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Some Integral Equations with Modified Argument

by Dr. Maria Dobritoiu

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Preface

The theory of integral equations is an important part in applied mathematics. The first books with theme of study, the integral equations appeared in the 19th century and early 20th century, and they have been authored by some of the famous mathematicians: N. Abel (1802-1829), A. Cauchy (1789-1857), E. Goursat (1858-1936), M. Bocher (1867-1918), David Hilbert (1862-1943), Vito Volterra (1860-1940), Ivar Fredholm (1866-1927), E. Picard (1856-1941), T. Lalescu (1882-1929). The first treatise in this field appeared in 1910 (T. Lalescu 1911, M. Bocher 1912, D. Hilbert 1912, V. Volterra 1913) (see I.A. Rus [100]). In the 20th century, the theory of integral equations had a spectacular development, both in terms of mathematical theories that may apply, and in terms of effective approximation of solutions.

The main methods that apply to the study of integral equations are: fixed point methods, variational methods, iterative methods and numerical methods. In this book was applied a fixed point method by applying the contraction principle. By this approach, the study of an integral equation represents the development of a fixed point theory, which contains the results on existence and uniqueness of the solution, the integral inequalities (lower-solutions and upper-solutions), the theorems of comparison, the theorems of data dependence of the solution (continuous data dependence and the differentiability of the solution with respect to a parameter) and an algorithm for approximating its solution.

The integral equations, in general, and the integral equations with modified argument, in particular, have been the basis of many mathematical models from various fields of science, with high applicability in practice, e.g., the integral equation from theory of epidemics and the Chandrasekhar's integral equation.

In this book, the Picard operators technique has been used for all the stages of this type of study.

This book is a monograph of integral equations with modified argument and contains the results obtained by the author in a period that began in the years of study in college and ended up with years of doctoral studies, both steps being carried out under the scientific coordination of Prof. Dr. Ioan A. Rus from Babes-Bolyai University of Cluj-Napoca. It is addressed to all who are concerned with the study of integral equations with modified argument and of knowledge of results and/or of obtaining new results in this area. The book is useful, also, to those concerned with the study of mathematical models governed by integral equations, generally, and by integral equations with modified argument, in particular.

Finally, we mention several authors of the used basic treatises having the theme of integral equations: T. Lalescu, I. G. Petrovskii, K. Yosida, Gh. Marinescu, A. Haimovici, C. Corduneanu, Gh. Coman, I. Rus, G. Pavel, I. A. Rus, W. Walter, D. Guo, V. Lakshmikantham, X. Liu, W. Hackbusch, D. V. Ionescu, Şt. Mirică, V. Mureşan, A. D. Polyanin, A. V. Manzhirov, R. Precup, I. A. Rus, M. A. Şerban, Sz. András.

I dedicate this book to my parents *Ana* and *Alexandru*.

Dr. Maria Dobrițoiu
University of Petroșani, Romania

Acknoledgements

This book was published with the support of WSEAS. I would like to expresss my sincere thanks to the WSEAS Team for their support in entering the results obtained in the study of integral equations with modified argument into the international scientific circuit. I wholeheartedly appreciate their help. Thank you WSEAS.

The Author

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Overview of the book

The integral equations, in general, and those with modified argument, in particular, form an important part of applied mathematics, with links with many theoretical fields, specially with practical fields. The first papers that treated the integral equations had as authors renowned mathematicians, such as: N. H. Abel, J. Liouville, J. Hadamard, V. Volterra, I. Fredholm, E. Goursat, D. Hilbert, E. Picard, T. Lalescu, E. Levi, A. Myller, F. Riez, H. Lebesgue, G. Bratu, H. Poincaré, P. Levy, E. Picone. T. Lalescu was the author of the first book about integral equations (Bucharest 1911, Paris 1912).

This book is a study of some of the integral equations with modified argument and it focuses mostly on the study of the following integral equation with modified argument

$$x(t) = \int_a^b K(t,s,x(s),x(g(s)),x(a),x(b))ds + f(t) \;, \quad t \in [a,b] \;, \tag{1}$$

where $K : [a,b]\times[a,b]\times\boldsymbol{B}^4 \to \boldsymbol{B}$, $f : [a,b] \to \boldsymbol{B}$, $g : [a,b] \to [a,b]$, and $(\boldsymbol{B},+,\boldsymbol{R},|\cdot|)$ is a Banach space.

Starting with the Fredholm integral equation with modified argument

$$x(t) = \int_a^b K(t,s,x(s),x(a),x(b))ds + f(t) \;, \quad t \in [a,b] \;, \tag{2}$$

which is a mathematical model from the turbo-reactors industry, we have also considered a modification of the argument through a continuous function $g : [a,b] \to [a,b]$, thus obtaining the integral equation with modified argument (1). It is an example of a nonlinear Fredholm integral equation with modified argument.

The integral equations (1) and (2) have been studied by the author, laying down the conditions of existence and uniqueness of the solution, the conditions of the continuous data dependence of the solution, and also, of differentiability of the solution with respect to a parameter and the conditions of approximating the solution, and the obtained results were published in papers [2], [22], [23], [24], [26], [29], [31], [33], [34], [35], [37], [38].

The book contains results of existence and uniqueness, of comparison, of data dependence, of differentiability with respect to a parameter and of approximation for the solution of the integral equation with modified argument (1) and a few results related to the solution of a well known equation from the epidemics theory.

Chapter 1, entitled "*Preliminaries*", that has eight paragraphs, is an introductory chapter which presents the notations and a few classes of operators that are used in this book, the basic notions and the abstract results of the fixed point theory and also, the notions from the Picard operators theory on L-spaces and the fiber contractions principle.

There are also presented the quadrature formulas (the trapezoids formula, the rectangles formula and Simpson's quadrature formula) that were used for the calculus of the integrals that appear in the terms of the successive approximations sequence from the obtained method of approximating the solution of the integral equation (1).

The seventh paragraph contains a very brief overview of Fredholm and Volterra nonlinear integral equations and the basic results regarding the existence and uniqueness of the solutions of these equations (see [10]).

In the eighth paragraph there are presented two mathematical models governed by functional-integral equations: an integral equation from physics and a mathematical model of the spreading of an infectious disease.

The first model refers to equation (2), and the results of existence and uniqueness, data dependence and approximation of the solution (theorems 1.8.1, 1.8.2 and 1.8.3), presented in this paragraph, were obtained by the author and published in the papers [2], [22], [23], [24], [26] and [29].

The presentation of the mathematical model of the spreading of an infectious disease, which refers to the following equation from the epidemics theory

$$x(t) = \int_{t-\tau}^{t} f(s, x(s))ds \ , \qquad (3)$$

contains results obtained by K.L. Cooke and J.L. Kaplan [18], D. Guo, V. Lakshmikantham [42], I. A. Rus [88], [93], Precup [73], [75], R. Precup and E. Kirr [78], C. Iancu [47], [48], I. A. Rus, M. A. Şerban and D. Trif [114].

The fiber generalized contractions theorem 1.5.2, theorem which is a result obtained by I.A. Rus in paper [100], was used to lay down theorem 1.5.3 in this chapter, theorem that was published in paper [27].

Chapter 2, entitled "*Existence and uniqueness of the solution*" has five paragraphs. Three of them contain the conditions of existence and uniqueness of the integral equation with modified argument (1), in the space $C([a,b],\boldsymbol{B})$ and in the sphere $\overline{B}(f;r) \subset C([a,b],\boldsymbol{B})$, in a general case and in two particular cases for $\boldsymbol{B}$: $\boldsymbol{B} = \boldsymbol{R}^m$ and $\boldsymbol{B} = \boldsymbol{l}^2(\boldsymbol{R})$. In order to prove these results, the following theorems have been used: *the Contraction Principle* 1.3.1 and *Perov's theorem* 1.3.4.

The fourth paragraph of this chapter contains three examples: two integral equations with modified argument and a system of integral equations with modified argument and for each of these examples the conditions of existence and uniqueness, which were obtained by using some of the results presented in the previous paragraphs, are given.

In the fifth paragraph was studied the existence and uniqueness of the solution of the integral equation with modified argument

$$x(t) = \int_{\Omega} K(t, s, x(s), x(g(s)), x|_{\partial\Omega})ds + f(t) \ , \quad t \in \overline{\Omega} \ , \qquad (4)$$

where $\Omega \subset \boldsymbol{R}^m$ is a bounded domain, $K : \overline{\Omega} \times \overline{\Omega} \times R^m \times R^m \times C(\partial\Omega, R^m) \to R^m$, $f : \overline{\Omega} \to R^m$ and $g : \overline{\Omega} \to \overline{\Omega}$. This equation is a generalization of the integral equation (1).

Some of the author's results that are presented in this chapter, were published in papers [31] and [37].

Chapter 3, entitled "*Gronwall lemmas and comparison theorems*" has three paragraphs. Several Gronwall lemmas, comparison theorems and a few examples for the integral equation with modified argument (1) are presented. These results represent the properties of the solution of this integral equation. In order to prove the results presented in this chapter, the following theorems were used: *the abstract Gronwall lemma* 1.4.1 and *the abstract comparison lemmas* 1.4.4 and 1.4.5. The third paragraph of this chapter contains examples which are applications of the results given in the first two paragraphs. These results were obtained by the author and published in the papers [35] and [38].

In chapter 4, entitled "*Data dependence*", which has four paragraphs, the author present the theorems of data dependence, the differentiability theorems with respect to *a* and *b* (limits of integration), and theorems of differentiability with respect to a parameter, of the solution of the integral equation with modified argument (1) and also, a few examples.

In order to prove the results presented in this chapter, the following theorems were used: *the abstract data dependence theorem* 1.3.5 and *the fiber generalized contractions theorem* 1.5.2. These results were published in the papers [31], [33], [34] and [37].

In chapter 5, entitled "*Numerical analysis of the Fredholm integral equation with modified argument* (2.1)", following the conditions of one of the existence and uniqueness theorems given in the second chapter, a method of approximating the solution of the integral equation (1) is given, using the successive approximations method. For the calculus of the integrals that appear in the successive approximations sequence, the following quadrature formulas were used*: the trapezoids formula*, *Simpson's formula* and *the rectangles formula*.

This chapter has five paragraphs. The first paragraph presents the statement of the problem and the conditions under which it is studied. In paragraphs 2, 3 and 4 there are presented the results obtained related to the method of approximating the solution of the integral equation (1). The results obtained in paragraphs 2, 3 and 4 are used in the fifth paragraph to approximate the solution of an integral equation with modified argument, given as example.

The MatLab software was used to calculate the approximate value of the integral which appears in the general term of the successive approximations sequence, with trapezoids formula, rectangles formula and Simpson's formula; for each of these cases was obtained the approximation of the solution of the integral equation given as example. In appendices 1, 2 and 3 one can find the results obtained by these programs written in MatLab.

Some of the results obtained by the author for equation (1), that were presented in this chapter, were published in paper [31]. The results obtained for the numerical analysis of equation (2) were published in the papers [22], [23], [24] and [26].

Chapter 6, entitled "*An equation from the theory of epidemics*", has four paragraphs and contains the results obtained through a study of the solution of the integral equation (3), using the Picard operators. This study was carried out by the author in collaboration with I.A. Rus and M.A. Şerban, and the results obtained, refering to the existence and uniqueness of the solution in a subset of the space $C(\boldsymbol{R},I)$, lower and upper solutions, data dependence and differentiability of the solution of the integral equation (3), with respect to a parameter, are published in paper [36].

The bibliography used to write this book contains several important basic treatises from the theory of integral equations, scientific papers on this topic, of some known authors and scientific articles which contains the author's own results.

Each of the six chapters has its own bibliography and all these references are listed in a bibliography at the end of the book.

The basic treatises used for the study in this book are the following: T. Lalescu [56], I. G. Petrovskii [69], K. Yosida [129], Gh. Marinescu [59] and [60], A. Haimovici [45], C. Corduneanu [20], Gh. Coman, I. Rus, G. Pavel and I. A. Rus [15], D. Guo, V. Lakshmikantham and X. Liu [43], W. Hackbusch [44], C. Iancu [48], D. V. Ionescu [49] and [50], V. Lakshmikantham and S. Leela [55], Şt. Mirică [61], D. S. Mitrinović, J. E. Pečarić and A. M. Fink [62], V. Mureşan [65], B. G. Pachpatte [66], A. D. Polyanin and A. V. Manzhirov [72], R. Precup [74] and [81], I. A. Rus [88], [89], [95], [106], I. A. Rus, A. Petruşel and G. Petruşel [109], D. D. Stancu, Gh. Coman, O. Agratini and R. Trîmbiţaş [119], D. D. Stancu, Gh. Coman and P. Blaga [120], M. A. Şerban [124], Sz. András [6].

This book is a monograph of some of the integral equations with modified argument and it contains the results on which the author had been working, starting with the university years and ending with the years of Ph.D. studies, under the the scientific coordination of professor Ioan A. Rus from the "Babeş-Bolyai" University of Cluj-Napoca, Romania.

The purpose of this book is to help those who wish to study the integral equations with modified argument, to learn about these results and to obtain new results in this field.

This book is also useful for those who would like to study the mathematical models governed by integral equations, in general, and integral equations with modified argument, in particular.

Subject index

Index of authors

Dr. Maria Dobrițoiu is associate professor at the Department of Mathematics, University of Petrosani, Romania.

She graduated from the Faculty of Mathematics at the "Babes-Bolyai"University of Cluj-Napoca in 1978 and then, in 1979, she completed one year of specialization in Numerical Analysis. In 2000 she majored in Management at the Faculty of Science from the University of Petrosani.

In 2008 she received a Ph.D. in Mathematics at the "Babes-Bolyai" University of Cluj-Napoca. Fields of interest: Differential equations and integral equations, Numerical Analysis, Programming.

This book is a monograph of integral equations with modified argument and contains the results obtained by the author within a period that begins from her college years until to her doctoral studies, both carried out under the scientific coordination of Prof. Dr. Ioan A. Rus from the"Babes-Bolyai" University of Cluj-Napoca. This monograph is addressed to all who are interested of the study of integral equations with modified argument, in order to improve their knowledge, or to obtain new results in the field. The book is also useful to those concerned with the study of mathematical models governed by integral equations in general, and by integral equations with modified argument in particular.

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