Solving Multi Objective Linear Programming Problems Using Intuitionistic Fuzzy Optimization Method: A Comparative Study

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***Abstract*—The paper aims to give computational algorithm to solve a multi objective linear programming problem using intuitionistic fuzzy optimization method. It also includes some basic properties of intuitionistic fuzzy set and operations on it. The development of algorithm is based on principle of optimal decision set obtained by intersection of various intuitionistic fuzzy decision sets which are obtained corresponding to each objective function. Further, as the intuitionistic fuzzy optimization technique utilizes degree of belonging and degree of non-belonging, we made a comparative study of linear and nonlinear membership function for belonging and non-belonging to see its impact on optimization and to get insight in such optimization process. The developed algorithm has been illustrated by a numerical example.**

***Index Terms*—Intuitionistic fuzzy set, multi objective linear programming, membership function, non-membership function.**

I. INTRODUCTION

In several optimization problems, it has been observed that a small violation in given constraints or conditions may lead to more efficient solution to the problem. Such situations appear in frequent way in real life modeling, as a matter of fact in optimization problems; many times it is not practical to fix accurate parameters as many of these are obtained through approximation or through some kind of human observation. For example in a production optimization problem, it is not necessary that all the produced are of good quality and are completely sellable on a fixed price. There is possibility that some of the products may be defective and are not sellable on the fixed price. Further prices of raw material as well as market price of finished product may vary depending on its surplus/deficiency in the market due to some uncontrollable situations. Thus it is evident that prices and/or productions are not purely deterministic but in general these are imprecise or nondeterministic and thus such problems of optimization are to be dealt with help of some non-classical methods.

Modeling of most of real life problems involving optimization process turns out to be multi objective programming problem in a natural way. Such multi objective programming problems may in general comprise of conflicting objectives. For example, if we consider a problem of agricultural production planning, the optimal model should have the objectives of maximizing the profit and minimizing the inputs and cost of cultivation. Thus these objectives are conflicting in nature and hence solution of such problems are in general compromise solutions which satisfy each objective function to a degree of satisfaction and a concept of belonging and non-belonging arises in such situations. It was Zimmermann [1], [2] who first used the fuzzy set introduced by Zadeh [3] for solving the fuzzy multi objective mathematical programming problem. Optimization in fuzzy environment was further studied and was applied in various areas by many researchers such as Tanaka [4], Luhandjula [5], Sakawa[6] etc. A brief review of studies of various research workers on optimization under uncertainty can be found in work of Sahinidis [7].

In view of growing use of fuzzy set in modeling of problems under situations when information available is imprecise, vague or uncertain, various extension of fuzzy sets immerged. In such extensions, Atanassov [8], [9] introduced the intuitionistic fuzzy sets as a powerful extension of fuzzy set. Atanassov in his studies emphasized that in view of handling imprecision, vagueness or uncertainty in information both the degree of belonging and degree of non-belonging should be considered as two independent properties as these are not complement of each other. This concept of membership and non-membership was considered by Angelov [10] in optimization problem and gave intuitionistic fuzzy approach to solve optimization problems. Jana and Roy [11] studied the multi objective intuitionistic fuzzy linear programming problem and applied it to transportation problem. Luo [12] applied the inclusion degree of intuitionistic fuzzy set to multi criteria decision making problem. Further many workers such as Mahapatra *et al.*, [13], Nachammai[14] and Nagoorgani [15] etc. have also studied linear programming problem under intuitionistic fuzzy environment. Recently Dubey *et al.*, [16], [17] studied linear programming problem in intuitionistic fuzzy environment using intuitionistic fuzzy number and interval uncertainty in fuzzy numbers.

The motivation of the present study is to give computational algorithm for solving multi objective linear programming problem by intuitionistic fuzzy optimization approach. We also aim to study the impact of various type of membership and non-membership functions in such optimization process and thus have made comparative study of linear membership and non-membership function with that of nonlinear function for membership and non-membership. The study has been organized in continuing

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sections as: Section two contains the preliminaries and basic principle of intuitionistic fuzzy optimization needed for developing algorithm. Section three contains two computational algorithms and the algorithm has been implemented on an illustration in section four and the result obtained has been placed in section five followed by references.

II. Preliminaries

A. Multi Objective Linear Programming Problem

In general, a multi objective optimization problem with p objectives, q constraints and n decision variables, is follows as

$$\max\left[\!\left[z = \{z_1\right]\!\right] z_2 \ldots\ldots\ldots\ldots z_p \}$$

Such that $g_j(x) \leq 0$, i=1.2……q

$$.X_i \geq 0,\ i=1,\ 2\ldots\ldots\ldots\ldots n \qquad (1)$$

where $X=\{X_1, X_2 \ldots\ldots.. X_n\}$

1) Complete solution

x^0 is said to be a complete optimal solution for problem (1) if there exist $x^0 \in X$ such that $f_k(x^0) \geq f_k(x)$, $k=1,\ 2,\ldots\ldots\ldots, p$, for all $x \in X$.

However, in general such complete optimal solutions that simultaneously maximize all of the multiple-objective function do not exist specially the objective functions are conflicting in nature. Thus instead of a complete optimal solution a solution concept, called Pareto optimality was introduced in multi-objective programming.

2) Pareto-optimality

$x^0 \in X$ is said to be a Pareto optimal solution for (1) if there does not exist another $x \in x$ such that $f_k(x^0) \leq f_k(x)$ for all $p=1,2,\ldots\ldots., p$ and $f_j(x^0) < f_j(x)$ for at least one $j \in \{1,\ 2,\ldots\ldots, p\}$.

B. Intuitionistic Fuzzy Sets

Let X be a non-empty set and $I=[0,\ 1]$, then an IFS $\tilde{A}$ is defined as a set $\tilde{A} = \{< x, \mu_{\tilde{A}}(x), \nu_{\tilde{A}}(x) >: x \in X\}$ where $\mu_{\tilde{A}}: X \to I$ and $\nu_{\tilde{A}}: X \to I$ denotes the degree of belonging and the degree of non-belonging with $0 \leq \mu_{\tilde{A}}(x) + \nu_{\tilde{A}}(x) \leq 1$ for each $x \in X$.

Further, every fuzzy set A on a non-empty set X with membership function $\mu_{\tilde{A}}$ is obviously AN IF with $\nu_{\tilde{A}}(x) = 1 - \mu_{\tilde{A}}(x)$ and so IFS is a generalization of a fuzzy set.

Here union and intersection of two intuitionistic fuzzy sets are defined as

$$\tilde{A} \cap \tilde{B} = \{\ [x,\ \min(\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x)),$$
$$\max(\nu_{\tilde{A}}(x),\ \nu_{\tilde{B}}(x))] \mid x \in X\ \}$$

$$\tilde{A} \cup \tilde{B} = \{\ [\ x,\ \max(\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x)),$$
$$\min(\nu_{\tilde{A}}(x),\ \nu_{\tilde{B}}(x))\ ] \mid x \in X\ \}.$$

Fuzzy Optimization Technique Max- min approach Zimmermann first used the max- min operator given by Bellman and Zadeh [18] to solve Multi Objective Linear Programming (MOLP) problems and considered the problem (1) as:

Find X

Such that $Z_k(x) \tilde{\geq} g_k$, $k = 1, 2, \ldots\ldots., p.$

$$g_j(x) \leq 0,\ i=1.2\ldots\ldots q$$
$$X \geq 0 \qquad (2)$$

where g_k, $\forall x$, denote goals and all objective functions are assumed to be maximized. Here objective functions are considered as fuzzy constraints. To establish membership functions of objective functions, we could first obtain the table of positive ideal solution (PIS). Under the concept of min-operator, the feasible solution set is defined by interaction of the fuzzy objective set. This feasible solution set is then characterized by its membership $\mu_D(x)$ which is:

$$\mu_D(x) = \min\ (\mu_1(x), \ldots\ldots\ldots, \mu_k(x)).$$

Further, a decision maker makes a decision with a maximum μ_D value in the feasible decision set. The decision solution can be obtained by solving the problem of maximize $\mu_D(x)$ subject to the given constraintsi.e.

Max [min $\mu_k(x)$]

Such that $g_j(x) \leq 0$, i=1.2……q

Now, if suppose $\alpha = \min_k \mu_k(x)$ be the overall satisfactory level of compromise, then we obtain the following equivalent model

Max α

Such that $\mu_k(x) \geq \alpha,\ \forall k$,

$$g_j(x) \leq 0,\ j=1.2\ldots\ldots q X \geq 0 \qquad (3)$$

C. Intuitionistic Fuzzy Optimization Technique

Consider the intuitionistic fuzzy optimization problem as generalization of the above problem a under taken by Angelov [3]

$$\min\ f_i(x), \qquad i=1,2,\ldots.. p$$
$$g_j(x) \leq 0, \qquad j=1,2,\ldots\ldots. q \qquad (4)$$

where, x is decision variables, $f_i(x)$ denotes objective functions, $g_j(x)$ denotes the constraint functions, p and q denote the number of objective functions and constraints respectively.

The optimal solution of this problem must satisfy all constraints exactly. Thus an analogous fuzzy optimization model of the problem the degree of acceptance of objectives and constraints are maximized as:

$$\widetilde{\min}\ f_i(x), \qquad i=1,2,\ldots\ldots, p$$

$$g_j(x) \lesssim 0, \qquad j = 1, 2, \ldots\ldots. q \tag{5}$$

where $\widetilde{\min}$ denotes fuzzy minimization and $\lesssim$ denotes fuzzy inequality.

For solution of this system (5), Bellman and Zadeh [4] used fuzzy set maximize for the degree of membership of the objectives and constraints as

$$\max \mu_k(x), \quad x \in X, \; k = 1, 2, \ldots.., p+q \;\; 0 \le \mu_k(x) \le 1 \tag{6}$$

where, $\mu_k(x)$ denotes the degree of satisfaction to respective fuzzy sets.

It is important to understand that in fuzzy set the degree of non-membership is complement of membership, hence maximization of membership function will automatically minimize the non-membership. But in intuitionistic fuzzy set degree of rejection is defined simultaneously with the degree of acceptance and both these degree are not complementary each other, hence IFS may give more general tool for describing this uncertainty based optimization model.

Thus, intuitionistic fuzzy optimization (IFO) model for problem(3) is given as

$$\max_x \{\mu_k(x)\}, \quad x \in X \quad k = 1, 2, \ldots\ldots.., p+q$$

$$\min_x \{\nu_k(x)\}, \quad k = 1, 2, \ldots\ldots p+q$$

Such that

$$\nu_k(x) \ge 0, \quad k = 1, 2, \ldots\ldots.. p+q$$

$$\mu_k(x) \ge \nu_k(x), \qquad k = 1, 2, \ldots\ldots., p+q$$

$$\mu_k(x) + \nu_k(x) \le 1, \;\; k = 1, 2, \ldots\ldots., p+q \tag{7}$$

where, $\mu_k(x)$ denotes the degree of acceptance of x to the k^{th} IFS and $\nu_k(x)$ denotes the degree of rejection of x from the k^{th} IFS. These IFS include intuitionistic fuzzy objectives and constraints.

Now the decision set $\tilde{D}$ a conjunction of intuitionistic fuzzy objectives and constraints is defined as

$$\tilde{F} \cap \tilde{C} = \left\{ \begin{array}{l} [x, \min(\mu_{\tilde{F}}(x), \; \mu_{\tilde{C}}(x)), \\ \max(\nu_{\tilde{F}}(x), \; \nu_{\tilde{C}}(x))] \,| \end{array} x \in X \right\} \tag{8}$$

where, $\tilde{F}$ is integrated intuitionistic fuzzy objective and $\tilde{C}$ denotes integrated intuitionistic fuzzy constraints and is defined as:

$$\tilde{F} = \left\{ [x, \mu_{\tilde{F}}(x), \nu_{\tilde{F}}(x)] | x \in X \right\} = \bigcap_{i=1}^{p} \tilde{F}^{(i)}$$

$$= \{ [x, \min_{i=1}^{p} \mu_i^f(x), \max_{i=1}^{p} \nu_i^f(x)] | x \in X \}$$

$$\tilde{C} = \{ [x, \; \mu_{\tilde{C}}(x), \; \nu_{\tilde{C}}(x)] \,|\, x \in X \} = \bigcap_{j=1}^{q} \tilde{C}^{(j)}$$

$$= \{ [x, \; \min_{j=1}^{q} \mu_j^g(x), \; \max_{j=i}^{q} \nu_j^g(x)] \,|\, x \in X \}$$

Further, the intuitionistic fuzzy decision set (IFDS) denoted as $\tilde{D}$:

$$\tilde{D} = \tilde{F} \cap \tilde{C} = \left\{ (x, \; \mu_{\tilde{D}}(x), \; \nu_{\tilde{D}}(x)) \,|\, x \in X \right\} \tag{9}$$

$$\mu_{\tilde{D}}(x) = \min[\mu_{\tilde{F}}(x), \; \mu_{\tilde{C}}(x)] = \min_{k=1}^{p+q} \mu_k(x) \tag{10}$$

$$\nu_{\tilde{D}}(x) = \max[\nu_{\tilde{F}}(x), \nu_{\tilde{C}}(x)] = \max_{k=1}^{p+q} \nu_k(x) \tag{11}$$

where, $\mu_{\tilde{D}}(x)$ denotes the degree of acceptance of IFDS and $\nu_{\tilde{D}}(x)$ denotes the degree of rejection of IFDS.

Now for the feasible solution the degree of acceptance of IFDS is always less than or equal to the degree of acceptance of any objective and constraint and the degree of rejection of IFDS is always more than or equal to the degree of rejection of any objective and constraint, i, e.

$$\mu_{\tilde{D}}(x) \le \mu_k(x), \;\; \nu_{\tilde{D}}(x) \ge \nu_k(x),$$
$$\forall \;\; k = 1, 2, \ldots\ldots, p+q$$

Thus the above system can be transformed to the following system of inequalities:

$$\begin{array}{c} \alpha \le \mu_k(x), \qquad k = 1, \ldots.., p+q \\ \beta \ge \nu_k(x), \qquad k = 1, \ldots\ldots, p+q \\ \alpha + \beta \le 1 \\ \alpha \ge \beta, \;\; \beta \ge 0 \\ x \in X \;\; . \end{array} \tag{12}$$

where, α denotes the minimum acceptable degree of objective(s) and constraints, and β denotes the maximum degree of rejection of objective(s) and constraints.

Now using the Intuitionistic fuzzy optimization the problem (1) is transformed to the linear programming problem given as:

$$\text{Maximize} \quad (\alpha - \beta)$$

$$\begin{array}{ll} \text{Subject to} & \alpha \le \mu_k(x), \qquad k = 1, \ldots\ldots., p+q \;, \\ & \beta \ge \nu_k(x), \qquad k = 1, \ldots\ldots.., p+q \;, \\ & \alpha + \beta \le 1 \;, \\ & \alpha \ge \beta \;, \\ & \beta \ge 0, \\ & x \in X. \end{array} \tag{13}$$

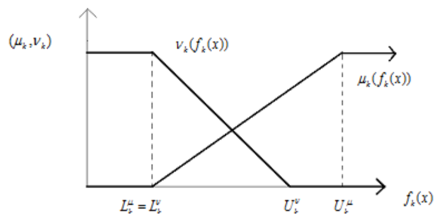

Fig. 1.The illustration of non-linear membership and non-linear non-membership functions.

Now this linear programming problem can be easily solved by a simplex method to give solution of multi-objective linear programming problem (1) by intuitionistic fuzzy optimization approach.

Fig. 1 illustrates the linear membership and linear non-membership functions.

III. Computational Algorithm

A. Algorithm I (Linear Membership Function)

Step 1. Taking the first objective function from set of k objectives of the problem and solve it as a single objective subject to the given constraints. Find value of objective functions and decision variables.

Step 2. From values of these decision variables compute values of remaining (k-1) objectives.

Step 3. Repeat the Step 1 and Step 2 for remaining (k-1) objective functions.

Step 4. Tabulate values of objective functions thus obtained from Step 1 and Step 2 and Step 3 to form to form a table known as PIS.

Step 5. From Step 4 obtain the lower bounds and upper bounds for each objective functions.

TABLE I: Positive Ideal Solution (PIS)

	f_1 f_2 f_3 f_k				X
max f_1	f_1^*	$f_2(X_1)$	$f_3(X_1)$	.... $f_k(X_1)$	X_1
max f_2	$f_1(X_2)$	f_2^*	$f_3(X_2)$.	.... $f_k(X_2)$	X_2
max f_3	$f_1(X_3)$	$f_2(X_3)$	f_3^*	.... $f_k(X_3)$	X_3
:	:				:
:	:				:
max f_k	$f_1(X_k)$	$f_2(X_k)$	$f_3(X_k)$	.... $f_k^*(X_k)$	X_k
	$f_1^{'}$ $f_2^{'}$ $f_3^{'}$			$f_k^{'}$	

where f_k^* and $f_k^{'}$ are the maximum, minimum values respectively.

Step 6. Set $U_k^\mu = \max(Z_k(X_r))$ and $L_k^\mu = \min(Z_k(X_r))$, $1 \le r \le p$ for membership and for non-membership functions $U_k^\nu = U_k^\mu - \lambda(U_k^\mu - L_k^\mu)$ and $L_k^\nu = L_k^\mu$, $0 < \lambda < 1$.

Step 7. Use following linear membership function $\mu_k(f_k(x))$ and non-membership function $\nu_k(f_k(x))$ for each objective functions:

$$\mu_k(f_k(x)) = \begin{cases} 0 & if \quad f_k(x) \le L_k^\mu \\ \dfrac{f_k(x) - L_k^\mu}{U_k^\mu - L_k^\mu} & if \quad L_k^\mu \le f_k(x) \le U_k^\mu \\ 1 & if \quad f_k(x) \ge U_k^\mu \end{cases}$$

$$\nu_k(f_k(x)) = \begin{cases} 0 & if \quad f_k(x) \ge U_k^\nu \\ \dfrac{U_k^\nu - f_k(x)}{U_k^\nu - L_k^\nu} & if \quad L_k^\nu \le f_k(x) \le U_k^\nu \\ 1 & if \quad f_k(x) \le L_k^\mu \end{cases}$$

Step 8. Now theintuitionistic fuzzy optimization method for MOLP problem (1) with linear membership and non membership functions gives a equivalent linear programming problem as :

Maximize $(\alpha - \beta)$

Subject to $\alpha \le \mu_k(f_k(x))$,

$$\begin{aligned} &\beta \ge \nu_k(f_k(x)) \ , \\ &\alpha + \beta \le 1 \ , \\ &\alpha \ge \beta \ , \\ &\beta \ge 0, \\ &g_j(x) \le b_j \ , x \ge 0 , \\ &k = 1,2,\dots p; \quad j = 1,2,\dots,q . \end{aligned} \tag{14}$$

Step 9. The above linear programming problem(14) can be easily solved by olve the above a simplex method.

B. Algorithm II (Nonlinear Membership Function)

Repeat steps 1 to step 6 and construct table of positive ideal solutions.

Step 7. Assume that solutions so for computed by algorithm follow hyperbolic function for membership and exponential for non-membership function given as

$$\mu_k(f_k(x)) = \begin{cases} 0, & f_k(x) < L_k^\mu \\ 1 - Exp\left\{-\psi \dfrac{f_k(x) - L_k^\mu}{U_k^\mu - L_k^\mu}\right\} & , \quad L_k^\mu \le f_k(x) \le U_k^\mu \\ 1, & f_k(x) \ge U_k^\mu \ and \ \psi \to \infty \end{cases}$$

$$\nu_k(f_k(x)) = \begin{cases} 1, & f_k(x) \le L_k^\nu \\ \dfrac{1}{2} + \dfrac{1}{2}\tanh\left(\delta_k . \dfrac{U_k^\nu + L_k^\nu}{2} - f_k(x)\right), & L_k^\nu \le f_k(x) \le U_k^\nu \\ 0, & f_k(x) \ge U_k^\nu \end{cases}$$

where ψ, δ_k are non zere parameters precribed by the decision maker.

Further, the intuitionistic fuzzy optimization technique for MOLP problem (1) with the exponential membership and hyperbolic non membership functions gives the following linear programming problem :

Maximize $(\alpha - \beta)$

Subject to $\alpha \le \mu_k(f_k(x))$,

$$\begin{aligned} &1 - Exp\left\{-\psi \frac{f_k(x) - L_k^\mu}{U_k^\mu - L_k^\mu}\right\} \ge \alpha \\ &\beta \ge \nu_k(f_k(x)) \ , \\ &\frac{1}{2} + \frac{1}{2}\tan h\left(\delta_k \frac{U_k^\nu + L_k^\nu}{2} - f_k(x)\right) \le \beta , \\ &\alpha + \beta \le 1 \ , \\ &\alpha \ge \beta \ , \\ &\beta \ge 0, \\ &g_j(x) \le b_j \ , x \ge 0 \\ &k = 1,2,\dots p; \quad j = 1,2,\dots,q . \end{aligned} \tag{15}$$

For solution convenience the above problem (15) is transformed to

Maximize $\gamma - \eta$

Subject to $f_k(x) - \frac{\gamma(U_k^\mu - L_k^\mu)}{4} \geq L_k^\mu$, where $\gamma = -\log(1-\alpha$,

$$f_k(x) - \frac{\eta}{\delta_k} \geq \frac{U_k^\nu + L_k^\nu}{2},$$

where $\eta = -\tanh^{-1}(2\beta - 1)$, and $\psi = 4$, $\delta_k = \frac{6}{U_k^\nu - L_k^\nu}$

$$\gamma \geq \eta,$$
$$\gamma + \eta \leq 1,$$
$$\eta \geq 0, \tag{16}$$
$$g_j(x) \leq b_j \ , x \geq 0$$
$$k = 1,2,\ldots.p; \quad j = 1,2,\ldots.,q \ .$$

Which can be easily solved by a simplex method.

IV. Numerical Illustration

A. Production Planning Problem

Consider a park of six mechine types whose capacities are to be devoted to production of three products. A current capacity portfolio is available , measured in mechine hours per weak for each mechine capacity unit priced according to machine type.

Necessary data is summerized below Table II.

TABLE II: Physical Parameter Values

Machine type	Machi hours	Unit price ($100 per hour)	Products x_1	x_2	x_3
Milling machine	1400	0.75	12	17	0
Lathe	1000	0.60	3	9	8
Grinder	1750	0.35	10	13	15
Jig saw	1325	0.50	6	0	16
Drill press	900	1.15	0	12	7
Band saw	1075	0.65	9.5	9.5	4
Total capacity cost $4658.75					

TABLE III: Positive Ideal Solution

	f_1	f_2	f_3	X
Max f_1	8041.14	10020.33	9319.25	X_1
Max f_2	5452.63	10950.59	5903.00	X_2
Max f_3	7983.60	10056.99	9355.90	X_3

Let x_1, x_2, x_3 denote three products, then the complete mathematical formulation of the above mentioned problem as a Multi objective Linear Programming (MOLP) problem is given as:

$$\text{Max} \quad f_1(x) = 50x_1 + 100x_2 + 17.5x_3 \quad \text{(profit)}$$
$$\text{Max} \quad f_2(x) = 92x_1 + 75x_2 + 50x_3 \quad \text{(quality)}$$
$$\text{Ma} \quad f_3(x) = 25x_1 + 100x_2 + 75x_3 \ \text{(worker satifaction)}$$

Subject to the constraints

$$12x_1 + 17x_2 \leq 1400$$
$$3x_1 + 9x_2 + 8x_3 \leq 1000$$
$$10x_1 + 13x_2 + 15x_3 \leq 1750$$
$$6x_1 + 16x_3 \leq 1325$$
$$x_1, \quad x_2, \quad x_3 \geq 0. \tag{17}$$

Solution of the above problem is considered by the algorithm I and algorithm II mentioned in previous sections. For illustration of the procedures some of steps are shown as

Step 1. Solve a linear programming problem taking one objective

Maximize $f_1 = 50x_1 + 100x_2 + 17.5x_3$

Subject to the constraints

$$12x_1 + 17x_2 \leq 1400$$
$$3x_1 + 9x_2 + 8x_3 \leq 1000$$
$$10x_1 + 13x_2 + 15x_3 \leq 1750$$
$$6x_1 + 16x_3 \leq 1325 \tag{18}$$
$$12x_2 + 7x_3 \leq 900$$
$$9.5x_1 + 9.5x_2 + 4x_3 \leq 1075$$
$$x_1, \quad x_2, \quad x_3 \geq 0.$$

TABLE IV: Values of Optimal Decision Vectors

λ	x_1	x_2	x_3	α	β
	Intuitionistic fuzzy optimization Technique when membership and Non- memberships are linear.				
.1	65.2571	26.9187	49.8324	.5899	.4101
.2	58.4833	34.5907	47.6992	.8525	.1475
.3	65.2600	26.9155	49.8333	.7583	.2417
.4	65.2585	26.9172	49.8328	.8847	.1153
.5	66.1947	25.8441	49.2978	1.000	.0000
.6	71.1362	22.6184	44.8504	1.000	.0000
.7	71.7199	25.7841	35.6084	1.000	.0000
.8	75.3355	14.2823	45.3258	1.000	.0000
.9	82.1131	9.12270	46.1075	1.000	.0000
	Intuitionistic fuzzy optimization Technique when membership and Non- memberships are Non-linear				
λ	x_1	x_2	x_3	α	β
.1	49.8906	47.1360	42.5550	.6321	.3345
.2	64.6968	36.6846	41.7421	.6321	.0073
.3	62.1896	38.0097	41.8452	.6321	.0009
.4	62.8180	38.0109	41.5300	.6321	.0001
.5	62.8157	38.0125	41.8454	.6321	.0000
.6	62.8163	38.0120	41.8454	.6321	.0000
.7	59.7690	40.1631	42.0127	.6321	.0000
.8	62.8265	38.0048	41.8448	.6321	.0000
.9	62.8207	38.0087	41.8451	.6321	.0000

Optimal solution to this crisp linear programming problem is

$$x_1 = 44.93, \quad x_2 = 50.63, \quad x_3 = 41.77 \ ,$$
$$(f_1)_1 = 8041.14$$

Step 2. Withthese decision variables, computed values of other remaining objective functions are:

$$(f_2)_1 = 10020.33$$

$$(f_3)_1 = 9319.25$$

Step 3. Step 1 and Step 2 are repeated for other objective functions f_2, f_3.

Step 4. The Positive Ideal Solution (PIS) obtained are placed in Table III.

Step 5. Applying the solution algorithm I and algorithm II , the solutions of the mentioned MOLP are obtained. The problem is solved by linear membership and non membership and is also solved by nonlinear membership and nonmembership functionusing various values of λ α β the solutions thus obtained are placed in Table IV to have insight in the solution process. The feasibility of solutions in view of various satisfaction levels are depicted in the Table V.

TABLE V: VALUES OF OPTIMAL OBJECTIVE FUNCTIONS

	Intuitionistic fuzzy optimization Technique when membership and Non- memberships are linear.			
λ	max f_1	max f_2	max f_3	Total
0.1	6826.7920	10514.1757	8060.7275	25401.6952
0.2	7217.9710	10359.7261	8498.5925	26076.2896
0.3	6826.6328	10514.2475	8060.5475	25401.4278
0.4	6826.7190	10514.2120	8060.6425	25401.5735
0.5	6756.8565	10493.1099	7936.6125	25186.5789
0.6	6603.5320	10483.4304	7404.0250	24490.9874
0.7	6787.5520	10312.4583	7041.0375	24142.0478
0.8	5988.2065	10268.3285	6711.0525	22967.5875
0.9	5824.8063	10543.9827	6423.1600	22791.9490
	Intuitionistic fuzzy optimization Technique when membership and Non- memberships are non-linear.			
λ	max f_1	max f_2	max f_3	Total
0.1	7952.8425	10252.8852	9152.4900	27358.2177
0.2	7633.7868	10790.5556	8416.5375	26840.8793
0.3	7642.7410	10664.4307	8494.1000	26801.2717
0.4	7668.7650	10706.5735	8486.2900	26861.6285
0.5	7674.3295	10722.2519	8510.0475	26906.6289
0.6	7674.3095	10722.2696	8510.0125	26906.5916
0.7	7737.9823	10611.6155	8661.4875	27011.0853
0.8	7674.0890	10722.6380	8509.5025	26906.2295
0.9	7674.1942	10722.4119	8509.7700	26906.3761

TABLE VI: COMPARISON OF OPTIMAL SOLUTIONS OBTAINED BY VARIOUS METHODS

Decision variables & objective functions	Best Solution obtained by fuzzy optimization method with level of satis faction α=0.5309	Best Solution obtained by proposed intutionistic fuzzy optimization *alogirthm I*	Best Solution obtained by proposed intutionistic fuzzy optimization *alogirthm II*
x_1	65.2571	58.4833	49.8906
x_2	26.9187	34.5907	47.1360
x_3	49.8324	47.6992	42.5550
f_1	6826.7920	7217.9710	7952.8425
f_2	10514.1757	10359.7261	10252.8852
f_3	8060.7275	8498.5925	9152.4900
Sum of objectives	25401.6952	26076.2896	27358.2177

V. CONCLUSIONS

In view of comparing the intuitionistic fuzzy optimization with fuzzy optimization method, we also obtained the solution of the undertaken numerical problem by fuzzy optimization method given by Zimmermann [17] and took the best result obtained for comparison with present study. We considered the best solution obtained by the developed two algorithms and are placed in Table VI for comparison with each other and also to compare with the results obtained by fuzzy optimization method.

The objective of the present study is to give the effective algorithm for intuitionistic fuzzy optimization method for getting optimal solutions to a multi objective linear programming problem. The merit of the method lies with fact that it gives a set of solutions with various level of satisfaction to the decision makers. The decision makers may choose a suitable optimal solution according to the demand of the actual situation. Further the comparisons of results obtained for the undertaken problem clearly show the superiority of intuitionistic fuzzy optimization over fuzzy optimization. The results thus obtained also reveal that intuitionistic fuzzy optimization by proposed *algorithm II* using nonlinear membership and nonlinear non membership give a better result than intuitionistic fuzzy optimization *algorithm I* using linear membership function and linear non membership function.

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