Solving Non-Linear Equation by Newton-Raphson Method using Built-in Derivative Function in Casio *fx*-570ES Calculator

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Abstract: *This paper studied the difference between the used of built in derivative function of scientific calculator and self derivative function in solving non-linear equation by means of Newton-Raphson method. Since this method requires the derivative of the function, some basic differentiation skills are needed. However, functions especially highly non-linear functions are hardly to be differentiated. Students may find it is difficult and thus quit from it. Now, with the help of the built in derivative function in scientific calculator, the problem can be easily solved regardless of the analytical expression. Results obtained using built-in derivative function and exact derivative are compared and discussed. A total of 350 engineering students (2009) from School of Technology in Kolej Tunku Abdul Rahman who enrolled in subject ATGE3083 Mathemathics V have been taught using this method to solve non-linear equations. Analyses were done using Statistical Package for Social Science (SPSS) and the finding shows there is a great improvement where students more confident in solving non-linear problem using lesser time. Casio fx-570ES was used in this study as this model of calculator is widely used in Malaysia.*

1.   Introduction

Over the years, finding roots of non-linear equation have attracted researchers' intentions. Many methods have been introduced and give good results. However, some methods do guarantee results but convergence speed is low. Unlike Newton-Raphson method which provide good results and fast convergence speed under certain conditions. Many mathematical packages such as Matlab adopted this method in solving non-linear equation as well as systems non-linear equations. However, most of the packages have to be installed into personal computer and it is costly if compared to a scientific calculator.

Calculator is a tool for calculating as well as a tool for learning mathematics. Despite the myths of harmful consequences resulting from their use, calculators are a pedagogical tool of great value. Kissane (2006), has described in details the role of calculator. He stated that students using calculator for vital learning to display mathematics concepts and idea. Nevertheless, Rittle-Johnson and Alexander Kmicikewycz (2008), proved that students using calculators in mathematics were be able to practice more problems and had fewer errors in calculation.

2.   Problem Statement

Most of the students dislike mathematics especially numerical analysis due to large iterative calculations. Students get frustrated and give up in solving it. To overcome the problem, scientific calculators provide ways to students to shorten up the time in solving problems. However, some researchers claim that this will affect students' understanding in mathematics. This paper is to study the effectiveness of using scientific calculator in solving non-linear equations by means of Newton-Raphson method either by self-derivatives or built-in derivative function.

3.   Research Questions

This study is to determine the effectiveness of using scientific calculator Casio *fx*-570ES in finding the roots of non-linear equations by the means of Newton-Raphson's method using manual derivative and built-in derivative function.

1.   Is there a difference between manual derivative and built-in derivative function in solving non-linear equation?
2.   Does this help students in solving non-linear equation in more convenient way using built-in derivative function in Casio *fx*-570ES?
3.   Does this help students in solving non-linear equation faster using built-in derivative function in Casio *fx*-570ES?

4.   Research Hypothesis

1.   By using Casio *fx*-570ES scientific calculator, there is a difference of solution between manual derivatives and built-in derivative function in solving non-linear equation using Newton-Raphson method.
2.   Solving non-linear equation by built-in derivative function in Casio fx-570ES using Newton-Raphson method does provide convenient way to student.
3.   Solving non-linear equation by built-in derivative function in Casio fx-570ES using Newton-Raphson method uses lesser time.

5.   Methods

In this study, a total of 372 engineering students, from School of Technology in Kolej Tunku Abdul Rahman at Kuala Lumpur, who enrolled in ATGE3083 Mathematics V in second semester year 2009/2010 were selected. 22 students were randomly selected out from the population to set up the reliability of the test. The distributions of the remaining students were as follow: 170 students from Diploma in Mechanical Engineering (DME2), 20 students from Diploma in Material Engineering (DMT2), 10 students from Diploma in Telecommunication (DTE2), 85 students from Diploma in Electrical Engineering (DEL2) and 65 students from Diploma in Mechatronics Engineering (DMH2).

Students were first taught with traditional ways in the first lecture where students solved the equation step by step based on Newton-Raphson formulae. During second lecture, they were taught solving non-linear equations using Casio *fx*-570ES (manual derivative) and lastly, they were taught solving non-linear equations using Casio *fx*-570ES (built-in derivative). Later tests were conducted in their tutorial class respectively. Each test was given 15 minutes to complete. Total time for each student to complete the test was recorded once they submitted the test to their invigilator. Test 1 was done using traditional method where student solved the equation as taught in the lecture and calculator was used for simple operator calculations. Test 2 was done one week after Test 1 was conducted where students need to differentiate the equations manually and keyed into calculator to perform iterative calculation. Lastly, students sat for Test 3 where they used Built-In differentiation function inside calculator and solved it iteratively.

The mean marks and mean duration took to complete the test were recorded and analyzed by using Statistical Package for the Social Science (SPSS). In order to test the hypothesis regarding whether there is any difference and improvement in marks and duration for two methods, the alpha value which is 0.05 is compared with the *p* value obtained.

6.   Newton-Raphson Method

In numerical analysis, Newton–Raphson (James, 2008) method also known as Newton's methods is one of the well-known approximation methods in solving non-linear equations. Newton's method is well-known for its fast converge speed; especially when the initial guess is sufficiently closed to the root.

The general form of Newton Methods are given as

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}, \quad i=0,1,2\ldots$$

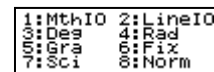where function $f(x)$ is the non-linear equation and its derivative $f'(x)$.

6.1   Example

By using Newton Raphson Method, find the root of equation for $f(x) = \cos(x^2) - x\sin(x)$. Choose $x=1$ as initial guess and the algorithm shall stop at $|x_{i+1} - x_i| < 0.0005$
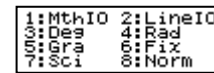
From the question, it is known that $f(x) = \cos(x^2) - x\sin(x)$ and $f'(x) = -2x\sin(x^2) - x\cos(x) - \sin(x)$.

6.1.1   Algorithm of Casio fx-570ES for Newton-Raphson Method (Manual Derivatives)
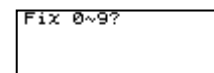
Step 1:   First, set the calculator into radian mode and fix mode into 4 decimal places. Press [SHIFT] [MODE] [4]   for radian mode since the function is a trigonometric function.

```
1:MthIO  2:LineIO
3:Deg    4:Rad
5:Gra    6:Fix
7:Sci    8:Norm
```

Press [SHIFT] [MODE] [6] for fix mode.

```
1:MthIO  2:LineIO
3:Deg    4:Rad
5:Gra    6:Fix
7:Sci    8:Norm
```
```
Fix 0~9?
```

Press [SHIFT] [MODE] [4]   for 4 decimal places.

Step 2:   Input the iteration formula into the calculator which is as follow:

$$x = x - \frac{\cos(x^2) - x\sin(x)}{-2x\sin(x^2) - x\cos(x) - \sin(x)}$$

Press [ALPHA] [)] [ALPHA] [CALC] [ALPHA] [)] [−] [▤]

```
X=X-□/□
```

Shift the cursor up to the numerator box and key in the function,
Press [cos] [ALPHA] [)] [x²] [)] [−] [ALPHA] [)] [sin] [ALPHA] [)] [)]

```
◄s(X²)-Xsin(X)
        □
```

Shift the cursor down to the denominator box and key in the function,
Press [−] [2] [ALPHA] [)] [sin] [ALPHA] [)] [x²] [)] [−] [ALPHA] [)] [cos] [ALPHA] [)] [)] [−] [sin] [ALPHA] [)] [)]

```
)-Xsin(X)
Xcos(X)-sin(X)I
```

Step 3:   Press [CALC] and screen as shown below is displayed to ask for input of initial guess.

```
X?
        0.0000
```

Press ① ☰

Step 4: Press ☰ Ans ☰ ⓪ · ⑨ ⓪ ① ⑦ to update the
value *x*.

Continue to update the new *x* until the iteration value obtained is equal for next iteration. This means that $f(x_i) \approx 0$ which satisfy our stopping criteria.

SOLUTION:

Table 1 Iterative Table for Manual Derivative

i	x_i	$\left|x_{i+1} - x_i\right|$
0	1	
1	0.9017	0.0983
2	0.8942	0.0075
3	0.8941	0.0001

Since, $\left|x_3 - x_2\right| < 0.0005$, hence x_3 is the root. One of the roots for $f(x) = \cos(x^2) - x\sin(x)$ is $x^* = 0.8941$.

6.1.2 Algorithm of Casio fx-570ES for Newton-Raphson Method (Built-In Derivatives)

Step 1: First, set the calculator into radian mode and fix mode into 4 decimal places.
Press SHIFT MODE ④ for radian mode since the function is a trigonometric function.
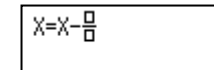
Press SHIFT MODE ⑥ for fix mode.

Press SHIFT MODE ④ for 4 decimal places.

Step 2: Input the iteration formula into the calculator which is as follow:

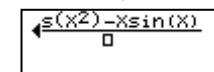$$x = x - \frac{\cos(x^2) - x\sin(x)}{\dfrac{d}{dx}\left(\cos(x^2) - x\sin(x)\right)\Big|_{x=x}}$$

Press ALPHA) ALPHA CALC ALPHA) ☐ ⊟ ☐

Shift the cursor up to the numerator box and key in the function,

Press COS ALPHA) x^2) ⊟ ALPHA) sin ALPHA))
Shift the cursor down to the denominator box and key in the function,

Press SHIFT ☐ and get the screen as shown

Shift the cursor into the derivative function and key in the function *f(x)*.

Shift the cursor into x input and press [ALPHA] [)]

Step 3: Press [CALC] and screen as shown below is displayed to ask for input of initial guess.

Press [1] [=]

Step 4: Press [=] [0] [·] [9] [0] [1] [7] to input the next value into new

Continue to update the new x until the iteration value obtained is equal for next iteration. This means that $f(x_i) \approx 0$ which satisfy our stopping criteria.

SOLUTION:

Table 2 Iterative Table For Built-In Derivative

i	x_i	$\left|x_{i+1} - x_i\right|$
0	1	
1	0.9017	0.0983
2	0.8942	0.0075
3	0.8941	0.0001

Since, $\left|x_3 - x_2\right| < 0.0005$, hence x_3 is the root. One of the roots for $f(x) = \cos(x^2) - x\sin(x)$ is $x^* = 0.8941$.

From Table 1 and 2, both methods give same root of equation which is 0.8941. However, there is a tiny difference in $f'(x_i)$ as shown in Table 3. This is because the built-in function use the concept of central difference approximation, where the derivative is computed by perturbed the x. Thus, this may cause difference in calculation especially if the function $f(x)$ is highly non-linear. It may cause the number of iteration will be increased and yet more time consuming.

Table 3 Difference of $f'(x_i)$ between manual derivative and built-in derivative

i	Manual Derivative, Mx_i	Built-In Derivative, Bx_i	Error, $\left|Mx_i - Bx_i\right|$
0	-3.0647	-3.0647	0
1	-2.6538	-2.6537	0.0001
2	-2.6219	-2.6220	0.0001

7. Result and Analysis
7.1 Students' Mean Scores

This research was carried out and was measured using three tests. A total of 22 students from five courses were randomly selected to sit for all three tests before used of calculator in solving non-

linear equations were introduced to them. These 22 students played as control group where there will not take the treatment test. Levels of difficulty of all three tests are similar. Table 4 shows the correlation score of the 22 students in the three tests.

Table 4 Correlations score between Traditional, Manual Derivatives and Built in Derivatives.

		Test 1	Test 2	Test 3
Test 1	p-value, Sig. (2-tailed)	-	0.863	0.949
	N	22	22	22

As shown in table 4, the p-value (Sig. 2-tailed) is greater than 0.05 which means that the means score for Test 1, Test 2 and Test 3 have no significant changes. In other words, the levels of difficulty of the test are similar.

Table 5 Means score and standard deviation of students' marks for test 1, 2 and 3 respectively.

Test		DME2	DMT2	DTE2	DEL2	DMH2	TOTAL
	N	170	20	10	85	65	350
1	Mean	5.89	6.45	5.60	6.60	5.87	6.08
	Stand. Dev	2.36	2.24	2.63	2.91	2.29	2.49
2	Mean	7.75	7.85	8.10	7.54	7.56	7.76
	Stand. Dev	1.37	0.99	1.03	1.52	1.37	1.26
3	Mean	8.45	8.48	8.30	8.70	8.33	8.45
	Stand. Dev	0.69	0.71	0.57	0.82	0.59	0.68

From Table 5, it is found that the mean score for all five courses has increased very well and the standard deviation has decreased significantly. Hence, the students' performances in solving the non-linear equations increased after treatment was given to them. The maximum marks for each test is 10%.

To analyze students' performance, paired sample t-test is carried out for each different course. The results of the analyses are shown in Table 6 and Table 7.

Table 6 Paired sample t-test for mean of score of 5 different courses between Traditional Method and Built-In Derivatives

		Paired Differences							Sig. (2-tailed)
					95% Confidence Interval of the Difference				
		Mean	Std. Deviation	Std. Error Mean	Lower	Upper	t	df	
Pair 1	test1dme - test2dme	-1.86	2.90	0.22	-2.30	-1.43	-8.39	169.00	0.00
Pair 2	test1dmt - test2dmt	-1.40	2.48	0.55	-2.56	-0.24	-2.53	19.00	0.02
Pair 3	test1dte - test2dte	-2.50	2.80	0.89	-4.50	-0.50	-2.82	9.00	0.02
Pair 4	test1del - test2del	-0.94	2.57	0.28	-1.50	-0.39	-3.38	84.00	0.00
Pair 5	test1dmh - test2dmh	-1.60	2.93	0.36	-2.33	-0.87	-4.40	64.00	0.00

Table 7 Paired sample t-test for mean of score of 5 different courses between Self-Derivative and Built-In Derivatives

		Paired Differences							
				Std. Error Mean	95% Confidence Interval of the Difference		t	df	Sig. (2-tailed)
		Mean	Std. Deviation		Lower	Upper			
Pair 1	test2dme - test3dme	-0.72	0.84	0.06	-0.85	-0.60	-11.19	169.00	0.00
Pair 2	test2dmt - test3dmt	-0.60	0.60	0.13	-0.88	-0.32	-4.49	19.00	0.00
Pair 3	test2dte - test3dte	-0.60	0.84	0.27	-1.20	0.00	-2.25	9.00	0.05
Pair 4	test2del - test3del	-0.79	1.00	0.11	-1.00	-0.57	-7.26	84.00	0.00
Pair 5	test2dmh - test3dmh	-0.83	1.07	0.13	-1.10	-0.57	-6.27	64.00	0.00

As resulted in Table 6, the p-value obtained from courses DME, DEL and DMH are 0, while DMT and DTE is 0.02. At the significant level of 5%, there is sufficient evidence to support the hypothesis that the mean score has increased since the p-value is less than the significant value 0.05.

In Table 7, the p-value obtained from courses DME, DMT, DEL and DMH are 0 and DTE is 0.025. At the significant level of 5%, there is sufficient evidence to support the hypothesis that the mean score has increased since the p-value is less than the significant value 0.05. Obviously, students' mean score increase significantly after treatment was given to them in solving non-linear equations.

7.2 Duration taken

Table 8 shows the mean of time and standard deviation of time taken by student to complete test 1, 2 and 3 respectively. Each test was given 15 minutes to complete. Time was recorded once students submitted their test to their invigilator.

Table 8 Mean time and standard deviation time taken by student to complete test 1, 2 and 3 respectively

Test		DME2	DMT2	DTE2	DEL2	DMH2	TOTAL
	N	170	20	10	85	65	350
1	Mean	12.42	10.40	11.50	10.53	9.89	10.95
	Stand. Dev	1.42	1.54	1.58	1.39	1.20	1.43
2	Mean	5.49	5.60	5.80	5.56	5.37	5.46
	Stand. Dev	0.94	0.99	1.14	1.01	0.99	1.05
3	Mean	4.99	4.85	4.50	4.59	4.58	4.70
	Stand. Dev	0.69	0.75	0.53	0.70	0.66	0.67

From Table 8, it is found that the mean time for all five courses has decreased significantly. This means that majority students able to solve non-linear equations in a shorter time. Students take

lesser time in solving the non-linear equations after treatment was given to them. The maximum time for each test is 15 minutes.

To analyze students' performance, paired sample t-test is carried out for each different courses. The results of the analyses are shown in Table 9 and Table 10.

Table 9 Paired sample t-test for mean of time of 5 different courses between Traditional Method and Built-In Derivatives

| | | Paired Differences | | | | | | | |
| | | | | | 95% Confidence Interval of the Difference | | | | |
		Mean	Std. Deviation	Std. Error Mean	Lower	Upper	t	df	Sig. (2-tailed)
Pair 1	test1dme - test2dme	6.93	1.83	0.14	6.65	7.21	49.41	169.00	0.00
Pair 2	test1dmt - test2dmt	4.80	1.96	0.44	3.88	5.72	10.94	19.00	0.00
Pair 3	test1dte - test2dte	6.20	2.35	0.74	4.52	7.88	8.35	9.00	0.00
Pair 4	test1del - test2del	4.96	1.52	0.16	4.64	5.29	30.21	84.00	0.00
Pair 5	test1dmh - test2dmh	4.52	1.58	0.20	4.13	4.92	23.05	64.00	0.00

Table 10 Paired sample t-test for mean of time of 5 different courses between Self-Derivative and Built-In Derivatives

| | | Paired Differences | | | | | | | |
| | | | | | 95% Confidence Interval of the Difference | | | | |
		Mean	Std. Deviation	Std. Error Mean	Lower	Upper	t	df	Sig. (2-tailed)
Pair 1	test2dme - test3dme	0.50	1.18	0.09	0.32	0.68	5.53	169.00	0.00
Pair 2	test2dmt - test3dmt	0.75	1.25	0.28	0.16	1.34	2.68	19.00	0.01
Pair 3	test2dte - test3dte	1.30	1.57	0.50	0.18	2.42	2.62	9.00	0.03
Pair 4	test2del - test3del	0.98	1.23	0.13	0.71	1.24	7.29	84.00	0.00
Pair 5	test2dmh - test3dmh	0.78	1.12	0.14	0.51	1.06	5.62	64.00	0.00

As resulted in Table 9, the p-value obtained from all five courses is 0. At the significant level of 5%, there is sufficient evidence to support the hypothesis that the mean time has decreased since the p-value is less than the significant value 0.05.

In Table 10, the p-value obtained from courses DME, DEL and DMH are 0 while DMT and DTE is 0.005 and 0.015 respectively. At the significant level of 5%, there is sufficient evidence to support

the hypothesis that the mean time has decreased since the p-value is less than the significant value 0.05. Obviously, students' mean time decrease significantly after treatment was given to them in solving non-linear equations.

8. Discussion

From the results obtained and analyzed, it can be concluded that by using Casio *fx*-570ES scientific calculator in solving non-linear equations by means of Newton-Raphson give a remarkable and significant improvement not only in score obtained, but as well total time in solving problems. Nevertheless, from the feedback of the students, they are now very confident in solving any non-linear equations by using scientific calculator with the means of Newton-Raphson method.

Table 11 Mistakes did by students during test 1, 2 and 3

Test	DME2			DMT2			DTE2			DEL2			DMH2		
	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3
Number of students do not set mode of calculator into radians mode	20	5	3	1	0	0	2	1	2	5	0	1	0	0	0
Number of students having problems in differentiating the function	20	15	0	2	0	0	2	1	0	5	2	0	5	1	0
Number of students do not have sufficient time to complete the test	25	0	0	3	0	0	3	0	0	8	0	0	5	0	0
Total	65	20	3	6	0	0	7	2	2	18	2	1	10	1	0

From Table 11, the number of students made mistakes and had problem in differentiating function had decreased drastically. This shows that with the help of Casio *fx*-570ES, the percentage of making error in solving non-linear equations had been reduced nevertheless the total time taken to solve the problem no longer a nightmare to students.

9. Conclusions and Discussions

From this study, the effectiveness of using scientific calculator in solving non-linear equations using Newton-Raphson method is clearly stated. This not only reflects on the improvement on students' score, while this also reduces the time in differentiating and solving non-linear equations. With the help of built-in derivative function in calculator, students are now able to solve any complicated non-linear functions within a shorter time rather than spending times in differentiating the functions at any place at any time. This research also shows that the common mistakes made by the participants had been reduced after they were taught the technique to solve the problem using the calculator.

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