENTS

Multiplying Powers with the Same Base

Property: When multiplying powers with the same base, add the exponents.

y ³ • y ⁴ = y ⁷	Since the bases are the same (y), you can
	add the exponents: 3+4 = 7.

Power of a Power Property

Property: To find the power of a power, multiply the exponents.

(a3)5 = a15

Multiply the exponents.

Power of a Product Property

Property: To find the power of a product, find the power of each factor and multiply.

Think of it as distributing the exponent to each factor!

 $(2xy)^3 = 2^3x^3y^3 = 8x^3y^3$ **↓ ↓↓** 8 x³y³

 $2^3 = 8$. x^3y^3 cannot be combined because the bases are not the same.

Power of Quotient Property

Property: To find the power of a quotient, raise the numerator to the power, and the denominator to the power. Then divide.

$$\left(\frac{2}{3}\right)^2 = \frac{2^2}{3^2} = \frac{4}{3^2}$$

Zero Exponents Any number (except 0) to the zero power is equal to 1. $4^0 = 1$ y⁰=1 $10^{\circ} = 1$ $22^0 = 1$

The Rule for Negative Exponents:



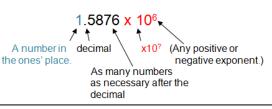
**In this problem, only the x contains the negative exponent, so we only take the reciprocal of x².

Multiplying Monomials Example

(3x ² y ³ z) ² (-3xy ⁴ z)	Original Problem
(3x ² y ³ z) ² (-3xy ⁴ z)	The first monomial is raised to
*	the second power. Every
(9x ⁴ y ⁶ z ²) (-3xy ⁴ z)	constant and variable must be raised to the second power. **The second monomial is not raised to a power, so leave it as is!
(9x ⁴ y ⁶ z ²) (-3xy ⁴ z) = -27	Multiply your coefficients.
(9x ⁴ y ⁶ z ²) (-3xy ⁴ z) = -27x ⁵ y ¹⁰ z ³	Multiply the variables with like bases. (Add the exponents.)
$(3x^2y^3z)^2(-3xy^4z) = -27x^5y^{10}z^3$	Final Answer.

Simplifying Monomials Example $9x^2y^2$ Original Problem $2x^2y^3$ v^4 3*x* $9x^2y^2$ Step 1: Multiply the $18x^4y^5$ $2x^2y$ numerators. Add the $y^{\overline{4}}$ 3xexponents of like bases. Step 2: Multiply the denominators. **There $9x^2y^2$ $18x^4y^5$ $2x^2y^3$ v^4 3x $3xy^4$ are no like bases, so we can't add the exponents. <mark>18</mark>x⁴y⁵ Step 3: Divide the coefficients, if possible. $3xy^4$ $\frac{18x^4y^5}{3xy^4} =$ $6x^3y$ Step 4: Subtract the exponents of like bases. $\frac{x^4}{x} = x^3 \text{ and } \frac{y^5}{y^4} = y$ Final Answer! $2x^2y$ $9x^2y$ 6x³y $y^{\overline{4}}$ 3x

Scientific notation must always be written with the same components as the following model:



Polynomials – Quick Reference

Subtracting Polynomials
You must remember to use Keep Change Change.
If you have a subtraction sign preceding a set of parenthesis , then you must rewrite the problem as an addition problem. We are going to ADD the OPPOSITE
Subtraction sign & parenthesis $(2x - 6) - (3x^2 + 2x - 6)$ Rewritten as: $(2x - 6) + (-3x^2 - 2x + 6)$
$\begin{array}{cccc} (2x-6) & - & (3x^2+2x-6) \\ & \uparrow & \uparrow \\ \\ Keep & Change & Change the \\ the & to & sign of \\ same & Addition & every term \\ & \downarrow & \downarrow & \downarrow & \downarrow \\ \end{array}$
$(2x - 6) + -3x^2 - 2x + 6$
Multiplying Polynomials
We must use our laws of exponents in order to multiply polynomials.
2a ² b ² (a ³ + 3ab – b ³) Original Problem
$2a^{2}b^{2}(a^{3}) + 2a^{2}b^{2}(3ab) + 2a^{2}b^{2}(-b^{3})$ Distribute $2a^{2}b^{2}$ through the parenthesis.
and add the exponents
like bases for each term
Solution:
$2a^{5}b^{2} + 6a^{3}b^{3} - 2a^{2}b^{5}$
Using FOIL
(3x -4) (2x +1) Original Problem
Multiply the First terms:
$(3x - 4)(2x + 1)$ $(3x)(2x) = 6x^2$
6x ²
Multiply the Outside terms:
(3x - 4) (2x + 1) $(3x)(1) = 3x$
6x ² +3x
(3x - 4) (2x + 1) Multiply the Inside terms: (-4)(2x) = -8x
(3x - 4)(2x + 1) $(-4)(2x) = -8x$
$(3x - 4) (2x + 1) (-4)(2x) = -8x$ $6x^2 + 3x - 8x$
(3x - 4)(2x + 1) $(-4)(2x) = -8x$
$(3x - 4) (2x + 1) (-4)(2x) = -8x$ $6x^{2} + 3x - 8x$ $(3x - 4) (2x + 1) (-4)(1) = -4$ Multiply the Last terms: (-4)(1) = -4
$(3x - 4) (2x + 1) (-4)(2x) = -8x$ $6x^{2} + 3x - 8x$ $(3x - 4) (2x + 1) (-4)(1) = -4$ $6x^{2} + 3x - 8x - 4$ Multiply the Last terms: (-4)(1) = -4
$(3x - 4) (2x + 1) (-4)(2x) = -8x$ $6x^{2} + 3x - 8x$ $(3x - 4) (2x + 1) (-4)(1) = -4$ Multiply the Last terms: (-4)(1) = -4

Factoring-Quick Reference

Finding the GCF

Find the GCF for: $30x^3 + 5x^2 - 25x$

Step 1: Look at the coefficients

Ask yourself: Is there a number that I can divide 30, 5, and 25 by evenly? Yes, 5! (30/5 = 6) (5/5=1) (-25/5 = -5)

Step 2: Look at the variable(s).

Ask yourself: Can I factor out a variable from EVERY term? Yes! Each term has at least one x, therefore, I can factor out x.

Step 3: Identify the GCF:

The GCF for this polynomial is: 5x. (You can divide every term by 5x evenly (without creating a fraction).

Factoring Using the GCF

Factor the greatest common factor from: $3x^4y^3 + 12x^3y - 18x^2y^2$

Solution:

Step 1: Look at the coefficients. What is the GCF for 3, 12, 18? $3x^4y^3 + 12x^3y - 18x^2y^2$

(What is the greatest number that can be divided into all evenly?)

3 is the GCF (for the coefficients).

Step 2: Look at the variable.

Can I factor out a variable for EVERY term? Yes! Each term contains at least one x^2 and $y.(x^2y)$ $3x^4y^3 + 12x^3y - 18x^2y^2$

Step 3: Identify the GCF.

The GCF is $3x^2y$. Now we are going to divide EVERY term by $3x^2y$. (Most students do this mentally, but I am going to write it out to show you the process.)

 $\frac{3x^4y^3}{3x^2y} + \frac{12x^3y}{3x^2y} - \frac{18x^2y^2}{3x^2y}$

 $x^2y^2 + 4x - 6y$ (the result after dividing)

Step 4: Write appropriately in factored form.

 $3x^2y(x^2y^2 + 4x - 6y)$ \uparrow GCF Result after dividing each term by the GCF

Factored Form: $3x^2y(x^2y^2 + 4x - 6y)$

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Factoring by GroupingFactor the following polynomial:
$$x^2y^2 + 2x^3 + 4x^2y^2 + 8x^2$$
Step 1: Separate the polynomial into 2 or more groups according to common factors: Identify the common factor for each group. In this problem in need to rewrite the problem with common factors idee by side.) $x^3y^3 + 4x^3y^3 + 2x^3 + 8x^2$ $x^4y^3 + 4x^3y^3 + 2x^3 + 8x^2$ $x^4y^3 + 4x^3y^3 + 2x^3 + 8x^2$ $x^4x^4 + x + 4 + c$ Result after dividingStep 2: Divide each term by the common factor: $x^3y^3 + 4x^3y^3 + 2x^2 + 8x^2$ $x + 4 + x + 4 + c$ Result after dividingStep 3: Write appropriately in factored form. $x^3y^3 (x+4) + 2x^2 (x+4)$ Common ResultCommon Result Common ResultFactor after dividing factor after dividingFactor after dividing factor after dividingCommon factor: New these two terms have a common factor. The common factors (x+4). We can factor (x+4) and we are left with: $x + 4) (x^2y^3 + 2x^2)$ This is the final answer in factored form.(() () ()Step 1: We are going to form two binomials, so write two sets of parenthesis:() () () ()Step 2: What can we multiply together to get x?? (The first term of the chronomial is multiplied together to get 2? (The first term of the chronomial is multiplied together to get 2? (The miss term ender to take the sign in front of the term with it, therefore, the middle term is -10. $x^2 - 10x + 21$ Out numbers that we add together to get 21. (You must emember to take

Step 4: Complete the binomials.

(x - <mark>3</mark>) (x -7)

Your Solution

Quadratic Equations – Quick Reference

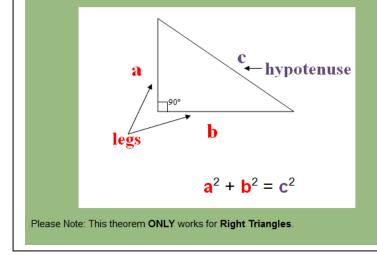
What is a Quadratic Equation?		
ax ² + bx + c		
ax ² +bx +c = 0	$2x^{2} + 3x + 4 = 0$ $\uparrow \qquad \uparrow \qquad \uparrow$ coefficients constant	
a and b are coefficients and c is a constant. The one factor that identifies these expressions as quadratic is the exponent 2. The first term must always be a x ² , and a cannot be 0.		
Solving Simple Quadratic Equations		
x ² -4=77	Our goal is to get x by itself on the left hand side of the equation. We must get rid of the -4 (first) then the exponent 2.	
$x^2 - 4 + 4 = 77 + 4$	Add 4 to both sides of the equation.	
x ² = 81	Simplify: 77 + 4 = 81	
$\sqrt{x^2} = \pm \sqrt{81}$	Take the square root of both sides. (Remember to use the \pm sign.)	
x = ± 9	There are 2 solutions. X is equal to positive 9 and negative 9.	

If $a^2 = b$, then $a = \sqrt{b}$ or $a = -\sqrt{b}$ This can also be written as: $a = \pm \sqrt{b}$

Read as: a = "plus or minus" the square root of b.



In any right triangle, the sum of the squares of the legs (2 shorter sides) is equal to the square of the hypotenuse (the longest side).



Solving Equations by Factoring		
Solve: x ² – 7x +2 = -10 ← Our equation is not equal to 0.		
x ² -7x+2 + 10 = -10 + 10	Before we can factor, we must set our equation equal to 0.	
	Add 10 to both sides.	
$x^2 - 7x + 12 = 0$	Now our equation is equal to 0. I can factor.	
(x-4)(x-3) = 0	Factor: $x^2 - 7x + 12$	
x-4=0 or $x-3=0x=4$ or $x=3$	Set both factors equal to 0. (The zero-factor property)	
Check: $x^2 - 7x + 2 = -10$ $4^2 - 7(4) + 2 = -10$ -10 = -10	Substitute the two solutions into the original equation. 4 works! When I substituted I	
	got an answer of -10.	
$x^2 - 7x + 2 = -10$ $3^2 - 7(3) + 2 = -10$ -10 = -10	3 works! When I substituted I got an answer of -10.	

The Quadratic Formula

Given any quadratic equation:

 $ax^{2} + bx + c = 0$

We can substitute the values for a, b, & c into the following formula and solve

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

