

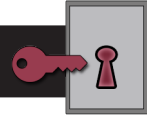
BONUS ACTIVITY (ONLINE ONLY)



Solving Word Problems

Learning skills: defining the problem, defining knowns and validating

WHY



Mathematical word problems (or story problems) require you to take real-life situations and find solutions by translating the given information into equations with unknowns. Since very few problems in life are clear cut with simple steps and easily defined numbers, knowing how to set up and solve (word) problems is something beneficial to know. Although you may be anxious when you see a word problem, having a strategy will help you to organize your information and resources so that you can become a good problem solver.

LEARNING OBJECTIVES



1. Read a word problem and write a statement defining the problem.
2. Correctly set up mathematical equations based on the information provided in the problem.
3. Apply the Methodology for Solving Word Problems.

PERFORMANCE CRITERIA



CRITERION #1: answers to the Critical Thinking Questions

Attributes:

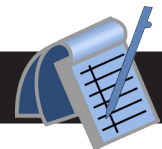
- a. thorough and complete
- b. accurate or correct

CRITERION #2: solution to the In-Class Exercise

Attributes:

- a. documentation of the steps of the methodology for solving word problems
- b. final solution is correct

PLAN



1. Read through the Methodology for Solving Word Problems.
2. Study the example problems which illustrate the methodology.
3. Answer the Critical Thinking Questions.
4. Solve the In-Class Exercise. Exchange your solution to the problem with another team. Perform an assessment of the other team's solution and documentation of the methodology for solving word problems. Indicate strengths, areas for improvement and insights gained.

Methodology for Solving Word Problems

1. Read and define the problem.
2. Identify the given information.
3. Decide what information is relevant to the problem.
4. Decide what is (are) the key unknown value(s) or variable(s).
5. Model the problem.
 - Begin with an equation containing the most important unknown value.
 - Write additional equations for any unknown variables.
 - Continue writing equations until the number of equations equals the number of unknowns.
6. Evaluate the model; solve the equations from Step 5.
7. Validate the solution.

Helpful Tips

1. Sometimes it is necessary to draw upon previously learned common knowledge that is not explicitly stated in the problem. You might even have to look up a formula.
2. One relationship that is useful to know is that two consecutive integers are expressed as x for the first one and $x+1$ for the second one. Two consecutive even or odd integers would be x and $x+2$.
3. If the sum of two numbers is given, then the two numbers can be expressed as x and $SUM - x$.
4. Always convert percents to decimal form for calculations.
5. If quantities are to be combined or compared, there will be *a quantity and a value multiplied together*.
 - the quantity can be how many objects, how much money, etc.
 - the value can be expressed as a percent, a cost per item, or some ratio that compares the items to the total

Example 1 – Solving Word Problems

A car leaves a town traveling at 40 miles per hour. Two hours later, a second car leaves the same town, on the same road, traveling at 60 miles per hour. In how many hours will the second car pass the first car?

Step 1 *Define the problem.*

How many hours will it take for the second car to pass the first car?

Step 2 *Identify the given information.*

- the speed of the first car is 40 miles per hour,
- the speed of the second car is 60 miles per hour,
- the first car starts two hours sooner than the second car, and
- both cars are traveling on the same road.

Step 3 *Decide which information is relevant.*

All the information from Step 2 is relevant to the defined problem.

Step 4 *Decide which is the key unknown value or variable.*

The key variable is *time*, which will be labeled t . This is known by looking at what needs to be solved. Since the first car starts two hours ahead of the second, $t+2$ must be the time of the first car, and t is the time of the second car.

time (car 1) = $t+2$ time (car 2) = t

Step 5 *Model the problem.*

We need to use a formula which relates distance traveled with speed and elapsed time. In this case, distance = rate \times time. We have two cars that travel the same distance.

So distance (car 1) = distance (car 2) is the same as

rate \times time (car 1) = rate \times time (car 2)

$$40 \times (t + 2) = 60 \times t$$

Step 6 *Evaluate the model.*

$$40(t + 2) = 60t$$

$$40t + 80 = 60t$$

$$80 = 60t - 40t$$

$$80 = 20t$$

$$4 = t \quad \text{the second car will pass the first car in 4 hours}$$

Step 7 *Validate the solution.*

The second car will pass the first car in 4 hours. This means that the first car will have been traveling for 6 hours.

$$\text{Car 1} = 40 \text{ miles per hour} \times 6 \text{ hours} = 240 \text{ miles}$$

$$\text{Car 2} = 60 \text{ miles per hour} \times 4 \text{ hours} = 240 \text{ miles}$$

$$40(t + 2) = 60t \quad \text{at } t = 4$$

$$40(4 + 2) = 60(4)$$

$$40(6) = 60(4)$$

$$240 = 240$$

Example 2 – Solving Word Problems

The Johnsons used 8 more gallons of gasoline for their family car in May than in April, and twice as much gas in June as in April. If the Johnsons used 68 gallons in three months, how much gas was used in the family car during each month?

- Step 1 How many gallons of gas were used by the Johnsons for their family car in each of three months (April, May, and June)?
- Step 2 In May, 8 gallons more of gas were used than in April.
In June, twice as many gallons of gas were used as in April.
68 total gallons of gas were used during the three months.
- Step 3 All information in Step 2 is relevant.
- Step 4 Number of gallons of gas used in April, May and June are all key unknown values.
- Step 5 Let A = gallons of gas used in April
 M = gallons of gas used in May
 J = gallons of gas used in June
 $M = A + 8$
 $J = A \times 2$
 $68 = A + M + J$
- Step 6 $68 = A + M + J$
 $68 = A + (A+8) + (A \times 2)$ *substituting for M and J*
 $68 - 8 = A + A + 2A$
 $60 = 4A$
 $15 = A$ *15 gallons of gas were used in April*
 $M = A + 8 = 15 + 8 = 23$ *23 gallons of gas were used in May*
 $J = A \times 2 = 15 \times 2 = 30$ *30 gallons of gas were used in June*
- Step 7 $68 = A + M + J$
 $68 = 15 + 23 + 30$
 $68 = 68$

CRITICAL THINKING QUESTIONS



1. Why is it important to define the problem before doing any of the other steps in the Solving Word Problem Methodology?
2. Do you need to use all the information given in a word problem? How do you know if information is relevant?
3. What strategy would you give someone to help them identify and define the problem?
4. How are the key unknown values related to the statement of the problem?

5. What mathematical equation, in terms of A and B, can be substituted for each of the following phrases?

B is 10 less than A

A is 500 more than twice B

B is 20 less than three times A

In 5 years, A will be half as old as B will be in 3 years

IN-CLASS EXERCISE

Solve the following word problem. Document your use of the methodology for solving word problems.

There are 91 cars in the junkyard. There are twice as many Fords as there are Buicks and 7 more Dodges than Buicks. How many Buicks, Fords, and Dodges are there in the junkyard?