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Solving Mathematics Word Problems through Reflection

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Abstract: This paper offers a model for mathematics word problem solving. This integrated model consists of 4 main steps namely Entry, Analyze, Attack and Review. The author will demonstrate how the reflection in the step of "Analyze" will help the pupils to make judgment on their chosen mathematical operations for solving the non-routine mathematics word problem. We stress that in order to be successful in mathematics word problem solving, we need to reflect on three perspectives namely personal experiences, meanings of mathematical operations or symbols and the problem itself. We conjecture that these three perspectives are the main factors for pupils to be able to solve the mathematics word problem correctly. In this study, nine Year 6 pupils were involved. They are required to answer five mathematics word problems in a task then a follow-up interview will be conducted on a voluntary basis in order to explore why they have solved those problems in a particular way.

Keywords: Mathematics Word Problem Solving, Reflection, Personal Experiences, Mathematical Operations, Symbols

1.0 Introduction

According to Desoete & Roeyers (2005), many research studies have been conducted based on human thinking but still not able to get a model that can be applied in all situations. Most of the educators agree that the ability of problem solving is an objective of instruction in primary education (Lester, 2013). Tambychik and Meerah (2010) concluded in their research that pupils need to master the skills to recall and memorize in order to success in mathematics problem solving. In this study, we are interested to look at the effect of reflection and examine how reflection helps pupils to make judgment on their chosen mathematical operations for solving non-routine mathematics word problems.

After reviewing a few models of problem solving, we managed to create an alternative model for solving mathematics word problem. This model is a product of blending three mathematics problem solving models such as Polya's model, John Mason's model and Alan Schoenfeld's model. This integrated model consists of 4 main steps namely Entry, Analyze, Attack and Review. In this paper, we will focus on the step "Analyse". We conjecture that there are three objects of reflection needed to be successful in mathematics word problem solving namely personal experiences, meanings of mathematical operation or symbols and the problem itself. In line with the research done by Gasco et al. (2014), the importance to understand the

problem, an appropriate method of solution and the skills to create mental models were the crucial parts in word problems solving.

The ways of the question presented in the examinations have changed since KSSR (Kurikulum Standard Sekolah Rendah) was implemented in the Malaysia school curriculum. More questions are in the form of word problems. Mathematics word problem is difficult for pupils in two aspects. First, the understanding of the question that involves extraction of information and knowing the request of the problem. Second, the difficulty to formulate the word problem to a model in particular mathematical model that can be solved. Non-routine problems are problems with no readily available procedures for finding the solutions in pupils' mind therefore pupils need to build their own ways to solve the problems.

2.0 Theoretical framework

In this paper, we propose a model of problem solving which was developed through the blending of three problem solving models. This integrated model has a unique characteristic that focuses on the objects of reflection. This model also supports individuals who want to assess their own understanding regarding the meaning of the given mathematics problem. Understanding the meaning of the problem that we want to cope with is very crucial in order to solve that particular problem (Evans, 2012).

Gasco et al (2014) stated that the purpose of learning problem solving is to make pupils competent in thinking, persistence in the process and confident when faced unfamiliar environment or situations. This matter indirectly will develop the skills of creative and critical thinking while the pupils learn mathematics. Nowadays, the concept of teaching needs to be revised and we must focus more on the problem solving skills in order to help pupils to master the concept of mathematics better. Thinking process is a very unique thing for every individual and different individuals will have their own ways of looking at the same thing.

The previous teaching and learning process will affect how pupils develop their own knowledge (Isoda, 2012) but the knowledge can be supportive or problematic to them (Chin, 2014). Teachers play an important role in shaping the ways of pupils think, therefore we need to make improvement in the learning process by focusing on the thinking process that may eventually guide the pupils to successfully solve the mathematics problems. Figure 1 shows an overview of the stages involve in the proposed model that was adapted from Lester (2013). The dark arrows in Figure 1 represent the successful performed activities from the previous "box" to produce an effect on the next "box". Dashed arrows denote the activity of comparing two boxes in order to explore the relationships between them.

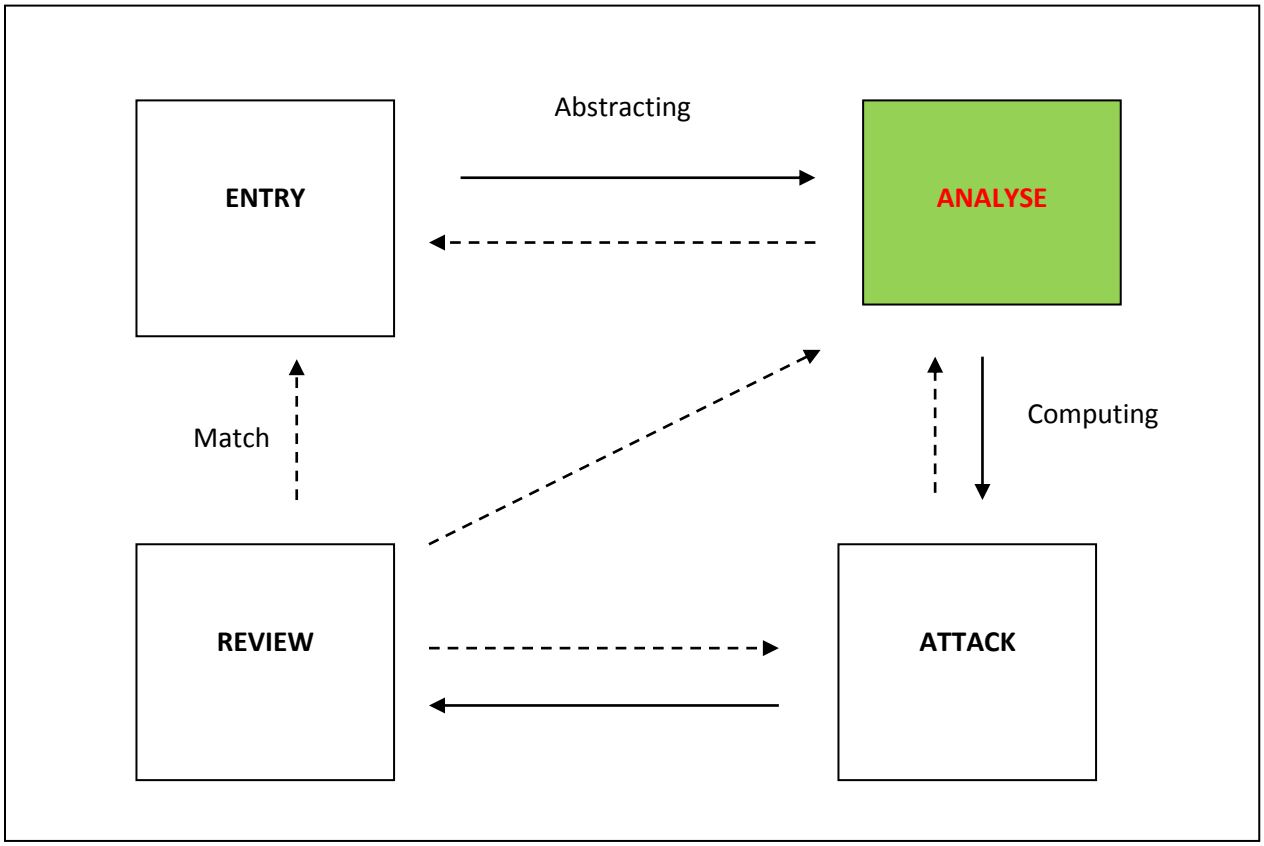


Fig. 1 Mathematics word problem solving model

In this paper, we will focus on the step "Analyse" because this is a critical step that involves the formulation of mathematical model in order to solve a mathematics word problem mathematically. We conjecture that in the step "Analyse", objects of reflection are the main factors in understanding how a pupil formulates a particular mathematical model. In this case, the objects of reflection that we are referring to are mathematical operations or symbols, personal experiences and understanding of the mathematics words problem. Chin (2012) and Chin (2014) proposed the notions of supportive and problematic conceptions in making sense of mathematics.

Supportive conceptions refer to those conceptions that work in an old context and continue to work in a new context. On the other hand, problematic conceptions refer to those conceptions that work in an old context and don't work in a new context. As an illustration, the arithmetic operation for multiplication can be conceived as repeated addition when it involves the multiplication of two natural numbers. However, when two fractions are involved in multiplication then the meaning of multiplication must be conceived as "of". Therefore it is very important for a pupil to be able to reflect on the meaning of an arithmetic operation for a particular context.

This paper will show some empirical evidence on the pupils' chosen problem solving strategies as a consequence of reflection. According Mason (2002), pupils will use their mental imagery power to re-enter significant moment from previous in order to learn from it as an experience and this shows the effect of personal experience in the thinking process of problem solving. The understanding of a mathematics words problem also plays a significant role in problem solving. A pupil needs to understand the problem so that he/she can extract the necessary information needed to solve the problem.

There are many types of problem solving skills and each type of skill has its own strengths and weaknesses. Sometimes, we don't even need to do any mathematical calculation in order to solve a problem. Take for instance, if you are required to calculate what is 1/3 – 1/4, we can use the standard mathematical procedure to solve it by converting these fractions into fractions with common denominator then we find the difference between these two numerators. On the other hand, we can also solve this problem by drawing a bar with twelve equal parts then shade the parts for 1/3 (i.e. 4 parts) and ¼ (i.e. 3 parts). Lastly we will notice that the answer should be 1 part out of the 12 parts.

The first instance might be a reflection of personal experience in using the standard mathematical procedure for solving problems involving fractions. The second instance might be a reflection on the meaning of mathematical symbols i.e. 1/3 – ¼ that leads to the drawing of a bar. This shows that the roles of reflection on different objects might lead to different problem solving strategies. Table 1 gives a brief description for all the steps in this model.

Table 1 Mathematics word problem solving

The proposed Words Problem Solving Model	**Action**
Entry	- Read the question (Scanning), tick the numbers or important information.
Analyse	- Re-read the question (Trimming) - Relate the information with mathematic terms. - Choose the operation (+, -, ×, ÷) } REFLECTION
Attack	- Doing the solution
Review	- Used other calculation to check the similarities (if any). - Check if all the information were being used.

3.0 Method

3.1 Participant

There were 9 pupils who had participated this research. All of them were year 6 pupils of a primary school at the Beaufort District. This was a school with low attainment in mathematics in particular mathematics words problem solving. The participants were chosen randomly based on the namelist obtained from this school.

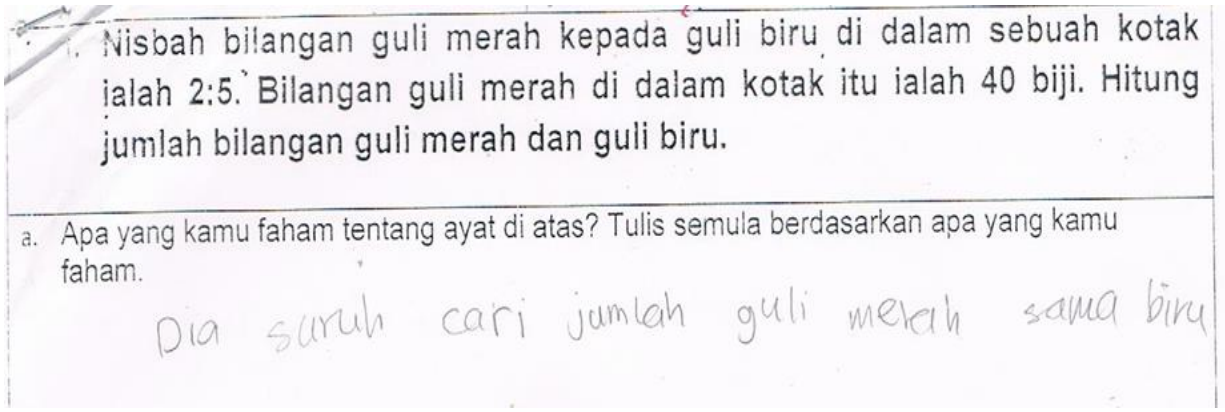
3.2 Procedure and Material

The data of this research were collected through a questionnaire that consisted of 5 mathematics word problems. Then, one participant was interviewed on a voluntary basis to gain more insight regarding the thinking process. Researchers conducted the intervention for 4 weeks and gave 3 different tests during this intervention using different types of word problem solving questions. All the materials were prepared based on the Malaysia KSSR curriculum standard but the questions were totally created by the researchers. Pupils were not allowed to discuss among each other while answering the items in the questionnaire.

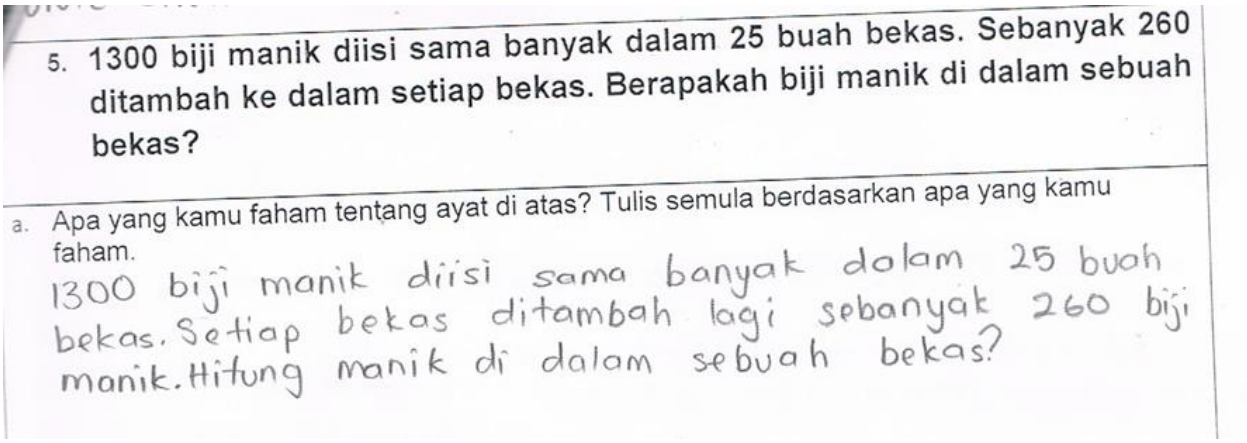
4.0 Results and Discussion

4.1 Reflect on the instructions given in the question.

Fig. 2 Participant A's response for Item No.1



Based on the questions in the questionnaire, we were able to classify them into 3 categories. The first category of questions is to guide the pupils to reflect on the given questions. In this category of questions, participants were asked to comment on their understanding for the given items then they were required to rephrase the items based on their understanding. Figure 2 shows that participant A focused on the things that are requested by the item but not on the information given.


5. 1300 biji manik diisi sama banyak dalam 25 buah bekas. Sebanyak 260 ditambah ke dalam setiap bekas. Berapakah biji manik di dalam sebuah bekas?

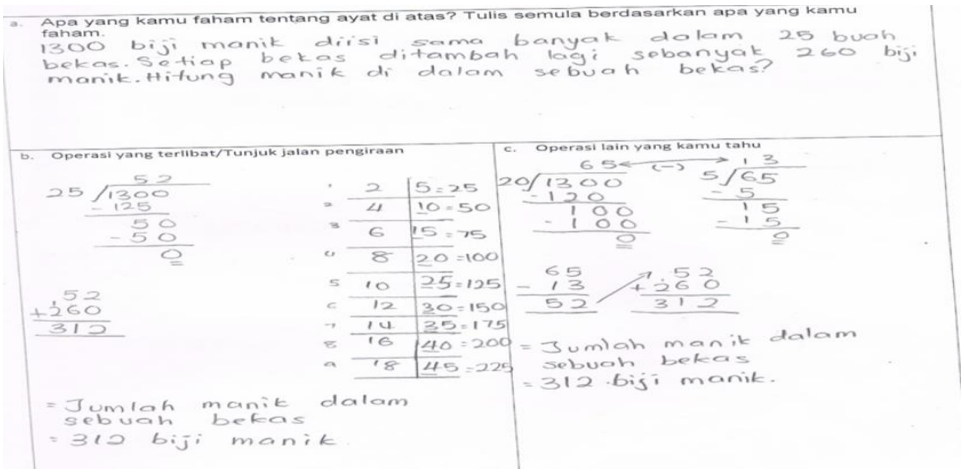
a. Apa yang kamu faham tentang ayat di atas? Tulis semula berdasarkan apa yang kamu faham.

1300 biji manik diisi sama banyak dalam 25 buah bekas. Setiap bekas ditambah lagi sebanyak 260 biji manik. Hitung manik di dalam sebuah bekas?

Fig. 3 Participant D's response for Item No.3

On the other hand, Figure 3 shows that participant D was able to rewrite the item in a way that could provide more details to the original item. He detailed the sequence of actions and rephrased the item based on his understanding on the item. Participant D not only focused on the things requested by the item but also on the information given by the item. This means that the participant reinterpreted the item based on his understanding. Apparently, he had reflected on the given item.

4.2 Reflect on the meanings of mathematical operation


a. Apa yang kamu faham tentang ayat di atas? Tulis semula berdasarkan apa yang kamu faham.
1300 biji manik diisi sama banyak dalam 25 buah bekas. Setiap bekas ditambah lagi sebanyak 260 biji manik. Hitung manik di dalam sebuah bekas?

b. Operasi yang terlibat/Tunjuk jalan pengiraan

52
25 / 1300
- 125
50
- 50
0

52
+ 260
312

: 2 | 5 = 25
2 4 | 10 = 50
3 6 | 15 = 75
4 8 | 20 = 100
5 10 | 25 = 125
6 12 | 30 = 150
7 14 | 35 = 175
8 16 | 40 = 200
9 18 | 45 = 225

= Jumlah manik dalam sebuah bekas
= 312 biji manik

c. Operasi lain yang kamu tahu

65 ← (÷) → 13
20 / 1300
- 120
100
- 100
0

5 / 65
- 5
15
- 15
0

65
- 13
52

52
+ 260
312

= Jumlah manik dalam sebuah bekas
= 312 biji manik.

Fig. 4 Participant D's response for Item No. 2 (Question no. 2)

Based on Figure 4, Participant D used the arithmetic operation of division initially as he wanted to know how many beads can be distributed equally into 25 containers if he has 1300 beads. Based on the information from the item, he knew that each container will be added another 260 beads later therefore he used the arithmetic operation of addition. Finally he got the answer as 312 beads in each container. Besides that, Participant D also computed the multiplication table for 2 and 5 (See Figure 5 and Figure 6). He got 104 when 2x52 and 260 when 5x25. By using some sorts of manipulation skills that he had memorized then he was able to get 1300 as the result of 25x52 by using 104 and 260 (See Figure 5). This also shows that he was using his past experience when applying those manipulation skills.

We did a follow-up interview with participant D to gain further insight on his thinking. We asked why he had used division for that item. He responded that "hmm…because the item said 1300 beads were distributed evenly into 25 containers therefore I related the idea of distributed evenly as the operation of division…" He further elaborated that he also used the addition operation in a later stage because the item said that additional beads were put into those containers. We also asked participant D why he had used the operation of multiplication, he responded that "I was using multiplication to check my answer and I knew that division is the inverse of multiplication"

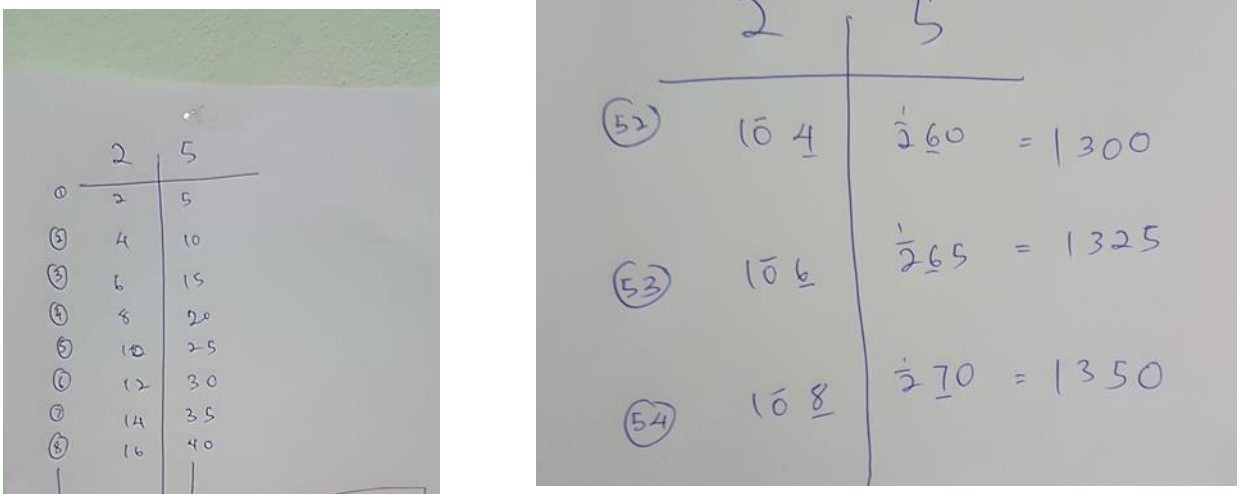


Fig. 6 Pattern of multiplication

Fig. 5 Multiplication table

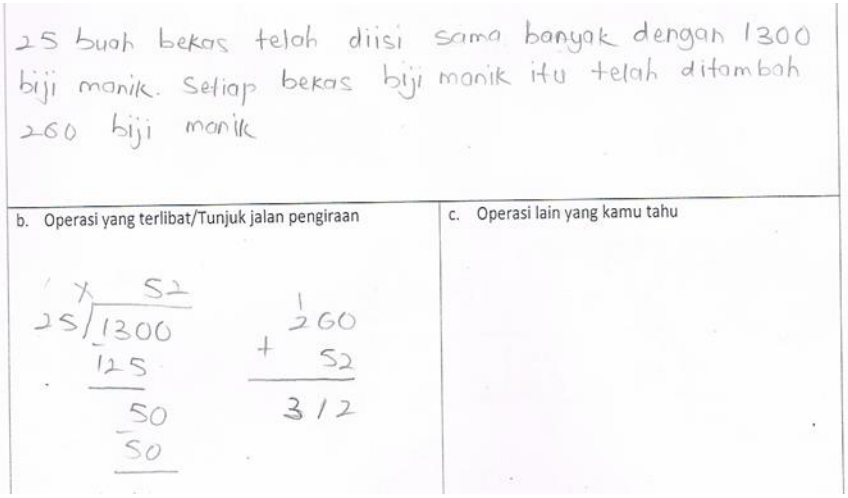


Fig. 7 Participant H's response for Item No.2

Participant H gave a similar response with participant D however he didn't provide alternative solving strategy for Item No. 2.

4.3 Reflect on our own previous experience

Based on Figure 5, we can see that the way that participant D computed the answer for 25x52 was quite unique. He computed the answers for 2x52 and 5x52 then by using some memorized manipulation skills he managed to get the answer for 25x52 as 1300. Apparently he was referring to his past experience of using those manipulation skills in order to get the answer.

5.0 Conclusion

This paper has shown some empirical evidence regarding how the objects of reflection had influenced the ways of participants in solving the problems. Three objects of reflection were identified in this study namely personal experience, meanings of mathematical operations or symbols and the problem itself. We want to stress that mathematics educators should guide the pupils on what to reflect during the process of problem solving so that the pupils can solve the given problems successfully.

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References

Chin, K. E. (2012). Making sense of mathematics through perception, operation & reason: the case of trigonometry functions. In *Proceedings of the 36th Conference of the International Group for the Psychology of Mathematics Education,* (pp. 1–8).

Chin, K. E. (2014). Supportive and problematic aspects in mathematical thinking over the longer term. In *Proceedings of the 38th conference of the international group for the psychology of mathematics education and the 36th conference of the North American chapter of the psychology of mathematics education*. Vancouver, Canada.

Desoete, A., & Roeyers, H. (2005). Cognitive skills in mathematical problem solving in Grade 3, 119–138. http://doi.org/10.1348/000709904X22287

Evans, B. R. (2012). Problem Solving Abilities and Perceptions in Alternative Certification Mathematics Teachers.

Gasco, J., Villarroel, J. D., Zuazagoitia, D., Sciences, E., Sciences, E., & Country, B. (2014). Different Procedures for Solving Mathematical Word Problems in High School, *7*(7), 77–84. http://doi.org/10.5539/ies.v7n7p77

Isoda, M. (2012). Introductory Chapter : Problem Solving Approach to Develop Mathematical Thinking, *1*, 1–28.

Lester, F. K. (2013). Thoughts About Research On Mathematical Problem- Solving Instruction. *The Mathematics Enthusiast*, *10*(1&2), 245–278. http://doi.org/ISSN 1551-3440

mason, J. (2002). Exploiting Mental Imagery in Teaching & Learning Mathematics. In *Actas ProfMat 2002* (pp. 75–81).