Solving Equations with Integers

Properties for solving equations:

Addition property
  - the same number can be added to each side of an
    equation without changing the equation

Subtraction property
  - the same number can be subtracted from each
    side of an equation without changing the equation

Division property
  - each side of an equation can be divided by the
    same number without changing the equation

Addition property:

- We can use the **addition property** to undo subtraction *by adding the same number to each side of the equation.*

  Example:

  $$X - 7 = 2$$ The original equation

  $$X - 7 + 7 = 2 + 7$$ Here we add "7" to both sides to undo the subtraction

  $$X = 9$$ We now have the equation solved for "X"

Subtraction property:

- We can use the **subtraction property** to undo addition *by subtracting the same number from each side of the equation.*

Example:

$X + 6 = 13$	The original equation
$X + 6 - 6 = 13 - 6$	Here we subtract "6" from both sides to undo the addition
$X = 7$	We now have the equation solved for "X"

Division property:

- We can use the **division property** to undo multiplication *by dividing each side of the equation by the same number.*

Example:

$5y = -30$	The original equation
$\dfrac{5y}{5} = \dfrac{-30}{5}$	Here we divide each side by "5" to undo the multiplication
$y = -6$	We now have the equation solved for "X"

Application of the properties in word problems

Steps in solving word problems:

Step 1: Identify what the problem asks us to find and choose a variable to represent the unknown number (X).

Step 2: Find two verbal expressions for the same value.

Step 3: Translate the verbal expressions into a mathematical equation.

Step 4: Solve the equation by using our properties.

Example: Negative fifty-six equals negative eight times a number. Find the number.

Following our steps, we first must identify what we are looking for and then assign it a variable.

To identify what we are looking for, search the problem for words and phrases like:

"Find", "What is", "What was", "How much", etc.

In this problem, the last sentence asks us to "*Find* the number." So this is what we are looking for and we will let the number be represented by the variable "X."

Our next step is to find two verbal expressions that have the same value.

When looking for our expressions, we want to search the problem for words and phrases like:
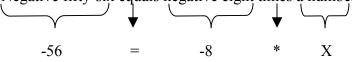
"is", "equals", "is equal to", etc.

Looking at the first sentence in our problem, we can find our two expressions.

Negative fifty-six *equals* **negative eight times a number**.

"Negative fifty-six" and "negative eight times a number" are our two expressions since they are joined by the word "equals."

Our third step is to now translate these verbal expressions into a mathematical equation.

Negative fifty-six equals negative eight times a number.

$$-56 \qquad = \qquad -8 \qquad * \qquad X$$

Performing the multiplication we give us the following equation to work with.

$$-56 = -8X$$

Now that we have our equation, we can move to the last step, which is to solve the equation using the properties we have covered.

$$-56 = -8X \qquad \text{This is our equation to solve.}$$

$$\frac{-56}{-8} = \frac{-8X}{-8} \qquad \text{Since we have multiplication involved in our equation (-8X), we will use the division property to undo the multiplication.}$$

$$7 = X$$ After performing the division, we have our answer. The number is 7.

Example: The high temperature today is 7º lower than the high temperature yesterday. The high temperature today is -13°C. What was the high temperature yesterday?

Remember our first step is to identify what we are looking for and assign it a variable.

The last sentence in our problem asks us "***What was*** the high temperature yesterday?"

So in this problem yesterday's high temperature is what we are looking for and will assign it the variable "T."

Now we must look for our verbal expressions to use in our equation.

The first sentence in our problem reads:

The high temperature today _is_ 7º lower than the high temperature yesterday.

"The high temperature today" and "7º lower than the high temperature yesterday." are our expressions since they are joined by the word "is."
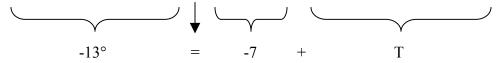
The second sentence in the problem also provides us with the information needed to complete our equation using the first sentence. Without this information we would have two unknown values.

The high temperature today _is_ -13° C.

We can now setup our equations.

We will replace the phrase "The high temperature today" with the value given in the second sentence of -13° C.

The high temperature today is 7° lower than the high temperature yesterday.

-13°   =   -7   +   T

Since the 7 is followed by the phrase "lower than" we must put a negative sign before the 7. Remember that the phrase "lower than" indicates subtraction. We can now rearrange the equation so that the variable "T" is first.

$$-13 = T - 7$$

Now all that is left is for us to solve the equation.

$$-13 = T - 7$$ We start with our original equation.

$$-13 + 7 = T - 7 + 7$$ Since our equation involves subtraction, we must use the addition property to undo it.

$$-6 = T$$ Performing the addition we get our answer of "-6."
So the high temperature yesterday was -6° C.