UNIT 4

Sound and Waves

Chapter 11

Harmonic Motion

Introduction to Chapter 11

The motion we have studied so far has been from one place to another. In this chapter we will investigate harmonic motion, which is motion that repeats in cycles. From the orbit of the Earth to the rhythmic beating of your heart, harmonic motion is fundamental to our understanding of nature.

Investigations for Chapter 11

11.1	**Harmonic Motion**	*How do we describe the back and forth motion of a pendulum?*

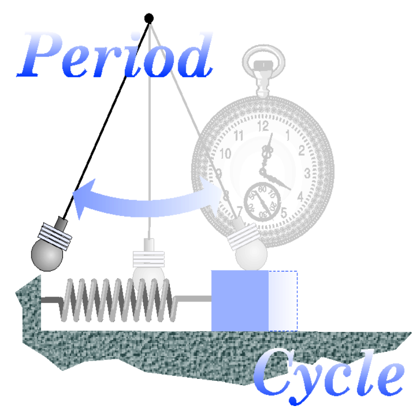
The pendulum is an ideal start for investigating harmonic motion. The objective for this Investigation is to design a clock that can keep accurate time using a pendulum.

11.2	**Graphs of Harmonic Motion**	*How do we make graphs of harmonic motion?*

Graphs tell us much about straight-line motion. This Investigation will apply graphing techniques to oscillators. Learn how to read a heartbeat from an EKG and how to read the seismogram of a powerful earthquake!

11.3	**Simple Mechanical Oscillators**	*What kinds of systems oscillate?*

Many things in nature oscillate. Guitar strings, trees in the wind, and stretched rubber bands are all examples of oscillators. In this Investigation we will construct several simple oscillators and learn how to adjust their frequency and period.



Learning Goals

In this chapter, you will:

- ✔ Learn about harmonic motion and how it is fundamental to understanding natural processes.
- ✔ Use harmonic motion to keep accurate time using a pendulum.
- ✔ Learn how to interpret and make graphs of harmonic motion.
- ✔ Construct simple oscillators.
- ✔ Learn how to adjust the frequency and period of simple oscillators.
- ✔ Learn to identify simple oscillators.

Vocabulary

amplitude	harmonic motion	period	system
cycle	hertz	periodic motion	
frequency	oscillator	phase	

11.1 Harmonic Motion

As you watch moving things, you see two different kinds of motion. One kind of motion goes from one place to another. This is called *linear motion*. The concepts of distance, time, speed, and acceleration come from thinking about this kind of motion.

The second kind of motion is motion that repeats itself over and over. We call motion that repeats over and over harmonic motion and that is what you will learn about in this section. The word comes from *harmony* which means "multiples of." Swinging back and forth on a swing is a good example of harmonic motion (figure 11.1). Many moving things have both kinds of motion. A bicycle moves forward but the wheels and pedals go around and around in harmonic motion (figure 11.2).

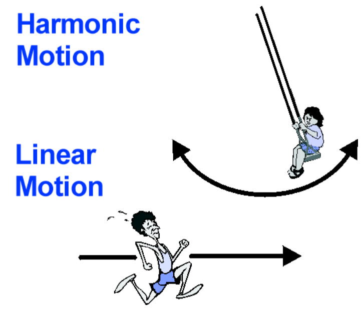
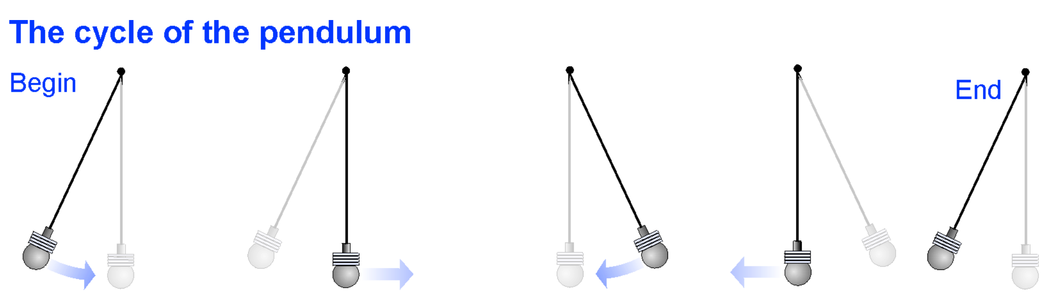


Figure 11.1: *Linear motion goes from one place to another without repeating. Harmonic motion repeats over and over the same way.*

Cycles, systems, and oscillators

What is a cycle? The cycle is the building block of harmonic motion. A cycle is a unit of motion that repeats over and over. All harmonic motion is a repeated sequence of cycles. The cycle of the pendulum is shown below.



Finding the cycle When investigating harmonic motion we start by identifying the basic cycle. A cycle has a beginning and ending. Between beginning and end, the cycle has to include all the motion that repeats. The cycle of the pendulum is defined by where we choose the beginning. If we start the cycle when the pendulum is all the way to the left, the cycle ends when the pendulum has returned all the way to the left again. If we choose the cycle correctly, the motion of the pendulum is one cycle after the next with no gaps between cycles.

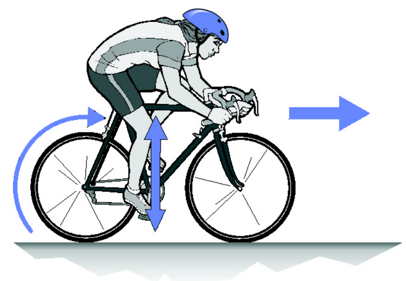


Figure 11.2: *Real-life situations can include both linear motion and harmonic motion.*

Harmonic motion in nature

Choosing a system

In science we often refer to a system. A system is a group we choose that includes all the things we are interested in. Choosing the system helps us concentrate on what is important and exclude what is not important. For the pendulum, the system is the hanger, string, and weight. We don't need to include the floor or the table, since these are not directly important to the motion.

We might choose the system differently depending on what we want to investigate. If we wanted to see how gravity affected the pendulum, we would have to include Earth's gravity as part of the system.

An oscillator is a system with harmonic motion

A system that shows harmonic motion is called an **oscillator**. The pendulum is an example of an oscillator. So is your heart and its surrounding muscles (figure12.4). Oscillators can be very small. The electrons in the atom make harmonic motion, so an atom is an oscillator. Oscillators can also be very large. The solar system is an oscillator with each of the planets in harmonic motion around the sun. We are going to study oscillators using simple models, but what we learn will also apply to more complex systems, like a microwave communications satellite.

Earth is part of several systems in harmonic motion

Earth is a part of several oscillating systems. The Earth/sun system has a cycle of one year, which means Earth completes one orbit around the sun in a year. The Earth/moon system has a cycle of approximately one month. Earth itself has several different cycles (figure 11.4). It rotates around its axis once a day making the 24-hour cycle of day and night. There is also a wobble of Earth's axis that cycles every 22,000 years, moving the north and south poles around by hundreds of miles. There are cycles in weather, such as the El Nino and La Nina oscillations in ocean currents that produce fierce storms every decade or so. Much of the planet's ecology depends on cycles.

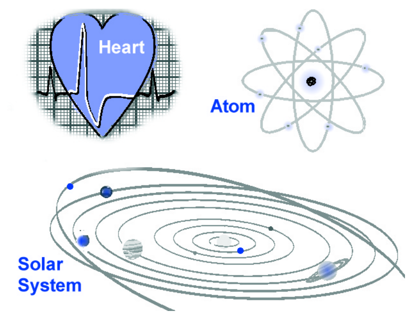


Figure 11.3: *The pendulum is an oscillator. Other examples of oscillators are an atom, your beating heart, and the solar system.*

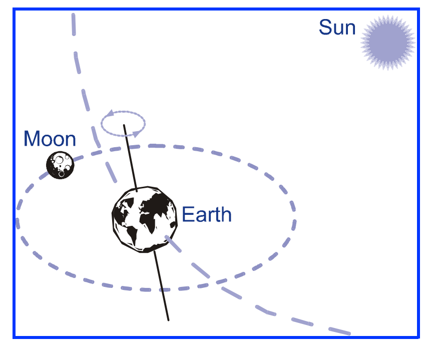


Figure 11.4: *The Earth/sun/moon system has many different cycles. The year, month, and day are the result of orbital cycles.*

Harmonic motion in art and music

Music comes from oscillations Both light and sound come from oscillations. Music and musical instruments are oscillators that we design to create sounds with specific cycles that we enjoy hearing. Sound is an oscillation of the air. A moving speaker pushes and pulls on the air creating a small oscillation in pressure (figure 11.5). The oscillation travels to where it hits your eardrum. Your vibrating eardrum moves tiny bones in the ear setting up more oscillations that are transmitted by nerves to the brain. There is harmonic motion at every step of the way, from the musical instrument to the perception of sound by your brain.

Color comes from oscillations We see colors in light waves, which are oscillations of electricity and magnetism. Faster oscillations make blue light while slower oscillations make red light. When painting a picture, each color of paint contains different molecules that absorb and reflect different colors of light. The colors you see come from the interaction between the oscillations of light and the oscillations of the electrons in the pigment molecules.

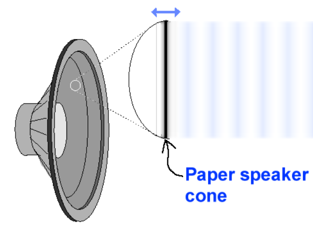


Figure 11.5: *A moving speaker oscillates back and forth, making sound that you can hear.*

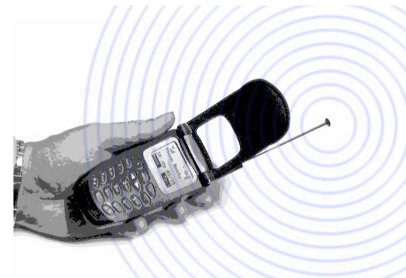


Figure 11.6: *The cordless phone you use has an electronic oscillator at millions of cycles per second.*

Harmonic motion in technology

Oscillators are used in communications Almost all modern communication technology relies on fast electronic oscillators. Cell phones use oscillators that make more than 100 million cycles each second (figure 11.6). FM radio uses oscillators between 95 million and 107 million cycles per second. When you tune a radio you are selecting the frequency of the oscillator you want to listen to. Each station sets up an oscillator at a different frequency. Sometimes, you can get two stations at once when you are traveling between two radio towers with nearly the same frequency.

Oscillators are used to measure time The cycles of many oscillators always repeat in the same amount of time. This makes harmonic motion a good way to keep time. If you have a pendulum that has a cycle one second long, you can count time in seconds by counting cycles of the pendulum. Grandfather clocks and mechanical watches actually count cycles of oscillators to tell time (figure 11.7). Even today, the world's most accurate clocks keep time by counting cycles of light from a cesium atom oscillator. Modern atomic clocks are so accurate they lose only one second in 1,400,000 years!

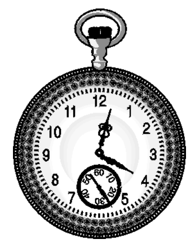


Figure 11.7: *Clocks and watches use oscillators to keep time. This works because many oscillators have precisely stable cycles.*

Investigating harmonic motion

Period is the time for one cycle

What makes harmonic motion useful for clocks is that each cycle takes the same amount of time. The time for one cycle is called the **period**. Some clocks have a pendulum with a period of exactly two seconds. The gears in the clock cause the minute hand to move 1/60 of a turn for every 30 swings of the pendulum. The period is one of the important characteristics of all harmonic motion (figure 11.8).

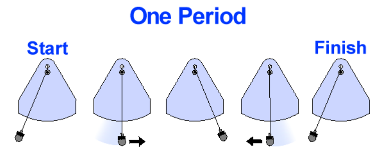


Figure 11.8: *The period is the time it takes to complete one cycle.*

Frequency is the number of cycles per second

Frequency is closely related to period. The **frequency** of an oscillator is the number of cycles it makes per second. Every day, we experience a wide range of frequencies. FM radio uses frequencies between 95 million and 107 million cycles per second (the FM standing for frequency modulation) (figure 11.9). Your heartbeat probably has a frequency between one-half and two cycles per second. The musical note "A" has a frequency of 440 cycles per second. The human voice contains frequencies mainly between 100 and 2,000 cycles per second.

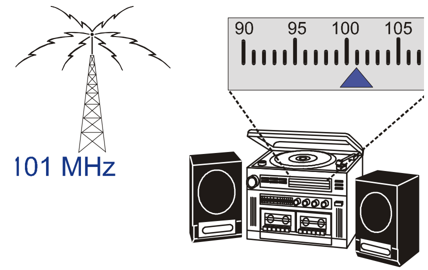


Figure 11.9: *When you tune a radio to receive a station, you are matching frequencies between receiver and transmitter.*

Frequency is measured in hertz

The unit of one cycle per second is called a **hertz**. A frequency of 440 cycles per second is usually written as 440 hertz, or abbreviated 440 Hz. The Hz is a unit that is the same in English and metric. When you tune into a station at 101 on the FM dial, you are actually setting the oscillator in your radio to a frequency of 101 megahertz, or 101,000,000 Hz. You hear music when the oscillator in your radio is exactly matched to the frequency of the oscillator in the transmission tower connected to the radio station.

Period and frequency

$$T = \frac{1}{f} \qquad f = \frac{1}{T}$$

Period (seconds) → T; Frequency (hertz) → f

Frequency (hertz) → f; Period (seconds) → T

Frequency and period are inversely related. The period is the time per cycle. The frequency is the number of cycles per time. If the period of a pendulum is 1.25 seconds, its frequency is 0.8 cycles per second (0.8 Hz). If you know one, you can calculate the other.

Example:

Calculate the frequency of a pendulum that has a period of 1/4 second.

Solution:

(1) You are asked for frequency.

(2) You are given the period.

(3) The relationship you need is F=1/T.

(4) Plug in numbers.

F = 1 / (0.25 sec)

= 4 Hz

Amplitude

Amplitude describes the size of a cycle

Another important characteristic of a cycle is its size. The period tells how long the cycle lasts. The **amplitude** describes how big the cycle is. The diagram below shows a pendulum with small amplitude and large amplitude. With mechanical systems (such as a pendulum), the amplitude is often a distance or angle. With other kinds of oscillators, the amplitude might be voltage or pressure. The amplitude is measured in units appropriate to the kind of oscillation you are describing.

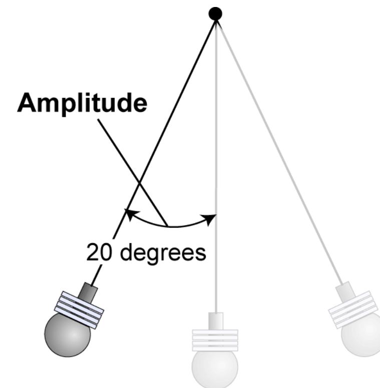
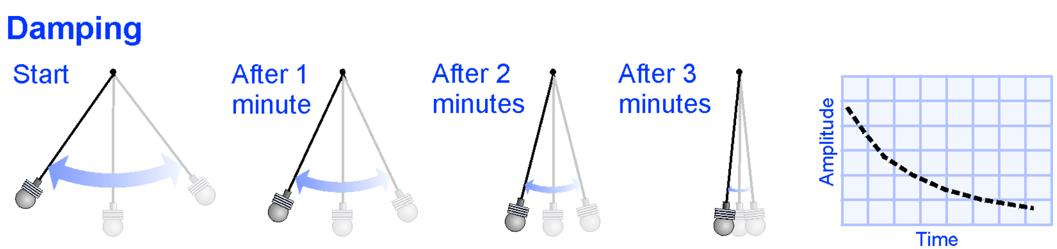


Figure 11.10: *A pendulum with an amplitude of 20 degrees swings 20 degrees away from the center.*

How do you measure amplitude?

The amplitude is the maximum distance the motion moves away from the average. For a pendulum, the average is at the center. The pendulum spends as much time to the right of center as it does to the left. For the pendulum in figure 11.10, the amplitude is 20 degrees, because the pendulum moves 20 degrees away from center in either direction.



Damping

Friction slows a pendulum down, as it does all oscillators. That means the amplitude slowly gets reduced until the pendulum is hanging straight down, motionless. We use the word **damping** to describe the gradual loss of amplitude of an oscillator. If you wanted to make a clock with a pendulum, you would have to find a way to keep adding energy to counteract the damping of friction.

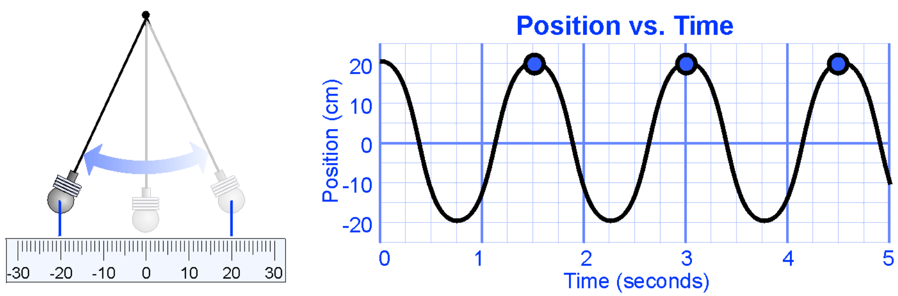
11.2 Graphs of Harmonic Motion

Harmonic motion graphs show cycles. This is what makes them different from linear motion graphs (figure 11.11). The values of the period and amplitude can be read from the graphs. If you know the period and amplitude, you can quickly sketch a harmonic motion graph.

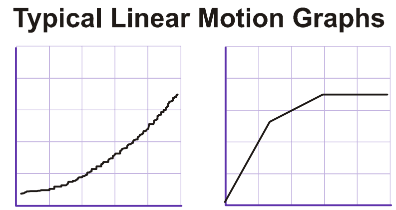
Reading harmonic motion graphs

Cycles and time Most graphs of harmonic motion show how things change with time. The pendulum is a good example. The diagram below shows a graph of position vs. time for a pendulum. The graph shows repeating cycles just like the motion. Seeing a pattern of cycles on a graph is an indication that harmonic motion is present.

Using positive and negative numbers Harmonic motion graphs often use positive and negative values to represent motion on either side of center. We usually choose zero to be at the equilibrium point of the motion. Zero is placed halfway up the *y*-axis so there is room for both positive and negative values. The graph alternates from plus to minus and back. The example graph below shows a pendulum swinging from +20 centimeters to -20 centimeters and back. The amplitude is the maximum distance away from center, or 20 centimeters.



Harmonic graphs repeat every period Notice that the graph (above) returns to the same place every 1.5 seconds. No matter where you start, you come back to the same value 1.5 seconds later. Graphs of harmonic motion repeat every period, just as the motion repeats every cycle. Harmonic motion is sometimes called periodic motion for this reason.



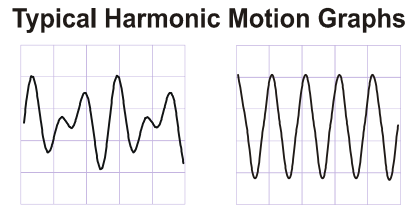


Figure 11.11: *Typical graphs for linear motion (top) and harmonic motion (bottom). Harmonic motion graphs show cycles.*

Determining amplitude and period from a graph

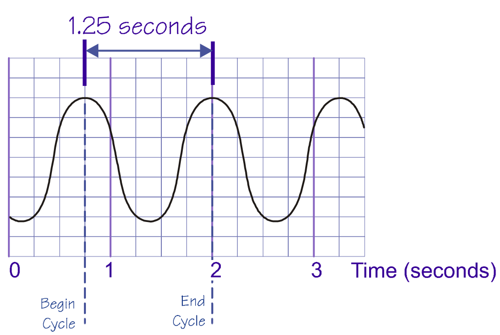
Calculating amplitude from a graph

The amplitude is half the distance between the highest and lowest points on the graph. For the example in figure 11.12, the amplitude is 20 centimeters, as illustrated by the calculation below. The difference between the highest and lowest value of the graph is the *peak-to-peak* value.

$$\text{Amplitude} = \tfrac{1}{2}(\text{high} - \text{low}) = \tfrac{1}{2}(20 - (-20))$$
$$= 20\ \text{cm}$$

Calculating period from a graph

To get the period from a graph, start by identifying one complete cycle. The cycle must begin and end in the same place on the graph. Figure 11.13 shows how to choose the cycle for a simple harmonic motion graph and for a more complex one. Once you have identified a cycle, you use the time axis of the graph to determine the period. The period is the time difference between the beginning of the cycle and the end. Subtract the beginning time from the ending time, as shown in the example below.



$$\text{Period} = (\text{ending time} - \text{beginning time})$$
$$= (2.0\ \text{sec} - 0.75\ \text{sec})$$
$$= 1.25\ \text{seconds}$$

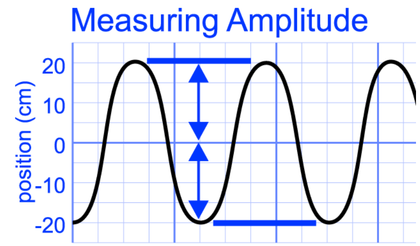


Figure 11.12: *The amplitude of a wave is one-half the peak-to-peak distance. In this graph of harmonic motion, the amplitude of the wave is 20 centimeters.*

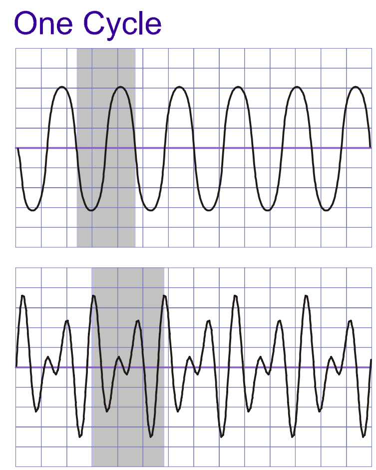


Figure 11.13: *The cycle is the part of the graph that repeats over and over. The gray shading shows one cycle for each of the graphs above. Before you can find the period, you need to identify the cycle.*

Circles and harmonic motion

Circular motion

Circular motion is very similar to harmonic motion. For example, a turning wheel returns to the same position every 360 degrees. Rotation is a cycle, just like harmonic motion. One key difference is that cycles of circular motion *always* have a length of 360 degrees. It does not matter how big the wheel is, each full turn is 360 degrees.

Figure 11.14 shows a shadow of a peg on a rotating wheel. As the wheel rotates, the shadow of the peg goes back and forth on the wall. If we make a graph of the position of the shadow, we get a harmonic motion graph. The period of the cycle is exactly the time it takes the wheel to turn 360 degrees.

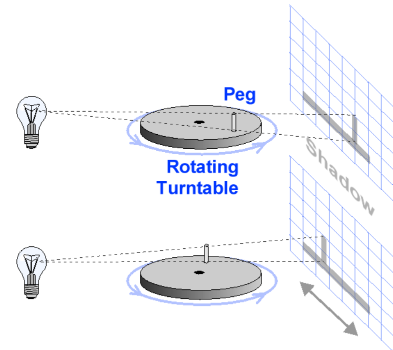


Figure 11.14: *The shadow of a peg moves back and forth on the wall as the turntable rotates. The shadow itself appears to be in harmonic motion.*

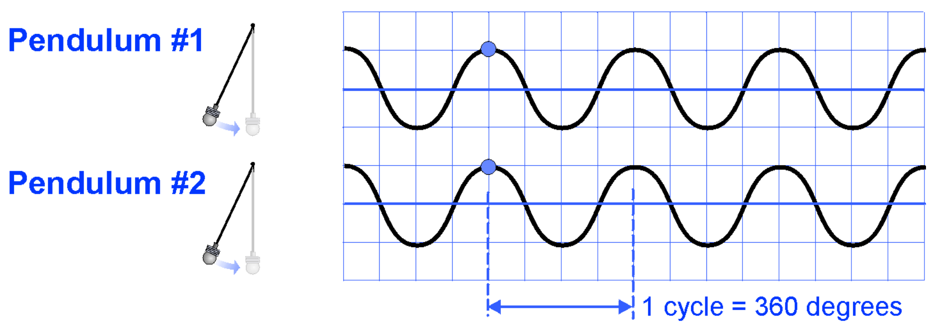
The phase of an oscillator

We often use degrees to tell us where we are within the cycle of an oscillator. For example, how would you identify the moment when the pendulum was one-tenth of the way through its cycle? If we let one cycle be 360 degrees, then one-tenth of that cycle is 36 degrees. Thirty-six degrees is a measure of the **phase** of the oscillator. The word "phase" means where the oscillator is in the cycle.

What do we mean by "in phase"?

The concept of phase is important when comparing one oscillator with another. Suppose we have two identical pendulums, with exactly the same period. If we start them together, their graphs would look like the picture below. We describe the two pendulums as being *in phase* because cycles are aligned. Each oscillator is always at the same place at the same time.

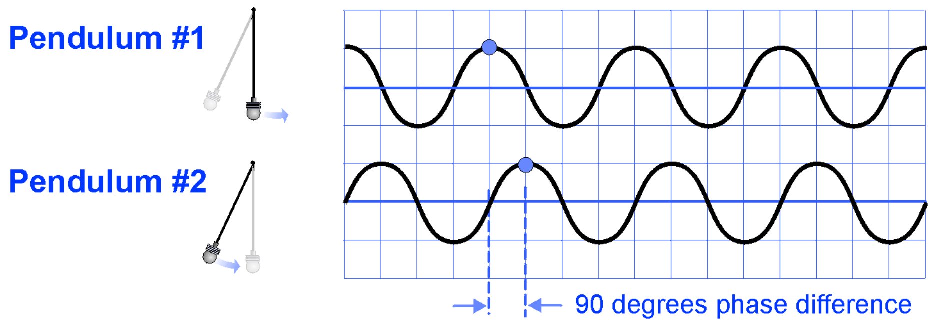
Both pendulums in phase



Out of phase by 90 degrees

If we start the first pendulum swinging a little before the second one, the graphs look like the diagram below. Although, they have the same cycle, the first pendulum is always a little ahead of the second. The graph shows the lead of the first pendulum as a phase difference. Notice that the top graph reaches its maximum 90 degrees *before* the bottom graph. We say the two pendulums are *out of phase* by 90 degrees, or one-fourth of a cycle.

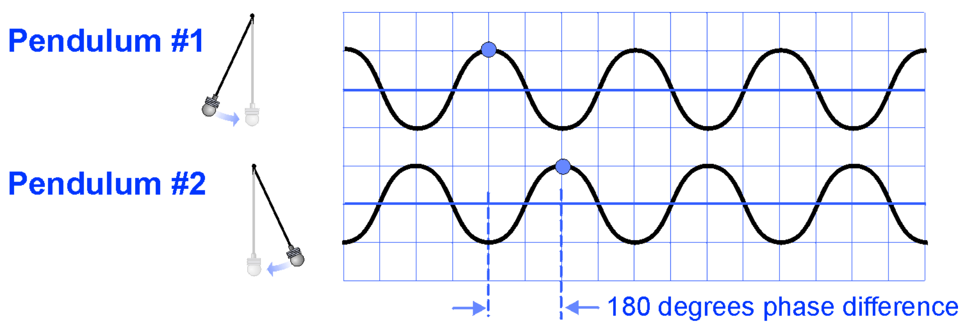
90 degrees out of phase



Out of phase by 180 degrees

When they are out of phase, the relative motion of the oscillators may differ by a little or by as much as half a cycle. Two oscillators 180 degrees out of phase are one-half cycle apart. The next diagram (below) shows that the two pendulums are always on opposite sides of the cycle from each other. The concepts of in phase and out of phase will be very important to our Investigations with waves and sound.

180 degrees out of phase



11.3 Simple Mechanical Oscillators

Harmonic motion is so common that it would be impossible to list all the different kinds of oscillators you might find. Fortunately, we can learn much about harmonic motion by looking at just a few examples. Once we understand some basic oscillators, we will have the experience needed to figure out more complex ones.

Examples of oscillators

The pendulum

The simplest pendulum is a weight hanging from a string. The weight swings back and forth once it is pulled away and released. The force that always pulls the pendulum back to center comes from its weight (figure 11.15). If you swing a pendulum to one side, the string causes it to lift slightly.

The period of a pendulum does not change much, even when its amplitude is changed. This is because two opposite effects occur. First, if you make the amplitude large, the pendulum has a greater distance to travel, which increases the period. But remember that by releasing it from a high position, it also starts with more energy. More energy means the pendulum goes faster and higher speed decreases the period. The effect of higher speed almost exactly cancels the effect of longer swing distance.

A mass on a spring

If you have ever been in a car with worn-out shock absorbers, you have experienced another common oscillator. The system of a car and shock absorbers is an example of a mass on a spring. Springs resist being extended or compressed. Figure 11.16 shows how the force from a spring always acts to return to equilibrium. A mass attached to a spring adds inertia to the system. If the mass is given an initial push, the mass/spring system oscillates.

A vibrating string

Vibrating strings are used in many musical instruments. A stretched rubber band is a good example. If you pull the rubber band to one side, it stretches a bit. The stretching creates a restoring force that tries to pull the rubber band back straight again. Inertia carries it past being straight and it vibrates. Vibrating strings tend to move much faster than springs and pendulums. The period of a vibrating string can easily be one-hundredth of a second (0.01) or shorter.

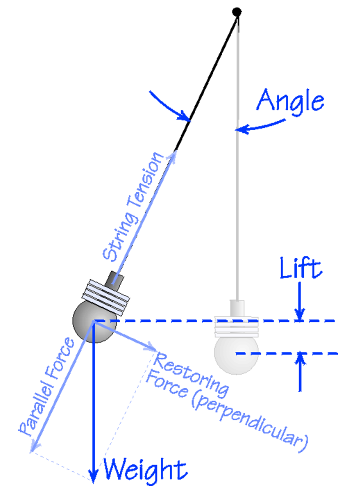


Figure 11.15: *The forces acting on the pendulum. The weight (gravity) points straight down.*

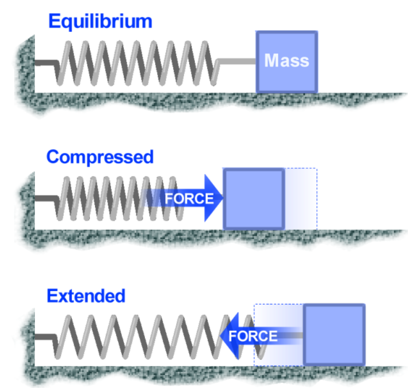


Figure 11.16: *A mass on a spring oscillator. When the spring is compressed or extended, it pushes the mass back toward equilibrium.*

Chapter 11 Review

Vocabulary review

Match the following terms with the correct definition. There is one extra definition in the list that will not match any of the terms.

Set One

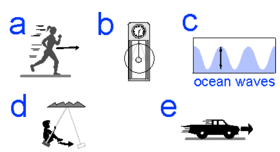
1. harmonic motion	a. A system in harmonic motion
2. cycle	b. A unit of one cycle per second
3. system	c. Back and forth or repeating motion
4. oscillator	d. A part of motion that repeats over and over
5. hertz	e. A group of things we think are important to consider when analyzing something
	f. Motion that goes from one point to another without repeating

Set Two

1. period	a. The number of cycles per second
2. frequency	b. The size of a cycle
3. amplitude	c. A way to identify where an oscillator is in its cycle
4. damping	d. The time it takes to complete one cycle
5. phase	e. Any process that causes cycles to get smaller and smaller in amplitude
	f. A unit of one cycle per second

Concept review

1. Name three objects or systems around you that have cycles.
2. What is the relationship between frequency and period?
3. Which pictures show only periodic motion?



   a. A girl running a race.
   b. The swinging pendulum of a clock.
   c. An ocean wave rising and falling.
   d. A boy swinging.
   e. A car moving down the street.
4. If the length of the rope on a swing gets longer, the period of the swing will:
   a. Get longer.
   b. Get shorter.
   c. Stay the same.
   d. I need more information to answer.

5. In a pendulum experiment, the ____________ is the maximum angle that the pendulum swings away from center. (Pick one.)

    a. cycle

    b. amplitude

    c. period

    d. speed

6. Oscillations have something to do with the answers to which of the following questions? (You can pick more than one.)

    a. What color is it?

    b. How much mass does it have?

    c. How long is it?

    d. How loud is that noise?

    e. What radio station is this?

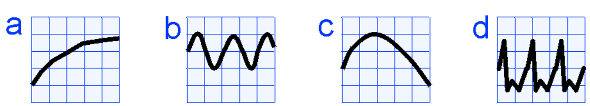
7. A clock is made using a pendulum to count the time. The weight at the bottom of the pendulum can be adjusted to make the length of the pendulum longer or shorter. The clock runs too fast, meaning it counts 50 minutes as one full hour. What should you do to correct the clock?

    a. Move the weight upward, making the pendulum shorter.

    b. Move the weight downward, making the pendulum longer.

    c. Add more weight to make the pendulum heavier.

8. Which of the graphs clearly shows harmonic motion? You may choose more than one.



9. The measurement of 2.5 seconds could be:

    a. The frequency of an oscillator.

    b. The period of an oscillator.

    c. The mass of an oscillator.

    d. The system where we find an oscillator.

10. A measurement of 1 meter could be:

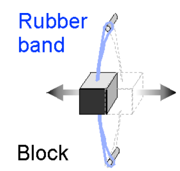
    a. The frequency of an oscillator.

    b. The amplitude of an oscillator.

    c. The mass of an oscillator.

    d. The system where we find an oscillator.

11. An oscillator is made with a rubber band and a block of wood, as shown in the diagram. What happens to the oscillator if we make the block of wood heavier?



    a. The frequency increases.

    b. The period increases.

    c. The frequency stays the same.

    d. The period stays the same.

12. If the amplitude of a pendulum is doubled, which of the following will be true?

    a. It will swing twice as far away from center.

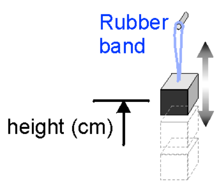
    b. Its period will be twice as long.

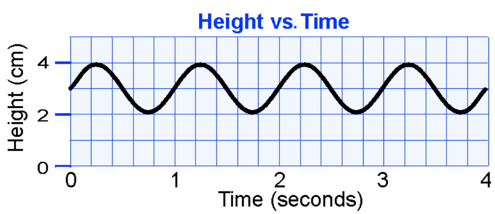
    c. Its frequency will be twice as high.

    d. It must have twice the mass.

Problems

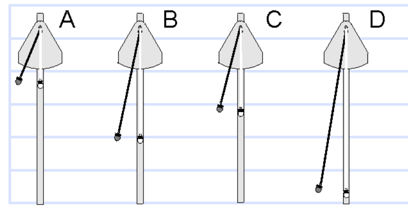
1. A person's heartbeat is measured to be 65 beats per minute. What is the period between heartbeats in seconds?
   a. 65 seconds
   b. 65 Hertz
   c. 0.92 seconds
   d. 1.08 seconds
2. A pendulum has a period of 1.5 seconds. How many cycles do you have to count to make one minute?
3. A string vibrates back and forth 30 times per second. What is the period of the motion of the string?
4. The graph shows the motion of an oscillator that is a weight hanging from a rubber band. The weight moves up and down. Answer the following questions using the graph.





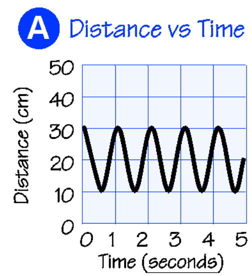
   a. What is the period?
   b. What is the amplitude?
   c. If you counted for 5 seconds, how many cycles would you count?

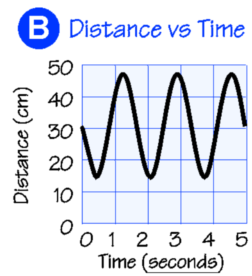
5. Four different groups of students make measurements of the period of a pendulum. Each group hands in a lab with no names on it. Can you tell which lab group was working with which pendulum? Match the letter of the pendulum to the number of the lab group.

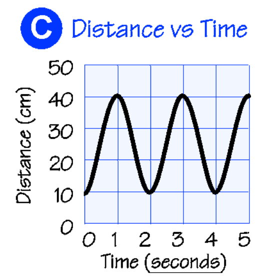


Group	Period	Pendulum
1	1.0 sec	
2	1.2 sec	
3	1.4 sec	
4	1.7 sec	

Questions 6, 7, and 8 refer to the three graphs below. Distance in these graphs means displacement of the oscillator.







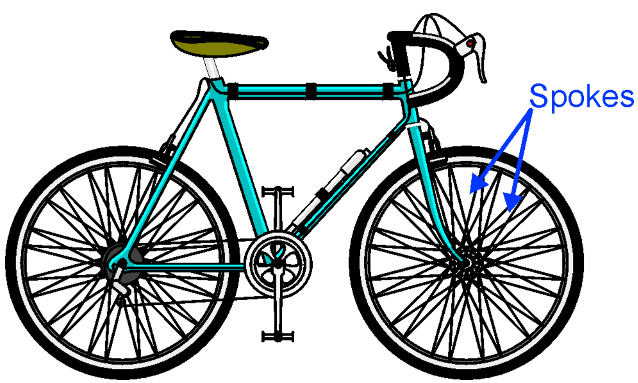
6. Which graph shows exactly 3 cycles?
7. Which graph has a period of 2 seconds?
8. Which graph has an amplitude of 10 centimeters?

Applying your knowledge

1. What is the period of the rotation of the Earth that gives us day and night?

2. An animal research scientist films a small bird and counts 240 beats of the bird's wings in 2 minutes. What is the frequency of the motion of the bird's wings?

3. The Earth, moon, and sun make a system with many cycles. Give at least two examples of cycles that relate to the Earth, moon, or sun and also give the period of each example you find.

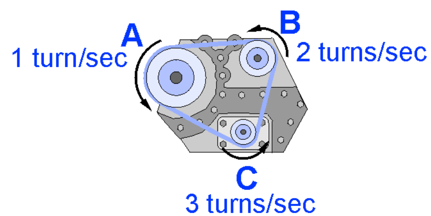


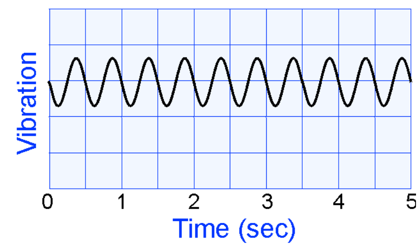
4. You invent a bicycle speedometer that counts how many spokes of the wheel pass by each second. You ride your bicycle to test the speedometer and measure 2,160 spokes pass in one minute.

   a. What is the frequency of spokes passing your sensor in hertz?

   b. The wheel has 36 spokes. How many turns per minute does the wheel make?

5. The human heart is both strong and reliable. As a demonstration of how reliable the heart is, calculate how many times your heart beats in one day. Start by measuring the frequency of your pulse in beats per minute and use the result for your calculation.





6. Frequency can be a clue to finding problems in engines before they cause serious damage. Suppose the engine has three spinning parts (A, B, C), each turning at a different speed. Since the speed of each part is different, the frequency of each is also different. If one part starts to wear out, it will vibrate more than it should. By looking at the frequency of vibration for the whole engine, you can spot which part is the problem by looking for vibrations at its characteristic frequency.
The graph shows the vibration of the whole engine, including all three spinning parts. From the graph, can you tell which part is making too much vibration, and is therefore likely to fail?