SOME ASPECTS OF ALGORITHM PERFORMANCE AND MODELING IN TRANSIENT THERMAL ANALYSIS OF STRUCTURES

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The status of an effort to increase the efficiency of calculating transient temperature fields in complex aerospace vehicle structures is described. The advantages and disadvantages of explicit and implicit algorithms are discussed. A promising set of implicit algorithms with variable time steps, known as the GEAR package is described. Four test problems, used for evaluating and comparing various algorithms, have been selected and finite-element models of the configurations are described. These problems include a Space Shuttle frame component, an insulated cylinder, a metallic panel for a thermal protection system, and a model of the Space Shuttle Orbiter wing. Results generally indicate a preference for implicit over explicit algorithms for solution of transient structural heat transfer problems when the governing equations are "stiff" (typical of many practical problems such as insulated metal structures).

NOMENCLATURE

- C capacitance matrix
- DT time step size
- h_n n-th time step
- K conductivity matrix
- Q thermal load vector
- R residual of the system of equations generated by the implicit method
- t time
- t_n n-th time point
- T vector of temperatures
- α thermal diffusivity or coefficient in Adams-Moulton formula
- β coefficient in backward difference method

INTRODUCTION

An effort is in progress at the NASA Langley Research Center to improve capability to predict and optimize the thermal-structural behavior of aerospace vehicle structures. The focus of this activity is on space transportation vehicles such as the Space Shuttle Orbiter. A principal task is to reduce the computing effort for obtaining transient temperature fields. Current activity is focused on evaluation and comparison of explicit and implicit solution algorithms.

In reviewing current literature, a preference is evident among researchers for implicit algorithms for solution of stiff[1] sets of ordinary differential equations (2-7). Many engineering analysts, however, prefer to use the longer-established explicit algorithms. A partial explanation for this dichotomy is that the full power of the implicit approach has not been transferred from researchers to engineering analysts. In the explicit algorithms, the time step is limited (often severely) in order for the technique to be stable. In the implicit algorithms, there is no stability-imposed limitation on step size. The step size is limited by solution accuracy only, so that implicit algorithms can, in general, use much larger time steps than explicit algorithms. Because a single explicit time step is computationally faster than a single implicit time step, the key to the advantageous use of implicit algorithms is to use the largest possible time step size. As presently implemented in production thermal analysis computer programs, implicit algorithms generally require a user-specified fixed time step (8-11). The step size must be determined by trial and error.

The strategy being advocated in the solution of large problems by implicit methods is to use algorithms with variable step size and order and to automatically select both throughout the solution process (12-15). A promising set of algorithms, developed for the purpose of implementing the aforementioned

[1] Stiff sets or ordinary differential equations are characterized by solutions with widely varying time-constants. The typical case is when the solution to the homogeneous problem has very small time constants compared to those of the forcing function (1).

strategy, is denoted the GEAR algorithms (13-14). A version of the GEAR algorithms well-suited to heat transfer analysis denoted GEARIB has been recently installed in the SPAR finite-element thermal analyzer (8) for testing.

The purpose of the present paper is to describe some ongoing evaluations and demonstrations of the use of explicit and implicit algorithms for transient thermal analysis of structures using the finite-element method. A Shuttle frame test article, an insulated cylinder, a metallic multiwall thermal protection system panel, and a model of the Shuttle Orbiter wing are analyzed using SPAR. Comparisons between implicit and explicit algorithms are presented. The performance of the GEARIB algorithms and especially the value of variable step size and order is demonstrated. For benchmark checks, the cylinder is also analyzed with the MITAS lumped parameter program (16). It is a characteristic of thermal analysis by finite-element and lumped-parameter techniques that modeling affects the stiffness. Since stiffness is one of the key factors in the performance of implicit and explicit algorithms, the paper contains a study of the effects of modeling on the performance of the explicit and implicit algorithms. The present work focuses on the implicit and explicit algorithms implemented in production programs such as SPAR and MITAS. The authors do not evaluate but are aware of and hereby recognize recent developments underway which are still at the research stage. These include, for example, the mixed implicit-explicit techniques (17) and the use of quasi-Newton methods to solve the nonlinear algebraic equations associated with implicit algorithms (18).

NATURE OF ALGORITHMS USED IN TRANSIENT THERMAL ANALYSIS

A transient heat transfer problem when discretized by finite-element, finite-difference, or similar techniques, is governed by the following system of equations

$$C\dot{T} = Q(T,t) - K(T,t)T = F(T,t) \quad T(0) \text{ given} \qquad (1)$$

where F is generally a nonlinear function. It is usually impractical to obtain an analytical solution to eq. (1) so that numerical integration methods are used. The simplest numerical integration technique is the Euler method which uses the first two terms in a Taylor series to predict T at time t_{n+1} as

$$\begin{aligned} T(t_{n+1}) &= T(t_n) + h_n \dot{T}(t_n) \\ &= T(t_n) + h_n C^{-1} F(T(t_n),t_n) \end{aligned} \qquad (2)$$

Euler's method is an example of an explicit integration technique, so-named because $T(t_{n+1})$ is given explicitly in terms of known quantities. Another approach is the backward-difference method which is an example of an implicit method. In this approach

$$\begin{aligned} T(t_{n+1}) &= T(t_n) + h_n \dot{T}(t_{n+1}) \\ &= T(t_n) + h_n C^{-1} F(T(t_{n+1}), t_{n+1}) \end{aligned} \qquad (3)$$

Eq. (3) is a system of implicit equations for $T(t_{n+1})$, which is generally nonlinear. The explicit algorithm is therefore easier to implement but must be bounded to avoid numerical instability (unbounded propagation of numerical errors during the solution). Implicit techniques are generally stable and thus can take larger time steps which are determined from accuracy considerations.

Most practical transient thermal analysis problems in flight structures have the following characteristics which profoundly affect the choice of a solution method:

(1) The thermal response may be divided into regions of slowly and rapidly varying temperatures. Steep transients accompany initial conditions or sudden changes in the heat load.

(2) The rapidity of variation of the transient portion of the temperature history is proportional to the quantity L^2/α where L is a characteristic conduction length and α is thermal diffusivity. During the transient, time steps much smaller than L^2/α must be taken no matter what type of integration technique is used.

During a period of slowly-varying temperature large time steps may be taken by implicit integration techniques but explicit techniques must still use time steps which are less than L^2/α. When L^2/α values for some elements in the structure are small compared to the time scale of the slower temperature variation, the problem is stiff. It follows that stiff problems are usually best solved by implicit methods. The effort involved in solving a system such as eq. (3) is usually cost-effective if a small number of large time steps are used.

The Euler method and the backward-difference method are presented as representatives of a large class of explicit and implicit techniques, respectively. Higher-order methods (i.e., multistep) typically use more previous information to predict the temperature at the current time but the stability properties of explicit multistep methods are similar to those of the Euler method. Most explicit methods are unstable for time steps much larger than L^2/α. Accordingly, thermal analysis computer programs generally select the explicit time step automatically based on the stability requirement. For implicit methods, the analyst is left to select the implicit time step and order without a great deal of guideline information and usually several trial runs are needed. There is an emerging consensus that the approach to take for integrating stiff systems of ordinary differential equations would be to use implicit methods which automatically select the order and the step size based on desired accuracy. One package denoted the GEARIB algorithms has these features and is discussed next.

THE GEARIB ALGORITHMS

Several software packages based on the work of Gear have been developed for general use (13). The package most appropriate for application to finite-element thermal analysis is denoted GEARIB. This package is intended to solve systems of ordinary differential equations of the form

$$C(T,t)\dot{T} = F(T,t) \qquad (4)$$

The package employs two classes of implicit multistep methods, Adams-Moulton and backward difference. For nonstiff equations, the Adams-Moulton method of order one through twelve is used. This method has the general form

$$T(t_{n+1}) = T(t_n) + h_n \sum_{i=0}^{q} \beta_i \dot{T}(t_{n+1-i}) \quad (5)$$

where q is the order. For stiff equations, the backward difference algorithms of orders one through five are used. These algorithms have the general form

$$T(t_{n+1}) = h_n \beta_o \dot{T}(t_{n+1}) + \sum_{i=1}^{q} \alpha_i T(t_{n+1-i}) \quad (6)$$

The coefficients α_i and β_i are given in (15). The user selects the class of methods (Adams-Mouton or backward differences), and as described in (13) GEARIB automatically selects the appropriate time step and the order based on a user-specified error tolerance.

Use of the GEARIB algorithms is illustrated using the backward difference option. Applied to eq. (4), eq. (6) gives

$$R = C[T(t_{n+1}) - \sum_{i=1}^{q} \alpha_i T(t_{n+1-i})]$$

$$- h_n \beta_o F(T(t_{n+1}), t_{n+1}) = 0 \quad (7)$$

This system of nonlinear algebraic equations is solved by the modified Newton's method. That is

$$T^{i+1}(t_{n+1}) = T^i(t_{n+1}) - [\frac{\partial R}{\partial T}]^{-1} R \quad (8)$$

where

$$[\frac{\partial R}{\partial T}] = C - \beta_o h_n J$$

and $J = \partial F/\partial T$ is the Jacobian of the system at a previous time point. Methods used in GEARIB for computing J are described in (13) and (19).

DESCRIPTION OF TEST PROBLEMS AND RESULTS[2]

Insulated Shuttle test frame

A Shuttle Orbiter frame component analyzed and tested under transient heating as described in (20) is shown in figure 1 and consists of an aluminum frame surrounded by insulation. The principal purpose of the study of the configuration as discussed in (20) was to evaluate the thermal performance of the insulation during a simulated Shuttle flight. A secondary purpose was to evaluate the adequacy of thermal analysis techniques applicable to the Shuttle.

[2] Additional details of the test problems are given in (19). All calculations were performed on the Langley CDC Cyber 173 computer.

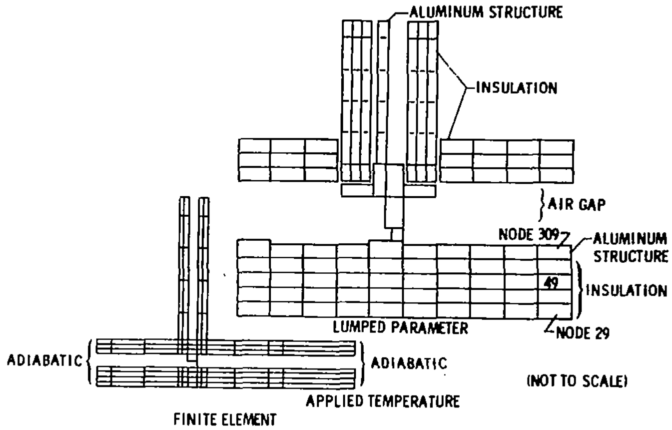

Fig.1 Finite-element and lumped parameter models of Shuttle frame

The lumped parameter model from (20) consists of a two-dimensional section of a symmetric half of the structure and contains 118 nodes (see figure 1). The unknown temperatures are located at the centroids of the lumps. The lumped parameter model was converted to a finite-element model for analysis using the SPAR program (8). The corresponding SPAR finite-element model contains 149 grid points located at the ends or corners of the elements. The model contains 148 elements including one-dimensional elements which account for conduction in the aluminum structure and radiation across the air gap and two-dimensional elements which model conduction in the insulation and across the gap. The difference in numbers of elements and grid points is due to the different modeling approaches of the two methods.

Minor modifications were made to the finite-element model following the conversion. These consisted of eliminating or consolidating some extremely thin or short finite elements in the aluminum structure in order to reduce the stiffness of the equations and to increase the allowable time step for the explicit solution algorithm. The properties of the aluminum structure are functions of temperature and the properties of the insulation are functions of temperature and pressure. The pressure dependence is treated in SPAR as time dependence using the pressure vs. time variation from the trajectory data for the simulated flight conditions. The applied heating is specified by tabulations of temperatures at the outer surface of the insulation.

The temperature history for the frame was computed using explicit (Euler) and implicit techniques (Crank-Nicholson and backward differences) and GEARIB. Comparisons of solution times are given in Table I. The explicit procedure using a time step of 0.16 s required 1723 s of CPU time. This time step was controlled by conduction through most of the aluminum elements along the center and front of the frame. Solution time using the Crank-Nicholson algorithm varied from 475 s to 65 s as the time step was varied between 1.0 and 50 s. The solution times for backward differences were close to those of Crank-Nicholson and are not shown. The GEARIB algorithm used time steps from 50 to 170 s and the solution time was 54 s. As indicated in Table I(b), there is very little loss of accuracy in either the structure or insulation temperatures with increased time step size.

TABLE I.- PERFORMANCE OF VARIOUS ALGORITHMS FOR TRANSIENT THERMAL ANALYSIS OF SHUTTLE FRAME

(a) Solution Time Comparison

Explicit		Implicit			
Euler		Crank-Nicholson		GEARIB	
Time Step (s)	Solution Time (s)	Time Step (s)	Solution Time (s)	Time Step (s)	Solution Time (s)
0.16	1723	1 10 25 50	475 249 106 65	50-170	54

(b) Effect of Time Step on Accuracy of Implicit Algorithms

Step Size (s)	Temp. of Node 309** at 1200 s		Temp. of Node 49** at 1200 s	
	K	°F	K	°F
1.0	442.1	335.7	477.0	398.6
10.0	442.0	335.6	476.9	398.5
25.0	439.4	331.6	475.6	396.0
50.0	437.9	328.3	474.8	394.7
0.16*	442.1	335.7	477.0	398.6
50-170***	443.1	337.5	477.9	400.3

*Explicit Algorithm
**See figure 1
***GEARIB

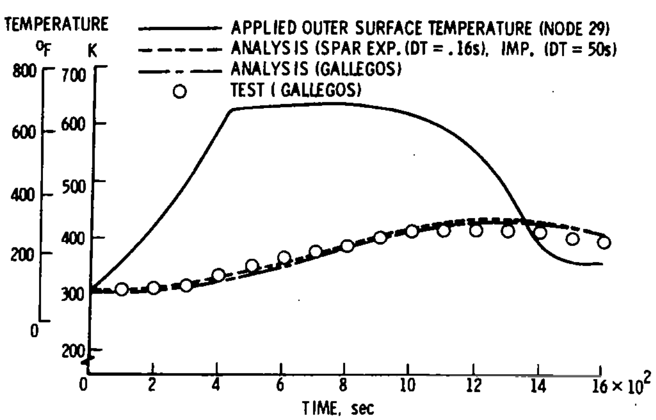

Fig.2 Temperature history in outer structural surface of Shuttle frame (Node 309)

The accuracy of the solutions by the various techniques is further assessed in figure 2 which displays temperature histories at a point in the outer layer of the aluminum structure corresponding to node 309 (see figure 1). The solid line in figure 2 represents the applied temperatures at the outer surface of the insulation (node 29). The dotted line shows temperatures obtained by the SPAR analysis. The SPAR temperatures are plotted as a single curve since there is little difference between the results. The dashed-dot line shows analytical results from the lumped parameter analysis of (20) which are also in close agreement with the SPAR temperatures. The circular symbols represent test data from (20). The closeness of all the results indicates that the models are adequate to simulate the temperature history in the test article.

Multiwall thermal protection system panel

The next example problem is one which grew out of a study of the thermal performance of a titanium multiwall thermal protection system (TPS) panel which is under study for future use on space transportation systems (21). The configuration as depicted in figure 3(a) consists of alternating layers of flat and dimpled sheets fused at the crests to form a sandwich. The representation of a typical dimpled sheet is shown in figure 3(b). For the purpose of this analysis, it is assumed that the heat load does not vary in directions parallel to the plane of the panel. This assumption in addition to the regular geometry of the structure leads to the modeling simplification wherein only a triangular prismatic section of the panel needs to be modeled; fig. 3(a). The intersection of this prism with a typical dimpled layer is indicated by the shaded triangle in fig. 3(b).

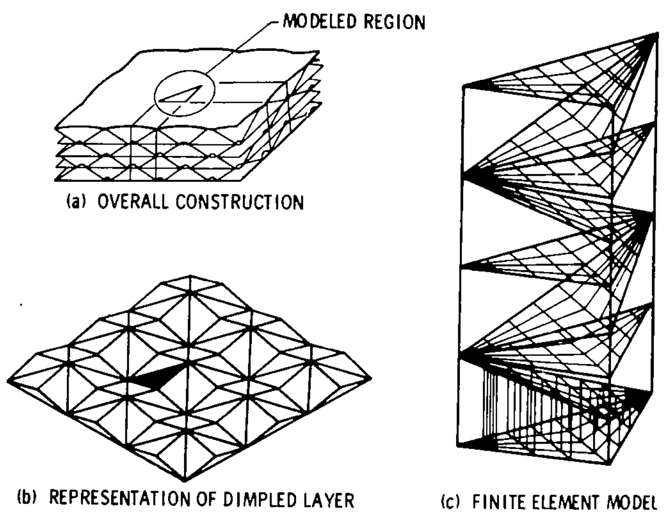

Fig.3 Multiwall thermal protection system panel

The finite-element model shown in fig. 3(c) contains 333 grid points located on nine titanium sheets (five horizontal and four inclined). The model contains 288 triangular and quadrilateral metal conduction elements, 264 solid air conduction elements which account for gas conduction between the layers and 544 triangular and quadrilateral radiation elements which account for radiation heat transfer between adjacent horizontal and inclined sheets. Thermal properties of titanium and air are functions of temperature. Radiation exchange (view) factors were computed and supplied to SPAR using the TRASYS II computer program (22).

The temperature history of the panel in response to an imposed transient temperature at the outer surface of the panel was computed for 3200 s. Results were obtained with SPAR using explicit, Crank-Nicholson, backward difference and GEARIB algorithms. Solution-time comparisons are presented in Table II. The explicit algorithm required a time step of .007 s. This time step was dictated by conduction of heat through the short heat paths between the vertices of adjacent triangular layers and indicates that this is an extremely stiff problem. Required solution time for the explicit algorithm was estimated to be 98368 s.

TABLE II.- COMPARISON OF ALGORITHMS FOR TRANSIENT THERMAL ANALYSIS OF TITANIUM MULTIWALL TPS (3200 s temperature history)

Explicit		Implicit			
Euler		Crank-Nicholson		GEARIB	
Time Step (s)	Solution Time (s)	Time Step (s)	Solution Time (s)	Time Step (s)	Solution Time (s)
.007	98368*	1 5	28412** 6352	1.0-113	2754

*Extrapolated value based on 12296 s for 400 s of temperature history

**Extrapolated value based on 8879 s for 1000 s of temperature history

The Crank-Nicholson solution was carried out using time steps of 1 and 5 s which led to solution times of 28412 s and 6352 s, respectively. Backward difference was used with the same time steps and had the same solution times. GEARIB took time steps ranging between 1.0 and 113 seconds and required a solution time of 2754 seconds. This example shows again advantages of using implicit algorithms in general and the GEARIB algorithms in particular for thermal analysis of stiff problems. A plot of typical temperature histories for a point midway through the panel and the primary structure is shown in figure 4 along with the applied outer surface temperature. The results were obtained by the implicit algorithm with a time step of 5 s and are identical to results using a time step of 1 s and GEARIB.

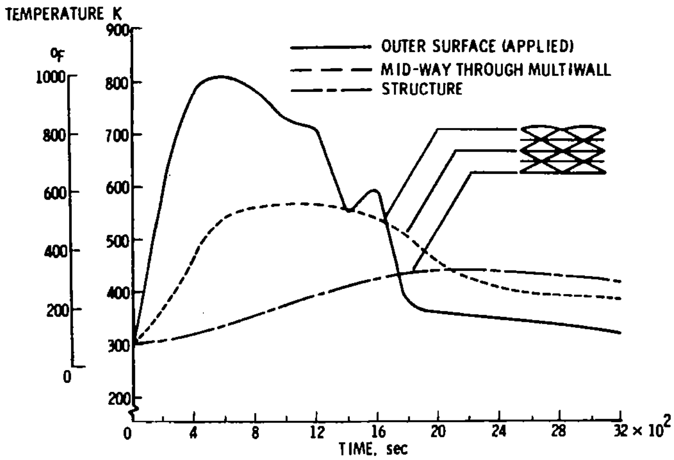

Fig.4 Transient temperatures in titanium multiwall TPS panel

Space Shuttle orbiter wing

The SPAR thermal model of the Shuttle orbiter wing (figure 5) consists of a relatively coarse model of the structure (327 grid points) augmented by layers of insulation attached to the upper and lower surfaces. The structure is modeled by rod, triangular and quadrilateral elements (K21, K31, K41 SPAR elements). The insulation on each surface is modeled by six layers of one-dimensional conduction elements (K21). Use of these elements neglects lateral heat transfer in the insulation--a reasonable assumption since the temperature gradients through the insulation are at least an order of magnitude greater than the lateral temperature gradients. The complete model contains 2289 grid points, 1400 one- and two-dimensional elements in the structure and 1962 one-dimensional elements in the insulation. Thermal properties of the aluminum structure are temperature-dependent; thermal properties of the insulation are temperature- and time-dependent.

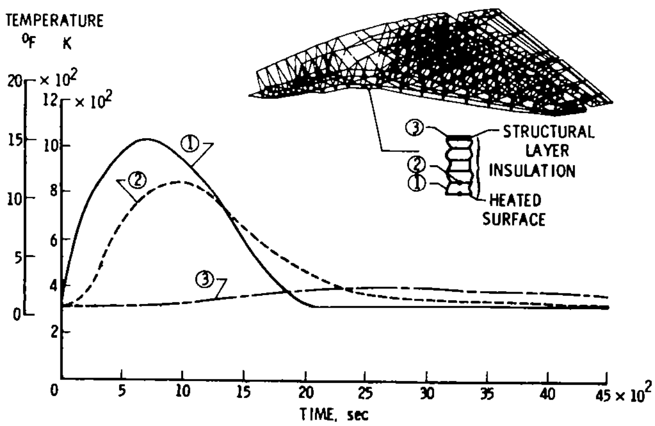

Fig.5 Transient temperatures in Shuttle orbiter wing

TABLE III.- COMPARISON OF ALGORITHMS FOR TRANSIENT THERMAL ANALYSIS OF SPACE SHUTTLE ORBITER WING (4500 s temperature history)

Explicit		Implicit			
Euler		Crank-Nicholson		GEARIB	
Time Step (s)	Solution Time (s)	Time Step (s)	Solution Time (s)	Time Step (s)	Solution Time (s)
10	2288	10	11730	1-528	557

For the purpose of this analysis, the applied heating on the wing is represented by a time-dependent temperature applied to the external surface of the insulation on the under side of the wing. The shape of this curve shown as the solid line in figure 5 is roughly indicative of atmospheric reentry heating. The temperature history of the wing for 4500 seconds was computed using the explicit, Crank-Nicholson, backward difference and GEARIB algorithms. Solution time comparisons are shown in Table III along with the time steps used to obtain comparable accuracy. The explicit algorithm used a time step of 10 seconds--in fact stability requirements actually permitted a time step of over 100 seconds but the step size was dictated by accuracy and the need to periodically update temperature-dependent material properties and not by stability requirements. The large permitted time step is due to the coarse modeling of the structure which did not include the thin, high-conducting or radiating elements present in the previous models. The implicit algorithms (Crank-Nicholson and backward difference produced the same results) were used with a time step of 10 s and required about five times as much computer time as the explicit algorithm. The GEARIB algorithms performed very well for this problem. By adaptively varying the time step from 1.0 second early in the temperature history to as large as 528 seconds toward the end, GEARIB required only 557 seconds to complete the solution. Figure 5 shows the temperature histories of a point on the structure and a point in the insulation 1/5 of the distance through the insulation of a typical cross section through the wing. The explicit, implicit, and GEARIB algorithms produced essentially the same results.

EFFECT OF MODELING ON ALGORITHM PERFORMANCE

This section of the paper describes a study of how modeling details can affect the performance of transient solution algorithms--especially explicit algorithms. Also, the influence of alternate ways of including the nonlinear effects of temperature-dependent material properties is studied. The structure chosen for the study is an insulated cylindrical shell shown in figure 6. The cylinder is 18 m (720 in.) in length and 4.5 m (180 in.) in diameter. The aluminum is 0.25 cm (0.1 in.) thick and the insulation is 5.0 cm (2.0 in.) thick. The outer surface of the insulation is heated over a region which consists of one-third the length and half the circumference.

Three finite-element models are used in the study. Due to symmetry, only half the cylinder is modeled in each case. In model I, solid (K81) elements are used exclusively--39 along the cylinder length, 4 around the circumference, and 3 through the depth (2 elements in the insulation and 1 in the structure). The outer surface has quadrilateral elements (K41) which receive the heat load and radiation elements (R41) which radiate to space. Model I contains 800 grid points and 650 elements. In model II, the solid elements in the structural layer are replaced by quadrilateral elements (K41) in which temperatures do not vary through the thickness. This is generally a good assumption for thin metal structures. Model II has an extra layer of solid elements in the insulation in order to preserve the number of grid points in the model at 800. In model III, the insulation is modeled with one-dimensional conductors (K21). This model neglects lateral heat conduction but as mentioned previously in connection with the Shuttle wing model, this effect is small for the class of insulated flight structures of interest in the present work.

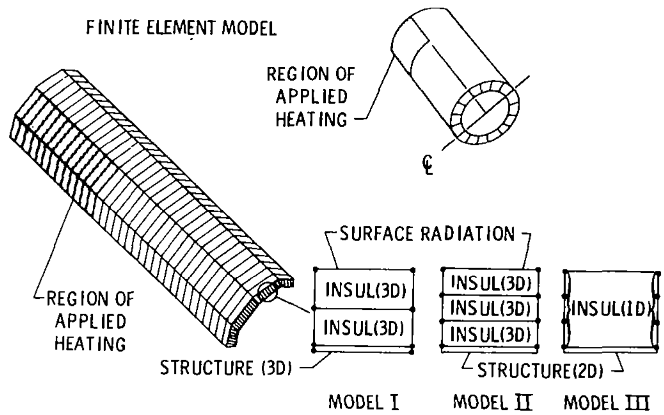

Fig.6 Finite-element models of insulated cylinder

Another aspect of the effect of modeling is comparison of results from finite-element and lumped-parameter models. To investigate this, the MITAS lumped parameter computer program (16) was applied to the analysis of the cylinder. The finite-element model I was converted to a lumped-parameter model by use of the CINGEN program (23). The resulting lumped-parameter model contained 625 nodes as compared to 800 grid points in the finite-element model. Recall the unknown MITAS temperatures are located only at the centroids of each lump.

TABLE IV.- EFFECT OF MODELING ON SOLUTION TIMES FOR INSULATED CYLINDER PROBLEM

Problem and Model / Algorithm	SPAR (Ref. 8)			MITAS (Ref. 16)
	Model I	Model II	Model III	lumped parameter model
Explicit (time step)	10107 (.06)	1518 (2.4-10)	279 (3.3-10)	226 (10)
Implicit* (DT=10 s)	1880	1920	536	320
GEARIB (time step)	1779 (1.0-83)	1707 (5-106)	266 (2-133)	

*Backward differences and Crank-Nicholson

The first 2000 seconds of the temperature history in the cylinder in response to a time-dependent heat load were computed in each model. The explicit

(Euler) and implicit (backward difference) algorithms were used for all models and in addition GEARIB was used for the three SPAR models. Solution times are shown in Table IV. Model I is extremely stiff as evidenced by the small time step of 0.06 seconds required for stability of the explicit algorithm. The high stiffness is due to the use of K81 elements to model the metal layer. In model II, the stiffness has been essentially eliminated by replacing the 3-D elements modeling the metal by 2-D elements. In this model, the explicit technique is faster than backward difference and GEARIB. In model III, due to low stiffness again, the explicit algorithm is faster than the implicit but GEARIB is slightly faster than the explicit technique. It is observed that in models I and II, GEARIB despite using much larger time steps was only marginally faster than the implicit method. This is due to the different ways of handling the temperature-dependent material properties. In the explicit and implicit methods, the properties are represented as being piecewise constant within time intervals specified by the user (by the input quantity TI) in SPAR. Material properties are evaluated at the beginning of each interval and the conductivity and capacitance matrices are regenerated at those times. Results for models I, II, and III in Table IV were obtained using TI = 20 s. In GEARIB, the material properties vary continuously and the residual R must be evaluated each time an iteration in solving eq. (8) is taken. The residual evaluation is much more costly in computer time than the regeneration of the conductance and capacitance matrices. This extra effort is the price paid for higher accuracy. However, this burden only shows up in problems which utilize solid (K81) elements due to the extreme cost of regenerating the matrices for those elements (note model III does not contain K81 elements). A way to eliminate the burden (for thermally isotropic elements) has been identified and is easily implemented. The method is to generate the matrices only once for unit values of the appropriate property and simply scale the matrices by the property whenever it is updated.

MITAS computation times are shown in the last column of Table IV. Because none of the SPAR models is equivalent to the MITAS model in terms of the number of unknown temperature or nodal connections, no direct comparison of MITAS and SPAR solution times is appropriate. However, some trends evident in Table IV are noted. The MITAS model is not particularly stiff as evidenced by the large time step used in the explicit solution technique. SPAR models II and III which begin to resemble the MITAS model in certain respects are also less stiff and favor explicit algorithms. It is noted that the way MITAS treats temperature-dependent material properties is by the scaling method cited above.

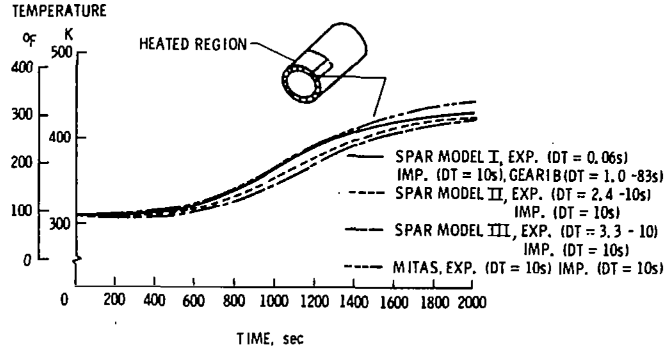

Fig.7 Effects of choice of algorithm and model changes on temperature history of insulated cylinder. Model I: all 3-D elements. Model II: insulation - 3-D, metal - 2-D. Model III: insulation - 1-D, metal - 2-D.

Figure 7 contains comparisons of temperature histories of a point in the cylinder. Model II is considered to be the best of the models (recall the additional insulation elements used) and thus the temperatures represented by the dotted line are thought to be the most accurate. These results are bracketed by results from model I and MITAS (from above) and by model III (from below). There are negligible differences between temperatures from the implicit and explicit solutions for any given model. Results from models II and III are different from that of model I because of the extra layer of elements through the insulation. The MITAS temperature history agrees well with that of model I (on which the MITAS model is based) except for some differences beginning at 1400 s.

CONCLUDING REMARKS

This paper discusses the status of an effort to obtain increased efficiency in calculating transient temperature fields in complex aerospace vehicle structures. Explicit solution techniques which require minimal computation per time step and implicit techniques which permit larger time steps because of better stability are reviewed. A promising set of implicit solution algorithms having variable time steps and order, known as the GEARIB package, is described. Four test problems for evaluating the algorithms have been selected and finite-element models of each one are described. The problems include a Shuttle frame component, an insulated cylinder, a metallic panel for a thermal protection system, and a model of the Space Shuttle Orbiter wing. Calculations were carried out using the SPAR finite-element program and the MITAS-lumped parameter program. Results generally indicate that implicit algorithms are more efficient than explicit algorithms for solution of transient structural heat transfer problems when the governing equations are stiff. Stiff equations are typical of many practical problems such as insulated metal structures and are characterized by widely differing time constants and cause explicit methods to take small time steps. As evidenced by their excellent performance in solving the test problems, the GEARIB algorithms offer high potential for providing increased computational efficiency in the solution of stiff problems. Studies were also made of the effect on algorithm performance of different models of the same cylinder test problem. These studies revealed that the stiffness of the problem is highly sensitive to modeling details and that careful modeling can reduce the stiffness of the resulting equations to the extent that explicit methods may become advantageous.

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