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SOME THOUGHTS ON THE MODELLING OF EROSION AND DEPOSITION OF COHESIVE SEDIMENTS

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Some thoughts on the modelling of erosion and deposition of cohesive sediments

INTRODUCTION

The description of the sediment exchange between the bed and the water column probably is the weakest part of sediment transport models. The present report briefly reviews the modelling of erosion and deposition and some critical remarks and suggestions are made. This report is a contribution to Task D.5 of the COSINUS project.

BASIC CONCEPTS

A sediment transport model solves the sediment mass balance equation, which can be written as:

$$\frac{\partial C}{\partial t} + U_j \frac{\partial C}{\partial x_j} = \frac{\partial}{\partial x_j}\left(\varepsilon_s \frac{\partial C}{\partial x_j} \right) - \frac{\partial}{\partial z}\left(w_s C \right) \qquad \textbf{(1)}$$

where: C = the sediment concentration by mass, U = the (Reynolds-averaged) flow velocity, obtained as solution of a hydrodynamic model, ε_s = the eddy diffusivity (= ν_t/σ_t, with ν_t the eddy viscosity, obtained from a turbulence closure model, and σ_t the turbulent Schmidt number, usually given as an empirical function of a Richardson number (Toorman, 2000a)), and w_s = the representative settling velocity.

The computational domain is formed by a defined body of water. Boundary conditions are given in terms of sediment fluxes, i.e. exchange rates of sediment with the continuum at the other side of the boundary. The bottom boundary of the computational domain is formed by the bed surface. For cohesive sediment beds, it is not always evident to define it. Physically the best definition is given by the locus of points where the effective stress in the bed becomes zero.

Exchange with the bed happens in the form of a sink through deposition, or as a source through erosion. The bed boundary condition is then obtained by replacing the vertical sediment flux S by the sum of the erosion and the deposition fluxes, S_E and S_D respectively:

$$\left(\varepsilon_s \frac{\partial C}{\partial z} - w_s C \right)_{bed} = S_{bed} = S_E - S_D \qquad \textbf{(2)}$$

Since real sediments are graded, i.e. they have a grain size distribution, erosion of fine particles with low settling velocities can happen simultaneously with deposition of coarse particles with higher settling velocities. However, most of the presently used cohesive sediment transport models only consider one fraction. Therefore, the net flux is the net result of the processes erosion and deposition, and “erosion” then implies for the model an increase of sediment concentration in the water column (i.e. an upward, ingoing flux), and “deposition” a decrease (i.e. a downward, outgoing flux).

EROSION

The erosion flux traditionally is modelled with an empirical closure which evaluates the bed shear stress, generated by the hydrodynamics due to friction between flowing water and bed, against the erosion strength (or critical stress for erosion). Erosion modelling requires two assumptions. The first on the erosion law, the second on the erosion strength, i.e. how the erosion resistance is related to the bed properties as a function of depth and time.

Basically, two types of erosion law are found in the literature. A typical erosion rate equation for surface erosion is of the form:

$$S_E = M_E \max\left(0, \left((\tau_b/\tau_{cE})^n - 1\right)^m\right) \quad \textbf{(3)}$$

where: M_E = erosion rate parameter; τ_b = (flow induced) bottom shear stress; τ_{cE} = critical (or threshold) stress for erosion (or bottom erosion strength). Usually $m = n = 1$, which yields the well known empirical "Partheniades" formula, proposed by Ariathurai (1974), based on data from Partheniades (1962) (McAnally & Mehta, 2001). This law has been confirmed by several laboratory experiments (e.g. Mehta & Partheniades, 1973).

For soft mud another form has been proposed by Parchure & Mehta (1985):

$$S_E = M_E \exp\left(\alpha \sqrt{\tau_b - \tau_{cE}}\right) \qquad (\tau_b > \tau_{ce}) \quad \textbf{(4)}$$

This model is used only for the erosion of freshly deposited sediment. A typical value of τ_{cE} in this case is 0.07 Pa (e.g. Pathirana, 1994).

Erosion rate

Erosion rates are quite difficult to be measured accurately. They are obtained from erosion experiment data as the slope of the concentration as a function of time. The major problem is to know at the same time the correct bed shear stress.

Usually, in a model a constant value for M_E is taken. However, the erosion rate parameter M_E is expected to be proportional to the bed surface concentration, as the amount that can be eroded cannot exceed the available amount. This idea has also been proposed recently by Sanford (2000).

Bed shear stress

The accurate determination of the bed shear stress is crucial to the calibration of erosion laws.

The bed shear stress in laboratory flumes can be determined using different methods. A first method is based on extrapolation of the velocity profile, assuming validity of the law of the wall (Torfs, 1995). Alternatively, it can be calculated from the energy slope (Torfs, 1995). These methods are very sensitive to measurement errors, particularly in the low range of bed shear stresses at which erosion of cohesive sediments starts (usually < 1 Pa). By assuming proportionality between bed shear stress and discharge squared, a channel characteristic can be determined (Toorman & Luycks, 1997). The latter method seems to yield more reliable results.

A major problem in erosion experiments is the difference between the ideal flat bottom, for which also the in-situ erosion devices are designed, and the real eroding bed. At present, there is no way to account for this.

Furthermore, it is now better understood how sediment-turbulence interaction modifies the law of the wall and causes drag reduction for fine particle suspensions (Toorman, 2000a). Erosion calibration tests do not account for these effects. Consequently, bed shear stresses may be underestimated, up to a factor 3 at saturation conditions (Toorman, 2000c).

Therefore, the interpretation of erosion data obtained after the top layer has started to erode, has to be done with great care.

Erosion strength

The erosion strength traditionally is correlated to a measure of strength of the bed at various densities.

The most popular correlation is the one with the vane shear strength. Various empirical relationships between shear strength and density have been proposed, i.e. a power law:

$$\tau_y = a_1 C^{n_1} \qquad \textbf{(5)}$$

and an exponential law:

$$\tau_y = \exp(a_2 C - b_2) \qquad \textbf{(6)}$$

A new alternative formulation has been proposed by Toorman (1995), which accounts much better for the curvatures observed in the experimental data:

$$\tau_y = a_3\left(e^{C/C_3 - 1} - 1\right) \qquad (C > C_3) \qquad \textbf{(7)}$$

Moreover this formulation accounts for the fact that there is no structure below the concentration C_3, which makes this form physically more realistic. Figure 1 shows that this curve fit gives the best approximation over the total range.

There are other measures for the strength of a sediment bed, e.g. penetration resistance (van Kessel, 1997), the effective stress or the shear moduli. The various strength parameters yield different results, but seem to be linearly related with each other (Merckelbach, 2000). They need to be correlated to the actual erosion strength, which can only be determined from the critical bed shear stress. Simultaneous measurement of bed strength and erosion resistance seems not possible at present.

Erosion tests only provide more or less reliable information on the erosion characteristics of the top layer. For medium and long-term simulations, the critical stress for erosion needs to be determined for various layers of the same bed, and for layers which will be deposited in the future. Furthermore, one should account for the fact that the bed strength may change, e.g. when it is unloaded due to erosion of the top layer. The variation in space and time of effective stresses can, in principle, be estimated with a detailed bed dynamics model, such as is being developed at KUL (Toorman *et al.*, 2000). But these type of models are too detailed for use on estuarine scales.

The problem of erosion modelling is illustrated by the intercomparison exercise of Working Group F of the MASTII G8M Project (Hamm *et al.*, 1996). Various well-documented flume experiments, carried out at SOGREAH (Viguier *et al.*, 1994), have been simulated. Data was available on vane shear strength as a function of density and of density profiles. An assumption had to be made regarding the relationship between this shear strength and the erosion strength.

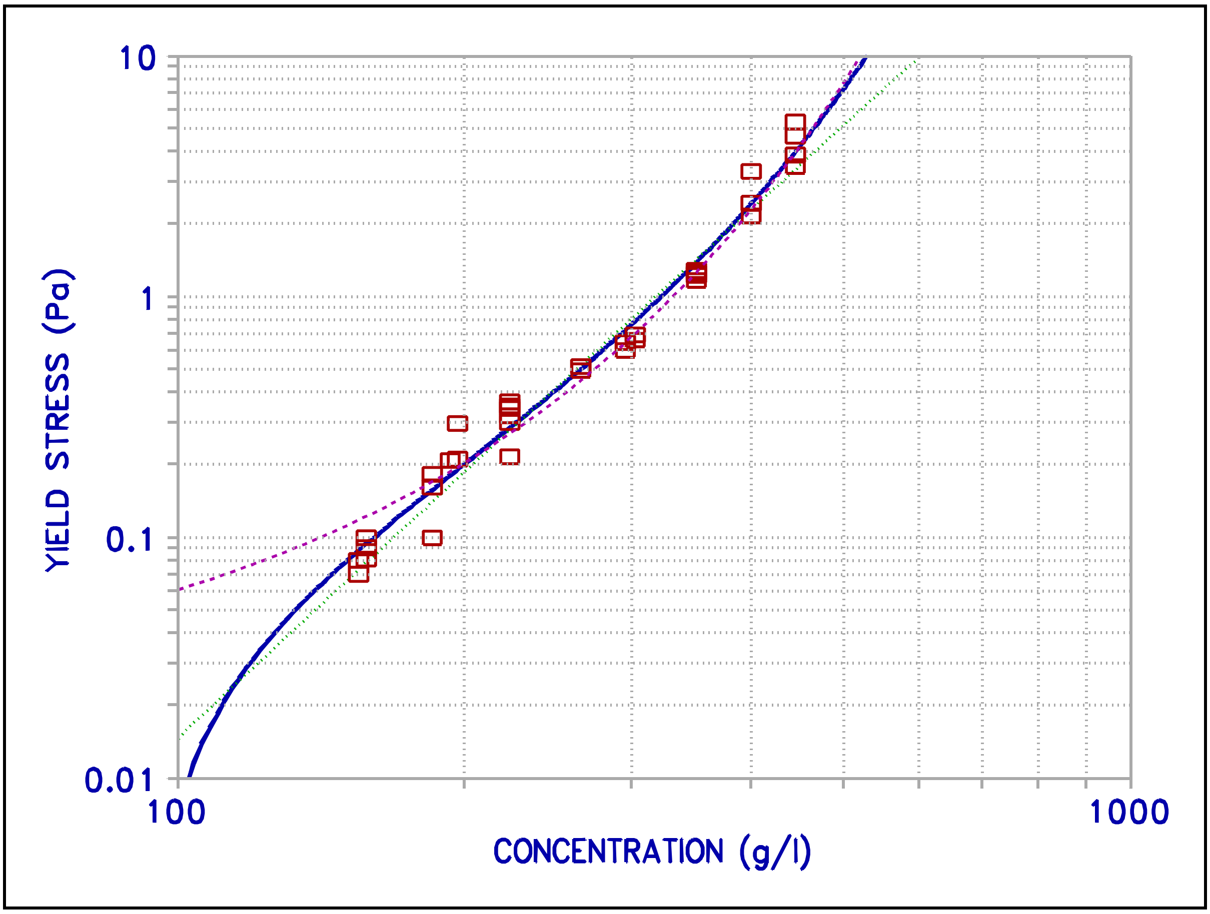

Figure 1: Yield stress versus concentration. □ = classical rheological measurement (after Viguier *et al.*, 1994); curve fits: **--** eq.(5) (regression of the logarithmic values gives: a_1 = 1.22 10^{-9}, n_1 = 3.57 with correlation factor R = 0.972), **...** eq.(6) (regression: a_2 = 0.0121, b_2 = 3.965 with R = 0.972) and — eq.(7) (regression: a_3 = 0.0938 Pa, C_3 = 93.215 g/l).

Application of any erosion law with τ_{cE} equal to the vane shear strength proved unsuccessful. In order to understand the problem better, two approaches were followed.

Jakobsen & Deigaard (1995) did not use the vane shear strength data but an analytical approximation of the bed density profiles and estimated the strength-concentration relationship, obtaining the stress from assuming a logarithmic velocity profile and the concentration from a mass balance at the equilibrium moments. The results of their analysis suggested time-dependence, which they empirically implemented in their erosion strength closure.

Toorman (1995) re-analysed the input data by application of inverse modelling, assuming an erosion law and erosion strength closure, the bed density profile was computed, showing significant differences with the original data. This seemed to confirm the suspicion that the given density profiles, determined in settling columns, were not representative for the flume conditions.

In the field, the problem is worse, because in general information on the bed density profiles is not available, and highly spatially variable.

Bulk erosion

Bulk erosion is observed in laboratory experiments, increasingly with consolidation time (e.g. Migniot, 1968; Toorman & Luyckx, 1997). Little is known on the occurrence of bulk erosion in the field. There are indications that it happens. For instance, tile shaped mud blocks have been found on layered mud flats, which apparently were broken off and carried away by currents (Silva Jacinto & Le Hir, 2001; photos have been shown at INTERCOH'98). Figure 2 shows cracking in an over-consolidated clay layer and broken off clumps of clay (possibly from a clay layer dumped during former construction works of a small naval harbour) in the IJzer estuary (Belgium). Another photograph of an eroded "mud pebble" (probably from a laboratory test) is found in the famous paper of Migniot (1968, figure 44). It is very difficult to draw conclusions from laboratory observations because usually the bed has a completely unnatural strength profile and history.

Figure 2: Erosion of over-consolidated clay in the IJzer estuary. Left: bank erosion. Right: the mobile stone-like clay humps. (*Photos by Jaak Monbaliu, K.U. Leuven*).

Hence, ***if*** (!) bulk erosion would be important, it may be possible that the transport mode might be dominated by bed load transport. Laboratory erosion experiments at KUL indicate that under these conditions bed load transport may be 30 times more important than suspended load transport (Toorman & Luyckx, 1997).

DEPOSITION

The rate at which particles deposit equals the settling flux w_sC at the bottom. It is evident that the amount of sediment that becomes part of the bed cannot be larger than the amount which settles, i.e. $S_D < w_sC$. Therefore, it is generally assumed that the deposition flux can be written as:

$$S_D = p_D w_s C \qquad \textbf{(8)}$$

where: p_D is the fraction of the particles that stick to the bed surface, i.e. the deposition probability. The remaining fraction 1-p_D remains mobile in the water column, and can possibly be transported as bed load .

The deposition probability traditionally is modelled with an empirical closure which evaluates the bed shear stress against a critical stress for deposition. Hence, the deposition law becomes (Krone, 1962):

$$S_D = \max\left(1 - \frac{\tau_b}{\tau_{cD}}, 0\right) w_s C \qquad \textbf{(9)}$$

It is very important to realize that Krone's original work follows from the interpretation of depth-averaged data, i.e. the settling velocity and concentration are the depth-averaged values. Therefore, the meaning of p_D in Krone's work is different: it is the fraction of particles that is no longer kept in suspension. In the present context of 3D models, Krone's traditional deposition law may not be so evident to be used. It is important to distinguish between settled particles which stick to the bed and subsequently create a new top layer on the bed surface, and those that remain mobile and can be transported as bed load.

The meaning of the critical stress for deposition actually is not so evident. It implies that a minimal energy is required, i.e. the turbulent kinetic energy, generated by shear production near the bed surface, in order to keep particles in suspension. Deposition occurs when a suspension is oversaturated (Cellino & Graf, 1999). Within Task A of the COSINUS project, conditions for saturation have been determined (Toorman, 2000b). It is found that oversaturation occurs as soon as the vertical gradient of the flux Richardson number Rf becomes negative, which corresponds to a critical value of $Rf_{sat} = 0.25$. The corresponding sediment load per unit area at saturation can then be determined:

$$L_{sat} \cong \frac{h\, S\, Rf_{sat}}{\sigma_{t,sat}\, \Delta\rho_s/\rho_s} \left(\frac{u_*}{w_s}\right)^2 \ln\left(\frac{h}{z_0} - 1\right) \qquad \textbf{(10)}$$

with z_0 = the roughness height. Equation (10) shows that the saturation load is proportional to the energy slope S and water depth h and the squared ratio of shear velocity to settling velocity.

According to Sanford & Halka (1993), numerical models perform better when no treshold is considered for deposition. This makes sense physically if one considers the fraction of the settling flux $w_s C$ which does not stick to the bed as a part that is immediately "eroded". It is then possible to include the fraction that does not stick to the bottom in the erosion flux, i.e.:

$$S_0 = S_E - S_D = \left(S_E + (1 - p_D) w_s C\right) - w_s C = S_E^{/} - w_s C \qquad \textbf{(11)}$$

If the non-sticking fraction of depositing particles should be included into the erosion law, a contribution without critical erosion stress should be added.

CHARACTERISATION OF MUD PARTICLES

Comments on the use of fractal dimension as characteristic

The computation of the terminal settling velocity according to Stokes' law requires the knowledge of two characteristics of the particle: its size (i.e. the equivalent diameter of a sphere settling at the same rate) and its density. Since cohesive sediment particles are flocs, the required characteristics ideally need to be generated by a flocculation model.

Recently the use of the fractal dimension as characteristic to correlate mechanical properties of cohesive sediments has been proposed by Kranenburg (1994) and is strongly advocated by the cohesive sediment research community in Delft (e.g. Winterwerp, 1999). The basic assumption is that flocs are self-similar, i.e. the floc structure is independent on the size. Hence, the floc density can then be replaced by the fractal dimension as second characteristic. The relationship between fractal number and volume concentration φ_f is given by (Huang, 1994, Kranenburg, 1994):

$$\varphi_f = \left(\frac{D_p}{D}\right)^{3-n_F} \qquad \textbf{(12)}$$

where: D = floc diameter, D_p = the size of the primary particle, n_F = fractal dimension. Hence, the floc density if given by:

$$\rho_f = \rho_w + (\rho_s - \rho_w)\left(\frac{D_p}{D}\right)^{3-n_F} \qquad \textbf{(13)}$$

The settling velocity can then be written as (Winterwerp, 1999):

$$w_s = \frac{f_{shape}}{f_{drag}}\,\frac{(\rho_s - \rho_w)\,g}{18\,\mu}\,D_p^{3-n_F}\,D^{n_F-1} \qquad \textbf{(14)}$$

where: μ = fluid viscosity.

However, ***flocs are <u>not</u> self-similar***. Indeed, when flocs break up into smaller aggregates, these smaller aggregates generally are expected to have a more compact structure, higher floc density and thus higher fractal number. This corresponds to Krone's conceptual model of levels of aggregation (Krone, 1986). Winterwerp's model assumes a constant fractal dimension.

Some settling velocity data suggest that settling velocities increase faster with D then predicted by (14) assuming a constant n_F, at least in the particle size range $100 < D < 1000$ µm (e.g. fig. 4.3 in Winterwerp, 1999). This, strange enough, implies an increase of n_F with D. Possibly, some data may actually refer to non-cohesive particles (as recorded e.g. in the COSINUS Tamar field experiment).

Similarly, during consolidation the fractal dimension increases. This has been confirmed recently by the analysis of consolidation data by Sills (2000). A consolidation model which is based on one characteristic fractal dimension for the bed cannot be realistic. A more complex relationship between fractal dimension and D seems to be appropriate, instead of a constant value.

Small variations in fractal dimension result in large floc density changes:

$$\Delta\rho_f = \Delta n_F \frac{\mathrm{d}\rho_f}{\mathrm{d}n_F} = -(\rho_s - \rho_w)\,\varphi_f \ln\left(\frac{D_p}{D}\right) \qquad \textbf{(15)}$$

E.g., consider a floc with $D/D_p = 25$, $n_F = 2$ and assume $\rho_s = 2650$ kg/m^3 and $\rho_w = 2650$ kg/m^3 . The corresponding floc density, according (13), is 1066 kg/m^3. A change of n_F with 0.1 yields a change in floc density of roughly 20 kg/m^3, i.e. a change of the floc volume fraction with roughly 30% and a similar change in settling velocity.

With this in mind, and because a floc density is easier to "understand" than a fractal dimension, the author prefers to work with floc density as second characteristic of a floc (the first being floc diameter) instead of fractal number. ***Moreover, the chemical community is abandoning the use of the fractal dimension to characterise floc structures because it gives a wrong representation*** (Prof. J. Mewis, pers. comm.).

Density of eroded particles

The particles that are eroded in general are aggregates again. The aggregate density can be estimated from the assumption that the bed surface corresponds to a maximum packing of equivalent spheres:

$$\varphi_a = \frac{\Delta\rho_a}{\Delta\rho_s} = \frac{1}{\varphi_{\max}}\frac{\Delta\rho_b}{\Delta\rho_s} \qquad \textbf{(16)}$$

Hence, for a bed of density 1200 kg/m^3, assuming $\Delta\rho_s = 2650$ kg/m^3 and $\varphi_{max} = 0.65$, the aggregate volume fraction is expected to be of the order 19%, having a floc density of 1308 kg/m^3, which is higher than the bed density. This also implies a higher fractal dimension than for the bed surface, i.e. the eroded aggregates are more compact than the bed surface, which is as expected.

Bed surface density of fresh deposit

The inverse reasoning can be used to determine the bed surface density after deposition (provided that the number of particles that have settled during a time step is large enough to cover the area). As the bed by definition is a bed when effective stresses develop, it requires that the deposited aggregate particles form a bed of "maximum" compaction. Considering the depositing aggregate as an equivalent sphere, and the maximum packing as 65% by volume, the bed surface density can be related to the floc density of the aggregate by the same equation as above.

See also Dearnaley *et al.* (2000) for a discussion on how this can be reconciled with experimental data.

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