# Sophomore PSAT Training Packet 

Math Department Answer Key

# PSAT/NMSOT® ${ }^{\circledR}$ <br> Practice Test \#1 

## Answer <br> Explanations

# Math Test - No Calculator Answer Explanations 

Question 1
A babysitter earns $\$ 8$ an hour for babysitting 2 children and an additional $\$ 3$ tip when both children are put to bed on time. If the babysitter gets the children to bed on time, what expression could be used to determine how much the babysitter earned?
A) $8 x+3$, where $x$ is the number of hours
B) $3 x+8$, where $x$ is the number of hours
C) $x(8+2)+3$, where $x$ is the number of children
D) $3 x+(8+2)$, where $x$ is the number of children

Item Difficulty: Easy
Content: Heart of Algebra
Correct Answer: A

Choice A is the correct answer. Let $x$ be the number of hours that the babysitter worked. Since the babysitter earns money at a rate of $\$ 8$ per hour, she earned $8 x$ dollars for the $x$ hours worked. If the babysitter gets both children to bed on time, the babysitter earns an additional $\$ 3$ tip. Therefore, the babysitter earned a total amount of $8 x+3$ dollars.

Choice $B$ is incorrect since the tip and the rate per hour have been interchanged in the expression. Choices $C$ and $D$ are incorrect since the number of children is not part of how the babysitter's earnings are calculated.

Question 2

$$
3(x+y)=y
$$

If $(x, y)$ is a solution to the equation above and $y \neq 0$, what is the ratio $\frac{x}{y}$ ?
A) $-\frac{4}{3}$
B) $-\frac{2}{3}$
C) $\frac{1}{3}$
D) $\frac{2}{3}$

Item Difficulty: Medium
Content: Passport to Advanced Math
Correct Answer: B
Choice B is the correct answer. We can find the ratio $\frac{x}{y}$ by rearranging the equation. Multiplying out the expression on the left side of the equation yields $3 x+3 y=y$. Then, subtracting $3 y$ from both sides of the equation gives $3 x=-2 y$. Finally, dividing both sides of this equation by $3 y$ (note that $y \neq 0$ ) gives $\frac{x}{y}=-\frac{2}{3}$

Choices $\mathrm{A}, \mathrm{C}$, and D are incorrect; they could result from errors during algebraic transformations of the equation $3(x+y)=y$.

Question 3

$$
\begin{aligned}
& \frac{1}{2} x-\frac{1}{4} y=10 \\
& \frac{1}{8} x-\frac{1}{8} y=19
\end{aligned}
$$

Which ordered pair $(x, y)$ satisfies the system of equations above?
A) $(-112,-264)$
B) $(64,88)$
C) $\left(\frac{232}{3}, \frac{224}{3}\right)$
D) $(288,536)$

Item Difficulty: Medium
Content: Heart of Algebra
Correct Answer: A
Choice A is the correct answer. First, we clear the fractions from the two given equations by multiplying both sides of the first equation by 4 and then both sides of the second equation by 8 (note that the new equations are equivalent to the original ones). Thus the system becomes $\left\{\begin{array}{l}2 x-y=40 \\ x-y=152\end{array}\right.$. Subtracting side by side the second equation from the first eliminates the variable $y$, $(2 x-y)-(x-y)=40-152$, leaving an equation with just one variable, $x$. Solving this equation gives $x=-112$. Substituting -112 for $x$ into the equation $x-y=152$ gives $y=-264$. Therefore, $(-112,-264)$ is the ordered pair that satisfies the system of equations given.

Choices B, C, and D are incorrect since the ordered pair in each choice does not satisfy both equations in the system. For example, the ordered pair of choice B, $(64,88)$, does not satisfy equation $\frac{1}{8} x-\frac{1}{8} y=19$ because $\frac{1}{8}(64)-\frac{1}{8}(88) \neq 19$.

Question 4


Note: Figure not drawn to scale.

Triangle $A B C$ above is isosceles with $A B=A C$ and $B C=48$. The ratio of $D E$ to $D F$ is 5:7. What is the length of $\overline{D C}$ ?
A) 12
B) 20
C) 24
D) 28

Item Difficulty: Medium Content: Additional Topic in Math Correct Answer: D

Choice D is the correct answer. The base angles, $\angle B$ and $\angle C$, of isosceles triangle $A B C$ are congruent. Additionally, $\triangle B E D$ are both right angles and therefore are congruent. Because $\triangle B E D$ and $\triangle C F D$ have two corresponding pairs of angles that are congruent, they are similar. Consequently, the corresponding sides of the similar triangles are proportional. So $\frac{B D}{D C}=\frac{D E}{D F}$, and since $\frac{D E}{D F}=\frac{5}{7}$, it follows that $\frac{B D}{D C}=\frac{5}{7}$. If we let $B D=5 x$, then $D C=7 x$. Since
$B D+D C=B C$ and $B C=48$, it follows that $5 x+7 x=48$. Solving this equation for $x$ gives $x=4$, and so $D C$ is $7(4)=28$.

Alternatively: Due to the similarity of $\triangle B E D$ and $\triangle C F D$, one can conclude that $\frac{B D}{D C}=\frac{5}{7}$, and so $D C$ must be greater than half of $B C$, which is 24 . Of the choices given, only one satisfies this condition, namely 28 . If $D C=28$, then $B D=48-28=20$, confirming that $\frac{B D}{D C}=\frac{20}{28}=\frac{5}{7}$. Therefore, the length of $\overline{D C}$ must be 28 .

Choices A, B, and C are incorrect because each of the values for $D C$ would result in $B C$ being less than 48 units long.

Question 5
In a certain game, a player can solve easy or hard puzzles. A player earns 30 points for solving an easy puzzle and 60 points for solving a hard puzzle. Tina solved a total of 50 puzzles playing this game, earning 1,950 points in all. How many hard puzzles did Tina solve?
A) 10
B) 15
C) 25
D) 35

Item Difficulty: Medium
Content: Heart of Algebra
Correct Answer: B
Choice B is the correct answer. Let $x$ and $y$ be the number of easy and hard puzzles, respectively, that Tina solved. Since she solved a total of 50 puzzles, it follows that $x+y=50$. She earned a total of 1,950 points, so it must also be true that $30 x+60 y=1,950$. Dividing both sides of this equation by 30 gives $x+2 y=65$. Subtracting the first equation, $x+y=50$, from the second equation, $x+2 y=65$, gives $y=15$. Therefore, Tina solved 15 hard puzzles.

Alternatively: Let $x$ be the number of easy puzzles Tina solved. Then, $50-x$ is the number of hard puzzles she solved. And since she earned a total of 1,950 points, it must be true that $30 x+60(50-x)=1,950$. Solving this equation for $x$ gives $x=35$, and so $50-x=15$. Therefore, Tina solved 15 hard puzzles.

Choices $A$ and $C$ are incorrect because if the number of hard puzzles Tina solved were as they indicate, the total number of points she would earn will not be 1,950. The incorrect answer in choice D could be the result of interchanging the number of hard puzzles and easy puzzles.

Question 6

$$
2 x^{2}+7 x-15=0
$$

If $r$ and $s$ are two solutions of the equation above and $r>s$, which of the following is the value of $r-s$ ?
A) $\frac{15}{2}$
B) $\frac{13}{2}$
C) $\frac{11}{2}$
D) $\frac{3}{2}$

Item Difficulty: Medium
Content: Passport to Advanced Math
Correct Answer: B
Choice B is correct. This equation can be solved using the quadratic formula or factoring. The quadratic formula approach is left as an exercise for students. We will show first how to solve this equation using simple factoring and then will show how to solve it using both the structure of the equation and factoring.

Since $7 x=10 x-3 x$, the given equation can be rewritten as $2 x^{2}+(10 x-3 x)-15=0$. Regrouping the terms so that the left side of the equation is in the factored form gives $(2 x-3)(x+5)=0$, from which it follows that $2 x-3=0$ or $x+5=0$. Thus, the quadratic equation has solutions $\frac{3}{2}$ and -5 . Since $r$ and $s$ are solutions to the quadratic equation and $r>s$, we can conclude that $r=\frac{3}{2}$ and $s=-5$; therefore, $r-s=\frac{3}{2}-(-5)=\frac{13}{2}$.

Alternatively: Multiplying the original equation by 2, we can rewrite it in terms of $2 x$ as follows: $(2 x)^{2}+7(2 x)-30=0$. Since the two numbers whose sum is -7 and whose product is -30 are -10 and 3 , the equation will be factored as $(2 x-3)(2 x+10)=0$, generating $\frac{3}{2}$ and -5 as solutions. Since $r$ and $s$ are solutions to the quadratic equation and $r>s$, we can conclude that $r=\frac{3}{2}$ and $s=-5$; therefore, $r-s=\frac{3}{2}-(-5)=\frac{13}{2}$.

Choices A, C, and D are incorrect and could result from calculating the value of expressions given in terms of the solutions $r$ and $s$, but are not equivalent to the
difference $r-s$ of these solutions. For example, $\frac{15}{2}$ is the value of $-r s$, not the value of $r-s$.

Question 7

To cut a lawn, Allan charges a fee of $\$ 15$ for his equipment and $\$ 8.50$ per hour spent cutting a lawn. Taylor charges a fee of $\$ 12$ for his equipment and $\$ 9.25$ per hour spent cutting a lawn. If x represents the number of hours spent cutting a lawn, what are all the values of x for which Taylor's total charge is greater than Allan's total charge?
A) $x>4$
B) $3 \leq x \leq 4$
C) $4 \leq x \leq 5$
D) $x<3$

## Item Difficulty: Medium

Content: Heart of Algebra
Correct Answer: A
Choice A is the correct answer. If $x$ represents the number of hours spent cutting the lawn, the total fee that Allan charges is $8.5 x+15$ dollars and the total fee that Taylor charges is $9.25 x+12$ dollars. To find all of the values of $x$ for which Taylor's total fee is greater than Allan's total fee, we solve the inequality $9.25 x+12>8.5 x+15$, which simplifies to $0.75 x>3$, and so $x>4$.

Alternatively: Since Taylor's hourly rate charge is higher than Allan's, it can be concluded that after a certain amount of hours, Taylor's total charge will always be greater than Allan's total charge. Thus the inequality that represents all possible values of $x$ for which this occurs will be of the form $x>a$ for some value $a$. Of the choices given, only $x>4$ is in this form. Lastly, one can confirm that Taylor and Allan charge the same amount when $x=4$. Therefore, choice A is correct.

Choice B is incorrect because Allan's total charge is greater than Taylor's total charge when $x<4$. Choice $C$ is incorrect because Allan's total charge and Taylor's total charge at $x=4$ are exactly the same, and Taylor's total charge is greater than Allan's total charge also for values of $x$ greater than 5. Choice D is incorrect because Allan's total charge is greater than Taylor's charge when $x$ is less than 3.

Question 8

$$
n=456-3 T
$$

The equation above is used to model the relationship between the number of cups, $n$, of hot chocolate sold per day in a coffee shop and the average daily temperature, $T$, in degrees Fahrenheit. According to the model, what is the meaning of the 3 in the equation?
A) For every increase of $3^{\circ} \mathrm{F}$, one more cup of hot chocolate will be sold.
B) For every decrease of $3^{\circ} \mathrm{F}$, one more cup of hot chocolate will be sold.
C) For every increase of $1^{\circ} \mathrm{F}$, three more cups of hot chocolate will be sold.
D) For every decrease of $1^{\circ} \mathrm{F}$, three more cups of hot chocolate will be sold.

Item Difficulty: Medium
Content: Heart of Algebra
Correct Answer: D
Choice D is the correct answer. According to the model, if the average daily temperature is $T$ degrees Fahrenheit, then the number of cups of hot chocolate sold per day in the coffee shop would be $456-3 T$. If the temperature decreases by $1^{\circ} \mathrm{F}$, then the number of cups of hot chocolate sold per day in the coffee shop would be $456-3(T-1)$, which can be rewritten as $(456-3 T)+3$. Therefore, for every $1^{\circ} \mathrm{F}$ drop in the average daily temperature, the coffee shop sells three more cups of hot chocolate.

Choices $A$ and $B$ are incorrect because the change in the average daily temperature and the change in the number of cups of hot chocolate have been interchanged. Choice C is incorrect because, according to the model, the higher value of daily temperature corresponds to a lower, not higher, number of cups of hot chocolate sold.

Question 9
A truck enters a stretch of road that drops 4 meters in elevation for every 100 meters along the length of the road. The road is at 1,300 meters elevation where the truck entered, and the truck is traveling at 16 meters per second along the road. What is the elevation of the road, in meters, at the point where the truck passes $t$ seconds after entering the road?
A) $1,300-0.04 t$
B) $1,300-0.64 t$
C) $1,300-4 t$
D) $1,300-16 t$

Item Difficulty: Medium
Content: Heart of Algebra
Best Answer: B

Choice $B$ is the correct answer. Since the truck is traveling at 16 meters per second along the road, the distance it has traveled $t$ seconds after entering the road is $16 t$ meters. Since the elevation of the road drops 4 meters for every 100 meters along the length of the road, it follows that for $16 t$ meters along the road, the elevation drops $\frac{4}{100} \times 16 t$ or $0.64 t$. Therefore, the elevation of the road at the point where the truck passes $t$ seconds after entering the road is $1,300-0.64 t$ meters.

Choice A is incorrect because $\frac{4}{100} t$ would be the number of meters that the elevation drops $t$ seconds after the truck enters the road if its speed were 1 meter per second. Choice C is incorrect because $4 t$ meters does not give the number of meters the elevation of the road drops. Choice $D$ is incorrect because the drop rate of 4 meters for every 100 meters along the road is not used.

Question 10

If $f(x-1)=2 x+3$ for all values of $x$, what is the value of $f(-3)$ ?
A) -7
B) -5
C) -3
D) -1

Item Difficulty: Medium
Content: Passport to Advanced Math
Correct Answer: D

Choice D is correct. Since $f(x-1)=2 x+3$ for all values of $x$, $f(-3)=f(-2-1)=2(-2)+3$ and so the value of $f(-3)$ is -1 .

Alternatively: $2 x+3$ can be rewritten as $2(x-1)+5$ and since $f(x-1)=2(x-1)+5$ for all values of $x$, it follows that $f(x)=2 x+5$ for all values of $x$. Substituting -3 for $x$ in this equation gives $f(-3)=2(-3)+5=-1$.

Choices $A, B$, and $C$ are incorrect because $f$ is a function, and there is one and only one value for $f(-3)$, which as shown above is -1 . Therefore, neither of the choices, $-7,-5$, or -4 can be the value of $f(-3)$.

Question 11
Which of the following is equivalent to $(s-t)\left(\frac{s}{t}\right)$ ?
A) $\frac{s}{t}-s$
B) $\frac{s}{t}-s t$
C) $\frac{s^{2}}{t}-s$
D) $\frac{s^{2}}{t}-\frac{s}{t^{2}}$

Item Difficulty: Medium
Content: Passport to Advanced Math
Correct Answer: C

Choice C is the correct answer. Using the distributive property to expand the given expression gives $s\left(\frac{s}{t}\right)-t\left(\frac{s}{t}\right)=\frac{s^{2}}{t}-s$.

Choices $A, B$, and $D$ are incorrect. In each of these choices, at least one of the products in the expansion is not correct. For example $s\left(\frac{s}{t}\right)=\frac{s^{2}}{t}$, not $\frac{s}{t}$, and $t\left(\frac{s}{t}\right)=s, \operatorname{not} s t$ or $\frac{s}{t}$.

Question 12

$$
p(x)=3\left(x^{2}+10 x+5\right)-5(x-k)
$$

In the polynomial $p(x)$ defined above, $k$ is a constant. If $p(x)$ is divisible by $x$, what is the value of $k$ ?
A) -3
B) -2
C) 0
D) 3

Item Difficulty: Medium
Content: Passport to Advanced Math
Correct Answer: A
Choice A is the correct answer. If polynomial $p(x)$ is divisible by $x$, then $x$ must be a factor of the polynomial, or equivalently, the constant term of the polynomial must be zero. Multiplying out on the right side of the equation gives
$p(x)=3 x^{2}+30 x+15-5 x+5 k$, which can be rewritten as $p(x)=3 x^{2}+25 x+(5 k+15)$. Hence, $5 k+15=0$, and so $k=-3$.

Choices B, C, and D are the not correct answers because if the value of $k$ were as indicated in those choices, then $x$ would not be a factor of the polynomial $p(x)$, and so $p(x)$ would not be divisible by $x$.

Question 13

In the $x y$-plane, if the parabola with equation $y=a x 2+b x+c$, where $a, b$, and $c$ are constants, passes through the point $(-1,1)$, which of the following must be true?
A) $a-b=1$
B) $-b+c=1$
C) $a+b+c=1$
D) $a-b+c=1$

Item Difficulty: Hard
Content: Passport to Advanced Math
Correct Answer: D
Choice $D$ is the correct answer. If the graph of a parabola passes through the point $(-1,1)$, then the ordered pair $(-1,1)$ must satisfy the equation of the parabola.
Thus, $1=a(-1)^{2}+b(-1)+c$, which is equivalent to $a-b+c=1$.

Choices $A, B$, and $C$ are incorrect and could result from misinterpreting what it means for the point $(-1,1)$ to be on the parabola or from common calculation errors while expressing this fact algebraically.
These are the directions students will see in the test for the Student-Produced Response questions.

Question 14
For what value of $h$ is $24=\frac{h}{10}-6$ ?

Item Difficulty: Easy
Content: Heart of Algebra
Correct Answer: 300

The correct answer is 300. To solve the given equation for $h$, first add 6 to both sides of the equation to get $30=\frac{h}{10}$. Then multiply both sides of this equation by 10 to yield $h=300$.

Question 15

What is the value of $a$ if $(2 a+3)-(4 a-8)=7$ ?
Item Difficulty: Medium
Content: Heart of Algebra
Correct Answer: 2

The correct answer is 2 . The equation given can be rewritten as $2 a+3-4 a+8=7$, which is equivalent to $-2 a+11=7$, and so $a=2$.

Question 16

If $x$ is not equal to zero, what is the value

$$
\text { of } \frac{4(3 x)^{2}}{(2 x)^{2}} ?
$$

Item Difficulty: Medium
Content: Passport to Advanced Math
Correct Answer: 9
The correct answer is 9 . Multiplying out the given expression gives $\frac{4\left(9 x^{2}\right)}{4 x^{2}}$. Since $x \neq 0$, dividing both the numerator and the denominator of the fraction by $4 x^{2}$ simplifies the expression to 9 .

Question 17
If $x-2$ is a factor of $x^{2}-b x+b$, where $b$ is a constant, what is the value of
$b$ ?
Item Difficulty: Hard
Content: Passport to Advanced Math
Correct Answer: 4
The correct answer is 4 . If $x-2$ is a factor of $x^{2}-b x+b$, where $b$ is a constant, then $x^{2}-b x+b$ can be written as the product $(x-2)(x-a)$ for some real number $a$. Expanding $(x-2)(x-a)$ gives $x^{2}-2 x-a x+2 a$, which can be rewritten as $x^{2}-(2+a) x+2 a$. Hence, $x^{2}-(2+a) x+2 a=x^{2}-b x+b$ is true for all values of $x$. Consequently, the coefficients of like terms on each side of the equation must be the same: $2+a=b$ and $2 a=b$. Solving this system gives $b=4$.

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Alternatively: Since $x-2$ is a factor of $x^{2}-b x+b$ and $(x-2)^{2}=x^{2}-4 x+4$, one can correctly conclude that the value of $b$ is 4 .

## Math Test - Calculator Answer Explanations

Question 1

Tyra subscribes to an online gaming service that charges a monthly fee of $\$ 5.00$ and $\$ 0.25$ per hour for time spent playing premium games. Which of the following functions gives Tyra's cost, in dollars, for a month in which she spends $x$ hours playing premium games?
A) $C(x)=5.25 x$
B) $C(x)=5 x+0.25$
C) $C(x)=5+0.25 x$
D) $C(x)=5+25 x$

Item Difficulty: Easy Content: Heart of Algebra Correct Answer: C

Choice C is the correct answer. Tyra pays $\$ 0.25$ per hour for time spent playing premium games, so for the month in which she spends $x$ hours playing premium games, she pays $0.25 x$ dollars for playing the premium games. She also pays an additional $\$ 5$ monthly fee. Therefore, Tyra's cost, in dollars, for the month in which she spends $x$ hours playing premium games is given by the function $C(x)=5+0.25 x$.

Choice A is incorrect because Tyra is not charged $\$ 5.25$ per hour for time playing premium games. Choice $B$ is incorrect because the charge per hour has been interchanged with the monthly fee. Choice D is incorrect because $25 x$ is the charge for playing premium games in cents, not in dollars.

Question 2

A grocery store sells a brand of juice in individual bottles and in packs of 6 bottles. On a certain day, the store sold a total of 281 bottles of the brand of juice, of which 29 were sold as individual bottles. Which equation shows the number of packs of bottles, $p$, sold that day?
A) $p=\frac{281-29}{6}$
B) $p=\frac{281+29}{6}$
C) $p=\frac{281}{6}-29$
D) $p=\frac{281}{6}+29$

Item Difficulty: Easy Content: Heart of Algebra Correct Answer: A

Choice A is the correct answer. Since the store sold a total of 281 bottles, 29 of which were sold individually, it follows that $281-29$ bottles were sold in packs of 6 bottles. Therefore, the number of packs of bottles, $p$, sold that day in the store is $p=\frac{281-29}{6}$.

Choice $B$ is incorrect. Adding the number of bottles sold individually, 29, to the total number of bottles sold, 281, does not give the number of bottles that were sold in packs of 6 . Choices C and D are incorrect and could result from dividing all of the bottles into groups of 6 (incorrectly assuming that all 281 bottles of juice were sold in packs of 6), and either subtracting the 29 bottles sold individually from that result, as in choice $C$, or adding the 29 bottles to that result, as in choice $D$.

Question 3


The line graph above shows the monthly rainfall from March to October last year in Chestnut City. According to the graph, what was the greatest change (in absolute value) in the monthly rainfall between two consecutive months?
A) 1.5 inches
B) 2.0 inches
C) 2.5 inches
D) 3.5 inches

## Item Difficulty: Medium

Content: Probability and Data Analysis
Correct Answer: C
Choice C is the correct answer. The greatest change (in absolute value) in monthly rainfall could be an increase or a decrease in monthly rainfall. The table below shows the approximate changes in monthly rainfall in Chestnut City last year between each of the two consecutive months.

| Consecutive months | Change in monthly <br> rainfall (inches) |
| :--- | :--- |
| March to April | 0.5 |
| April to May | 1 |
| May to June | 0.5 |
| June to July | 1.5 |
| July to August | 0.5 |
| August to September | 1 |
| September to October | 2.5 |

Of the values on the right column, the greatest is from September to October, which is a change of 2.5 inches.

Choices $A, B$, and $D$ are incorrect because they contain values that either do not represent any of the changes in monthly rainfall between two consecutive months or that are not the greatest change.

Question 4

A rectangle has perimeter $P$, length $\ell$ and width $w$. Which of the following represents $\ell$ in terms of $P$ and $w$ ?
A) $\ell=P-w$
B) $\ell=\frac{2 P-w}{2}$
C) $\ell=\frac{P-2 w}{2}$
D) $\ell=2 P-2 w$

Item Difficulty: Medium
Content: Passport to Advanced Math
Correct Answer: C
Choice $C$ is the correct answer. The perimeter of a rectangle is the sum of the four sides and can be calculated using the formula $P=2 \ell+2 w$, where $\ell$ is the length and $w$ is the width of the rectangle. Subtracting $2 w$ from both sides of the equation gives
$P-2 w=2 \ell$, and then dividing by 2 yields $\ell=\frac{P-2 w}{2}$.
Choice A is incorrect. This choice does not use the fact that the perimeter of a rectangle is the sum of two length and two widths. Choice B and D are incorrect. In each of these choices, the equation incorrectly doubles the perimeter.

Question 5
Which ordered pair $(x, y)$ satisfies the system of equations shown below?

$$
\begin{aligned}
& 2 x-y=6 \\
& x+2 y=-2
\end{aligned}
$$

A) $(-6,2)$
B) $(-2,2)$
C) $(2,-2)$
D) $(4,2)$

Item Difficulty: Medium
Content: Heart of Algebra Correct Answer: C

Choice C is the correct answer. To eliminate $y$, the first equation in the system can be multiplied by 2 and then the equations can be added as shown below.

$$
\begin{array}{r}
4 x-2 y=12 \\
x+2 y=-2 \\
\hline 5 x+0=10
\end{array}
$$

Since the result is $5 x=10$, it follows that $x=2$. Substituting 2 for $x$ into the equation $x+2 y=-2$ gives $2+2 y=-2$ and so $y=-2$. Therefore, $(2,-2)$ is the solution to the system given.

Alternatively: Use the substitution method to solve the system. For example, the first equation can be rewritten as $y=2 x-6$. Substituting $2 x-6$ for $y$ in the second equation gives $x+2(2 x-6)=-2$, and so $x=2$. Finally, substituting 2 for $x$ in $y=2 x-6$ gives $y=-2$, leading to the same solution of the system, namely $(2,-2)$.

Choice B is incorrect. The value for $x$ and the value for $y$ have been reversed in the ordered pair. Choices A and D are incorrect. The ordered pair in each of these choices does not satisfy at least one of the equations in the system. For example, the ordered pair $(4,2)$ does not satisfy the equation $x+2=-2$, since $4+2(2) \neq-2$.

Question 6

A soda company is filling bottles of soda from a tank that contains 500 gallons of soda. At most, how many 20-ounce bottles can be filled from the tank? (1 gallon = 128 ounces)
A) 25
B) 78
C) 2,560
D) 3,200

## Item Difficulty: Easy

Content: Probability and Data Analysis
Correct Answer: D
Choice D is the correct answer. Since 1 gallon equals 128 ounces, 500 gallons equal $(500)(128)=64,000$ ounces. Therefore, the maximum number of 20 -ounce bottles that can be filled with the soda from the tank is $\frac{64,000}{20}=3,200$.

Choice A is incorrect and could result from dividing 500 (the number of gallons contained in the tank) by 20 (the capacity of one bottle, in ounces). The gallons need to be converted into ounces first, and then the result can be divided by 20. Choices $B$ and $C$ are incorrect because they do not give the maximum number of 20-ounce bottles that can be filled from the soda in the tank.

Question 7

A car traveled at an average speed of 80 miles per hour for 3 hours and consumed fuel at a rate of 34 miles per gallon. Approximately how many gallons of fuel did the car use for the entire 3-hour trip?
A) 2
B) 3
C) 6
D) 7

## Item Difficulty: Medium <br> Content: Probability and Data Analysis <br> Correct Answer: D

Choice D is the correct answer. Since the car traveled at an average speed of 80 miles per hour, the distance the car traveled during 3 hours is $(80)(3)=240$ miles.

The car consumed fuel at a rate of 34 miles per gallon, so the car used $\frac{240}{34}$ gallons of fuel, which is approximately 7 gallons of fuel.

Choices A, B, and C are incorrect. For each of these choices, the amount of fuel is not enough to travel the entire 240 miles.

Question 8

What is the slope of the line in the $x y$-plane that passes through the points

$$
\left(-\frac{5}{2}, 1\right) \text { and }\left(-\frac{1}{2}, 4\right) \text { ? }
$$

A) -1
B) $-\frac{2}{3}$
C) 1
D) $\frac{3}{2}$

Item Difficulty: Medium
Content: Heart of Algebra
Correct Answer: D
Choice D is the correct answer. In the $x y$-plane, the slope $m$ of a line that passes through the points $\left(x_{1}, y_{1}\right)$ and $\left(x_{2}, y_{2}\right)$ is the change in $y$ over the change in $x$ (rise over run), which is expressed by the formula $m=\frac{y_{2}-y_{1}}{x_{2}-x_{1}}$. Thus, the slope of the line through the points $\left(-\frac{5}{2}, 1\right)$ and $\left(-\frac{1}{2}, 4\right)$ is $\frac{4-1}{-\frac{1}{2}-\left(-\frac{5}{2}\right)}$, which simplifies to $\frac{3}{2}$.

Choices A and C are incorrect because the change in $y$ and the change in $x$ do not have the same magnitude. Choice $B$ is incorrect; the fraction $-\frac{2}{3}$ is the negative reciprocal of the slope of the line through the points $\left(-\frac{5}{2}, 1\right)$ and $\left(-\frac{1}{2}, 4\right)$.

Question 9


The scatterplot above shows the widths and the heights of 12 types of rectangular envelopes. What is the width, in inches, of the envelope represented by the data point that is farthest from the line of best fit (not shown)?
A) 2
B) 5
C) 7
D) 12

Item Difficulty: Medium
Content: Probability and Data Analysis
Correct Answer: C
Choice C is the correct answer. The data point that is farthest from the line of best fit is located at $(7,4)$, which means that this point represents a type of envelope that is 7 inches wide and 4 inches high.

Choices $A$ and $B$ are incorrect because none of the data points with width 2 or width 5 is the farthest from the line of best fit. Choice $D$ is incorrect because the scatterplot does not contain any points with width 12 inches.

Question 10

A high school basketball team won exactly 65 percent of the games it played during last season. Which of the following could be the total number of games the team played last season?
A) 22
B) 20
C) 18
D) 14

## Item Difficulty: Medium

Content: Probability and Data Analysis
Correct Answer: B
Choice B is the correct answer. The number of games won by the basketball team must be a whole number. Since $65 \%$ is equivalent to $\frac{13}{20}$, it follows that, of the choices given, the total number of games the team played last season can only be 20. Multiplying $\frac{13}{20}$ by each of the other answer choices does not result in a whole number.

Choices A, C, and D are incorrect because $65 \%$ of each of the numbers in the choices results in non-whole numbers.

Question 11

$$
110 x+y=1,210
$$

A coffee shop is running a promotion where a number of free coffee samples are given away each day. The equation above can be used to model the number of free coffee samples, $y$, that remain to be given away $x$ days after the promotion began. What does it mean that $(11,0)$ is a solution to this equation?
A) During the promotion, 11 samples are given away each day.
B) It takes 11 days during the promotion to see 1,210 customers.
C) It takes 11 days during the promotion until none of the samples are remaining.
D) There are 11 samples available at the start of the promotion.

Item Difficulty: Medium
Content: Heart of Algebra
Correct Answer: C

Choice C is the correct answer. Since $x$ represents the number of days after the promotion began and $y$ represents the remaining number of coffee samples, the fact that the ordered pair $(11,0)$ is a solution to the given equation means that it takes 11 days during the promotion until none of the samples are remaining.

Choice A is incorrect; if 11 samples were given away each day, then the coefficient of $x$ in the equation would be 11. Therefore, this is not the correct interpretation of $(11,0)$ as a solution to the equation. Choice $B$ is incorrect; the total number of free coffee samples given away during 11 days of the promotion was 1,210 . But the number of customers who were in the store during those days need not be 1,210. Choice D is incorrect; according to the given equation, there were 1,210, not 11, samples available at the start of the promotion.

Question 12
Which scatterplot shows a negative association that is not linear? (Note: A negative association between two variables is one in which higher values of one variable correspond to lower values of the other variable, and vice versa.)
A)

B)

C)

D)


## Item Difficulty: Medium

Content: Probability and Data Analysis
Correct Answer: B
Choice B is the correct answer. Of the choices given, only the scatterplots in A and B show a negative association between variables $x$ and $y$, and of these two associations, the one depicted in choice $B$ is not linear.

Choice $A$ is incorrect. The association depicted in this scatterplot is negative, but it can also be linear. Choice C is incorrect. The association depicted in this scatterplot is not linear. However, for $x$ greater than 10 , the association between $x$ and $y$ is positive. Choice D is incorrect. There is no clear association between $x$ and $y$ in this scatterplot.

Question 13


The histogram above shows the distribution of the heights, in meters, of 26 pyramids in Egypt. Which of the following could be the median height of the 26 pyramids represented in the histogram?
A) 44 meters
B) 48 meters
C) 63 meters
D) 77 meters

Item Difficulty: Medium
Content: Probability and Data Analysis
Correct Answer: B
Choice $B$ is the correct answer. The median of a data set is the middle value when the data points are sorted in either ascending or descending order. When the number of the data points is even, then the median is the mean of the two middle values of the sorted data. Hence, the median height of the 26 pyramids is the mean
of the 13th and 14th tallest pyramids. Since the number of pyramids that are less than 30 meters high is 5 and the number of pyramids that are less than 60 meters high is 17 , the median height of the 26 pyramids must be between 45 and 60 meters. Therefore, of the choices given, only 48 meters could be the median height of the 26 pyramids.

Choices A, C, and D are incorrect because the median height of the 26 pyramids cannot be less than 45 meters or greater than 60 meters.

## Questions 14-16 refer to the following information.

A survey of 170 randomly selected teenagers aged 14 through 17 in the United States was conducted to gather data on summer employment of teenagers. The data are shown in the table below.

|  | Have a <br> summer job | Do not have a <br> summer job | Total |
| :--- | :---: | :---: | ---: |
| Ages 14-15 | 20 | 69 | 89 |
| Ages $16-17$ | 39 | 42 | 81 |
| Total | 59 | 111 | 170 |

Question 14
Which of the following is closest to the percent of those surveyed who had a summer job?
A) $22 \%$
B) $35 \%$
C) $47 \%$
D) $53 \%$

Item Difficulty: Medium
Content: Probability and Data Analysis
Correct Answer: B
Choice $B$ is the correct answer. The number of teenagers surveyed in the data is 170. Of those surveyed, a total of 59 teenagers had a summer job; thus, the percent of those teenager surveyed who had a summer job is $\frac{59}{170}=0.347$, which rounds to $35 \%$.

Choice A is incorrect. This choice, $22 \%$, is the approximate percent $\left(\frac{20}{89} \approx 0.22\right)$ of teenagers aged 14 to 15 who had summer jobs. But that is not precisely what is
asked in this question. Choices $C$ and $D$ are incorrect and may be the result of calculating relative frequencies that are different from what the problem asks.

Question 15
In 2012 the total population of individuals in the United States who were between 14 and 17 years old (inclusive) was about 17 million. If the survey results are used to estimate information about summer employment of teenagers across the country, which of the following is the best estimate of the total number of individuals between 16 and 17 years old in the United States who had a summer job in 2012?
A) $8,200,000$
B) $3,900,000$
C) $2,000,000$
D) 390,000

Item Difficulty: Hard
Content: Probability and Data Analysis
Correct Answer: B

Choice B is the correct answer. In 2012, the total population of individuals in the United States who were between 14 and 17 years old (inclusive) was about 17 million, which is $10^{5}$ times the size of the survey sample, 170 . Since of those surveyed, 39 teenagers aged 16 to 17 had a summer job, it follows that the best estimate of the total number of individuals aged 16 to 17 in the United States who had a summer job in 2012 was $39 \times 10^{5}=3,900,000$.

Choices A, C, and D are incorrect and are likely the result of either conceptual or calculation errors made.

Question 16
Based on the data, how many times more likely is it for a 14 year old or a 15 year old to NOT have a summer job than it is for a 16 year old or a 17 year old to NOT have a summer job? (Round the answer to the nearest hundredth.)
A) 0.52 times as likely
B) 0.65 times as likely
C) 1.50 times as likely
D) 1.64 times as likely

Item Difficulty: Hard
Content: Probability and Data Analysis
Correct Answer: C

Choice C is the correct answer. According to the data shown in the table, 69 out of 89 teenagers aged 14 to 15 did not have summer jobs. So for a 14 - or 15 -year-old, the likelihood of not having a summer job is $\frac{69}{89}$. And since 42 out of 81 teenagers aged 16 to 17 did not have a summer job, the likelihood that a 15 - or 16-year-old not having a summer job is $\frac{42}{81}$. Therefore, a 14 - or 15 -year-old is $\frac{69}{89} \div \frac{42}{81}=\frac{1,863}{1,246}=1.49518$, or about 1.50 , times more likely to not have a summer job.

Choice $A$ is incorrect. This choice could result from calculating the likelihood that a teenager aged 16 to 17 will not have a summer job $\left(\frac{42}{81}\right)$. Choice $B$ is incorrect. This choice could result from calculating the likelihood that a teenager aged 14 through 17 will not have a summer job is $\left(\frac{111}{170}\right)$. Choice $D$ is incorrect. This choice could result from calculating the ratio of the number of teenagers aged 14 to 15 who do not have a summer job (69) to the number of teenagers aged 16 to 17 who do not have a summer job (42). If the total number of those surveyed in the two different groups were the same, this result would be correct. But the sizes of the two groups are different; therefore, the result obtained is incorrect.

Question 17

Percent Protein in Five Food Products


The graph above shows the amount of protein supplied by five different food products, $\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{D}$, and E , as a percentage of their total weights. The costs of 10 grams of products A, B, C, D, and E are $\$ 2.00, \$ 2.20, \$ 2.50$, $\$ 4.00$, and $\$ 5.00$, respectively. Which of the five food products supplies the most protein per dollar?
A) A
B) B
C) C
D) E

Item Difficulty: Medium
Content: Probability and Data Analysis
Correct Answer: C
Choice C is the correct answer. The table below organizes the information in the graph and the additional data needed to answer the question.

| Food <br> product | Cost of <br> 10 grams <br> of <br> product | Amount <br> of <br> product <br> (in <br> grams) | Percent <br> protein | Amount of <br> protein <br> (in grams) | Protein <br> per dollar <br> (in grams/dollar) |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $A$ | $\$ 2.00$ | 10 | $10 \%$ | $0.1(10)=1$ | $\frac{10(0.1)}{2}=0.5$ |
| $B$ | $\$ 2.20$ | 10 | $15 \%$ | $0.15(10)=$ <br> 1.5 | $\frac{10(0.15)}{2.2}=0.68$ |
| $C$ | $\$ 2.50$ | 10 | $20 \%$ | $0.2(10)=2$ | $\frac{10(0.2)}{2.5}=0.8$ |
| $D$ | $\$ 4.00$ | 10 | $25 \%$ | $0.25(10)=$ <br> 2.5 | $\frac{10(0.25)}{4}=0.625$ |
| $E$ | $\$ 5.00$ | 10 | $30 \%$ | $0.3(10)=3$ | $\frac{10(0.3)}{5}=0.6$ |

According to the table, food product $C$ provides the most protein per dollar (0.8).
Choices A, B, and D are incorrect. For each choice, the protein per dollar for each of the food products is less than 0.8 grams of protein per dollar.

Question 18


In quadrilateral $A B C D$ above, $\overline{B C}$ is parallel to $\overline{A D}$, and $A B=C D$. If $B C$ and $A D$ were each doubled and $B E$ was reduced by 50 percent, how would the area of ABCD change?
A) The area of ABCD would be decreased by 50 percent.
B) The area of ABCD would be increased by 50 percent.
C) The area of $A B C D$ would not change.
D) The area of ABCD would be multiplied by 2 .

Item Difficulty: Medium
Content: Passport to Advanced Math
Correct Answer: C

Choice C is the correct answer. Quadrilateral $A B C D$ is a trapezoid, and the formula for the area of a trapezoid is $A=\frac{1}{2} h\left(b_{1}+b_{2}\right)$, where $b_{1}$ and $b_{2}$ are the bases of the trapezoid ( $B C$ and $A D$ ) and $h$ is the height $(B E)$. If the bases ( $B C$ and $A D$ ) are each doubled and the height $(B E)$ is reduced by $50 \%$, then the area of the new trapezoid $A B C D$ would be $\frac{1}{2}\left(\frac{h}{2}\right)\left(2 b_{1}+2 b_{2}\right)$, which after multiplying out becomes $\frac{1}{2} h\left(b_{1}+b_{2}\right)$, the same as the area of the original trapezoid. Therefore, the area of the trapezoid would not change.

Choice $A$ is incorrect. This choice does not take into account the changes to the bases, $B C$ and $A D$. Choice B is incorrect. This choice could result from incorrectly interpreting the impact of doubling the bases on the area of $A B C D$ as a $100 \%$ increase and the impact of reducing the height by $50 \%$ as a $50 \%$ decrease, resulting
in a combined $100 \%-50 \%=50 \%$ increase of the area. Choice $D$ is incorrect. This choice does not take into account the change to height, $B E$.

Question 19
Boyd grows only tomatoes and raspberries in his garden. Last year, he grew 140 pounds of tomatoes and 60 pounds of raspberries. This year, the production, by weight, of tomatoes declined by 20 percent, and the production, by weight, of raspberries declined by 50 percent. By what percentage did the total yield, by weight, of Boyd's garden decline?
A) 29 percent
B) 30 percent
C) 35 percent
D) 70 percent

Item Difficulty: Hard
Content: Probability and Data Analysis
Correct Answer: A
Choice A is the correct answer. Since Boyd's production of tomatoes declined by $20 \%$ and the production of raspberries declined by $50 \%$ from the previous year, this year, his tomato production was $140-0.2(140)=112$ pounds and his raspberry production was $60-0.5(60)=30$ pounds. The percent decline in the total yield is the decline in the number of pounds of tomatoes and raspberries divided by the original number of pounds of tomatoes and raspberries, which is
$\frac{28+30}{140+60}=0.29=29 \%$.
Choice B is incorrect. This choice is close to the answer, but rounding may have erroneously led to this answer. Choice C is incorrect. This choice, $35 \%$, may be a result of calculating the mean of $20 \%$ and $50 \%$. Choice D is incorrect. This choice is the approximate percent weight of the tomatoes and raspberries produced this year compared to the last year, but that's not what the problem asks for.

Question 20


The graph above shows the frequency distribution of a list of randomly generated integers between 0 and 10 . What is the mean of the list of numbers?
A) 3.0
B) 3.5
C) 4.25
D) 12.0

Item Difficulty: Medium
Content: Probability and Data Analysis
Correct Answer: C
Choice $C$ is the correct answer. There are 12 integers in the list, and some of them are repeated at the frequencies shown in the graph. So the mean of the list of numbers is the sum of the numbers (repeats included) divided by 12 . That is $\frac{0+1+2+3(3)+2(4)+6+7+8+10}{12}=4.25$.

Choice $A$ is incorrect; 3 is the mode, not the mean, of the list of numbers. Choice $B$ is incorrect; 3.5 is the median, not the mean, of the list of numbers. Choice $D$ is incorrect; 12 is the total number of the integers in the list.

Question 21


What is the minimum value of the function graphed on the $x y$-plane above, for $-4 \leq x \leq 6$ ?
A) $-\infty$
B) -4
C) -2
D) 1

Item Difficulty: Hard
Content: Passport to Advanced Math
Correct Answer: C
Choice C is the correct answer. The minimum value of a graphed function is the minimum $y$-value of all the points on the graph. For the graph shown, the minimum is at the left endpoint of the graph, the $y$-value of which is -2 .

Choice A is incorrect. If the graph would continue indefinitely downward, then the minimum value of the function would be negative infinity. However, the domain of the function is restricted $(-4 \leq x \leq 6)$ and the minimum value of the graph occurs at point $(-4,-2)$. Choice $B$ is incorrect; -4 is the $x$-value of the point on the graph where the minimum value of the function occurs. Choice D is incorrect because there are points of the graph below the $x$-axis; therefore, the minimum value of the function cannot be positive.

## Questions 22-24 refer to the following information.

In 1929, the astronomer Edwin Hubble published the data shown. The graph plots the velocity of galaxies relative to Earth against the distances of galaxies from Earth.


Hubble's data can be modeled by the equation $v=500 d$, where $v$ is the velocity, in kilometers per second, at which the galaxy is moving away from Earth and $d$ is the distance, in megaparsecs, of the galaxy from Earth. Assume that the relationship is valid for larger distances than are shown in the graph. (A megaparsec (Mpc) is $3.1 \times 10^{19}$ kilometers.)

Question 22
According to Hubble's data, how fast, in meters per second, is Galaxy Q
moving away from Earth?
A) $2 \times 10^{6} \mathrm{~m} / \mathrm{s}$
B) $5 \times 10^{5} \mathrm{~m} / \mathrm{s}$
C) $5 \times 10^{2} \mathrm{~m} / \mathrm{s}$
D) $2.5 \times 10^{2} \mathrm{~m} / \mathrm{s}$

Item Difficulty: Hard
Content: Probability and Data Analysis
Correct Answer: B
Choice B is the correct answer. The coordinates of the data point that represent Galaxy Q on the scatterplot are $(2.0,500)$, which means that Galaxy Q is at a distance of about 2.0 Mpc from Earth and moves away from Earth at a velocity of approximately $500 \mathrm{~km} / \mathrm{s}$. The question asks for the velocity in meters per second; therefore, kilometers ( km ) need to be converted into meters ( m ). Since 1 km is
equal to $1,000 m$, it follows that Galaxy Q is moving away from Earth at a velocity of $500 \times 1,000 \mathrm{~m} / \mathrm{s}$, or $5 \times 10^{5} \mathrm{~m} / \mathrm{s}$.

Choices $A, C$, and $D$ are incorrect and may result from an incorrect interpretation of the coordinates of the point that represents Galaxy Q on the scatterplot or an incorrect conversion of the units.

Question 23
There are four galaxies shown in the graph at approximately 0.9 Mpc from
Earth. Which of the following is closest to the range of velocities of these
four galaxies, in kilometers per second?
A) 100
B) 200
C) 450
D) 700

Item Difficulty: Hard
Content: Probability and Data Analysis
Correct Answer: D
Choice D is the correct answer. The velocities, in km/s, of the four galaxies shown in the graph at approximately 0.9 Mpc from Earth are about $-50,+200,+500$, and +650 . Thus, the range of the four velocities is approximately $650-(-50)=700 \mathrm{~km} / \mathrm{s}$.

Choices $A, B$, and $C$ are incorrect. The range of velocities is the difference between the largest and smallest velocity. Each of the answer choices A, B, and C are too small compared to the real value of the range.

Question 24
Based on the model, what is the velocity, in kilometers per second, of a galaxy that is 15 Mpc from Earth?
A) $7,500 \mathrm{~km} / \mathrm{s}$
B) $5,000 \mathrm{~km} / \mathrm{s}$
C) $1,100 \mathrm{~km} / \mathrm{s}$
D) $750 \mathrm{~km} / \mathrm{s}$

## Item Difficulty: Medium

Content: Heart of Algebra
Correct Answer: A
Choice A is the correct answer. The model indicates that the relationship between the velocities of the galaxies, in km/s, and their distance from Earth, in Mpc, is $v=500 \mathrm{~d}$. Therefore, the velocity of a galaxy that is 15 Mpc from Earth is $v=500$ (15) km/s, or $7,500 \mathrm{~km} / \mathrm{s}$.

Based on the model, the other choices are incorrect: Choice $B$ is the speed of a galaxy that is 10 Mpc from Earth. Choice C is the speed of a galaxy that is 2.2 Mpc from Earth. Choice $D$ is the speed of a galaxy that is 1.5 Mpc from Earth.

Question 25
Janice puts a fence around her rectangular garden. The garden has a
length that is 9 feet less than 3 times its width. What is the perimeter of
Janice's fence if the area of her garden is 5,670 square feet?
A) 342 feet
B) 318 feet
C) 300 feet
D) 270 feet

Item Difficulty: Hard
Content: Passport to Advanced Math
Correct Answer: A
Choice A is the correct answer. Let $w$ represent the width of Janice's garden and $3 w-9$ represent the length of Janice's garden. Since the area of Janice's garden is 5,670 square feet, it follows that $w(3 w-9)=5,670$, which after dividing by 3 on both sides simplifies to $w(w-3)=1,890$.

From this point on, different ways could be used to solve this equation. One could rewrite this quadratic equation in the standard form and use the quadratic formula to solve it. Another approach would be to look among integer factors of 1,890 and try to find two that differ from each other by 3 and whose product is 1,890 . The prime factorization of $1,890\left(2 \cdot 3^{3} \cdot 5 \cdot 7\right)$ can help with this. Two factors that satisfy the conditions above are 42 and 45 (note that $42=2.3 \cdot 7$ and $45=3^{2} \cdot 5$ ). The numbers -45 and -42 also satisfy the above conditions ( $w=-42$ ), but since $w$ represents the width of Janice's garden, the negative values of $w$ can be rejected. Thus $w=45$ feet, and so the length of the garden must be 3(45) - $9=126$ feet. Therefore, the perimeter of Janice's garden is $2(45+126)=2(171)=342$ feet.

Choice B is incorrect. This answer choice could result from incorrectly identifying the width of the garden as 42 feet instead of 45 feet. Choices C and D are incorrect; both answers would result in an area of the garden that is significantly smaller than 5,670 square feet. For example, if the perimeter of the garden were 270 feet, as in choice D, then $w+l=135$ feet, where $w$ represents the width and $l$ represents the length of the garden. So $l=135-w$. It is also given that $l=3 w-9$, which
implies that $135-w=3 w-9$. Solving this for $w$ gives $w=36$, and so $l=99$. The area of the garden would then be $36 \times 99$ square feet, which is clearly less than 5,600 square feet.

Question 26


Given the right triangle $A B C$ above, which of the
following is equal to $\frac{b}{a}$ ?
A) $\sin A$
B) $\sin B$
C) $\tan A$
D) $\tan B$

Item Difficulty: Hard
Content: Additional Topic in Math
Correct Answer: D

Choice D is the correct answer. Since the ratio $\frac{b}{a}$ involves only the legs of the right triangle, it follows that, of the given choices, the ratio can be equal to the tangent of one of the angles. In a right triangle, the tangent of an acute angle is defined as the ratio of the opposite side to the adjacent side of the angle. Side $b$ is opposite to angle $B$ and side $a$ is adjacent to angle $B$. Therefore, $\tan B=\frac{b}{a}$.

Choices $A$ and $B$ cannot be correct; the sine of an acute angle in a right triangle is defined as the ratio of the opposite side to the hypotenuse, and the ratio shown involves only the legs of the triangle. Choice C is incorrect. In the triangle $A B C$ shown, $\tan A=\frac{a}{b}$, not $\frac{b}{a}$.

Question 27


A system of inequalities and a graph are shown above. Which section or sections of the graph could represent all of the solutions to the system?
A) Section $R$
B) Sections Q and S
C) Sections $Q$ and $P$
D) Sections $Q, R$, and $S$

Item Difficulty: Hard
Content: Heart of Algebra
Correct Answer: A

Choice A is the correct answer. The solution set of the inequality $y \leq-x$ is the union of sections R and S of the graph. The solution set of the inequality $2 y>3 x+2$ is the union of sections R and Q of the graph. The solutions of the system consist of the coordinates of all the points that satisfy both inequalities, and therefore, section $R$ represents all the solutions to the system since it is common to the solutions of both inequalities.

Choices B, C, and D are incorrect because they contain ordered pairs that do not satisfy both of the inequalities.


The $x y$-plane above shows one of the two points of intersection of the graphs of a linear function and a quadratic function. The shown point of intersection has coordinates $(v, w)$. If the vertex of the graph of the quadratic function is at $(4,19)$, what is the value of $v$ ?

Item Difficulty: Medium
Content: Passport to Advanced Math
The correct answer is 6 .

Since the vertex of the graph of the quadratic function is at $(4,19)$, the equation of the parabola is of the form $y=a(x-4) 2+19$. It is also given that the parabola passes through point ( 0,3 ). This means that
$a(3-4)^{2}+19$, and so $a=-1$. So the graph of the parabola is $y=-(x-4) 2+19$.
Since the line passes through the points $(0,-9)$ and $(2,-1)$, one can calculate the slope of the line $\left(\frac{-1-(-9)}{2-0}=4\right)$ that passes through these points and write the equation of the line in the slope-intercept form as $y=4 x-9$.

The coordinates of the intersection points of the line and the parabola satisfy both the equation of the parabola and the equation of the line. Therefore, these coordinates are the solutions to the system of equations below:

$$
\begin{aligned}
& y=4 x-9 \\
& y=-(x-4)^{2}+19
\end{aligned}
$$

Substituting $4 x-9$ for $y$ into the second equation gives $4 x-9=-(x-4)^{2}+19$, which is equivalent to $x 2-4 x-12=0$. After factoring, this equation can be rewritten as
$(x-6)(x+2)=0$, and so $x=6$ or $x=-2$. Since point $(v, w)$ is on the right side of the $y$-axis, it follows that $v$ cannot be -2 . Therefore, $v=6$.

Question 29
In a college archaeology class, 78 students are going to a dig site to find and study artifacts. The dig site has been divided into 24 sections, and each section will be studied by a group of either 2 or 4 students. How many of the sections will be studied by a group of 2 students?

## Item Difficulty: Hard

Content: Heart of Algebra
The correct answer is 9 .

Let $x$ be the number of sections that will be studied by 2 students and $y$ be the number of sections that will be studied by 4 students. Since there are 24 sections that will be studied by 78 students, it follows that $x+y=24$ and $2 x+4 y=78$. Solving this system gives $x=9$ and $y=15$. Therefore, 9 of the sections will be studied by a group of 2 students.

Alternatively, if all 24 sections were studied by a group of 4 students, then the total number of students required would be $24 \times 4=96$. Since the actual number of students is 78, the difference $96-78=18$ represents the number of "missing" students, and each pair of these "missing" students represents one of the sections that will be studied by 2 students. Hence, the number of sections that will be studied by 2 students is equal to the number of pairs that 18 students can form, which is $\frac{18}{2}=9$.

## Questions 30 and 31 refer to the following information.

$$
\begin{array}{ll}
v=v_{0}-g t & \text { (speed-time) } \\
h=v_{0} t-\frac{1}{2} g t^{2} & \text { (position-time) } \\
v^{2}=v_{0}^{2}-2 g h & \text { (position-speed) }
\end{array}
$$

An arrow is launched upward with an initial speed of 100 meters per second ( $\mathrm{m} / \mathrm{s}$ ). The equations above describe the constant-acceleration motion of the arrow, where $v_{0}$ is the initial speed of the arrow, $v$ is the speed of the arrow as it is moving up in the air, $h$ is the height of the arrow above the ground, $t$ is the time elapsed since the arrow was projected upward, and $g$ is the acceleration due to gravity $\left(9.8 \mathrm{~m} / \mathrm{s}^{2}\right)$.

Question 30
What is the maximum height from the ground the arrow will rise to the nearest meter?

Item Difficulty: Hard
Content: Passport to Advanced Math
The correct answer is 510 .

As the arrow moves upward, its speed decreases continuously and it becomes 0 when the arrow reaches its maximum height. Using the position-speed equation and the fact that $v=0$ when $h$ is maximum gives $0=100^{2}-2 g h$. Solving for $h$ gives $h=\frac{100^{2}}{2(9.8)}$ meters, which to the nearest meter is 510 .

Alternatively, the maximum height can be found using the position-time equation. Substituting 100 for $v_{0}$ and 9.8 for $g$ into this equation gives $h=100 t-\frac{1}{2}(9.8) t^{2}$. Completing the square gives the equivalent equation
$h=-4.9\left(t-\frac{100}{9.8}\right)^{2}+4.9\left(\frac{100}{9.8}\right)^{2}$. Therefore, the maximum height from the ground the arrow will rise is $4.9\left(\frac{100}{9.8}\right)^{2}$ meters, which to the nearest meter is 510 .

Question 31

How long will it take for the arrow to reach its maximum height to the nearest tenth of a second?

Item Difficulty: Hard
Content: Passport to Advanced Math
The correct answer is 10.2 seconds (or $51 / 5$ seconds).
As the arrow moves upward, its speed decreases continuously, and it becomes 0 when the arrow reaches its maximum height. Using the speed-time equation and the fact that $v=0$ when $h$ is maximum, we get $0=100-9.8 t$.
Solving this equation for $t$ gives $t=\frac{100}{9.8}=10.2041$ seconds, which to the nearest tenth of a second is 10.2 .

## Answer Key

| Reading |  | Writing \& Language |  | Math Test - No Calculator |  | Math Test Calculator |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Q 1 | A | Q 1 | C | Q 1 | A | Q 1 | C |
| Q 2 | A | Q 2 | D | O 2 | B | Q 2 | A |
| Q 3 | B | Q 3 | C | Q 3 | A | Q 3 | C |
| Q 4 | C | Q 4 | D | O 4 | D | O 4 | C |
| O 5 | C | Q 5 | C | O 5 | B | Q 5 | C |
| Q 6 | B | Q 6 | A | Q 6 | B | Q 6 | D |
| Q 7 | B | 08 | B | 07 | A | 08 | D |
| 08 | D | 08 | B | 08 | D | 08 | D |
| Q 9 | B | 09 | B | O 9 | B | O 9 | C |
| O 10 | C | O 10 | D | Q 10 | D | Q 10 | B |
| Q 11 | B | Q 11 | D | Q 11 | C | Q 11 | C |
| Q 12 | A | Q 12 | C | Q 12 | A | Q 12 | B |
| Q 13 | A | Q 13 | B | Q 13 | D | Q 13 | B |
| Q 14 | D | O 14 | B | Q 14 | 300 | Q 14 | B |
| Q 15 | D | Q 15 | B | Q 15 | 2 | Q 15 | B |
| O 16 | A | O 16 | D | O 16 | 9 | O 16 | C |
| O 17 | D | Q 17 | A | Q 17 | 4 | Q 17 | C |
| Q 18 | C | Q 18 | A |  |  | Q 18 | C |
| O 19 | D | O 19 | D |  |  | O 19 | A |
| Q 20 | C | Q 20 | C |  |  | Q 20 | C |
| Q 21 | D | Q 21 | C |  |  | Q 21 | C |
| O 22 | A | O 22 | A |  |  | Q 22 | B |
| Q 23 | B | Q 23 | D |  |  | Q 23 | D |
| Q 24 | B | Q 24 | A |  |  | Q 24 | A |
| Q 25 | D | Q 25 | A |  |  | Q 25 | A |
| O 26 | D | O 26 | B |  |  | Q 26 | D |
| Q 27 | A | Q 27 | C |  |  | Q 27 | A |
| Q 28 | A | Q 28 | D |  |  | Q 28 | 6 |
| Q 29 | B | Q 29 | C |  |  | Q 29 | 9 |
| Q 30 | C | Q 30 | D |  |  | Q 30 | 510 |
| Q 31 | D | Q 31 | D |  |  | Q 31 | 10.2, $51 / 5$ |
| O 32 | C | Q 32 | D |  |  |  |  |
| Q 33 | B | Q 33 | A |  |  |  |  |
| O 34 | D | O 34 | C |  |  |  |  |
| Q 35 | C | Q 35 | A |  |  |  |  |
| Q 36 | D | O 36 | D |  |  |  |  |
| O 37 | B | Q 37 | D |  |  |  |  |
| O 38 | D | O 38 | C |  |  |  |  |
| Q 39 | B | Q 39 | B |  |  |  |  |
| Q 40 | D | Q 40 | C |  |  |  |  |
| O 41 | B | Q 41 | B |  |  |  |  |
| O 42 | A | Q 42 | B |  |  |  |  |
| O 43 | C | O 43 | A |  |  |  |  |
| Q 44 | C | Q 44 | D |  |  |  |  |
| O 45 | B |  |  |  |  |  |  |
| O 46 | C |  |  |  |  |  |  |
| Q 47 | A |  |  |  |  |  |  |

## Section 3: Math Test - No Calculator

## QUESTION 1.

Choice $\mathbf{D}$ is correct. Since $k=3$, one can substitute 3 for $k$ in the equation $\frac{x-1}{3}=k$, which gives $\frac{x-1}{3}=3$. Multiplying both sides of $\frac{x-1}{3}=3$ by 3 gives $x-1=9$ and then adding 1 to both sides of $x-1=9$ gives $x=10$.

Choices $A, B$, and $C$ are incorrect because the result of subtracting 1 from the value and dividing by 3 is not the given value of $k$, which is 3 .

## QUESTION 2.

Choice A is correct. To calculate $(7+3 i)+(-8+9 i)$, add the real parts of each complex number, $7+(-8)=-1$, and then add the imaginary parts, $3 i+9 i=12 i$. The result is $-1+12 i$.

Choices $\mathrm{B}, \mathrm{C}$, and D are incorrect and likely result from common errors that arise when adding complex numbers. For example, choice $B$ is the result of adding $3 i$ and $-9 i$, and choice $C$ is the result of adding 7 and 8 .

## QUESTION 3.

Choice C is correct. The total number of messages sent by Armand is the 5 hours he spent texting multiplied by his rate of texting: $m$ texts/hour $\times 5$ hours $=5 m$ texts. Similarly, the total number of messages sent by Tyrone is the 4 hours he spent texting multiplied by his rate of texting: $p$ texts/hour $\times 4$ hours $=4 p$ texts. The total number of messages sent by Armand and Tyrone is the sum of the total number of messages sent by Armand and the total number of messages sent by Tyrone: $5 m+4 p$.

Choice A is incorrect and arises from adding the coefficients and multiplying the variables of $5 m$ and $4 p$. Choice B is incorrect and is the result of multiplying $5 m$ and $4 p$. The total number of messages sent by Armand and Tyrone should be the sum of $5 m$ and $4 p$, not the product of these terms. Choice D is incorrect because it multiplies Armand's number of hours spent texting by Tyrone's rate of texting, and vice versa. This mix-up results in an expression that does not equal the total number of messages sent by Armand and Tyrone.

## QUESTION 4.

Choice B is correct. The value 108 in the equation is the value of $P$ in $P=108-23 d$ when $d=0$. When $d=0$, Kathy has worked 0 days that week. In other words, 108 is the number of phones left before Kathy has started work for the week. Therefore, the meaning of the value 108 in the equation is that Kathy starts each week with 108 phones to fix because she has worked 0 days and has 108 phones left to fix.

Choice A is incorrect because Kathy will complete the repairs when $P=0$. Since $P=108-23 d$, this will occur when $0=108-23 d$ or when $d=\frac{108}{23}$, not when $d=108$. Therefore, the value 108 in the equation does not represent the number of days it will take Kathy to complete the repairs. Choices C and D are incorrect because the number 23 in $P=108-23 P=108$ indicates that the number of phones left will decrease by 23 for each increase in the value of $d$ by 1 ; in other words, that Kathy is repairing phones at a rate of 23 per day, not 108 per hour (choice C) or 108 per day (choice D ).

## QUESTION 5.

Choice C is correct. Only like terms, with the same variables and exponents, can be combined to determine the answer as shown here:

$$
\begin{aligned}
& \left(x^{2} y-3 y^{2}+5 x y^{2}\right)-\left(-x^{2} y+3 x y^{2}-3 y^{2}\right) \\
& =\left(x^{2} y-\left(-x^{2} y\right)\right)+\left(-3 y^{2}-\left(-3 y^{2}\right)\right)+\left(5 x y^{2}-3 x y^{2}\right) \\
& =2 x^{2} y+0+2 x y^{2} \\
& =2 x^{2} y+2 x y^{2}
\end{aligned}
$$

Choices A, B, and D are incorrect and are the result of common calculation errors or of incorrectly combining like and unlike terms.

## QUESTION 6.

Choice A is correct. In the equation $h=3 a+28.6$, if $a$, the age of the boy, increases by 1 , then $h$ becomes $h=3(a+1)+28.6=3 a+3+28.6=$ $(3 a+28.6)+3$. Therefore, the model estimates that the boy's height increases by 3 inches each year.

Alternatively: The height, $h$, is a linear function of the age, $a$, of the boy. The coefficient 3 can be interpreted as the rate of change of the function; in this
case, the rate of change can be described as a change of 3 inches in height for every additional year in age.

Choices B, C, and D are incorrect and are likely to result from common errors in calculating the value of $h$ or in calculating the difference between the values of $h$ for different values of $a$.

## QUESTION 7.

Choice B is correct. Since the right-hand side of the equation is $P$ times the expression $\frac{\left(\frac{r}{1,200}\right)\left(1+\frac{r}{1,200}\right)^{N}}{\left(1+\frac{r}{1,200}\right)^{N}-1}$, multiplying both sides of the equation by the reciprocal of this expression results in $\frac{\left(1+\frac{r}{1,200}\right)^{N}-1}{\left(\frac{r}{1,200}\right)\left(1+\frac{r}{1,200}\right)^{N}} m=P$.

Choices A, C, and D are incorrect and are likely the result of conceptual or computation errors while trying to solve for $P$.

## QUESTION 8.

Choice C is correct. Since $\frac{a}{b}=2$, it follows that $\frac{b}{a}=\frac{1}{2}$. Multiplying both sides of the equation by 4 gives $4\left(\frac{b}{a}\right)=\frac{4 b}{a}=2$.

Choice A is incorrect because if $\frac{4 b}{a}=0$, then $\frac{a}{b}$ would be undefined. Choice B is incorrect because if $\frac{4 b}{a}=1$, then $\frac{a}{b}=4$. Choice D is incorrect because if $\frac{4 b}{a}=4$, then $\frac{a}{b}=1$.

## OUESTION 9.

Choice B is correct. Adding $x$ and 19 to both sides of $2 y-x=-19$ gives $x=2 y+19$. Then, substituting $2 y+19$ for $x$ in $3 x+4 y=-23$ gives $3(2 y+19)+4 y=-23$. This last equation is equivalent to $10 y+57=-23$. Solving $10 y+57=-23$ gives $y=-8$. Finally, substituting -8 for $y$ in $2 y-x=-19$ gives $2(-8)-x=-19$, or $x=3$. Therefore, the solution $(x, y)$ to the given system of equations is $(3,-8)$.

Choices A, C, and D are incorrect because when the given values of $x$ and $y$ are substituted in $2 y-x=-19$, the value of the left side of the equation does not equal -19 .

## QUESTION 10.

Choice A is correct. Since $g$ is an even function, $g(-4)=g(4)=8$.
Alternatively: First find the value of $a$, and then find $g(-4)$. Since $g(4)=8$, substituting 4 for $x$ and 8 for $g(x)$ gives $8=a(4)^{2}+24=16 a+24$. Solving this
last equation gives $a=-1$. Thus $g(x)=-x^{2}+24$, from which it follows that $g(-4)=-(-4)^{2}+24 ; g(-4)=-16+24$; and $g(-4)=8$.

Choices B, C, and D are incorrect because $g$ is a function and there can only be one value of $g(-4)$.

## QUESTION 11.

Choice $\mathbf{D}$ is correct. To determine the price per pound of beef when it was equal to the price per pound of chicken, determine the value of $x$ (the number of weeks after July 1) when the two prices were equal. The prices were equal when $b=c$; that is, when $2.35+0.25 x=1.75+0.40 x$. This last equation is equivalent to $0.60=0.15 x$, and so $x=\frac{0.60}{0.15}=4$. Then to determine $b$, the price per pound of beef, substitute 4 for $x$ in $b=2.35+0.25 x$, which gives $b=2.35+0.25(4)=3.35$ dollars per pound.

Choice A is incorrect. It results from using the value 1 , not 4 , for $x$ in $b=2.35+0.25 x$. Choice $B$ is incorrect. It results from using the value 2 , not 4 , for $x$ in $b=2.35+0.25 x$. Choice C is incorrect. It results from using the value 3 , not 4 , for $x$ in $c=1.75+0.40 x$.

## QUESTION 12.

Choice $\mathbf{D}$ is correct. Determine the equation of the line to find the relationship between the $x$ - and $y$-coordinates of points on the line. All lines through the origin are of the form $y=m x$, so the equation is $y=\frac{1}{7} x$. A point lies on the line if and only if its $y$-coordinate is $\frac{1}{7}$ of its $x$-coordinate. Of the given choices, only choice $D,(14,2)$, satisfies this condition: $2=\frac{1}{7}(14)$.

Choice $A$ is incorrect because the line determined by the origin $(0,0)$ and $(0,7)$ is the vertical line with equation $x=0$; that is, the $y$-axis. The slope of the $y$-axis is undefined, not $\frac{1}{7}$. Therefore, the point $(0,7)$ does not lie on the line that passes the origin and has slope $\frac{1}{7}$. Choices B and C are incorrect because neither of the ordered pairs has a $y$-coordinate that is $\frac{1}{7}$ the value of the $x$-coordinate.

## QUESTION 13.

Choice B is correct. To rewrite $\frac{1}{\frac{1}{x+2}+\frac{1}{x+3}}$, multiply by $\frac{(x+2)(x+3)}{(x+2)(x+3)}$.
This results in the expression $\frac{(x+2)(x+3)}{(x+3)+(x+2)}$, which is equivalent to the expression in choice B.

Choices A, C, and D are incorrect and could be the result of common algebraic errors that arise while manipulating a complex fraction.

## QUESTION 14.

Choice A is correct. One approach is to express $\frac{8^{x}}{2^{y}}$ so that the numerator and denominator are expressed with the same base. Since 2 and 8 are both
powers of 2 , substituting $2^{3}$ for 8 in the numerator of $\frac{8^{x}}{2^{y}}$ gives $\frac{\left(2^{3}\right)^{x}}{2^{y}}$, which can be rewritten as $\frac{2^{3 x}}{2^{y}}$. Since the numerator and denominator of $\frac{2^{3 x}}{2^{y}}$ have a common base, this expression can be rewritten as $2^{3 x-y}$. It is given that $3 x-y=12$, so one can substitute 12 for the exponent, $3 x-y$, giving that the expression $\frac{8^{x}}{2^{y}}$ is equal to $2^{12}$.

Choices B and C are incorrect because they are not equal to $2^{12}$. Choice D is incorrect because the value of $\frac{8^{x}}{2^{y}}$ can be determined.

## QUESTION 15.

Choice $\mathbf{D}$ is correct. One can find the possible values of $a$ and $b$ in $(a x+2)(b x+7)$ by using the given equation $a+b=8$ and finding another equation that relates the variables $a$ and $b$. Since $(a x+2)(b x+7)=15 x^{2}+c x+14$, one can expand the left side of the equation to obtain $a b x^{2}+7 a x+2 b x+14=15 x^{2}+c x+14$. Since $a b$ is the coefficient of $x^{2}$ on the left side of the equation and 15 is the coefficient of $x^{2}$ on the right side of the equation, it must be true that $a b=15$. Since $a+b=8$, it follows that $b=8-a$. Thus, $a b=15$ can be rewritten as $a(8-a)=15$, which in turn can be rewritten as $a^{2}-8 a+15=0$. Factoring gives $(a-3)(a-5)=0$. Thus, either $a=3$ and $b=5$, or $a=5$ and $b=3$. If $a=3$ and $b=5$, then $(\mathrm{a} x+2)$ $(b x+7)=(3 x+2)(5 x+7)=15 x^{2}+31 x+14$. Thus, one of the possible values of $c$ is 31 . If $a=5$ and $b=3$, then $(\mathrm{a} x+2)(b x+7)=(5 x+2)(3 x+7)=$ $15 x^{2}+41 x+14$. Thus, another possible value for $c$ is 41 . Therefore, the two possible values for $c$ are 31 and 41 .

Choice A is incorrect; the numbers 3 and 5 are possible values for $a$ and $b$, but not possible values for $c$. Choice B is incorrect; if $a=5$ and $b=3$, then 6 and 35 are the coefficients of $x$ when the expression $(5 x+2)(3 x+7)$ is expanded as $15 x^{2}+35 x+6 x+14$. However, when the coefficients of $x$ are 6 and 35 , the value of $c$ is 41 and not 6 and 35 . Choice C is incorrect; if $a=3$ and $b=5$, then 10 and 21 are the coefficients of $x$ when the expression $(3 x+2)(5 x+7)$ is expanded as $15 x^{2}+21 x+10 x+14$. However, when the coefficients of $x$ are 10 and 21 , the value of $c$ is 31 and not 10 and 21 .

## QUESTION 16.

The correct answer is $\mathbf{2}$. To solve for $t$, factor the left side of $t^{2}-4=0$, giv-$\operatorname{ing}(t-2)(t+2)=0$. Therefore, either $t-2=0$ or $t+2=0$. If $t-2=0$, then $t=2$, and if $t+2=0$, then $t=-2$. Since it is given that $t>0$, the value of $t$ must be 2 .

Another way to solve for $t$ is to add 4 to both sides of $t^{2}-4=0$, giving $t^{2}=4$. Then, taking the square root of the left and the right side of the equation gives $t= \pm \sqrt{4}= \pm 2$. Since it is given that $t>0$, the value of $t$ must be 2 .

## QUESTION 17.

The correct answer is $\mathbf{1 6 0 0}$. It is given that $\angle A E B$ and $\angle C D B$ have the same measure. Since $\angle A B E$ and $\angle C B D$ are vertical angles, they have the same measure. Therefore, triangle $E A B$ is similar to triangle $D C B$ because the triangles have two pairs of congruent corresponding angles (angleangle criterion for similarity of triangles). Since the triangles are similar, the corresponding sides are in the same proportion; thus $\frac{C D}{x}=\frac{B D}{E B}$. Substituting the given values of 800 for $C D, 700$ for $B D$, and 1400 for $E B$ in $\frac{C D}{x}=\frac{B D}{E B}$ gives $\frac{800}{x}=\frac{700}{1400}$. Therefore, $x=\frac{(800)(1400)}{700}=1600$.

## QUESTION 18.

The correct answer is 7. Subtracting the left and right sides of $x+y=-9$ from the corresponding sides of $x+2 y=-25$ gives $(x+2 y)-(x+y)=-25-(-9)$, which is equivalent to $y=-16$. Substituting -16 for $y$ in $x+y=-9$ gives $x+(-16)=-9$, which is equivalent to $x=-9-(-16)=7$.

## QUESTION 19.

The correct answer is $\frac{4}{5}$ or 0.8 . By the complementary angle relationship for sine and cosine, $\sin \left(x^{\circ}\right)=\cos \left(90^{\circ}-x^{\circ}\right)$. Therefore, $\cos \left(90^{\circ}-x^{\circ}\right)=\frac{4}{5}$. Either the fraction $\frac{4}{5}$ or its decimal equivalent, 0.8 , may be gridded as the correct answer.

Alternatively, one can construct a right triangle that has an angle of measure $x^{\circ}$ such that $\sin \left(x^{\circ}\right)=\frac{4}{5}$, as shown in the figure below, where $\sin \left(x^{\circ}\right)$ is equal to the ratio of the opposite side to the hypotenuse, or $\frac{4}{5}$.


Since two of the angles of the triangle are of measure $x^{\circ}$ and $90^{\circ}$, the third angle must have the measure $180^{\circ}-90^{\circ}-x^{\circ}=90^{\circ}-x^{\circ}$. From the figure, $\cos \left(90^{\circ}-x^{\circ}\right)$, which is equal to the ratio of the adjacent side to the hypotenuse, is also $\frac{4}{5}$.

## QUESTION 20.

The correct answer is $\mathbf{1 0 0}$. Since $a=5 \sqrt{2}$, one can substitute $5 \sqrt{2}$ for $a$ in $2 a=\sqrt{2} x$, giving $10 \sqrt{2}=\sqrt{2} x$. Squaring each side of $10 \sqrt{2}=\sqrt{2} x$ gives $(10 \sqrt{2})^{2}=(\sqrt{2} x)^{2}$, which simplifies to $(10)^{2}(\sqrt{2})^{2}=(\sqrt{2} x)^{2}$, or $200=2 x$. This gives $x=100$. Checking $x=100$ in the original equation gives $2(5 \sqrt{2})=\sqrt{(2)(100)}$, which is true since $2(5 \sqrt{2})=10 \sqrt{2}$ and $\sqrt{(2)(100)}=(\sqrt{2})(\sqrt{100})=10 \sqrt{2}$.

## Section 4: Math Test - Calculator

## QUESTION 1.

Choice B is correct. On the graph, a line segment with a positive slope represents an interval over which the target heart rate is strictly increasing as time passes. A horizontal line segment represents an interval over which there is no change in the target heart rate as time passes, and a line segment with a negative slope represents an interval over which the target heart rate is strictly decreasing as time passes. Over the interval between 40 and 60 minutes, the graph consists of a line segment with a positive slope followed by a line segment with a negative slope, with no horizontal line segment in between, indicating that the target heart rate is strictly increasing then strictly decreasing.

Choice A is incorrect because the graph over the interval between 0 and 30 minutes contains a horizontal line segment, indicating a period in which there was no change in the target heart rate. Choice $C$ is incorrect because the graph over the interval between 50 and 65 minutes consists of a line segment with a negative slope followed by a line segment with a positive slope, indicating that the target heart rate is strictly decreasing then strictly increasing. Choice D is incorrect because the graph over the interval between 70 and 90 minutes contains horizontal line segments and no segment with a negative slope.

## QUESTION 2.

Choice $\mathbf{C}$ is correct. Substituting 6 for $x$ and 24 for $y$ in $y=k x$ gives $24=(k)(6)$, which gives $k=4$. Hence, $y=4 x$. Therefore, when $x=5$, the value of $y$ is $(4)(5)=20$. None of the other choices for $y$ is correct because $y$ is a function of $x$, and so there is only one $y$-value for a given $x$-value.

Choices $\mathrm{A}, \mathrm{B}$, and D are incorrect. Choice A is the result of using 6 for $y$ and 5 for $x$ when solving for $k$. Choice B results from using a value of 3 for $k$ when solving for $y$. Choice D results from using $y=k+x$ instead of $y=k x$.

## QUESTION 3.

Choice $\mathbf{D}$ is correct. Consider the measures of $\angle 3$ and $\angle 4$ in the figure below.


The measure of $\angle 3$ is equal to the measure of $\angle 1$ because they are corresponding angles for the parallel lines $\ell$ and $m$ intersected by the transversal line $t$. Similarly, the measure of $\angle 3$ is equal to the measure of $\angle 4$ because they are corresponding angles for the parallel lines $s$ and $t$ intersected by the transversal line $m$. Since the measure of $\angle 1$ is $35^{\circ}$, the measures of $\angle 3$ and $\angle 4$ are also $35^{\circ}$. Since $\angle 4$ and $\angle 2$ are supplementary, the sum of the measures of these two angles is $180^{\circ}$. Therefore, the measure of $\angle 2$ is $180^{\circ}-35^{\circ}=145^{\circ}$.

Choice A is incorrect because $35^{\circ}$ is the measure of $\angle 1$, and $\angle 1$ is not congruent to $\angle 2$. Choice B is incorrect because it is the measure of the complementary angle of $\angle 1$, and $\angle 1$ and $\angle 2$ are not complementary angles. Choice C is incorrect because it is double the measure of $\angle 1$.

## QUESTION 4.

Choice C is correct. The description " $16+4 x$ is 10 more than 14 " can be written as the equation $16+4 x=10+14$, which is equivalent to $16+4 x=24$. Subtracting 16 from each side of $16+4 x=24$ gives $4 x=8$. Since $8 x$ is 2 times $4 x$, multiplying both sides of $4 x=8$ by 2 gives $8 x=16$. Therefore, the value of $8 x$ is 16 .

Choice A is incorrect because it is the value of $x$, not $8 x$. Choices B and D are incorrect; those choices may be a result of errors in rewriting $16+4 x=$ $10+14$. For example, choice $D$ could be the result of subtracting 16 from the left side of the equation and adding 16 to the right side of $16+4 x=10+14$, giving $4 x=40$ and $8 x=80$.

## QUESTION 5.

Choice D is correct. A graph with a strong negative association between $d$ and $t$ would have the points on the graph closely aligned with a line that has a negative slope. The more closely the points on a graph are aligned with a line, the stronger the association between $d$ and $t$, and a negative slope indicates a negative association. Of the four graphs, the points on graph D are most closely aligned with a line with a negative slope. Therefore, the graph in choice D has the strongest negative association between $d$ and $t$.

Choice A is incorrect because the points are more scattered than the points in choice D , indicating a weak negative association between $d$ and $t$. Choice B is incorrect because the points are aligned to either a curve or possibly a line with a small positive slope. Choice C is incorrect because the points are aligned to a line with a positive slope, indicating a positive association between $d$ and $t$.

## QUESTION 6.

Choice $\mathbf{D}$ is correct. Since there are 10 grams in 1 decagram, there are $2 \times 10=20$ grams in 2 decagrams. Since there are 1,000 milligrams in 1 gram, there are $20 \times 1,000=20,000$ milligrams in 20 grams. Therefore, 20,000 1 -milligram doses of the medicine can be stored in a 2 -decagram container.

Choice A is incorrect; 0.002 is the number of grams in 2 milligrams. Choice B is incorrect; it could result from multiplying by 1,000 and dividing by 10 instead of multiplying by both 1,000 and 10 when converting from decagrams to milligrams. Choice C is incorrect; 2,000 is the number of milligrams in 2 grams, not the number of milligrams in 2 decagrams.

## QUESTION 7.

Choice C is correct. Let $x$ represent the number of installations that each unit on the $y$-axis represents. Then $9 x, 5 x, 6 x, 4 x$, and $3.5 x$ are the number of rooftops with solar panel installations in cities A, B, C, D, and E, respectively. Since the total number of rooftops is 27,500 , it follows that $9 x+5 x+6 x+4 x+3.5 x=27,500$, which simplifies to $27.5 x=27,500$. Thus, $x=1,000$. Therefore, an appropriate label for the $y$-axis is "Number of installations (in thousands)."

Choices A, B, and D are incorrect and may result from errors when setting up and calculating the units for the $y$-axis.

## QUESTION 8.

Choice $\mathbf{D}$ is correct. If the value of $|n-1|+1$ is equal to 0 , then $|n-1|+1=0$. Subtracting 1 from both sides of this equation gives $|n-1|=-1$. The expression $|n-1|$ on the left side of the equation is the absolute value of $n-1$, and the absolute value can never be a negative number. Thus $|n-1|=-1$ has no solution. Therefore, there are no values for $n$ for which the value of $|n-1|+1$ is equal to 0 .

Choice A is incorrect because $|0-1|+1=1+1=2$, not 0 . Choice $B$ is incorrect because $|1-1|+1=0+1=1$, not 0 . Choice C is incorrect because $|2-1|+1=1+1=2$, not 0 .

## QUESTION 9.

Choice A is correct. Subtracting 1,052 from both sides of the equation $a=1,052+1.08 t$ gives $a-1,052=1.08 t$. Then dividing both sides of $a-1,052=1.08 t$ by 1.08 gives $t=\frac{a-1,052}{1.08}$.
Choices B, C, and D are incorrect and could arise from errors in rewriting $a=1,052+1.08 t$. For example, choice B could result if 1,052 is added to the
left side of $a=1,052+1.08 t$ and subtracted from the right side, and then both sides are divided by 1.08 .

## QUESTION 10.

Choice $\mathbf{B}$ is correct. Substituting 1,000 for $a$ in the equation $a=1,052+1.08 t$ gives $1,000=1,052+1.08 t$, and thus $t=\frac{-52}{1.08} \approx-48.15$. Of the choices given, $-48^{\circ} \mathrm{F}$ is closest to $-48.15^{\circ} \mathrm{F}$. Since the equation $a=1,052+1.08 t$ is linear, it follows that of the choices given, $-48^{\circ} \mathrm{F}$ is the air temperature when the speed of a sound wave is closest to 1,000 feet per second.

Choices A, C, and D are incorrect, and might arise from errors in calculating $\frac{-52}{1.08}$ or in rounding the result to the nearest integer. For example, choice C could be the result of rounding -48.15 to -49 instead of -48 .

## QUESTION 11.

Choice A is correct. Subtracting $3 x$ and adding 3 to both sides of $3 x-5 \geq 4 x-3$ gives $-2 \geq x$. Therefore, $x$ is a solution to $3 x-5 \geq 4 x-3$ if and only if $x$ is less than or equal to -2 and $x$ is NOT a solution to $3 x-5 \geq 4 x-3$ if and only if $x$ is greater than -2 . Of the choices given, only -1 is greater than -2 and, therefore, cannot be a value of $x$.

Choices B, C, and D are incorrect because each is a value of $x$ that is less than or equal to -2 and, therefore, could be a solution to the inequality.

## QUESTION 12.

Choice C is correct. The average number of seeds per apple is the total number of seeds in the 12 apples divided by the number of apples, which is 12 . On the graph, the horizontal axis is the number of seeds per apple and the height of each bar is the number of apples with the corresponding number of seeds. The first bar on the left indicates that 2 apples have 3 seeds each, the second bar indicates that 4 apples have 5 seeds each, the third bar indicates that 1 apple has 6 seeds, the fourth bar indicates that 2 apples have 7 seeds each, and the fifth bar indicates that 3 apples have 9 seeds each. Thus, the total number of seeds for the 12 apples is $(2 \times 3)+(4 \times 5)+(1 \times 6)+(2 \times 7)+(3 \times 9)=73$, and the average number of seeds per apple is $\frac{73}{12}=6.08$. Of the choices given, 6 is closest to 6.08 .

Choice A is incorrect; it is the number of apples represented by the tallest bar but is not the average number of seeds for the 12 apples. Choice B is incorrect; it is the number of seeds per apple corresponding to the tallest bar, but is not the average number of seeds for the 12 apples. Choice D is incorrect; a student might choose this by correctly calculating the average number of seeds, 6.08 , but incorrectly rounding up to 7 .

## OUESTION 13.

Choice C is correct. From the table, there was a total of 310 survey respondents, and $19 \%$ of all survey respondents is equivalent to $\frac{19}{100} \times 310=58.9$ respondents. Of the choices given, 59 , the number of males taking geometry, is closest to 58.9 respondents.

Choices A, B, and D are incorrect because the number of males taking geometry is closer to 58.9 than the number of respondents in each of these categories.

## QUESTION 14.

Choice C is correct. The range of the 21 fish is $24-8=16$ inches, and the range of the 20 fish after the 24 -inch measurement is removed is $16-8=8$ inches. The change in range, 8 inches, is much greater than the change in the mean or median.

Choice A is incorrect. Let $m$ be the mean of the lengths, in inches, of the 21 fish. Then the sum of the lengths, in inches, of the 21 fish is 21 m . After the 24 -inch measurement is removed, the sum of the lengths, in inches, of the remaining 20 fish is $21 m-24$, and the mean length, in inches, of these 20 fish is $\frac{21 m-24}{20}$, which is a change of $\frac{24-m}{20}$ inches. Since $m$ must be between the smallest and largest measurements of the 21 fish, it follows that $8<m<24$, from which it can be seen that the change in the mean, in inches, is between $\frac{24-24}{20}=0$ and $\frac{24-8}{20}=\frac{4}{5}$, and so must be less than the change in the range, 8 inches. Choice B is incorrect because the median length of the 21 fish is the length of the 11 th fish, 12 inches. After removing the 24 -inch measurement, the median of the remaining 20 lengths is the average of the 10th and 11th fish, which would be unchanged at 12 inches. Choice D is incorrect because the changes in the mean, median, and range of the measurements are different.

## QUESTION 15.

Choice A is correct. The total cost $C$ of renting a boat is the sum of the initial cost to rent the boat plus the product of the cost per hour and the number of hours, $h$, that the boat is rented. The $C$-intercept is the point on the $C$-axis where $h$, the number of hours the boat is rented, is 0 . Therefore, the $C$-intercept is the initial cost of renting the boat.

Choice B is incorrect because the graph represents the cost of renting only one boat. Choice C is incorrect because the total number of hours of rental is represented by $h$-values, each of which corresponds to the first coordinate of a point on the graph. Choice D is incorrect because the increase in cost for each additional hour is given by the slope of the line, not by the $C$-intercept.

## QUESTION 16.

Choice C is correct. The relationship between $h$ and $C$ is represented by any equation of the given line. The $C$-intercept of the line is 5 . Since the points $(0,5)$ and $(1,8)$ lie on the line, the slope of the line is $\frac{8-5}{1-0}=\frac{3}{1}=3$. Therefore, the relationship between $h$ and $C$ can be represented by $C=3 h+5$, the slope-intercept equation of the line.

Choices A and D are incorrect because each uses the wrong values for both the slope and intercept. Choice B is incorrect; this choice would result from computing the slope by counting the number of grid lines instead of using the values represented by the axes.

## QUESTION 17.

Choice $\mathbf{B}$ is correct. The minimum value of the function corresponds to the $y$-coordinate of the point on the graph that is the lowest along the vertical or $y$-axis. Since the grid lines are spaced 1 unit apart on each axis, the lowest point along the $y$-axis has coordinates $(-3,-2)$. Therefore, the value of $x$ at the minimum of $f(x)$ is -3 .

Choice A is incorrect; -5 is the smallest value for an $x$-coordinate of a point on the graph of $f$, not the lowest point on the graph of $f$. Choice C is incorrect; it is the minimum value of $f$, not the value of $x$ that corresponds to the minimum of $f$. Choice D is incorrect; it is the value of $x$ at the maximum value of $f$, not at the minimum value of $f$.

## QUESTION 18.

Choice $\mathbf{A}$ is correct. Since $(0,0)$ is a solution to the system of inequalities, substituting 0 for $x$ and 0 for $y$ in the given system must result in two true inequalities. After this substitution, $y<-x+a$ becomes $0<a$, and $y>x+b$ becomes $0>b$. Hence, $a$ is positive and $b$ is negative. Therefore, $a>b$.

Choice B is incorrect because $b>a$ cannot be true if $b$ is negative and $a$ is positive. Choice C is incorrect because it is possible to find an example where $(0,0)$ is a solution to the system, but $|a|<|b|$; for example, if $a=6$ and $b=-7$. Choice D is incorrect because the equation $a=-b$ could be true, but doesn't have to be true; for example, if $a=1$ and $b=-2$.

## OUESTION 19.

Choice B is correct. To determine the number of salads sold, write and solve a system of two equations. Let $x$ equal the number of salads sold and let $y$ equal the number of drinks sold. Since the number of salads plus the number of drinks sold equals 209, the equation $x+y=209$ must hold. Since each
salad cost $\$ 6.50$, each soda cost $\$ 2.00$, and the total revenue was $\$ 836.50$, the equation $6.50 x+2.00 y=836.50$ must also hold. The equation $x+y=209$ is equivalent to $2 x+2 y=418$, and subtracting each side of $2 x+2 y=418$ from the respective side of $6.50 x+2.00 y=836.50$ gives $4.5 x=418.50$. Therefore, the number of salads sold, $x$, was $x=\frac{418.50}{4.50}=93$.

Choices A, C, and D are incorrect and could result from errors in writing the equations and solving the system of equations. For example, choice $C$ could have been obtained by dividing the total revenue, $\$ 836.50$, by the total price of a salad and a soda, $\$ 8.50$, and then rounding up.

## QUESTION 20.

Choice $\mathbf{D}$ is correct. Let $x$ be the original price of the computer, in dollars. The discounted price is 20 percent off the original price, so $x-0.2 x=0.8 x$ is the discounted price, in dollars. The tax is 8 percent of the discounted price, so $0.08(0.8 x)$ is the tax on the purchase, in dollars. The price $p$, in dollars, that Alma paid the cashiers is the sum of the discounted price and the tax: $p=0.8 x+(0.08)(0.8 x)$ which can be rewritten as $p=1.08(0.8 x)$. Therefore, the original price, $x$, of the computer, in dollars, can be written as $\frac{p}{(0.8)(1.08)}$ in terms of $p$.

Choices A, B, and C are incorrect; each choice either switches the roles of the original price and the amount Alma paid, or incorrectly combines the results of the discount and the tax as $0.8+0.08=0.88$ instead of as (0.8)(1.08).

## QUESTION 21.

Choice C is correct. The probability that a person from Group Y who recalled at least 1 dream was chosen from the group of all people who recalled at least 1 dream is equal to the number of people in Group Y who recalled at least 1 dream divided by the total number of people in the two groups who recalled at least 1 dream. The number of people in Group Y who recalled at least 1 dream is the sum of the 11 people in Group Y who recalled 1 to 4 dreams and the 68 people in Group Y who recalled 5 or more dreams: $11+68=79$. The total number of people who recalled at least 1 dream is the sum of the 79 people in Group Y who recalled at least 1 dream, the 28 people in Group X who recalled 1 to 4 dreams, and the 57 people in Group X who recalled 5 or more dreams: $79+28+57=164$. Therefore, the probability is $\frac{79}{164}$.

Choice A is incorrect; it is the number of people in Group Y who recalled 5 or more dreams divided by the total number of people in Group Y. Choice $B$ is incorrect; it uses the total number of people in Group Y as the denominator of the probability. Choice D is incorrect; it is the total number of people in the two groups who recalled at least 1 dream divided by the total number of people in the two groups.

## QUESTION 22.

Choice B is correct. The average rate of change in the annual budget for agriculture/natural resources from 2008 to 2010 is the total change from to 2008 to 2010 divided by the number of years, which is 2 . The total change in the annual budget for agriculture/natural resources from 2008 to 2010 is $488,106-358,708=129,398$, in thousands of dollars, so the average change in the annual budget for agriculture/natural resources from 2008 to 2010 is $\frac{\$ 129,398,000}{2}=\$ 64,699,000$ per year. Of the options given, this average rate of change is closest to $\$ 65,000,000$ per year.

Choices A and C are incorrect; they could result from errors in setting up or calculating the average rate of change. Choice D is incorrect; $\$ 130,000,000$ is the approximate total change from 2008 to 2010, not the average change from 2008 to 2010.

## QUESTION 23.

Choice B is correct. The human resources budget in 2007 was $4,051,050$ thousand dollars, and the human resources budget in 2010 was $5,921,379$ thousand dollars. Therefore, the ratio of the 2007 budget to the 2010 budget is slightly greater than $\frac{4}{6}=\frac{2}{3}$. Similar estimates for agriculture/natural resources give a ratio of the 2007 budget to the 2010 budget of slightly greater than $\frac{3}{4}$; for education, a ratio of slightly greater than $\frac{2}{3}$; for highways and transportation, a ratio of slightly less than $\frac{5}{6}$; and for public safety, a ratio of slightly greater than $\frac{5}{9}$. Therefore, of the given choices, education's ratio of the 2007 budget to the 2010 budget is closest to that of human resources.

Choices A, C, and D are incorrect because the 2007 budget to 2010 budget ratio for each of these programs in these choices is further from the corresponding ratio for human resources than the ratio for education.

## QUESTION 24.

Choice $\mathbf{A}$ is correct. The equation of a circle can be written as $(x-h)^{2}+$ $(y-k)^{2}=r^{2}$ where $(h, k)$ are the coordinates of the center of the circle and $r$ is the radius of the circle. Since the coordinates of the center of the circle are $(0,4)$, the equation is $x^{2}+(y-4)^{2}=r^{2}$, where $r$ is the radius. The radius of the circle is the distance from the center, $(0,4)$, to the given endpoint of a radius, $\left(\frac{4}{3}, 5\right)$. By the distance formula, $r^{2}=\left(\frac{4}{3}-0\right)^{2}+(5-4)^{2}=\frac{25}{9}$. Therefore, an equation of the given circle is $x^{2}+(y-4)^{2}=\frac{25}{9}$.

Choice B is incorrect; it results from the incorrect equation $(x+h)^{2}+$ $(y+k)^{2}=r^{2}$. Choice C is incorrect; it results from using $r$ instead of $r^{2}$ in the equation for the circle. Choice D is incorrect; it results from using the incorrect equation $(x+h)^{2}+(y+k)^{2}=\frac{1}{r}$.

## QUESTION 25.

Choice $\mathbf{D}$ is correct. When the ball hits the ground, its height is 0 meters. Substituting 0 for $h$ in $h=-4.9 t^{2}+25 t$ gives $0=-4.9 t^{2}+25 t$, which can be rewritten as $0=t(-4.9 t+25)$. Thus, the possible values of $t$ are $t=0$ and $t=\frac{25}{4.9} \approx 5.1$. The time $t=0$ seconds corresponds to the time the ball is launched from the ground, and the time $t \approx 5.1$ seconds corresponds to the time after launch that the ball hits the ground. Of the given choices, 5.0 seconds is closest to 5.1 seconds, so the ball returns to the ground approximately 5.0 seconds after it is launched.

Choice A, B, and C are incorrect and could arise from conceptual or computation errors while solving $0=-4.9 t^{2}+25 t$ for $t$.

## OUESTION 26.

Choice B is correct. Let $x$ represent the number of pears produced by the Type B trees. Then the Type A trees produce 20 percent more pears than $x$, which is $x+0.20 x=1.20 x$ pears. Since Type A trees produce 144 pears, the equation $1.20 x=144$ holds. Thus $x=\frac{144}{1.20}=120$. Therefore, the Type B trees produced 120 pears.

Choice A is incorrect because while 144 is reduced by approximately 20 percent, increasing 115 by 20 percent gives 138 , not 144 . Choice C is incorrect; it results from subtracting 20 from the number of pears produced by the Type A trees. Choice D is incorrect; it results from adding 20 percent of the number of pears produced by Type A trees to the number of pears produced by Type A trees.

## OUESTION 27.

Choice C is correct. The area of the field is 100 square meters. Each 1-meter-by-1-meter square has an area of 1 square meter. Thus, on average, the earthworm counts to a depth of 5 centimeters for each of the regions investigated by the students should be about $\frac{1}{100}$ of the total number of earthworms to a depth of 5 centimeters in the entire field. Since the counts for the smaller regions are from 107 to 176 , the estimate for the entire field should be between 10,700 and 17,600 . Therefore, of the given choices, 15,000 is a reasonable estimate for the number of earthworms to a depth of 5 centimeters in the entire field.

Choice A is incorrect; 150 is the approximate number of earthworms in 1 square meter. Choice B is incorrect; it results from using 10 square meters as the area of the field. Choice D is incorrect; it results from using 1,000 square meters as the area of the field.

## QUESTION 28.

Choice C is correct. To determine which quadrant does not contain any solutions to the system of inequalities, graph the inequalities. Graph the inequality $y \geq 2 x+1$ by drawing a line through the $y$-intercept $(0,1)$ and the point $(1,3)$, and graph the inequality $y>\frac{1}{2} x-1$ by drawing a dashed line through the $y$-intercept $(0,-1)$ and the point $(2,0)$, as shown in the figure below.


The solution to the system of inequalities is the intersection of the shaded regions above the graphs of both lines. It can be seen that the solutions only include points in quadrants I, II, and III and do not include any points in quadrant IV.

Choices A and B are incorrect because quadrants II and III contain solutions to the system of inequalities, as shown in the figure above. Choice D is incorrect because there are no solutions in quadrant IV.

## QUESTION 29.

Choice $\mathbf{D}$ is correct. If the polynomial $p(x)$ is divided by $x-3$, the result can be written as $\frac{p(x)}{x-3}=q(x)+\frac{r}{x-3}$, where $q(x)$ is a polynomial and $r$ is the remainder. Since $x-3$ is a degree 1 polynomial, the remainder is a real number. Hence, $p(x)$ can be written as $p(x)=(x-3) q(x)+r$, where $r$ is a real number. It is given that $p(3)=-2$ so it must be true that $-2=p(3)=(3-3) q(3)+r=(0) q(3)+r=r$. Therefore, the remainder when $p(x)$ is divided by $x-3$ is -2 .

Choice A is incorrect because $p(3)=-2$ does not imply that $p(5)=0$. Choices B and C are incorrect because the remainder -2 or its negative, 2, need not be a root of $p(x)$.

## QUESTION 30.

Choice $\mathbf{D}$ is correct. Any quadratic function $q$ can be written in the form $q(x)=a(x-h)^{2}+k$, where $a, h$, and $k$ are constants and $(h, k)$ is the vertex of the parabola when $q$ is graphed in the coordinate plane. (Depending on the
sign of $a$, the constant $k$ must be the minimum or maximum value of $q$, and $h$ is the value of $x$ for which $a(x-h)^{2}=0$ and $q(x)$ has value $k$.) This form can be reached by completing the square in the expression that defines $q$. The given equation is $y=x^{2}-2 x-15$, and since the coefficient of $x$ is -2 , the equation can be written in terms of $(x-1)^{2}=x^{2}-2 x+1$ as follows: $y=x^{2}-2 x-15=\left(x^{2}-2 x+1\right)-16=(x-1)^{2}-16$. From this form of the equation, the coefficients of the vertex can be read as $(1,-16)$

Choices A and C are incorrect because the coordinates of the vertex $A$ do not appear as constants in these equations. Choice $B$ is incorrect because it is not equivalent to the given equation.

## QUESTION 31.

The correct answer is any number between 4 and 6, inclusive. Since Wyatt can husk at least 12 dozen ears of corn per hour, it will take him no more than $\frac{72}{12}=6$ hours to husk 72 dozen ears of corn. On the other hand, since Wyatt can husk at most 18 dozen ears of corn per hour, it will take him at least $\frac{72}{18}=4$ hours to husk 72 dozen ears of corn. Therefore, the possible times it could take Wyatt to husk 72 dozen ears of corn are 4 hours to 6 hours, inclusive. Any number between 4 and 6, inclusive, can be gridded as the correct answer.

## QUESTION 32.

The correct answer is 107. Since the weight of the empty truck and its driver is 4500 pounds and each box weighs 14 pounds, the weight, in pounds, of the delivery truck, its driver, and $x$ boxes is $4500+14 x$. This weight is below the bridge's posted weight limit of 6000 pounds if $4500+14 x<6000$. That inequality is equivalent to $14 x \leq 1500$ or $x<\frac{1500}{14}=107 \frac{1}{7}$. Since the number of packages must be an integer, the maximum possible value for $x$ that will keep the combined weight of the truck, its driver, and the $x$ identical boxes below the bridge's posted weight limit is 107 .

## QUESTION 33.

The correct answer is $\frac{5}{8}$ or .625. Based on the line graph, the number of portable media players sold in 2008 was 100 million, and the number of portable media players sold in 2011 was 160 million. Therefore, the number of portable media players sold in 2008 is $\frac{100 \text { million }}{160 \text { million }}$ of the portable media players sold in 2011. This fraction reduces to $\frac{5}{8}$. Either $\frac{5}{8}$ or its decimal equivalent, . 625 , may be gridded as the correct answer.

## QUESTION 34.

The correct answer is 96 . Since each day has a total of 24 hours of time slots available for the station to sell, there is a total of 48 hours of time slots
available to sell on Tuesday and Wednesday. Each time slot is a 30 -minute interval, which is equal to a $\frac{1}{2}$-hour interval. Therefore, there are a total of $\frac{48 \text { hours }}{\frac{1}{2} \text { hours/time slot }}=96$ time slots of 30 minutes for the station to sell on Tuesday and Wednesday.

## QUESTION 35.

The correct answer is $\mathbf{6}$. The volume of a cylinder is $\pi r^{2} h$, where $r$ is the radius of the base of the cylinder and $h$ is the height of the cylinder. Since the storage silo is a cylinder with volume $72 \pi$ cubic yards and height 8 yards, it is true that $72 \pi=\pi r^{2}(8)$, where $r$ is the radius of the base of the cylinder, in yards. Dividing both sides of $72 \pi=\pi r^{2}(8)$ by $8 \pi$ gives $r^{2}=9$, and so the radius of base of the cylinder is 3 yards. Therefore, the diameter of the base of the cylinder is 6 yards.

## QUESTION 36.

The correct answer is 3 . The function $h(x)$ is undefined when the denominator of $\frac{1}{(x-5)^{2}+4(x-5)+4}$ is equal to zero. The expression $(x-5)^{2}+$ $4(x-5)+4$ is a perfect square: $(x-5)^{2}+4(x-5)+4=((x-5)+2)^{2}$, which can be rewritten as $(x-3)^{2}$. The expression $(x-3)^{2}$ is equal to zero if and only if $x=3$. Therefore, the value of $x$ for which $h(x)$ is undefined is 3 .

## QUESTION 37.

The correct answer is 1.02. The initial deposit earns 2 percent interest compounded annually. Thus at the end of 1 year, the new value of the account is the initial deposit of $\$ 100$ plus 2 percent of the initial deposit: $\$ 100+\frac{2}{100}(\$ 100)=\$ 100(1.02)$. Since the interest is compounded annually, the value at the end of each succeeding year is the sum of the previous year's value plus 2 percent of the previous year's value. This is again equivalent to multiplying the previous year's value by 1.02 . Thus, after 2 years, the value will be $\$ 100(1.02)(1.02)=\$ 100(1.02)^{2}$; after 3 years, the value will be $\$ 100(1.02)^{3}$; and after $t$ years, the value will be $\$ 100(1.02)^{t}$. Therefore, in the formula for the value for Jessica's account after $t$ years, $\$ 100(x)^{t}$, the value of $x$ must be 1.02 .

## QUESTION 38.

The correct answer is 6.11. Jessica made an initial deposit of $\$ 100$ into her account. The interest on her account is 2 percent compounded annually, so after 10 years, the value of her initial deposit has been multiplied 10 times by the factor $1+0.02=1.02$. Hence, after 10 years, Jessica's deposit is worth $\$ 100(1.02)^{10}=\$ 121.899$ to the nearest tenth of a cent. Tyshaun made an initial deposit of $\$ 100$ into his account. The interest on his account is 2.5 percent compounded annually, so after 10 years, the value of his initial deposit
has been multiplied 10 times by the factor $1+0.025=1.025$. Hence, after 10 years, Tyshaun's deposit is worth $\$ 100(1.025)^{10}=\$ 128.008$ to the nearest tenth of a cent. Hence, Jessica's initial deposit earned $\$ 21.899$ and Tyshaun's initial deposit earned $\$ 28.008$. Therefore, to the nearest cent, Tyshaun's initial deposit earned $\$ 6.11$ more than Jessica's initial deposit.

