SOURCING STRATEGIES
IN A SUPPLY CHAIN

By

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This work is dedicated to my family, and especially my wife, Amy, for your love and support.

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The focus of this dissertation is on supply chain management (SCM), and more specifically on the upstream connection between a firm and its suppliers. My research examines single versus multiple supplier sourcing strategies under three specific scenarios. In general, my dissertation seeks to characterize when a buying firm should single source its requirements instead of employing a diversified purchasing policy under various commonly encountered operating scenarios.

First, the effects of upstream and downstream uncertainty on a firms sourcing strategy are examined. Our results show that order-splitting (i.e., choosing a multiple supplier strategy) is an optimal choice for the firm even when suppliers are completely heterogeneous in terms of their reliability and costs. Additionally, the choice of single versus multiple sourcing depends to some extent on supplier prices. This latter result motivates investigating our second scenario to gain insight into how alternate supplier pricing schemes may impact sourcing decisions.

The second scenario we examine is motivated not only through the results obtained under the stochastic supply setting described earlier but also through an understanding of the supplier selection and quantity allocation decisions made by

a major office products retailer located in Florida. The pricing schemes quoted by its suppliers tend to be either a constant price, a linearly discounted price, or a staged quantity discount (i.e., all-units and/or incremental discounted price) price. For each type of pricing scheme, we develop a unique optimization model where the objective is to minimize the sum of concave cost functions while satisfying the firms total requirements. We adapt existing branch and bound algorithms in order to identify the optimal number of suppliers who should receive an order.

Finally, we incorporate explicit diversification benefits (due to channel power leverage and price competition) into a newsvendor framework to analyze a firms sourcing decisions when suppliers are unreliable. Analysis reveals that a trade-off between the marginal benefit of diversification and the marginal cost of shifting allocated order quantities away from lower cost suppliers need to be assessed. Managerially, this model stresses the importance of consistency between a firms sourcing strategy and its corporate strategy.

CHAPTER 1
INTRODUCTION

1.1 Supply Chain Management-An Overview

A supply chain can be visualized as a network of firms servicing and being serviced by several other firms. However, it is conceptually easier to imagine a chain as a river, originating from a source, moving downstream and terminating at a sink. The supply chain extends upstream to the sourcing of raw materials and downstream to the afterlife activities of the product, such as disposal, recycling and remanufacturing. Regardless of magnitude, every supply chain can be visualized as consisting of sourcing stages, manufacturing stages and distribution stages.

Each of these stages plays both a primary (usually physical transformation or service creation) and a dual (market mediator) role. The approach taken to execute activities in support of both roles depends on the strategy of the supply chain, which in turn, is a function of the serviced products' demand pattern (Fisher, 1997). Depending upon the structure of the chain (in terms of products and processes employed), channel power can reside with the sourcing (e.g., monopolist supplier of key commodities such as oil), manufacturing (e.g., dominant producer of a unique product such as semiconductors), or distribution (e.g., key distributor of consumer items) stages in the supply chain. Relative power in the supply chain influences strategic positioning of each link in the chain. Thus, managing supply chains is a negotiation between the objectives of constituent's benefit at each stage and the impact of each constituent's objective to the overall objective of maximizing the benefit of the entire chain.

The contribution captured at each stage depends on the nature of the dealings between the buyer and supplier. The traditional model is characterized by an adversarial relationship where a buyer awards contracts to one or more competing suppliers based on price and other pertinent criteria. This paradigm has been widely criticized as short sighted by proponents of partnered buyer-supplier relationships. The partnered approach favors a smaller or even single supplier base for more supplier management initiatives to minimize inventory investments and encourage collaboration in, among other things, new product development.

Supply chain management (SCM) is the art and science of creating and accentuating synergistic relationships among the trading members that constitute supply and distribution channels. Supply chain managers strive to deliver desired goods or services on time to the appropriate place in the ordered quantity in the most effective and efficient manner. Usually this is achieved by negotiating a balance between conflicting objectives of customer satisfaction and cost efficiencies.

Each link in each supply chain represents an intersection where supply meets demand, and directing the product and information flows at these crossroads is at the core of SCM. The integral value proposition of an integrated supply chain is as follows. Total performance of the entire chain is enhanced when all links in the chain are simultaneously optimized as compared to the resulting total performance when each individual link is separately optimized. Supply chain performance as a whole hinges on achieving fit between the nature of the products it supplies, the competitive strategies of the interacting firms, and the overall supply chain strategy.

Coordination of the individual links in the chain is essential to achieve this objective. The ability of trading partners to jointly communicate in real time and the transactional ease of digital dealings allow web-connected firms to virtually integrate. The Internet and information technology in general facilitate the integration of multitudes of channel enterprises. On-line collaboration enables better informed economic decision making, reduces the costs of order placement, tracking and receipt, and enhances customer satisfaction.

Information technologies are a key driver of modern operational efficiency, and efficient operational execution is a driver of effective SCM. Selection of trading partners, location of facilities, manufacturing schedules, transportation routes and modes, and inventory levels and location are the fundamental operations decisions that run supply chains. These operational dimensions are the tributaries that pilot the chain downstream through its channel to end demand. Accurate and timely integrated information navigates the chain from source to sink.

A supply chain is a collection of multiple suppliers', manufacturers' and distributors' processes. Each process employs a distinct focus and a related dimension of excellence. Key issues in managing an entire supply chain relate to tactical and strategic analysis of coordinated decisions in logistics, manufacturing, distribution, and after sales activities of service and disposal or recycling; analyzing product strategies; and network design decisions.

The motivation for this research is derived from the debate as to the best number of suppliers to employ for satisfying a buyer's requirements. Further, the buyer considered is an intermediary in the supply chain and therefore must incorporate downstream demand into its sourcing decision. Essentially, the decisions analyzed address the question of whether a single sourcing strategy is optimal or not. To understand the relevance of strategic sourcing decisions, it must be understood how a firm's supply chain strategy is anchored to its sourcing strategy.

1.2 Strategic Issues in SCM

A supply chain is only as strong as its weakest link. How the chain defines strength is at the core of a supply chain's strategy, and therefore design. Is strength anchored in efficiency or responsiveness? Regardless of which strategic position is chosen, a firm's ability to maintain a competitive advantage will depend on how well it reinforces its firm level value proposition with functional and departmental strategic decision-making. By analyzing product demand characteristics and the supply chain's capabilities, and crafting a fit between them, an individual supply chain manager can be assured that the specific product and process strategy employed does not create dissonance within his firm and further throughout the entire supply chain.

1.2.1 Product Strategy

Achieving a tight fit between the competitive strategies of supply chain members and the supply chain itself is gained by evaluating the characteristics of the products serviced by the chain. "The root cause of the problems plaguing many supply chains is a mismatch between the type of product and the type of supply chain" (Fisher, 1997, p.106). Critical product attributes are (a) the demand pattern; (b) the life-cycle; (c) variety of offerings; and (d) the product delivery strategy. Fisher (1997) categorizes a product as being either functional (basic, predictable, long-lived, low profit margin) or innovative (differentiated, volatile, short-lived, high profit margin). Further, using the product life cycle argument, innovative products (if successful) will eventually evolve to become functional products. The types of supply chains needed to effectively service these two categories of products are quite distinct. An efficient or low cost supply chain is more appropriate for a functional product while a responsive or customer attuned supply chain better services an innovative product. Obviously, a spectrum of chain varieties exists between the end points of responsiveness and efficiency, and hence, most supply chains are hybrids which target responsiveness requirements for each product serviced while exploiting commonalities in servicing all products to gain economies of scope. Thus, the strategic position of a supply chain balances customer satisfaction demands and the firm's need for cost minimization.

Information technologies enable both efficient and responsive supply chains since they have the potential to provide immediate and accurate demand and order status information. Efficiency gains via information technologies are gleaned from decreased transactional costs resulting from order automation and easier access to information needed by chain members. Likewise, responsiveness gains can be obtained by a quicker response to customer orders. Hence, in practice, it seems to have become standard practice for all supply chains to utilize some form of information technology to enable not only a more efficient physical flow of their products but also to simultaneously improve their market mediation capability. However, the efficiency of physical flow primarily depends on a supply chain's infrastructure.

1.2.2 Network Design

In general, network design determines the supply chain's structure. The significant capital investments required in building such a structure indicate the relative long run or strategic importance of network decisions. Network decisions in a supply chain involve facility focus, facility location, capacity planning, and sourcing/distribution channels (Chopra and Meindl, 2001). Each network design decision impacts the firm's ability to provide value. Therefore, these decisions must incorporate their strategic influence into the analysis.

Facility focus relates to how network investments facilitate the supply chain strategy. If the facility in question is a manufacturing plant and the plant is set up to produce only a specific product type, the chain will be more efficient but less flexible than it would be if the plant produced multiple product types.

Facility location decisions are essential to a firm's strategy. The cost ramifications of a sub-optimal location decision could be substantial. Further, shutting down or moving a facility is significant not only in terms of financial resources, but also in terms of the impact on employees and communities. Other factors which should be considered are the available infrastructure for physical and information transportation, flexibility of production technologies employed, external or macroeconomic influences, political stability, location of competitors, availability of required labor and materials, and the logistics costs contingent on site selection.

Depending on the expected level of output for a facility, capacity allocations should be made so that idle time is minimal. Under-utilization results in lower return on investment and is sure to get the attention of company executives.
On the other hand, under allocating capacity (or large utilizations) will create a bottleneck or constricted link in the supply chain. This will result in unsatisfied demand and lost sales or increased costs as a result of satisfying demand from a non-optimal location. The capacity allocation decision is a relatively long-term commitment, which becomes more significant as sophistication and price of the production technology increase.

The most basic question of an enterprise is: Who will serve our needs and whose needs will we serve? This is a recurring question. Decisions regarding the suppliers to a facility and the demand to be satisfied by a facility determine the costs of material inputs, inventory, and delivery. Therefore, as forces driving supply and/or demand change, this decision must be reconsidered. The objective here is typically to match suppliers and markets to facilities in order to minimize not only the system-wide costs but also the customer responsiveness of the supply chain.

Each of these network design decisions is not made in isolation since there is a need to prioritize and coordinate their combined impact on the firm and its supply chain. In general, network configuration is the structure of the supply chain and it is within this structure that operations strategies and tactics are implemented to reinforce the overall strategy of the entire chain. Of particular relevance for this research is a firm's sourcing strategy.

1.3 Operational Issues in SCM

SCM has evolved from process reengineering efforts to coordinate and integrate production planning at the factory level in order to expand the scope of strategic fit (Chopra and Meindl, 2001). Positive results from these intra-functional efforts have extended the SCM philosophy throughout the enterprise. Further, process improvements at the firm level highlighted the need for suppliers and customers of supply chain managed firms to adopt an integrated SCM philosophy. Making a supply chain's linkages as frictionless as possible is the tactical goal of such an integrated philosophy. Key tactical coordination decisions for SCM relate to transportation, transformation, and information transmission.

1.3.1 Transportation

Transportation decisions impact product flow not only between supply chain members but also to the market place. In many supply networks, transportation costs account for a significant portion of total supply chain cost. In determining the mode(s) and route(s) to employ through the supply chain, transportation decisions seek to strike a balance between efficiency and responsiveness so as to reinforce the strategic position of the supply chain. For example, an innovative product's typically short life-cycle may warrant expensive air freight speed for a portion or all of its movement through the chain, while a commodity is generally transported by slow but relatively economical water or rail freight. Shipping via truck is also used frequently. Trucking is more responsive and more expensive than rail, and less responsive and less expensive than air. Most supply chains employ an intermodal strategy (e.g., raw materials are transported by rail or ship, components by truck, and finished goods by air).

A supply chain's transportation network decisions are inextricably linked to strategic network design decisions. Transportation network design choices drive routing decisions in the supply network. The major decisions are whether to ship directly to buyers or to a distribution center, and whether a routing scheme is needed. As consumers' expectations regarding merchandise availability and delivery become more instantaneous, the role of a supply chain's transportation network is more critical.

1.3.2 Transformation

"A transformation network links production facilities conducting work-in process inventories through the supply chain" (Erenguc, Simpson, and Vakharia, 1999, p.224). Suppliers linked to manufacturers linked to distribution systems can be viewed as a transformation network hinging on the manufacturer. Transforming supplies begins at the receiving stations of manufacturers. The configuration of manufacturing facilities and locations of transformation processes are determined by plant level design decisions. The manufacturing process strategy employed at a specific plant largely drives the decisions. While an assemble-to-order (ATO) plant may have very little investment in production, it requires larger investment in sub-assembly inventories. On the other hand, a make-to-stock (MTS) facility may have little or no investment in process inventories, it typically requires larger investments in raw materials and finished goods inventories. A make-to-order (MTO) facility may have significant investment in components and production facilities, with few raw materials and finished goods inventories. A product's final form can also take shape closer to the end consumer. To keep finished goods inventory costs as low as possible, and better match end demand, a supply chain may employ postponement to delay customizing end products.

Major design decisions such as facility configuration and transformation processes are considered longer term decisions. These decisions constrain the short to mid-term decisions addressed in a plant's aggregate plan. An aggregate plan is a general production plan that encompasses a specific planning horizon. Information required to develop an effective aggregate plan include accurate demand forecasts, reliable supply delivery schedules, and the cost trade-offs between production and inventory. Each supply chain member develops an aggregate plan to guide medium-term tactical decisions. To ensure that these individual plans support each other, the planning process must be coordinated. The degree and scope of coordination will depend on the economics of collaborative planning versus the costs of under-supply and over-supply. In general, a manufacturer should definitely involve major suppliers and buyers in aggregate planning. Whether this planning information trickles to other supply chain members (a key for the success of integrated supply chain management) will depend on the coordination capabilities of successive layers of members emanating from a collaborative planning center, which is often the major manufacturer.

The execution of the aggregate plan is a function of the information inputs into the aggregate plan. It is vital that these inputs be as accurate as possible throughout the entire supply chain. Integrated planning in a supply chain requires its members to share information. The initiator of integrated planning is typically an intermediary. To understand why, we must understand the dynamics of distribution.

1.3.3 Information Sharing

A distribution channel is typically composed of a manufacturer, a wholesaler, a distributor, and a retailer. The “bull-whip effect” is a classic illustration of dysfunction in such a channel due to the lack of information sharing. This effect is characterized by increasing variability in orders as the orders are transferred from the retailer upstream to the distributor, then to the wholesaler, and finally to the manufacturer. Distorted demand information induces amplifications in order variance as orders flow upstream. Therefore, the manufacturer bears the greatest degree of order variability. This is a major reason manufacturers initiate collaborative efforts with downstream channel members. To anticipate the quantity of product to produce and when, a manufacturer must compile demand forecasts from downstream supply chain members. Forecasting accuracy is paramount because it is the basis for effective and efficient management of supply chains.

A major challenge of SCM is to minimize costs and maintain flexibility in the face of uncertain demand. This is accomplished through capacity and inventory management. Similarly, marketers attempt to maximize revenues through demand management practices of pricing and promotion. Therefore, it is vital that marketing and operations departments collaborate on forecasts and share harmonious incentive structures. The degree of coordination between order acquisition, supply acquisition, and production process directly affects how smoothly a firm operates. Likewise, buyers', suppliers', and producers' coordination levels directly affect how smoothly the supply chain operates. Summarily, accurate information flows between channel members are essential for effective SCM.

1.4 Focus of this Research: Strategic Sourcing

The strategic importance of sourcing activities is inherent in purchasing's supply chain position. Purchasing activities link the firm to the upstream value system and allow a buying firm to obtain appropriate inputs from external suppliers. Procurement activities in large part support a firm's inbound logistics and are vital to value creation (Porter, 1985). A firm's sourcing strategy is therefore a key driver of an effective supply chain (value system).

Innovations in technology and increased global competition provide opportunities and challenges that drive firms to continuously evaluate and modify their sourcing strategies. Moreover, recent studies reveal that the long-term implications of poor supply chain management are far reaching, ultimately impacting both firm performance and market value. Since a typical manufacturing firm spends 55% of earned revenue on purchased materials (Leenders and Fearon, 1998), disruptions due to supply inadequacies could have a major impact on profitability. Hendricks and Singhal (2001) reinforce this by showing that in the 90 days prior and subsequent to a reported supply chain problem stemming from supplier glitches, the buying firm's average shareholder return typically decreases by 12%. Clearly, a manufacturer's operations strategy and financial livelihood rely on its chosen supplier pool and thus decisions with regard to suppliers are fundamental to successful enterprise.

A firm's sourcing strategy is characterized by three key interrelated decisions (Burke and Vakharia, 2002): (a) criteria for establishing a supplier base; (b) criteria for selecting suppliers (a subset of the base) who will receive an order from the firm; and (c) the quantity of goods to order from each supplier selected. To start with, criteria for developing a supplier base are typically based upon the firm's perception of the supplier's ability to fulfill the objectives of quality, quantity, delivery and price. While the supplier's price may be the most important criteria for profit maximization, the other dimensions can also affect the overall profitability of the firm. Scoring models are generally used to evaluate suppliers for inclusion in the base. In general, this approach ranks each supplier in terms of objectives and then, based on a relative weighting of each of the objectives, a total score for each potential supplier is derived. Next, by specifying a threshold score, all suppliers who achieve this threshold are included in the base. Objectives used in developing rankings vary across firms. For example, Sun Microsystems ranks its suppliers with a "scorecard" based on quality, delivery, technology, and supplier support (Holloway et al., 1996).

From the approved supply base, the specific subset of suppliers which will actually receive an order must be determined. Since all suppliers in the base meet the quality, delivery, and other objectives of the firm, dominant industry practice appears to base this decision primarily on cost considerations. While the supplier's price may be the most important criterion for profit maximization, some buying firms impose alternate criteria related to robust delivery reliability capabilities. Once the selected set of suppliers (a subset of the base) is determined, the firm must allocate product(s) requirements among them. For the allocation decision, supplier yields (in terms of percentage of “good” units), order quantity policies, lead times, and transportation costs are salient.

A firm may choose either a specialized (i.e., single supplier) or a generalized (i.e., multiple suppliers) strategic sourcing position, and allocation of requirements will seek to optimize the value to the firm from this decision-making process. Since it is the collective suppliers’ capabilities that can enable or limit supply chain performance at its inception, a firm’s sourcing strategy is vital to successful enterprise.

Single-sourcing strategies strive for partnerships between buyers and suppliers to foster cooperation and achieve shared benefits. The tighter coordination between buyer and supplier(s) required for successful just-in-time (JIT) inventory initiatives encourages supplier alliances to streamline the supply network and tends to shift supply relations toward single sourcing.

Managing more than one source is obviously more cumbersome than dealing with a single source. However, web-based SCM applications enable closer management of diverse suppliers, streamline supply chain processes and drive down procurement costs. For example, GM utilized Internet tools to purchase more of its total purchasing budget on-line, which resulted in a streamlined procurement process and decreased vehicle delivery times (Veverka, 2001). Other documented benefits of effectively utilizing web-based procurement tools include a reduction in price of materials, administrative costs, inventory costs, and purchase and fulfillment cycles. Consequently, firms that prefer single-sourcing for ease of management can embrace multiple-sourcing via information technology-based SCM applications as a more viable strategy to capture risk-pooling benefits. A shortcoming of Internet procurement tools is their limited capability to provide complex decision support for strategic sourcing decisions (Aberdeen Group, 1999). Nonetheless, the differential tactical or low-cost competitive advantages supported by single-sourcing over multiple-sourcing are diminished by the proliferation of Internet procurement capable firms.

Moreover, single-source dependency exposes the buying firm to a greater risk of supply interruption. Toyota's brake valve crisis in 1997 provides a recent example of realized supply risk resulting from a single sourcing strategy in a JIT inventory system. In 1997, Toyota's assembly plants were forced to shut down after a fire at Aisin's main plant. This single-supplier's particular facility provided 90% of all brake parts components and practically all brake valves for Toyota, before it was destroyed (Nishiguchi and Beaudet, 1998). It is estimated that the impact on Toyota's net income from this single event at Aisin was a decrease of $300 million. Thereafter, Toyota sought at least two suppliers for each part (Treece, 1997). Operationally, multiple-sourcing provides greater assurance of timely delivery, and greater upside volume flexibility due to the diversification of the firm's total requirements (Ramasesh, et. al., 1991).

Single-sourcing all of a firm's requirements also exposes the buying firm to hold-up risk. Land Rover's contractual problem with its only chassis supplier is an example of the operational difficulties this situation creates (Lester, 2002). Strategically, supplier power exerted on the buyer is weakened when the firm splits its total requirements among multiple sources. Multiple sourcing hedges the risks of creating a monopolistic (sole source) supply base and supplier forward integration (Newman, 1989). In sum, the choice of multiple-sourcing versus single-sourcing depends on the trade-off between the benefits of multiple-sourcing versus those of single-sourcing.

1.5 Organization of this Dissertation

To position this research in the broader area of supply chain management research, this chapter has provided an overview of supply chain management and strategic sourcing. The remainder of this proposal is organized as follows. Chapter two provides a comprehensive literature review of research in supply chain management that relates to supplier selection and requirements allocation decisions. In Chapters 3-5 we seek to characterize conditions under which a firm should choose either a specialized (i.e., single supplier) or a generalized (i.e., multiple suppliers) sourcing strategy under different scenarios. More specifically, chapter 3 incorporates the impact of variability in both supply and demand in addressing sourcing decisions; chapter 4 examines how the sourcing stategy is moderated when suppliers offer a variety of common quantity discount schemes; and chapter 5 integrates the impact of explicit diversification benefits on the sourcing decision. Chapter 6 provides a summary of key results from this research, and discussion of opportunities for future research.

1.6 Statement of Contribution

Research in the area of supply chain management continues to be active. This research focuses on the upstream connections of a buying firm to an approved suppplier base under various commonly encountered industrial operating conditions. We add to the existing body of knowledge in supply chain management by surveying closely related existing research, and developing and analyzing realistic models for strategic sourcing decision making. Our focus on optimal decision making in regard to the number of suppliers to requisition highlights the importance of risk mitigation through supplier diversification. In chapter 3 we illustrate this value to the firm by modeling the firm's purchase decision with both upstream and downstream uncertainty. we are able to analytically solve this sourcing problem for optimal solutions and show that in some scenarios cost dominates reliability.

Further, our analysis is uncommonly general as compared to existing work for this particular problem. In chapter 5 we explicitly consider strategic benefits of diversification with unreliable supply and downstream product demand uncertainty. This approach allows firm level flexibility in deriving the most strategically desired size of the selected supplier pool, and gauges the marginal cost trade off of selecting higher unit cost supply against the imputed benefits of supply diversification. Chapter 4 considers strategic behavior of capacitated suppliers via quantity discount pricing quotes. We develop constant, linear discount, incremental discount and all-units discount pricing schemes sourcing models. For the discount pricing models we find that they are concave minimization problems and combinatorial in nature. We develop an optimal branch and bound algorithm and well-performing heuristics to aid in strategic sourcing decision making in this operating environment. In all this research provides guidance for practitioners and academics concerned with supplier management in regards to the frequent decision making tasks of choosing the appropriate number of suppliers to source from and their respective order quantity allocations.

CHAPTER 2
LITERATURE REVIEW

2.1 Overview

The first section in this chapter describes the evolution of the role of sourcing from a purely transactional pursuit to one of strategic influence. Next, a review of literature examining the buyer-supplier relationship reveals the divide between proponents of single sourcing and multiple sourcing. The literature reviewed in the final section focuses on supplier qualification, supplier selection, and quantity allocation criteria.

2.2 Strategic Evolution of Sourcing

Ellram and Carr (1994) provide a history and review of literature related to sourcing's strategic importance. They note that even as the oil crisis of 1973-1974 highlighted the perils of raw material supply shortages, research on industrial buying behavior largely viewed the purchasing function as administrative. It was not until the 1980s when Porter's Five Forces model gained popularity that the strategic role of the interface between suppliers and buyers was better understood. This has led to more contemporary research investigating the strategic impact of sourcing as an integrative link between the firm and its suppliers.
As the interface between suppliers and the firm, purchasing's influence on firm performance increases as supplier contribution to the firm increases. Further, the inclusion of purchasing (sourcing) strategy in corporate strategy is more vital as global competition and the pace of technological change quickens. A firm's supplier management orientation is reflected in its contracting policies for external purchases. Cohen and Agrawal (1999) model the trade-offs between short term and long term contractual relationships. Short term contracts provide more flexibility

and avoid fixed investments, but also forgo improvement and price certainty benefits afforded from long term contracts. Their analysis reveals that short term contracting is optimal under a wide range of conditions. In a survey of supply managers they find that management intend to develop long term relationships, but often engage in short term contracting. This conflict over the optimal buyer-supplier relationship is not exclusive to the ranks of supply managers.

2.3 Buyer-Supplier Relationships

Research on the number of sources for fulfillment of product requirements is somewhat controversial. At one extreme, we have empirical evidence of many firms shrinking their supplier base per item and ordering the majority of total units required from a single source (e.g., Spekman (1988) and Pilling and Zhang (1992)). Further, the documented benefits of single-sourcing such as quantity discounts from order consolidation, reduced order lead times, and logistical cost reductions as a result of a scaled down supplier base. Hahn et al., (1986), and Bozarth et al., (1998) reinforce this evidence. In fact, Mohr and Spekman (1994) contend that single-sourcing performance benefits often outweigh the benefits of a price centric multiple-sourcing strategy. These benefits are also enhanced by incorporating performance improvement criteria in managing supplier relationships (Fawcett and Birou, 1992).

In contrast, Bhote (1987) observed that relationship management costs, in terms of time and capital, may outweigh the benefits of single-sourcing. The primary rationale driving this argument is that single-sourcing requires the firm and the supplier to develop a partnership based on trust. In line with this reasoning, McCutcheon and Stewart (2000) assert that the parties must achieve goodwill trust to have a successful partnership. Further, they conjecture that this level of trust is rarely attained. Adversarial history between channel members and competition for larger shares of a product's limited total margin drive distrust. A significant amount of research exists on procurement competitions where supply base size and order allocation is of concern. Elmaghraby (2000) and Minner (2003) both offer excellent overviews of the supply chain literature relevant to requirements allocation decisions.

2.4 Strategic Sourcing

As previously stated, a firm's sourcing strategy can be characterized by three key interrelated decisions (Burke and Vakharia, 2002): (a) criteria for establishing a supplier base; (b) criteria for selecting suppliers (a subset of the base) who will receive an order from the firm; and (c) the quantity of goods to order from each supplier selected. Literature related to these key decisions is now reviewed.

2.4.1 Qualification Criteria

Quality and delivery are strategically important supplier qualification criteria. In the context of these criteria, as a firm's supply management orientation increases, supplier and buyer performance increases (Shin, et al., 2000). This reinforces Dickson's (1966) survey of vendor selection criteria, which ranks quality and delivery, respectively, as the first and second most important supplier attributes. Similarly, in a survey of companies at different levels of the U.S auto industry, Choi and Hartley (1996) found that price is one of the least important factors for supplier selection across the supply chain, while consistency (defined as the marriage of quality and delivery) was the most important criterion. A review of prior research rates quality as the single *extremely* important criterion, while delivery is of *considerable* importance (Weber et al., 1991).

Measurements of quality are difficult to generalize across industries. Garvin (1987) outlines several dimensions of quality, including performance, features, reliability, conformance, durability, serviceability, aesthetics and perceived quality. In practice, a firm may use quality measures to screen out all suppliers who fail to meet some pre-specified minimal quality level. For example, Hillview Hospital utilized such a policy in eliminating potential suppliers for hand soap based upon the soap's ability to eradicate infection-causing germs (Maurer, 1997). Through a qualification process, an approved supply base can be assembled to ensure minimal capabilities on key dimensions.

2.4.2 Selection and Allocation

As noted earlier, supplier selection is concerned with identifying the subset of qualified suppliers who will be considered for order placement, and allocation focuses on splitting the required quantity between the selected suppliers. Obviously these decisions are interdependent and are also driven by the total delivered costs to the firm of an order quantity from each supplier. Pan (1989) proposes a linear programming model to optimally identify the number of suppliers and their respective quantity allocations to meet pre-specified product requirements. Other constraints incorporated are related to aggregate incoming quality, lead times, and service level. The overall objective is to minimize the price per unit as a weighted average of selected suppliers' prices. It is assumed that product requirements are deterministic and supply is reliable and unlimited. In reality, however, it is common for suppliers to quote alternative pricing schemes and uncertainty exists in both supply and demand markets. Since the focus of this dissertation is on incorporating these considerations in making supplier selection and quantity allocation decisions, prior research relating to this area are reviewed next.

Analytical studies on supplier selection and quantity allocation decisions show that in certain cases, multiple-sourcing, order-splitting, or diversification is preferable to single-sourcing. Horowitz (1986) provides an economic analysis of dual sourcing a single input at differing costs. It is shown that uncertainty in supply price and risk-aversion of the buyer motivate a firm to place positive orders from the high cost seller. Kelle and Silver (1990) investigate a continuous review inventory policy replenished by suppliers with stochastic delivery lead-times, and find that order-splitting among multiple sources reduces safety stock without increasing stockout probability. Ramasesh et al. (1991) also analyze a reorder point inventory model with stochastic supply lead-time, and find that in the presence of low ordering costs and highly variable lead-times, dual sourcing can be cost preferable.

Gerchak and Parlar (1990) examine second-sourcing in an EOQ context to reduce the effective yield randomness of a buying firm's purchase quantity. The benefits of diversification are traded-off against the costs of managing a larger supply base to determine whether second-sourcing is worthwhile. They also analyze the optimal number of identical sources. Rosenthal et al. (1995) introduce a mixed integer programming model for solving a supplier selection problem with bundling. The suppliers are capacitated, offer different prices, differing quality levels, and discount bundles. Agrawal and Nahmias (1997) examine a single period supplier selection and allocation problem with normally distributed supply and deterministic demand for a single product with fixed ordering costs. They are able to show that for two non-identical suppliers, the expected profit function is concave in the number of suppliers.

Parlar and Wang (1993) compare the costs of single versus dual-sourcing for a firm assuming that the overall objective is to minimize purchasing and inventory related costs. In their approach, they assume that actual incoming quantities are a function of a random variable representing the yield. Separately using an EOQ and newsboy based ordering policy, they are able to show that in certain cases dual-sourcing dominates single-sourcing. Both of these studies ignore the supplier capacity issue in making supplier selection and quantity allocation decisions. Further, Parlar and Wang (1993) note that supplier yields and demand uncertainty play a critical role in the analysis.

Other analytical studies similar to this research examine supplier selection and order allocation decisions with stochastic demand for the product purchased. Gallego and Moon (1993) employ Scarf's ordering rule for a distribution free optimal newsboy order quantity. They maximize profit against the worst possible distribution of demand with known mean and variance. Separate extensions incorporate a second purchasing opportunity, fixed ordering costs, random yields, and multiple items into the analysis. In particular, the case of random supplier yields assumes that each unit supplied has the same probability of being good, and the buyer pays for all units. Bassok and Akella (1991) introduce the Combined Component Ordering and Production Problem (CCOPP). The problem is one of selecting ordering and production levels of a component and a finished good for a single period with supply and demand uncertainty. In their model the distribution of supply depends on the order quantity given to a single source.

Anupindi and Akella (1993) consider a two supplier, single product procurement problem with stochastic supply and demand. They suggest that minimum order quantity policies of suppliers may affect their findings. Gurnani et al. (2000) simultaneously determine ordering and production decisions for a two component assembly system facing random finished product demand and random yield from two suppliers, each providing a distinct component. They also consider a joint supplier option and determine the value to the assembler of reliable component supply. Diversification occurs when positive orders are placed with the joint supplier and individual component supplier. They show that if there are not mismatched initial inventories of components, each component will be single sourced. Kim, et al. (2002) model a capacitated manufacturer's supply base configuration problem for multiple parts used to produce multiple products with independent stochastic demands. They develop an iterative algorithm to determine the manufacturer's end product mix and specify the suppliers who will be used to satisfy the parts requirements for each end product. In their model suppliers are capacitated and reliable. Yano and Lee (1995) also offer a review of the normative literature which addresses the random yield problem.

Order quantity or lot sizing decisions can be largely influenced by alternative supplier pricing schemes. Quantity discounts, especially all-units and incremental, are common pricing practices. In a quantity discount schedule, the range of potential purchase quantities is segmented by quantity break points. Order quantities that fall between two break points qualify for a specific unit price. Typically the larger the quantity ordered, the lower is the unit price. The key difference between an all-unit discount and incremental schedule is that in the former, all units ordered are supplied at the unit price, while in the latter; only the number of units in a specific break point are supplied at the unit price. Suppliers offer these types of schedules to encourage buyers to procure larger quantities and to reap the operating advantages associated with these larger quantities (such as economies of scale). Both the buyer and supplier can realize higher overall profits by making decisions jointly. SCM coordination research in part resides in the research efforts for joint optimal supplier pricing schemes and buyer lot size decisions that consider quantity discounts.

Operations literature takes a coordinated cost minimization approach for analyzing the impact of pricing schemes on supply chains. Munson and Rosenblatt (2001) analyze a three-level distribution channel for a single product where a supplier provides an order quantity discount schedule to a manufacturer. The manufacturer then proposes an order quantity discount schedule downstream to a retailer. The manufacturer solicits and proposes order quantity discounts upstream and downstream respectively, and is termed the channel captain for his dictatorial role. As such, the manufacturer retains all channel savings from coordinated quantity discounts. Numerical experiments provide a 40% increase in manufacturer savings from two way order quantity discounts over retailer only quantity discounts. Further, optimizing over both supplier and retailer simultaneously results in savings greater than the sum of optimizing over each individually.

Weng (1995) develops an integrated model of quantity discounts and channel coordination. In his model, operating costs are a function of purchase quantity and demand is a function of selling price. In this scenario, profit increases result from cost reductions and demand enhancement. However, a quantity discount scheme alone is not enough to achieve joint profit maximization. A fixed payment or franchise fee in concert with a quantity discount schedule is necessary to motivate system optimal decision making.

Abad (1988a, 1988b) also incorporates price dependent demand into lot-sizing models with alternate supply acquisition schemes. Optimal lot size and selling price are simultaneously solved under linear and constant price-elastic demand. A supplier offers an all-unit quantity discount in Abad (1988a) and an incremental quantity discount in (1988b). An iterative procedure is developed to handle the lot-size and selling price interdependency. Burwell et al. (1997) extend the work of Abad by developing an optimal lot sizing and selling price algorithm for a single item given supplier offered quantity and freight discounts and price dependent downstream demand. Numerical results indicate that the maximized profit with all-units dominates the maximized profits under incremental and mixed quantity and freight discounts.

It is clear from the above studies that order quantity discounts can influence decision making at each supply chain link regardless of analytical perspective. Benton and Park (1996) provide a thorough review of the lot-sizing literature with quantity discount considerations for time phased as well as non-time phased demands. Their taxonomy further classifies research based on the type of discount (all-units or incremental) and the perspective of the modeling effort (buyer or supplier-buyer). The scope of this dissertation is one of non-time phased demand from a buyer perspective. As such, the remainder of this section reviews literature with similar scope.

Benton (1991) uses Lagrangian relaxation to evaluate a purchasing manager's resource constrained order quantity decisions given alternative pricing schedules from multiple suppliers. The example decision maker has a limited budget and storage space for ten items offered by three vendors, each quoting three all-units discount levels for each item. The objective is to minimize total acquisition and inventory costs. The manager must choose a single supplier for all items. However, if multiple sourcing is allowed, the optimal objective is 8% lower than the single source optimum. Rubin and Benton (1993) use Lagrangian relaxation to formulate a separable dual problem. A branch and bound algorithm with a best bound branching rule is developed to close any duality gap. To allow multiple sourcing among the multiple items, a merged discount schedule is constructed which quotes the lowest price among all suppliers for each quantity of each item. A shortcoming of this model is that there are no order cost savings from consolidated delivery or order placement. As an extension to this work, Rubin and Benton (2003) analyze the same purchasing scenario, except that suppliers quote incremental quantity discount schedules instead of all-units. A similar solution methodology is employed. They note the best feasible solution obtained from their relaxation algorithm should be acceptable for use unless a large duality gap exists. Further, they observe that in numerical studies the primal cost minimization objective is rather flat. This implies that good solutions are rather easy to find, but fathoming is slow. Thus, an optimal solution search is probably more costly than beneficial.

Another Lagrangian relaxation based heuristic is developed by Guder et al. (1994) to solve a buyer's multiple item material cost minimization problem with incremental discounts offered by a single supplier. The buyer has a single resource constraint and demands for items are independent. The complexity of the problem lies in evaluating all feasible price level sequences. For large problems with n items and m price breaks, the optimal price level sequence can be obtained in O(nm). For 100 items with 8 price breaks each, numerical experiments result in precise solutions quickly. Their heuristic is adapted from Pirkul and Aras (1985) which analyzed the all-units version of this problem.

The majority of research on purchase decisions involving alternate pricing schemes assumes that demand for the purchased product is known. Modeling efforts that consider stochastic downstream demand and upstream quantity discounts are Jucker and Rosenblatt (1985), Khouja (1996a,b), and Lin and Kroll (1997). All of these analyze the implications of quantity discounts on the newsboy problem. Jucker and Rosenblatt (1985) develop a marginal analysis based algorithm to minimize total costs of purchasing and transportation with separate but simultaneous purchasing and transportation quantity discounts. Khouja (1996a,b) implements a bisection method to solve for a retailer's optimal order quantity with supplier offered all-units discounts and progressive multiple retail discounts. Lin and Kroll (1997) develop a solution technique for solving a risk constrained newsboy problem with all-units or incremental supplier pricing schemes.

As evidenced by the proliferation of research on integrating supply chains and adoption of SCM practices in industry, the importance of supplier management on a firm's competitive strategy is now widely accepted. A firm's supplier management approach is characterized by its sourcing strategy. This research provides managerial insight by examining the influence of alternate pricing schemes from suppliers, uncertainty in product supply and demand, and explicit treatment of diversification benefits on a firm's strategic sourcing decisions. Of particular interest is characterizing when it is optimal for a firm to single-source versus multiple-source.

Our modeling and analytical approaches to these problems of supplier selection and order quantity allocation position this research among the existing literature reviewed in this chapter.

CHAPTER 3
STRATEGIC SOURCING DECISIONS WITH STOCHASTIC SUPPLIER RELIABILITY

3.1 Introduction

Supplier sourcing strategies are a crucial factor driving supply chain success. While many firms utilize a single supplier for a particular item, others diversify their supply risk by sourcing from multiple suppliers. In particular, a firm's allocation decision determining an appropriate supplier set and order allotment impacts on all competitive dimensions for the delivery of finished goods to its customers, including cost, quality, reliability and flexibility. Benefits of a single supplier strategy have been touted in the popular press, such as JIT replenishment and increased quality levels. More recently, the benefits of supplier diversification as a possible defense against supply disruption has gained attention. In this chapter, we investigate the implications of uncertain supplier reliability on a firm's sourcing decisions in an environment with uncertain demand. In particular, we characterize circumstances under which a firm should diversify its orders amongst several suppliers to increase its total profit, rather than utilizing a single supplier sourcing strategy.

The modeling framework developed in this chapter is similar to Anupindi and Akella (1993). They analyze both single and multi-period models with uncertain supplier reliability and stochastic demand. In particular, in a two supplier setting, they characterize scenarios under which it would be optimal to source from one vs. two suppliers based on initial inventory levels. While their general results are derived for the two supplier case with no specific distributional assumptions, they also analyze specific scenarios utilizing an exponential demand distribution, and normal and gamma distributions to reflect supplier reliability. Parlar and Wang (1993) also consider a two supplier newsboy model with random yields. They obtain first order optimality conditions, and establish the concavity of the objective. Furthermore, they utilize a linear approximation to obtain optimal order quantities for each supplier.

Another closely related work is Agrawal and Nahmias (1997). They analyze a variation of the single period newsvendor model where demand is actually deterministic, but supplier reliabilities are normally distributed. In addition, they examine the optimal number of suppliers and corresponding order quantities assuming that a fixed order cost is incurred for each supplier with a positive order. Optimal policies for N suppliers are derived for the case of homogenous suppliers (i.e., all suppliers have similar reliability distributions and costs). For the case with heterogeneous supplier reliability and similar supplier costs, they show optimality conditions for two suppliers and conjecture that these also hold for cases with more than two suppliers.

3.2 Sourcing Model

In this section, we introduce the model notation and variables. First, we introduce the decision variables. Let N reflect the number of available suppliers. We assume that this set has been pre-qualified such that they all meet minimum sourcing standards set by the firm. The key decision variable in the model is to determine the number of units to purchase from supplier i, where ($i = 1, \ldots$,N). Although we do not include an explicit variable reflecting the optimal number of suppliers, it can be determined implicitly by identifying the number of suppliers with a non-zero order quantity.

Next, we introduce the supplier specific parameters. We assume that each supplier quotes a unit cost of c_i. In addition, the firm has some knowledge of the historical reliability in terms of the number of good units delivered for each supplier. We treat this quality or yield reliability for each supplier as a random variable, r_i. Let $g_i(r_i)$ denote the continuous probability density function associated with yield for each supplier i. We also assume that the density function is twice differentiable with $\bar{r}_i$ and σ_i representing the mean and standard deviation.

Finally, several firm specific parameters typically associated with a single-period newsboy framework are relevant here. We assume that the total demand is unknown but represented by a stochastic parameter x with a probability density function $f(x)$ and a cumulative distribution function denoted by $F(x)$. Also, the unit price (p), salvage value (s) and underage costs (u) are also assumed to follow standard assumptions associated with a newsboy model, including $p > c_i > s$. A complete list of the model variables and parameters is included in Table 3-1 below.

Table 3–1: Model Notation

Variable	Description
N	Number of suppliers available
q_i	Number of units to purchase from supplier i (decision variable)
x	Demand (random variable)
f(x)	Probability density function associated with demand
F(x)	Cumulative distribution function associated with demand
μ	Mean demand
a	Min value parameter for uniform demand distribution
b	Max value parameter for uniform demand distribution
c_i	Unit cost for supplier i
r_i	Yield for supplier i (random variable)
$g_i(r_i)$	Probability density function associated with yield for supplier i
$G_i(r_i)$	Cumulative distribution function associated with yield for supplier i
$\bar{r}_i$	Mean yield factor for supplier i
σ_i	Standard deviation of the yield factor for supplier i
V_i	Second moment of the yield factor for supplier i
p	Unit price
s	Unit salvage value
u	Unit underage cost
c_i	Unit cost for supplier i
Q	Total number of good units received
K_i	Critical ratio derived from cost for supplier i
b_i	Uniform demand adjusted by critical ration for supplier i
b’	Uniform demand adjusted critical ration for homogeneous suppliers

3.3 Model Development

The objective of the firm is to determine the appropriate order quantities for each supplier such that the expected profit associated with satisfying demand is maximized. Utilizing the framework from the traditional newsboy problem (Silver, Pyke and Peterson 1998), the objective function in Equation (3.1) maximizes the single period (season) expected profits for the firm. Note that we assume that the buying firm only pays suppliers for "good"' units delivered. In addition, a non-negativity constraint (Equation 3.2) is also included in our formulation.

Max $E(\pi) =$

$$\int_0^1 g_1(r_1) \int_0^1 g_2(r_2) ... \int_0^1 g_N(r_N) \left[\int_0^Q (px - \sum_{i=1}^{N} c_i r_i q_i + s(Q - x)) f(x) dx \right] dr_N ... dr_2 dr_1$$
$$+ \int_0^1 g_1(r_1) \int_0^1 g_2(r_2) ... \int_0^1 g_N(r_N) \left[\int_Q^\infty (pQ - \sum_{i=1}^{N} c_i r_i q_i - u(x - Q)) f(x) dx \right] dr_N ... dr_2 dr_1 \tag{3.1}$$

subject to

$$q_i \geq 0 \quad \forall i \tag{3.2}$$

where $Q = \sum_{i=1}^{N} r_i q_i$

3.4 Analysis

In this section, we describe the optimal supplier sourcing strategy under certain conditions. To make our analysis more tractable, we assume that demand is uniformly distributed with parameters $[a, b]$.[1] To support subsequent analysis, we first ensure that a key result holds.

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[1] Given that price per unit p is assumed to be fixed, it is reasonable to assume that demand around this fixed price is uniformly distributed.

Corollary 3.1: The expected profit function shown in Equation (3.1) is concave in the order quantities q_i for N suppliers when demand is uniformly distributed between $[a, b]$.

Proof: See APPENDIX A.

It is interesting to note that Agrawal and Nahmias (1997) were unable to prove concavity of the profit function for N heterogeneous suppliers with normally distributed reliability functions and deterministic demand. However, by assuming that demand is uniformly distributed, we can derive a closed form expression for optimal supplier order quantities making no distributional assumptions for supplier reliability.

We start our analysis for the general case of heterogeneous suppliers. Then we analyze the case where suppliers are homogeneous with respect to the reliability distributions but have differing cost structures. Next, we consider the case where the reverse is true (i.e., all suppliers have homogeneous cost structures but different reliability distributions). Finally, we analyze the case where all suppliers are completely homogeneous.

3.4.1 Heterogeneous Suppliers

When all suppliers have heterogeneous cost and reliability functions, a key result is that there is no one dominant sourcing strategy. That is, under certain circumstances, it will be optimal to place a single order with the lowest cost supplier; under other circumstances, an order is placed with a subset of the suppliers. However, it is possible to analytically determine the exact order quantities for each supplier as shown in Theorem 3.1 below.

<u>Theorem 3.1</u>: When suppliers are heterogeneous with respect to costs and reliability parameters, then the optimal sourcing quantity for each supplier i is:

$$q_i^* = \frac{\frac{\bar{r}_i}{\sigma_i^2}\left[\sum_{j=1}^{N}(b_i - b_j)\left(\frac{\bar{r}_j}{\sigma_j}\right)^2 + b_i\right]}{\sum_{j=1}^{N}\left(\frac{\bar{r}_j}{\sigma_j}\right)^2 + 1} \tag{3.3}$$

where $b_i = \ K_i(b-a)+a$, and $K_i = \frac{p+u-c_i}{p+u-s}$.

<u>Proof</u>: See APPENDIX A.

Of course, the result of Theorem 3.1 needs to be moderated to account for a negative order quantity (i.e.,, ${q_j}^*$ as computed above in Equation (3.3) is less than zero for some supplier j). Later, we analyze the case including the non-negativity constraints. We now focus on further characterizing the situation where a single supplier sourcing strategy is appropriate. Although the results of Theorem 3.1 do not provide guidelines into a generalizable optimal sourcing strategy, there are certain specific insights for the heterogeneous supplier case which can be obtained as discussed below.

<u>Corollary 3.2: </u> The firm will always order a positive quantity from the lowest cost supplier.

<u>Proof</u>: See APPENDIX A.

The key insight stemming from this result is that supplier cost structures dominate the reliability distributions for selection. Based on this, we can derive additional insights into optimal sourcing strategies for the firm. We first start by indexing suppliers ($i = 1, .., N$) in increasing order of costs (i.e., $c_1 \leq c_2 \leq c_3 \leq \ldots c_N$). Corollary 3.3 below specifies when a single supplier sourcing strategy is optimal for the firm.

Corollary 3.3: The firm will source all its requirements from the lowest cost supplier (i.e., use a single sourcing strategy) if and only if:

$$\left(\frac{\sigma_1}{\bar{r}_1}\right)^2 < \frac{b_1 - b_j}{b_j} = \frac{(c_j - c_1)(b-a)}{(p+u-c_j)b + (c_j - s)a} \; for \; j = 2, ..N \qquad (3.4)$$

Proof: See APPENDIX A.

The expression on the left hand side (LHS) of Equation (3.4) reflects the coefficient of variation for the first supplier, while the expression on the right hand side (RHS) of Equation (3.4) reflects the cost differential between the first and any of the j^{th} suppliers. Based on this result, a sensitivity analysis of the key parameters in the LHS and RHS of Equation (3.4) leads us to conclude that the single sourcing strategy is an optimal choice when:

1. The mean reliability of the low cost supplier is relatively high, $\bar{r}_1$;
2. The standard deviation of reliability for the low cost supplier is relatively low, σ_1;
3. The coefficient of variation for the reliability of the low cost supplier is relatively low, $\frac{\sigma_1}{\bar{r}_1}$;
4. The difference in costs between the low cost supplier (i.e., supplier 1) and the next highest cost supplier (i.e., supplier 2) is relatively large;
5. The minimum demand parameter is relatively high, a;
6. The maximum demand is relatively low, b;
7. The mean demand is relatively low, $\frac{a+b}{2}$;
8. The spread in demand is relatively high, $(b-a)$;
9. The unit price is relatively low, p;
10. The unit underage cost is relatively low, u;
11. The unit salvage value cost is relatively high, s.

First, consider the impact of supplier reliability on the supplier sourcing decision. When the coefficient of variation of the first supplier is relatively small, then the first supplier will optimally receive the complete order. Note that the reliability distribution is such that most values are less than 100%. Consequently, this necessitates that $\sigma \leq \bar{r} < 1$. For example, Agrawal and Nahmias (1997) assume that $3\sigma < \bar{r}$. Therefore, in many cases, the coefficient of variation in the left hand side of Equation (3.4) is fairly small such that the equation holds and a single supplier strategy is appropriate.

A surprising feature of the relationship shown in Equation (3.4) is that the expression is independent of the reliability distribution of suppliers 2,..,N. Specifically, the only parameter associated with every other supplier that impacts on the single vs. dual sourcing decision is the unit cost. Therefore, if a supplier has relatively higher costs than other suppliers, then he/she will not receive an order from the firm.

Another key parameter driving the single supplier decision is the demand distribution, as shown in parts (e)-(h) above. In particular, higher levels of mean demand lead a firm to diversify its total order and source from multiple suppliers. However, a counter-intuitive result concerns the impact of the variance or spread in the demand on the optimal sourcing policy. Note that the $(b - a)$ expression in the right hand side of Equation (3.4) is reflective of the variability of demand. From part (h), higher levels of variability in demand are associated with a single supplier strategy. Therefore, it appears that there are some interesting interactions between the uncertainty in demand and uncertainty in supply. When the uncertainty in demand is low, then it is optimal for a firm to hedge against the uncertainty in supply by diversifying its total orders amongst several suppliers. Conversely, when the uncertainty in demand is high, then the firm limits its financial risk and optimally relies only on the single lowest cost supplier.

Now, consider the case where Equation (3.4) does not hold, and a multiple supplier sourcing strategy is optimal. From Corollary 3.2, we know that the lowest cost supplier will always receive a positive order. Corollary 3.4 discusses the order quantity for the lowest cost supplier relative to the other suppliers by simply stating that, although the lowest cost supplier will receive an order, the actual order size may be lower than other higher cost suppliers. To illustrate, consider the case where a dual supplier strategy is optimal. In this situation, if the standard deviation of the first lowest cost supplier is fairly high, then the second higher cost supplier may receive a higher order.

Corollary 3.4: When a multiple supplier sourcing strategy is optimal, the lowest cost supplier will not necessarily receive the highest order quantity.

Proof: See APPENDIX A.

The logic used in the two supplier example follows through in determining an optimal subset of suppliers which receive a positive order quantity. Corollaries 3.5 and 3.6 summarize the importance of supplier cost in determining this optimal subset of suppliers.

Corollary 3.5: A higher cost supplier will never receive a positive order when a lower cost supplier's order quantity is equal to zero.

Proof: See APPENDIX A.

Corollary 3.6: The optimal subset of suppliers (n^*) receiving a positive order quantity is the lowest cost subset of suppliers such that the following relationships hold:

$$b_{n*} > \frac{\sum_{j=1}^{n*-1} b_j \left(\frac{\bar{r}_j}{\sigma_j}\right)^2}{1 + \sum_{j=1}^{n*-1} \left(\frac{\bar{r}_j}{\sigma_j}\right)^2} \quad and \quad b_{n*+1} \leq \frac{\sum_{j=1}^{n*} b_j \left(\frac{\bar{r}_j}{\sigma_j}\right)^2}{1 + \sum_{j=1}^{n*} \left(\frac{\bar{r}_j}{\sigma_j}\right)^2} \tag{3.5}$$

Proof: See APPENDIX A.

3.4.2 Heterogeneous Cost Suppliers

To extend the basic results derived from the analysis of heterogeneous suppliers, we now consider the situation where supplier reliability is homogeneous, but the costs are not. In particular, let each supplier have a cost of c_i and reliability parameters $r_i = r$, $g_i = g$, $G_i = G$, and $\sigma_i = \sigma$, $\forall i$. Theorem 3.2 characterizes the optimal solution under these conditions.

Theorem 3.2: When suppliers (a) have *heterogeneous* cost structures and (b) have *identical* reliability distributions, then there is no one dominant supplier sourcing strategy. The optimal order quantity for each supplier i is:

$$q_i^* = \frac{\bar{r}\left[\bar{r}^2 \sum_{j=1}^{N} (b_i - b_j) + b_i\sigma^2\right]}{N\bar{r}^2\sigma^2 + \sigma^4} \quad \forall i \tag{3.6}$$

Proof: See APPENDIX A.

While the firm optimally places an order with the lowest cost supplier, the orders for the remaining suppliers are determined via cost differentials and variance reduction. The results of Theorem 3.2 are similar to those of Theorem 3.1, in that the lowest cost supplier will receive a positive order, while the others may not.

Anupindi and Akella (1993) analyze the two supplier case with stochastic demand and supplier reliability where one supplier has a relative cost advantage. They show that when the initial on-hand inventory falls below some minimal level, it is optimal for the firm to source from both suppliers. In contrast, Theorem 3.2 and Equation (3.4) for a N supplier setting with uniform demand shows that, in the absence of capacity constraints, there are circumstances under which it is never optimal to order from more than one supplier when reliability distributions of suppliers are identical.

3.4.3 Heterogeneous Reliability Suppliers

In this case, we focus on a situation where supplier costs are roughly equivalent (i.e., $c_i = c \forall i$), but each supplier has a unique historical reliability function associated with the number of goods actually delivered. Theorem 3.3 characterizes the first order conditions of optimal sourcing strategy for this case.

Theorem 3.3: When suppliers (a) have *identical* cost structures and (b) have *unique* reliability distributions, then it is optimal to order from all suppliers. The optimal order quantity for each supplier i is:

$$q_i^* = \frac{\frac{\bar{r}_i}{\sigma_i^2} b'}{\sum_{j=1}^{N} \left(\frac{\bar{r}_j}{\sigma_j}\right)^2 + 1} \quad \forall i \tag{3.7}$$

where $b' = K(b-a) + a$, and $K = \frac{p+u-c}{p+u-s}$.

Proof: See APPENDIX A.

From the expressions for the optimal order quantity, the firm optimally orders different non-zero order quantities from all suppliers. Furthermore, it seems that the supplier reliabilities directly impact on the order quantities such that the buying firm realizes the diversification benefits. The order quantity for a particular supplier depends not only upon its unique reliability function, but also on the reliability of the other available suppliers. This optimal order quantity for an individual supplier increases in response to the following: (a) an increase in the mean reliability of that supplier, (b) a decrease in the standard deviation of reliability of that supplier, (c) a decrease in the mean reliability of other suppliers, and (d) an increase in the standard deviation of reliability of other suppliers. Consequently, the order quantity for the first supplier is adjusted for the uncertainty in the reliability of other suppliers. Further, a firm optimally sources from the entire supplier pool for this case.

3.4.4 Homogeneous Suppliers

Finally, consider the scenario where the suppliers are roughly equivalent in costs and reliability expectations. Let $c_i = c$, $g_i = g$, $G_i = G$, $r_i = r$, and $\sigma_i = \sigma$, $\forall i$. Theorem 3.4 characterizes the optimal solution under these conditions.

Theorem 3.4: When suppliers (a) have *identical* cost structures and (b) have *identical* reliability distributions, then it is optimal to order the same amount from all suppliers. The optimal order quantity for each supplier is:

$$q_i* = \frac{\bar{r}b'}{\sigma^2 + N\bar{r}^2} \quad \forall i. \tag{3.8}$$

Proof: See APPENDIX A.

This result indicates that it is optimal for the firm to diversify its supply base and place an equal order from every qualified supplier. Furthermore, it is obvious that the total quantity sourced by the firm increases in the number of available suppliers (N). This result concurs with those derived by Agrawal and Nahmias (1997) for a setting with normally distributed reliabilities and constant demand.

Corollary 3.7: Suppose m of the N suppliers (with $m < N$) have identical cost and reliability distributions. Then, the same order quantity should be placed with all m suppliers.

Proof: See APPENDIX A.

Comparing the results of Theorems 3.1-3.4, these advocate different sourcing strategies. From Theorems 3.1 and 3.2, if suppliers have unique unit costs, then a single supplier solution may be optimal. Conversely, from Theorems 3.3 and 3.4, a multiple supplier solution is always optimal such that the firm reaps the benefits of diversification. Hence, it appears that single sourcing strategies could be optimal only when there are differences in costs across suppliers. On the other hand, if costs across suppliers are identical then multiple sourcing strategies are an optimal choice regardless of the suppliers' reliability distributions.

In the next section, we conduct a numerical analysis to further explore some of the interactions between supplier reliability and firm demand and also to investigate the impact of minimum order quantities on optimal sourcing strategies.

3.5 Numerical Analysis

3.5.1 Experimental Design

We utilize a set of parameter values which satisfy the assumptions of the newsvendor model. For the buying firm, we let $p = 19$, $s = 2$, and $u = 6$. Demand is uniformly distributed over [300,700] and thus the corresponding mean demand (i.e., $(a + b)/2$) is 500 while the spread of demand (i.e., $(b - a)$) is 400. For these examples, we consider a possible supplier set (each meeting minimal qualification criteria) that consists of three suppliers. In addition, we assume that the supplier reliabilities are also uniformly distributed over the range $[M_i$ - L_i /2, $M_i + L_i$ /2] with a mean of M_i and a spread of L_i. Furthermore, we assume that all three suppliers have similar reliability distributions with a mean of 0.7 and a spread of 0.1. For the base case examples, we assume that the suppliers have not specified any minimal order quantities. Later, we investigate the impact of such constraints on the optimal solutions.

Three sets of examples are included which investigate the impact of changes in various parameters given that the supplier set has certain cost structures. The first two sets of examples illustrate the case where the suppliers have heterogeneous costs, while the third set of examples illustrates the case where the suppliers have homogeneous costs. Note that the heterogeneous suppliers are always rank ordered such that the first supplier has the lowest cost and the last supplier has the highest cost. For the first set of examples, the supplier costs are set at $c_1 = 6.75$, $c_2 = 7.00$, and $c_3 = 7.25$. For the second set of examples, the supplier costs are set at $c_1 = 6.95$, $c_2 = 7.00$, and $c_3 = 7.05$. For the third set of examples, the supplier costs are equal with $c_1 = c_2 = c_3 = 7.00$. The experimental design for the numerical examples is contained in Table 3-2.

We refer to the first example for each set (i.e., Examples 1, 2 and 3) as the base case example which reflects the parameter values discussed here. Within

Table 3–2: Description of Numerical Examples

Example	Cost Type	c_1,c_2,c_3	Parameter Changed	Parameter Value
1	Heterogeneous	[6.75,7,7.25]	Base Case 1	
1A	Heterogeneous	[6.75,7,7.25]	Mean Demand	(a+b)/2= 5200
1B	Heterogeneous	[6.75,7,7.25]	Spread Demand	(b-a) = 800
1C	Heterogeneous	[6.75,7,7.25]	Mean Reliability Supplier 1	M_1 = .5
1D	Heterogeneous	[6.75,7,7.25]	Spread Reliability Supplier 1	L_1 = .5
1E	Heterogeneous	[6.75,7,7.25]	Min Order Quantity Supplier 1	$minq_1$ =1000
2	Heterogeneous	[6.95,7,7.05]	Base Case 2	
2A	Heterogeneous	[6.95,7,7.05]	Mean Demand	(a+b)/2= 5200
2B	Heterogeneous	[6.95,7,7.05]	Spread Demand	(b-a) = 800
2C	Heterogeneous	[6.95,7,7.05]	Mean Reliability Supplier 1	M_1 = .5
2D	Heterogeneous	[6.95,7,7.05]	Spread Reliability Supplier 1	L_1 = .5
2E	Heterogeneous	[6.95,7,7.05]	Min Order Quantity Supplier 1	$minq_1$ =1000
3	Homogeneous	[7,7,7]	Base Case 3	
3A	Homogeneous	[7,7,7]	Mean Demand	(a+b)/2= 5200
3B	Homogeneous	[7,7,7]	Spread Demand	(b-a) = 800
3C	Homogeneous	[7,7,7]	Mean Reliability Supplier 1	M_1 = .5
3D	Homogeneous	[7,7,7]	Spread Reliability Supplier 1	L_1 = .5
3E	Homogeneous	[7,7,7]	Min Order Quantity Supplier 1	$minq_1$ = 300
3F	Homogeneous	[7,7,7]	Min Order Quantity All	minq = 300

each set of examples, we vary one specific parameter relative to those used in the base case. The first column in Table 3-2 contains the example reference with the corresponding number (i.e., 1-3) denoting the particular set the example belongs to, and the corresponding letter (i.e., A-F) denoting a parameter variation relative to the base case example for that set. The second column specifies the type of cost structure for the suppliers (i.e., heterogeneous vs. homogeneous), while the third column identifies the particular unit costs used for each example. Finally, the fourth and fifth columns describe the parameters and their corresponding values that are changing relative to the base case example. To illustrate, in Example 1A the three suppliers have heterogeneous costs ($c_1 = 6.75$, $c_2 = 7$, and $c_3 = 7.25$) and expected demand (i.e., $(a + b)/2 = 5200$) much higher than that of the base case.

3.5.2 Results

Table 3-3 contains a summary of the results of the numerical examples. The particular performance metrics included are the optimal order quantity for each supplier, the total units ordered, the number of suppliers which receive a positive order, and the firm profit. In the remainder of this section, we discuss highlights from these numerical examples.

Table 3–3: Results for Numerical Examples

Example	Parameter Changed	q1*	q2*	q3*	Total	n*	Profit
1	Base Case 1	880	0	0	880	1	$5,353
1A	Mean Demand	5619	1968	0	7587	2	$61,751
1B	Spread Demand	1048	0	0	1048	1	$4,604
1C	Mean Reliability Supplier 1	1231	0	0	1231	1	$5,335
1D	Spread Reliability Supplier 1	174	700	0	875	2	$5,218
1E	Min Order Quantity Supplier 1	0	874	0	874	1	$5,199
2	Base Case 2	803	73	0	876	2	$5,230
2A	Mean Demand	3259	2529	1798	7586	3	$61,183
2B	Spread Demand	1038	0	0	1038	1	$4,458
2C	Mean Reliability Supplier 1	759	333	0	1092	2	$5,220
2D	Spread Reliability Supplier 1	60	772	42	874	3	$5,202
2E	Min Order Quantity Supplier 1	0	802	72	874	2	$5,199
3	Base Case 3	292	292	292	876	3	$5,211
3A	Mean Demand	2529	2529	2529	7586	3	$61,158
3B	Spread Demand	346	346	346	1037	3	$4,430
3C	Mean Reliability Supplier 1	249	349	349	946	3	$5,210
3D	Spread Reliability Supplier 1	17	429	429	875	3	$5,208
3E	Min Order Quantity Supplier 1	300	288	288	875	3	$5,211
3F	Min Order Quantity All Suppliers	438	0	438	875	2	$5,208

First, consider the impact of the supplier cost structures as shown in the base case examples. Examples 1 and 2 confirm Theorem 3.2 for the case with heterogeneous costs and homogeneous reliabilities. In Example 1, the cost differentials are significant enough such that the firm optimally places a single order with the lowest cost supplier. In contrast, Example 2 shows the situation where the cost differentials are small enough such that the firm optimally places an order with the two lowest cost suppliers. Example 3 essentially illustrates the results shown in Theorem 3.4 for homogeneous suppliers. Specifically, the total order is equally split among all suppliers. A comparison of the optimal profit levels for these examples shows that the cost benefits associated with having the lowest cost supplier ($c_1=$ 6.75) available outweigh the diversification benefits. Specifically, the profit for Example 1 is higher than that of Examples 2 or 3.

Second, the impact of changes in demand on the optimal supplier sourcing strategies confirms the analytic results shown in Corollary 3.3. In Examples 1A and 2A, the optimal number of suppliers with a positive order increases in response to higher levels of mean demand. Therefore, if a firm anticipates a significant increase in demand, it should consider enlarging its supplier base. In Example 1B, the firm still sources from a single supplier, but the total order quantity is increased to buffer against demand uncertainty. In Example 2B, the firm decreases the number of suppliers which receive a positive order to only 1, but it increases its total order size to that single supplier. For the case with homogeneous suppliers, the optimal number of suppliers remains the same but the total order quantity increases in response to either an increase in the mean demand or an increase in demand variability. To summarize, the firm optimally decreases the number of suppliers receiving a positive order and increases the total order quantity in response to higher levels of demand uncertainty.

Third, consider the impact of supplier reliability on the optimal sourcing strategy. In general, it appears that the mean supplier reliability impacts the corresponding order quantity, but does not affect the optimal number of suppliers. The spread in supplier reliability, however, impacts on both the corresponding order quantities and the optimal number of suppliers when the supplier costs are heterogeneous. In Examples 1C, 2C, and 3C, when the mean reliability of the first supplier is reduced, the firm optimally lowers its order quantity to the first supplier but sources from the same number of suppliers. For the case with heterogeneous costs (i.e., Examples 1D and 2D), in response to an increase in the spread associated with the reliability for supplier 1, the firm optimally lowers its order quantity to the first supplier and increases the number of suppliers that receive positive orders. For the case with homogeneous costs (i.e., Example 3D), the order quantities are adjusted downward for the supplier with increased reliability uncertainty while the order quantities for other suppliers are adjusted upward. Recall that for the case with homogeneous supplier costs, it is always optimal to source from the full pool of suppliers. Therefore, an increase in supplier reliability uncertainty does not impact the optimal number of suppliers which receive positive orders.

Next, consider the impact of supplier dictated minimum order quantities on the firm's optimal sourcing strategy. Note that a minimum order quantity is essentially analogous to a fixed order cost associated with a particular supplier. Therefore, the minimum order quantity can be considered a proxy for costs associated with maintaining the buyer/supplier relationship. Examples 1E and 2E illustrate the situation where the lowest cost supplier also has a fairly high minimal order quantity (i.e., min q_1=1000). In this situation, the lowest cost supplier no longer receives an order from the firm. Instead, the firm optimally sources from an alternative set of suppliers which have slightly higher costs, thus forgoing the stringent minimum order quantity restrictions associated with the low cost supplier. Interestingly, the optimal number of suppliers remains the same as the base case for these examples. In Example 1E, the firm optimally sources the entire order from supplier 2 instead of supplier 1. In Example 2E, the firm optimally sources from suppliers 2 and 3 instead of suppliers 1 and 2. The total profit for both of these examples decreases relative to the corresponding base cases because the firm no longer sources from the lowest cost supplier.

The effect of the minimum order quantity is slightly different for the case with homogeneous suppliers. When one of the suppliers has a significant minimum order quantity, the firm still sources from all three suppliers for a total order quantity and profit which are essentially the same as the base case, (see Example 3E). However, the distribution of orders is changed such that the order to the restricted supplier is increased and the orders to the remaining suppliers are decreased. When all suppliers have a similar minimum order quantity which is significant, then the optimal number of suppliers receiving a positive order is reduced, and the results from Theorem 3.3 no longer hold. In Example 3F, the optimal number of suppliers is decreased such that the optimal order quantity for each supplier slightly exceeds the minimal restriction.

Finally, consider the performance of the homogeneous set of suppliers as a result of changes in supplier related factors. In Examples 3C-3F, the individual order quantities for each supplier changes in response to changes in the mean supplier reliability, the spread in supplier reliability, and supplier dictated minimum order quantities. However, the resulting profit remains very close to that of the base case shown in Example 3. For Examples 3D-3F, the total order quantity is also similar to the base case. Therefore, it appears that the scenario with homogeneous suppliers is fairly robust to changes in an individual supplier's reliability or minimum order quantities. This result may have some bearing on risk averse decision maker operating in an environment where consistency in performance and output is desirable.

3.6 Conclusions

This chapter provides structural and numerical results for determining an appropriate supplier sourcing strategy in the presence of upstream and downstream uncertainty. A firm's sourcing strategy is characterized by three interrelated decisions: (1) the criteria for qualifying as an approved vendor; (2) the supplier(s) selection from the approved base for order placement; and (3) the order quantities to place with each selected supplier. Our analytical results directly address the second and third decisions, while our numerical results offer managerial guidelines for making the first decision.

In the context of the supplier selection decision, our results are in line with observed practice. For example, Verma and Pullman (1996) find that while supply managers recognize the importance of quality, cost primarily drives their supplier selection decisions. In our model, a supplier's cost (and not its reliability) is the key factor which comes into play when a firm is deciding whether or not to place an order with that supplier. Consequently, the lowest cost supplier in the pre-qualified pool will always receive a positive order. An exception to this rule shown through numerical examples is when the lowest cost supplier has a restrictively high minimum order quantity. It follows that if all pre-qualified suppliers have similar costs, then it's optimal to place an order with all suppliers in the pool.

When addressing the second sourcing decision, a firm may decide to take an extreme approach and limit its supplier selection to only a single supplier. In the chapter, we derive a simple ratio to analytically determine whether or not a single supplier strategy is appropriate. This ratio reflects a trade-off between the first supplier's reliability <u>and</u> its cost advantage relative to other suppliers. Essentially, if the lowest cost supplier has a reliability distribution with a high mean and a low standard deviation, and has a large cost advantage, then a single supplier strategy is warranted.

Another key factor influencing the single supplier sourcing decision is the firm's anticipated demand. Both analytic and numerical results confirm that a single supplier strategy is favorable when the mean demand is low. However, if a firm anticipates a significant increase in demand, it should consider enlarging its supplier base even when the low cost supplier could provide the full order quantity. Surprisingly, an increase in the variability in demand also favors a single sourcing strategy. In this case, the firm limits its financial risk by sourcing only with the single lowest cost supplier when the firm anticipates great uncertainty in demand. Other factors contributing to a single supplier sourcing strategy are also discussed in the paper.

The third sourcing decision concerns how much to order from each selected supplier. Analytic expressions are developed which determine the optimal order quantity for each selected supplier under a variety of circumstances. The most general case is shown in Theorem 3.1 which addresses the situation where all suppliers have different costs and reliability functions. In this case, each supplier will receive an order amount based on its unit cost, mean reliability, and variance in reliability. Note that while the lowest cost supplier is guaranteed to receive a positive order, he/she won't necessarily receive the largest order. In contrast, we also analyze the situation where the suppliers are homogeneous in their costs and reliability functions in Theorem 3.4. In this situation, all pre-qualified suppliers receive an equivalent order quantity.

Finally, the first sourcing decision addresses what criteria a firm should use to pre-qualify a set of suppliers. While there are many factors impacting this decision that are not addressed here, we can extrapolate some insights concerning the appropriate number of suppliers that should be pre-qualified. The fixed costs of qualifying a supplier to ensure that it meets a minimal set of criteria based on quality, costs, and delivery can be exorbitant. It follows that a firm should judiciously target a set of suppliers which supports its anticipated sourcing strategy. For the set of numerical examples that we considered, sourcing only from a single supplier with very low costs resulted in a higher profit than other multi-supplier sourcing strategies. Therefore, if a firm anticipates that a single supplier has much lower costs, then it might be better off to focus on qualifying that single supplier to ensure that the supplier's reliability, quality and delivery are sufficient to meet demand. Essentially, additional efforts can be made to ensure consistent reliability of the lowest cost supplier.

However, if several suppliers are very close in cost, the company should consider qualifying them all such that they meet specific reliability criteria. Indeed, the numerical examples show that our results for the case with homogeneous suppliers are fairly robust with respect to changes in demand and supplier reliability. Specifically, when all suppliers in the pre-qualified pool have similar costs and reliability, then the total quantity ordered and the total firm profit is fairly constant even when demand and supplier reliability factors change.

There are several future areas of research related to this model which warrant further investigation. First, a more detailed model could be developed which addresses the first supplier sourcing decision, or appropriate criteria for qualifying suppliers. Second, we assume that demand is uniformly distributed to facilitate the development of simplified expressions. Other types of demand distributions could be explored to enhance the generalizability of the results derived here. Lastly, the focus of this model is on decision making at the buying firm; future research could incorporate the supplier's decision making process as well.

CHAPTER 4
IMPACT OF SUPPLIER PRICING SCHEMES AND CAPACITY ON SOURCING STRATEGIES

4.1 Introduction

Consider the following procurement process followed by a major office products retailer with headquarters in Florida, USA. The centralized purchasing organization (CPO) for the retailer is responsible for the procurement of all commodity type products which are sold through the retail outlets. Orders for these types of products are placed by the CPO on a periodic basis and in general, a four step process characterizes the procurement process. First, for each time period, the CPO aggregates the total estimated requirements for each product based on input from the retail outlets. Second, using a web-based interface (which is set-up to allow access only to pre-qualified suppliers), the CPO posts timing and quantity requirements for the commodity product(s). Third, suppliers respond by quoting prices and quantity limitations, if any, for delivering the products to the central warehouse maintained by the CPO. Finally, a CPO analyst analyzes the supplier submitted information and allocates requirements to suppliers.

The final step of this process for supplier selection and order placement was biased towards selecting a single supplier who could supply the entire set of requirements rather than an explicit focus on costs quoted by the suppliers. The primary motivation for such a strategy (i.e., single sourcing) was that it was easier to manage order receipts from one supplier at the central warehouse. In a more recent analysis of supplier responses to the CPO posting of requirements and timeing information, it was found that suppliers were starting to offer quote pricing schemes and thus, the supplier selection and order placement decision was becoming increasingly difficult.

This scenario motivates this chapter's analysis of supplier selection and order placement decisions in the presence of alternative supplier pricing schemes. It is well known that sourcing decisions in this setting are extremely complex and also require a frequent reassessment. For example, suppliers often offer discount schedules to induce larger purchases by offering progressively lower unit prices for progessively larger purchase quantities. Even for a single product's purchasing decision, if it is available from many vendors, each with various qualifying order sizes, identifying the optimal selected supplier set and corresponding quantity allocations is a difficult decision. Further complicating matters is that decisions must often be made quickly and with limited information due to time pressures (Rubin and Benton, 1993). This fast paced decision environment is not conducive to optimally solving combinatorial problems by complete enumeration. Therefore, heuristic procedures that produce optimal or near optimal feasible solutions are of significant value for decision makers.

More specifically, this chapter examines supplier selection and quantity allocation decisions for acquisition of a single product's total requirements from a pool of suppliers offering quantity discount schedules. We examine these decisions for environments where suppliers offer constant, linear discount, incremental discount, and all-units discount pricing schemes. The remainder of this paper is organized as follows. In the next section, we review the relevant prior literature. Section 3 develops our general sourcing model for each of the pricing schemes and due to the complexities associated with this model, we propose heuristics for obtaining feasible solutions very quickly. In Section 4, we analyze the performance of these heuristics through an extensive numerical analysis and this is followed by an application of our approach to data obtained from the office products retailer in

Section 5. A branch and bound algorithm for finding a minimum cost solution to the supplier selection and quantity allocation problem with incremental discounts is developed and stated in section 6. Finally, Section 7 discusses implications and conclusions of our research.

4.2 Sourcing Model

4.2.1 Preliminaries

Our analysis of the supplier sourcing decisions focuses on a single product, single period analysis of a system consisting of N suppliers ($i = 1, \ldots, N$) and a single buying firm. In general, we assume the following three-stage sequential decision framework for our analysis. At the first stage, the firm F communicates the total quantity of the single product (Q) which it will procure from the suppliers. Following this, in the next stage, each supplier i discloses a pricing scheme ($f_i(\cdot)$) and a related maximum quantity which it can provide to the firm (y_i). After receiving this information, the firm makes the supplier sourcing decision (q_i) for each supplier in the third stage.

We model the problem for a buying firm which is either a channel intermediary with a fixed quantity contract from a set of downstream firms or a manufacturer making procurement quantity decisions using automated materials planning systems (such as MRP). In both cases it is reasonable to assume that the buying firm can declare with reasonable certainty the total quantity Q to be procured from the suppliers. As noted earlier, zero fixed ordering costs for the buying firm are being assumed in line with our motivating example of the office products retailer.

There are three additional factors that need to be clarified in the context of our analysis. First, each supplier is an independent operator and hence, there are no opportunities for supplier collusion/collaboration in our setting. Second, the pricing scheme disclosed by each supplier (i.e., $f_i(\cdot)$) is all inclusive and includes the logistics/transportation cost. Finally, supply lead times are assumed to be relatively constant and thus, are not incorporated in our analysis. In the next section, we describe the alternative supplier pricing schemes (and in some cases, the related supplier capacity) parameters which are investigated in this paper.

4.2.2 Supplier Pricing Schemes and Capacity

There are several types of supplier pricing schemes which are used in the marketplace. These schemes are primary drivers for analyzing the sourcing decisions since they have a direct impact on firm-level profits. Further, supplier capacity also can drive some of these pricing schemes. On reviewing prior literature in the field, the four distinct supplier pricing schemes which we incorporate along with capacity consideration are:

1. *Constant Price*

   Under this scheme each supplier i discloses a constant price per unit (i.e., $f_i(q_i) = c_i$) it can provide the firm within the capacity range $[0, y_i]$.

2. *Linear Discount Price*

   Under this scheme each supplier i discloses a linearly declining price in the quantity q_i purchased from supplier i. Thus, $f_i(q_i) = a_i - b_i q_i$ (a_i, $b_i > 0$) within the capacity range $[0, y_i]$. We also assume that the linearly discounted pricing scheme disclosed is such that $a_i - b_i y_i > 0$ for each supplier i.

3. *Incremental Units Discounted Price*

   Under this scheme each supplier i discloses the traditional incremental units discounting scheme (Nahmias, 2001) which is dependent on the quantity q_i purchased from supplier i. To specify such a scheme, we first define $k = 1, \ldots, K_i$ as the index for discount classes offered by supplier i. Corresponding to each discount class k for a supplier i, define $[l_{ik}, u_{ik}]$ as the minimum and maximum quantities for the class such that $u_{iK_i} = y_i$. Based on these definitions, the pricing scheme can be specified as

$f_i(q_i) = \sum_{j=1}^{k-1} w_{ij}(u_{ij} - u_{ij-1}) + w_{ik}q_i$ if $l_{ik} \leq q_i \leq u_{ik}$ for $k = 1, \ldots, K_i$. It is assumed that $w_{i1} > w_{i2} > ... > w_{ik}$ for each supplier i.

4. *All Units Discount Price*

   Under this scheme each supplier i discloses the traditional all-units discounting scheme (Nahmias, 2001) which is dependent on the quantity q_i purchased from supplier i. As with the incremental units scheme, let $k = 1, \ldots, K_i$ as the index for discount classes offered by supplier i, and corresponding to each discount class k for a supplier i, define $[l_{ik}, u_{ik}]$ as the minimum and maximum quantities for the class, respectively, such that $u_{iK_i} = y_i$. Based on these definitions, the pricing scheme can be specified as $f_i(q_i) = v_{ik}q_i$ if $l_{ik} \leq q_i \leq u_{ik}$ for $k = 1, \ldots, K_i$. It is assumed that $v_{i1} > v_{i2} > \ldots > v_{iK_i}$ for each supplier i.

Next, we formulate the sourcing model for each pricing scenario and provide analytical/experimental insights into the sourcing strategy under each case.

4.3 Analysis and Insights

4.3.1 Constant Price

Given that $f_i(q_i) = c_i$ for each supplier i, our sourcing model for this case is as follows:

$$\text{Minimize } \mathrm{Z}_C = \sum_{i=1}^{n} c_i q_i \tag{4.1}$$

subject to:

$$\sum_{i=1}^{n} q_i = Q \tag{4.2}$$

$$0 \leq q_i \leq y_i \quad \forall i \tag{4.3}$$

Analysis of this model leads us to the following theorem which characterizes the optimal sourcing strategy for this case.

Theorem 4.1:Under constant supplier prices, the optimal sourcing policy for the firm is:

- Index suppliers in non-decreasing order of prices (i.e., $c_1 \leq c_2 \leq \ldots \leq c_n$).
- If $y_1 \geq Q$, then source the complete requirement Q from supplier, i.e., $q_i^* = Q$ and $q_j = 0 \; \forall \; j = 2, \ldots, n$.
- If $y_1 < Q$, then the following algorithm determines the optimal sourcing strategy for the firm:
  1. Set i=1.
  2. Order $q_i = \min\{Q, y_i\}$ units from supplier i. If $q_i = 0$, then set $q_j = 0$ $\forall j = i+1, \ldots, n$ and Stop, else goto 3.
  3. Set $Q = Q - q_i$, $i = i + 1$ and repeat 2.

Proof: See APPENDIX B.

The results of this theorem show that under this scenario, the optimal sourcing strategy will always choose to source from the lowest cost supplier. Further, multiple sourcing is optimal only when the lowest cost supplier does not have adequate capacity.

4.3.2 Linear Discount Price

Given that $f_i = a_i - b_i q_i$ for each supplier i, our sourcing model for this pricing scheme is:

$$\text{Minimize } Z_L = \sum_{i=1}^{n} (a_i - b_i q_i) q_i \tag{4.4}$$

subject to:

$$\sum_{i=1}^{n} q_i = Q \tag{4.5}$$

$$0 \leq q_i \leq y_i \quad \forall i \tag{4.6}$$

For the case where each supplier has adequate capacity (i.e., $y_i \geq Q \; \forall i$) to meet the aggregate requirement Q, it is obvious that the firm will choose to source the complete requirement Q from supplier j such that $a_j Q - b_j Q^2 = \min\{a_i Q - b_i Q^2 | 1 \leq i \leq n\}$. Thus, in this case, the single sourcing strategy is an optimal choice.

On the other hand, if there is at least one supplier i such that $y_i < Q$, there is no guarantee that this type of single sourcing strategy is optimal. However, by observing that the objective function is strictly concave in q_i, a general result which can characterize an optimal solution for our sourcing model under this pricing scheme is as follows.

Result 4.1: There exists at least one optimal solution to our sourcing model such that $q_i = 0$ or $q_i = y_i$ for all i suppliers except that there may be at most one supplier j for which $0 < q_j < y_j$.

Proof: See APPENDIX B.

Given that the optimal solution is difficult to obtain explicitly, we propose a heuristic that builds upon this result. The algorithm first starts by ranking suppliers in non-decreasing order of the total costs or average costs assuming that each supplier is given an order for the maximum he/she can supply or firm's requirements. Next, the first phase of our procedure considers all suppliers in this list in a sequential mannner. If the supplier capacity is less than the firm requirements, we place an order for the maximum quantity the supplier can deliver. Then we update the requirements for the firm based on this allocation, and again rank the remaining suppliers in non-decreasing order of total or average costs and the process repeats itself until the remaining requirements are zero. In the final phase of the heuristic, we consider switching the partial order quantity among all suppliers who have been allocated a positive order quantity. Details of our heuristic are as follows.

1. Define the active supplier set, Ω, as consisting of all suppliers.
2. For each supplier i included in Ω, compute $r_i = a_i - b_i(min\{Q, y_i\})$.
3. Rank suppliers in increasing order of r_i and index suppliers in this ranked list [1],[2],...,[N].
4. Set $j = 1$.

5. Set $q_{[j]} = min\{Q, y_{[j]}\}$. Remove supplier j from Ω.

6. Set $Q = Q - q_{[j]}$. If $Q > 0$ go to 2, else go to 7.

7. For all suppliers k, with $q_k > 0$, explore all possible improvements in the solution by switching the partial order quantity within this selected supplier set.

8. Store the best solution as Solution A

9. Repeat the above process, except at step 2, calculate $r_i = \min\{Q, y_i\} * (a_i - b_i(min\{Q, y_i\}))$, and in step 6, store the best solution as Solution B.

10. Choose solution A or Solution B based on the better objective function value.

To evaluate the solution quality of this heuristic, we carried out numerical experiments by randomly generating 30 test problems (APPENDIX B). For every problem, the supply base size, N, was set to 10, and the total requirements, Q, equals 2000 units. Further, for each supplier i, the pricing and capacity parameters were randomly generated from a uniform distribution as follows: base price, a_i $\tilde{\text{U}}$[0,200]; price elasticity, b_i $\tilde{\text{U}}$[0,1]; and capacity, y_i $\tilde{\text{U}}$[0,1000].

The results of this evaluation are presented in Table 4-1. The gap reported in this table is the percentage deviation of the heuristic solution from the optimal solution obtained using LINGO. As can be seen, in 20 of the 30 problems, the heuristic solution was optimal. For the remaining 10 problems, the worst case heuristic solution gap was 5.78%, while the average gap was 1.81%. Further all the heuristic solutions were obtained is less than 1 second using a PC (with 256 KB RAM, a Pentium III Processor, and 550 Mhz clock speed) while to obtain the optimal solutions using LINGO on a similar machines, the run times were between several minutes and a over 2 hours. This leads us to conclude that our heuristic provides good quality solutions in reasonable computation times.

Table 4–1: Linear Discount Heuristic Performance

Instance	Heuristic	Optimal	Gap %		Instance	Heuristic	Optimal	Gap %
1	88282.77	88282.77	0.00%		16	168636.10	168636.10	0.00%
2	103315.00	103315.00	0.00%		17	120547.20	120457.20	0.00%
3	128455.30	128455.30	0.00%		18	98735.40	98583.63	0.15%
4	89650.40	88850.70	0.90%		19	94898.40	94898.40	0.00 %
5	129274.70	127915.70	1.06%		20	174835.10	174835.10	0.00%
6	130886.20	128455.30	1.89%		21	39921.43	39921.43	0.00%
7	58198.44	58198.44	0.00%		22	45553.20	43856.09	3.87%
8	79593.48	79593.48	0.00%		23	89040.30	88585.22	0.51%
9	79593.48	79593.48	0.00%		24	111166.30	111166.30	0.00%
10	119205.40	119205.40	0.00%		25	66280.70	66051.12	0.35%
11	79593.48	79593.48	0.00%		26	86099.20	81393.94	5.78%
12	42926.90	41538.80	3.34%		27	53897.25	53897.25	0.00%
13	110474.80	110474.80	0.00%		28	119360.00	119360.00	0.00%
14	69594.40	69444.00	0.22%		29	55034.56	55034.56	0.00%
15	174675.70	174675.70	0.00%		30	195287.90	195287.90	0.00%
Mean Gap			0.60%		Worst Gap			5.78%

4.3.3 Incremental Units Discount Price

In this case, $f_{ik}(q_{ik}) = \sum_{j=1}^{k-1} w_{ij}(u_{ij} - u_{ij-1}) + w_{ik}(q_{ik} - l_{ik})$ if $l_{ik} \leq q_{ik} \leq u_{ik}$ for $k = 1, \ldots, K_i$. The sourcing model for this case is:

$$\text{Minimize } Z_{IU} = \sum_{i=1}^{n} \sum_{k=1}^{K_i} f_{ik}(q_{ik}) y_{ik} \tag{4.7}$$

subject to:

$$\sum_{i=1}^{n} \sum_{k=1}^{K_i} q_{ik} = Q \tag{4.8}$$

$$\sum_{k=1}^{Ki} y_{ik} \leq 1 \quad \forall i \tag{4.9}$$

$$l_{ik} y_{ik} \leq q_{ik} \leq u_{ik} y_{ik} \quad \forall i, k \tag{4.10}$$

$$y_{ik} \in [0, 1] \tag{4.11}$$

For the case where each supplier discloses a pricing scheme such that $u_{iK_i} \geq Q$ $\forall i$, to meet the aggregate requirement Q, it is trivial to show that the firm will choose to source the complete requirement Q from supplier j such that $w_{jk} =$

$\min_{1\leq i\leq n;1\leq k\leq K_i}\{w_{ik}|l_{ik} \leq Q \leq u_{ik}\}$. In line with the prior pricing schemes, the single sourcing strategy is an optimal choice for this situation.

As with the linear discount pricing scheme, the dominance of the single sourcing strategy is questionable when <u>every</u> supplier does not have adequate capacity to meet the aggregate requirements Q. In this case, again, we note that result 4.1 stated above for the linear discount pricing scheme still holds since our objective function is piecewise linear concave in q_i. Based on this, we propose the following heuristic algorithm for obtaining solutions to the sourcing model under this pricing scenario.

1. Define the active supplier set, Ω, as consisting of all suppliers.
2. For each supplier i included in Ω,
   - If $u_{iK_i} < Q$, compute $r_i = f_{iK_i}(u_{iK_i})/u_{iK}$, and set $q_i = u_{iK_i}$,
   - otherwise,
     - DO $k = 1, 2, ..., K_i$
     - If $l_{ik} \leq Q \leq u_{ik}$, $r_i = f_{ik}(Q)/Q$ and $q_i = Q$.
     - END
3. Rank suppliers in increasing order of r_i and index suppliers in this ranked list [1],[2],...,[N], and set $j = 1$.
4. Set $Q = Q - q_{[j]}$. Remove supplier j from Ω. If $Q > 0$ go to 2, else go to 5.
5. For all suppliers k, with $q_k > 0$, explore all possible improvements in the solution by switching the partial order quantity within this selected supplier set.
6. Store the best solution as Solution A
7. Repeat the above process, except at step 2, calculate in the first *if* statement $r_i = f_i(u_{iK_i})$, and in the DO loop $r_i = f_{ik}(Q)$; and in Step 6, store the best solution as Solution B.
8. Choose solution A or Solution B based on the better objective function value.

To evaluate the solution quality of this heuristic, we carried out numerical experiments by randomly generating 30 test problems (APPENDIX B). As with the linear discount pricing case, for every problem, the supply base size, N, was set to 10, and the total requirements, Q, equals 2000 units. Further, for each supplier i, the number of price breaks, upper and lower bounds of quantities for each price break, and unit prices for each price break were randomly generated from a uniform distribution as follows: base price, K_i $\tilde{\mathrm{U}}$[0,10]; $l_{i1} = 0$, and $l_{ik} = u_{ik} + 1$ for $k = 2, \ldots, K_i$; u_{i1} $\tilde{\mathrm{U}}$[0,100], and $u_{ik} = l_{ik}+$ U[0,100] for $k = 2, \ldots, K_i$; and w_{i1} $\tilde{\mathrm{U}}$[0,1] and $w_{ik} = w_{ik-1}-$ 0.05 for $k = 2, \ldots, K_i$.

Table 4–2: Incremental Units Discount Heuristic Performance

Instance	Heuristic	Optimal	Gap %		Instance	Heuristic	Optimal	Gap %
1	2944.16	2944.16	0.00%		16	2435.61	2435.61	0.00%
2	2620.90	2613.21	0.29%		17	3046.59	3046.59	0.00%
3	2937.55	2937.55	0.00%		18	3202.77	3202.77	0.00%
4	2680.84	2680.84	0.00%		19	2993.12	2993.12	0.00%
5	2477.86	2477.86	0.00%		20	2465.22	2465.22	0.00%
6	2592.35	2592.35	0.00%		21	2888.12	2888.12	0.00%
7	3306.59	3306.59	0.00%		22	2841.58	2841.58	0.00%
8	2964.60	2962.86	0.06%		23	3178.13	3178.13	0.00%
9	2696.40	2685.76	0.40%		24	3178.13	3178.13	0.00%
10	2574.28	2574.28	0.00%		25	2853.53	2853.53	0.00%
11	2670.19	2670.19	0.00%		26	2992.17	2992.17	0.00%
12	2365.78	2365.78	0.00%		27	2753.51	2753.51	0.00%
13	3017.81	3017.81	0.00%		28	2850.40	2850.40	0.00%
14	2964.46	2964.46	0.00%		29	2699.23	2699.23	0.00%
15	2546.91	2546.91	0.00%		30	2704.60	2704.60	0.00%
Mean Gap			0.03%		Worst Gap			0.40%

The results of this evaluation are presented in Table 4-2. The gap reported in this table is the percentage deviation of the heuristic solution from the optimal solution obtained using LINGO. As can be seen, in 27 of the 30 problems, the heuristic solution was optimal. For the remaining 3 problems, the worst case heuristic solution gap was 0.40%, while the average gap was 0.24%. As with the linear discount case, all the heuristic solutions were obtained is less than 1 second using a PC (with 256 KB RAM, a Pentium III Processor, and 550 Mhz clock speed) while to obtain the optimal solutions using LINGO on a similar machine, the run times were between several minutes and a over 2 hours. This leads us to conclude that our heuristic provides very good quality solutions fairly quickly.

4.3.4 All Units Discount Price

In this case, $f_{ik}(q_{ik}) = v_{ik}q_{ik}$ if $l_{ik} \leq q_{ik} \leq u_{ik}$ for $k = 1, \ldots, K_i$ for each supplier i. The sourcing model is now formulated as follows.

$$\text{Minimize } \mathrm{Z}_{AU} = \sum_{i=1}^{n} \sum_{k=1}^{K_i} v_{ik} q_{ik} y_{ik} \tag{4.12}$$

subject to:

$$\sum_{i=1}^{n} \sum_{k=1}^{K_i} q_{ik} = Q \tag{4.13}$$

$$\sum_{k=1}^{Ki} y_{ik} \leq 1 \quad \forall i \tag{4.14}$$

$$l_{ik} y_{ik} \leq q_{ik} \leq u_{ik} y_{ik} \quad \forall i, k \tag{4.15}$$

$$y_{ik} \in [0, 1] \tag{4.16}$$

For the case where each supplier has adequate capacity (i.e., $u_{iK_i} \geq Q$ $\forall i$) to meet the aggregate requirement Q, it is trivial to show that the firm will choose to source the complete requirement Q from a single supplier j such that $V_j^Q = \min_{1 \leq i \leq n}\{V_i^Q\}$. In this case, V_i^Q is the price per unit offered by supplier i for quantity Q.

Again, the dominance of the single sourcing strategy is questionable when every supplier does not have adequate capacity to meet the aggregate requirements Q. Further, in this case, our objective is discontinuous and thus, the result stated for the prior two cases (for a continuous concave objective function) do not necessarily apply. On the other hand, the property stated in the result is quite easy to incorporate in a heuristic and hence, the following heuristic was proposed to generate feasible solutions to our problem.

1. Define the active supplier set, Ω, as consisting of all suppliers.

2. For each supplier i included in Ω,

   - If $u_{iK_i} < Q$, compute $r_i = v_{iK_i}$ and set $q_i = u_{iK_i}$,
   - otherwise,
     - DO $k = 1, 2, ..., K_i$
     - If $l_{ik} \leq Q \leq u_{ik}$, $r_i = v_{ik}$ and $q_i = Q$.
     - END

   compute $r_i = max\{v_{iK_i}, v_{ij} | l_{ij} \leq Q \leq u_{ij}\}$.

3. Rank suppliers in increasing order of r_i and index suppliers in this ranked list [1],[2],...,[N], and set $j = 1$.
4. Set $Q = Q - q_{[j]}$. Remove supplier j from Ω. If $Q > 0$ go to 2, else go to 5.
5. For all suppliers k, with $q_k > 0$, explore all possible improvements in the solution by switching the partial order quantity within this selected supplier set.
6. Store the best solution as Solution A item Repeat the above process, except at step 2, calculate in the first *if* statement $r_i = v_{iK_i} u_{iK_i}$, and in the DO loop $r_i = v_{ik}Q$; and in Step 6, store the best solution as Solution B.
7. Choose solution A or Solution B based on the better objective function value.

To evaluate the solution quality of this heuristic, we carried out numerical experiments by randomly generating 30 test problems (APPENDIX B). The parameter setting for each problem are identical to those generated for the incremental units discount price case except that for each supplier i, the unit price $v_{ik} = w_{ik}$ for $k = 1, \ldots, K_i$.

The results of this evaluation are presented in Table 4-3. The gap reported in this table is the percentage deviation of the heuristic solution from the optimal solution obtained using LINGO. As can be seen, in 17 of 30 problems, the heuristic solution was optimal. For the remaining 13 problems, the worst case heuristic solution gap was 1.58%, while the average gap was 0.27%. All the heuristic solutions were obtained is less than 1 second using a PC (with 256 KB RAM, a Pentium III Processor, and 550 Mhz clock speed) while to obtain the optimal solutions using LINGO on a similar machine, the run times were between several minutes and a over 2 hours. This leads us to conclude that our heuristic provides fairly good quality solutions fairly quickly.

Table 4–3: All-Units Discount Heuristic Performance

Instance	Heuristic	Optimal	Gap %		Instance	Heuristic	Optimal	Gap %
1	2675.01	2675.01	0.00%		16	2116.36	2116.36	0.00%
2	2313.10	2308.95	0.18%		17	2753.34	2753.34	0.00%
3	2746.50	2736.80	0.35%		18	3023.82	3023.82	0.00%
4	2465.49	2465.49	0.00%		19	2756.77	2756.77	0.00%
5	2185.60	2181.90	0.17%		20	2245.90	2240.93	0.22%
6	2303.10	2267.30	1.58%		21	2529.90	2527.17	0.11%
7	3117.94	3117.94	0.00%		22	2497.80	2494.66	0.13%
8	2694.41	2694.41	0.00%		23	2931.00	2920.13	0.37%
9	2381.40	2380.86	0.02%		24	2161.84	2161.84	0.00%
10	2319.18	2319.18	0.00%		25	2578.03	2578.03	0.00%
11	2433.14	2433.14	0.00%		26	2650.97	2650.97	0.00%
12	2101.40	2099.66	0.08%		27	2530.91	2530.91	0.00%
13	2777.31	2777.31	0.00%		28	2523.10	2523.10	0.00%
14	2727.40	2721.61	0.21%		29	2493.43	2493.43	0.00%
15	2262.40	2259.67	0.12%		30	2461.70	2449.75	0.49%
Mean Gap			0.13%		Worst Gap			1.58%

4.3.5 Summary of Insights from Analysis

Based on our analysis, we can offer the following general conclusions. In regard to the firm's sourcing strategy, for any pricing scheme, the only time a single supplier sourcing strategy is preferred is when the lowest cost supplier has adequate capacity to meet the entire demand for the firm. In all other cases, a multiple sourcing strategy is the general choice. In regard to supplier quantity allocations, for the case where a multiple supplier sourcing strategy is preferred, we can determine the optimal supplier quantity allocations only when each supplier quotes a constant price. When suppliers quote quantity discount pricing schemes, the optimal quantity allocations are difficult to determine. However, we have developed efficient heuristics which can generate supplier quantity allocations which are close to optimal. We now turn to an application of our heuristic approaches for data obtained from an office products manufacturer.

4.4 Application

The Central Purchasing Organization (CPO) of a major office products retailer is frequently faced with complex sourcing decisions due to the presence of quantity discount schemes embedded within various suppliers' bids. Given the CPO's total quantity commitments and supplier bid data for two distinct commodity products we apply our modeling approach.

The data shared by the CPO is for two commodity products among hundreds of commodity stock keeping units (SKU) that must be frequently replenished for retail sale. Total quantity requirements for each product are aggregated from retail locations by the CPO. The CPO then posts a request for quotation (RFQ) for each product. Two distinct sets of suppliers quote on product A and product B respectively. The portfolio of bids for product A consists of six suppliers, four with constant price quotes and two with discount pricing. Product B's bid portfolio consists of eight suppliers, four with constant price quotes and four with discount pricing. All suppliers in both portfolios also provide limitations on their respective unit capacities. Tables 4-4 and 4-5 contain bid information for products A and B respectively.

Table 4–4: Product A Bid Information Data

TQC=9855	Price Break Min	Price Break Max	Unit Price
Supplier A1	0	1000	623
	1001	2100	534
	2101	3200	465
Supplier A2	0	2100	452
Supplier A3	0	2650	457
Supplier A4	0	1000	449
Supplier A5	0	700	654
	701	1920	494
Supplier A6	0	2200	453

From the bid information we generate a data set for each product's suppliers. Each data set is implemented twice. First it is assumed that quantity discount quotes are of the incremental variety. Secondly, an all-units quantity discount structure is assumed. Further, constant or fixed pricing can be modeled as an incremental or an all-units discount with only one discount level. Therefore, our heuristic can handle portfolios of supplier bids that consist of constant price quotes as well as more sophisticated bids with discount pricing.

Table 4–5: Product B Bid Information Data

TQC=7680	Price Break Min	Price Break Max	Unit Price
Supplier B1	0	1200	634
Suppler B2	0	600	875
	601	1300	800
	1301	2500	725
	2501	2900	634
Supplier B3	0	700	790
	701	1600	710
	1601	3000	620
Supplier B4	0	1460	621
Supplier B5	0	1275	625
Supplier B6	0	2600	632
Supplier B7	0	400	922
	401	1400	822
	1401	2000	722
	2001	2600	622
Supplier B8	0	800	821
	801	1750	700
	1751	2400	610

Using our heuristics we determine supplier selection and quantity allocation for this industry problem. Tables 4-6 and 4-7 provide a summary of heuristic solution performance versus LINGO's Global Solver optimal solutions for products A and B respectively.

For the incremental instances, the heuristic solution is optimal and the CPO should source from five suppliers for each product. In the all-units instances the heuristic arrives at near optimal solutions that vary from the optimal solutions in both the pool of selected suppliers and allocations among them. For example, it is optimal to source from five suppliers for product A and the heuristic selects

Table 4–6: CPO Product A Solutions Comparison

Incremental	q_{A1}	q_{A2}	q_{A3}	q_{A4}	q_{A5}	q_{A6}	Cost	Gap
Optimal	0	2100	2650	1000	1905	2200	4658920	
Heuristic	0	2100	2650	1000	1905	2200	4658920	0%
All Units	q_{A1}	q_{A2}	q_{A3}	q_{A4}	q_{A5}	q_{A6}	Cost	Gap
Optimal	2101	2100	2454	1000	0	2200	4493243	
Heuristic	2905	2100	2650	0	0	2200	4507675	0.32%

Table 4–7: CPO Product B Solutions Comparison

Incremental	q_{B1}	q_{B2}	q_{B3}	q_{B4}	q_{B5}	q_{B6}	q_{B7}	q_{B8}	Cost	Gap
Optimal	1200	0	1145	1460	1275	2600	0	0	4976485	
Heuristic	1200	0	1145	1460	1275	2600	0	0	4976485	0%
All Units	q_{B1}	q_{B2}	q_{B3}	q_{B4}	q_{B5}	q_{B6}	q_{B7}	q_{B8}	Cost	Gap
Optimal	0	0	3000	279	0	0	2001	2400	4741881	
Heuristic	0	0	3000	1460	820	0	0	2400	4743160	0.03%

only four. Optimally, supplier A4 is selected and allocated 1000 units by decreasing allocation to suppliers A1 and A3. For product B the heuristic and optimal solution consists of four selected suppliers. However, the optimal solution selects supplier B7 and not supplier B5 and our heuristic selects supplier B5 instead of B7. Also, the optimal allocation to supplier B4 is 279 units and our heuristic allocates 1460 units. Therefore, while the optimality gap is slight for the all-units instances, the sourcing strategy decisions of supplier selection and quantity allocation are quite different.

4.5 Optimal Algorithm for Incremental Quantity Discounted Sourcing

In this section we present a branch and bound algorithm for optimally solving our sourcing problem when all suppliers quote prices with incremental quantity discounts. The sourcing problem is exactly as described in section 4.3.3. As such we need to solve for the minimum of the sum of piecewise linear concave functions. Again, by leveraging Result 4.1, we search for a solution such that at most one supplier's allocation is not at a boundary of zero or his capacity. Since our sourcing problem with incremental quantity discounts is a special case of the separable piecewise linear concave cost allocation problem, we develop and outline an optimal algorithm for solving the separable piecewise linear concave cost allocation problem. The mathematical formulation of the problem is as follows:

(SPLCCAP): $Minimize \quad f(x)$

subject to $x \in D \bigcap C$

where $x = (x_1, \ldots, x_n) \in R^n$. Given are $f(x) = \sum_{i=1}^{n} f_i(x_i)$ and for each i, f_i concave and bounded on $[l_i, u_i]$; $C \equiv \prod_{i=1}^{n} C_i, C_i = [l_i, u_i]$, and $l_i, u_i \in R^+$; $D \equiv x : \sum_{i=1}^{n} x_i = Q, Q \in R^+$; and $D \bigcap C$ assumed to be bounded.

4.5.1 Algorithm Description

In this algorithm we branch if the solution at the current node, p allocates any x_i such that $l_i^p < x_i < u_i^p$. Branching corresponds to partitioning the subset of solutions in a hyper-rectangle $C^p \in C$ into two subsets of solutions in the two hyper-rectangles C^q and C^r where $C^q \bigcup C^r = C^p$. Bounding corresponds to determining a lower bound on the optimal value of (SPLCCAP), f^*, in C^p.

As the branch and bound tree is generated, the nodes are numbered accordingly. Let N^p, for $p = 0, 1, 2, \ldots$, denote node p, where N^0 is the root node and $C^0 = C$. At stage k of the algorithm nodes $2k$ and $2k - 1$ are generated. A hyper-rectangle $C^p = x_i : l_i^p \leq x_i \leq u_i^p, i = 1, 2, \ldots, n$ is associated with each node, N^p. The branching process to be discussed later determines the appropriate values for l_i^p and u_i^p for all i at each node, p. Let LB^p be a lower bound on the optimal value of f over $G \bigcap C^p$. The calculation and validity of LB^p is described shortly.

Each solution, $\bar{x}^p$, to a linear knapsack problem at each node, N^p, is a feasible solution to (SPLCCAP). Therefore the upper bound, UB^f, on f^* is the minimum value of f over all encountered feasible solutions, $\bar{x}^p$. The incumbent solution, $\bar{x}_{UB}$, is the feasible solution, $\bar{x}^p$ such that $UB_f = f(\bar{x}^p)$.

In the algorithm, a candidate list is maintained which includes nodes from which to branch. As the branch and bound tree grows, for each node, N^p that is created, if $LB^p < UB_f$, node N^p is added to the candidate list and a comparison is executed to determine whether UB_f can be updated. If $LB^p \geq UB_f$, node N^p is pruned. Further, if $LB^p < UB_f$, a separation variable is determined and from this separation variable node N^p is partitioned into two new nodes. For the new nodes, lower bounds are calculated and if necessary, pruning is performed and UB_f is updated. The process of node selection, partitioning and obtaining bounds on f^* is repeated until the candidate list becomes empty. When the candidate list is empty the algorithm terminates with an optimal solution, $\bar{x}_{UB}$ and $f^* = f(\bar{x}_{UB})$.

4.5.2 Computation and Validity of LB^p and UB_f

The algorithm determines bounds on each concave subproblem by solving a linear programming relaxation. For each concave term, $f_i(x_i)$ a linear under-estimator is constructed, $g_i(x_i)$ such that $g_i^p(l_i^p) = f_i(l_i^p)$ and $g_i^p(u_i^p) = f_i(u_i^p)$. In fact, $g_i(x_i)$ is the convex envelope of $f_i(x_i)$ over $[l_i^p, u_i^p]$. Also, from Falk and Soland (1969), it is well known that the convex envelope of $f(\bar{x}) = \sum_{i=1}^{n} f_i(x_i)$ over a rectangular set C^p is $g^p(\bar{x}) = \sum_{i=1}^{n} g_i^p(x_i)$ such that $x_i^p \in [l_i^p, u_i^p]$. Based on this, we know:

$g^p(\bar{x}) \leq f(\bar{x})$ if $x_i^p \in [l_i^p, u_i^p]$.

Therefore, the lower bound LB^p, on f^* over $D \bigcap C$ is given by the optimal solution to the linear knapsack problem (LKP^p).

(LKP^p) minimize $g^p(\bar{x})$

subject to $\bar{x} \in D \bigcap C$.

Let $LB^p = g^p(\bar{x^*})$, where $\bar{x^*}$ is an optimizer of (LKP^p). Thus from the discussion above and the fact that $(D \bigcap C^p) \subset (D \bigcap C)$, is a valid lower bound on f^*. Thus, at node N^p of the branch and bound tree, we solve the lower bounding subproblem (LKP^p).

From subproblems (LKP^p),$p = 0, 1, \ldots$, a solution $\bar{x^p}$ is produced by the algorithm. Since for each $p = 0, 1, \ldots$, $\bar{x^p} \in D \bigcap C$, then each $\bar{x^p}$ is a feasible solution to (SPLCCAP). The validity of the upper bound on f^*, UB_f, follows from the algorithm setting UB_f= min $f(\bar{x^p}) : p = 0, 1, \ldots$.

A lower bound on f^* over $D \bigcap C$, LB_f is now expressed. Let NB^k denote the set of nodes not branched from at stage k of the algorithm. A node $N^p \in NB^k$ due to fathoming or current inclusion in the candidate list. It can be shown that:

$LB_f = min_{N^p \in NB^k} LB^p$.

Therefore at each stage $k, k = 0, 1, \ldots$ of the algorithm $LB_f \leq f^* \leq UB_f$.

4.5.3 The Branching Process

At stage k of the algorithm, a node N^p is selected from the candidate list. Branching is invoked and node N^p is partitioned into two new nodes N^{2k-1} and N^{2k}. This is accomplished by first choosing a separation variable, x_s^p which satisfies $f_s(x_s^p) - g_s^p(x_s^p) > 0$ (i.e., $l_s^p < x_s^p < u_s^p$). Note If f_s consists of t_s linear intervals or price break points, BP_j, $(j = 1, \ldots, t_s)$, then $BP_{b-1} \leq x_s^p \leq BP_b$ for some $b \in 2, \ldots, t_s$. Then the interval $[l_s^p, u_s^p]$ is partitioned in the following manner to generate nodes N^{2k-1} and N^{2k} and their respective sets C^{2k-1} and C^{2k}.

$[l_s^{2k-1}, u_s^{2k-1}] = [l_s^p, BP_b]$ and $[l_s^{2k}, u_s^{2k}] = [BP_b, u_s^p]$

Recall that each f_i is piecewise linear concave over $[l_i, u_i]$. Therefore each f_i consists of t_i intervals (price break points) over $[l_i, u_i]$. Furthermore, since the branching rule guarantees that l_s^p and u_s^p for all $p = 0, 1, 2, \ldots$, correspond to price break points, there a finite number of hyper-rectangles that can be generated. Also, each invocation of the branching process will create two new hyper-rectangles. Due to these constructs the algorithm is finite.

4.5.4 Formal Statement of the Algorithm

1. Set $k = 0$ and solve (LKP^k). Set $UB_f = f(\bar{x^0})$ and $x_{UB} = \bar{x^0}$. If $LB^0 < UB_f$, add N^0 to the candidate list, set $k = 1$ and go to step 2. Otherwise, go to step 4.
2. If the candidate list is empty, go to step 4. Otherwise, go to step 3.

3. Remove any node N^p from the candidate list. If $LB^p \geq UB_f$ go to step 2. Otherwise, generate nodes N^{2k-1} and N^{2k} and find LB^{2k-1} and LB^{2k}. If necessary, update UB_f and $\bar{x}$. If $LB^j < UB_f$ for each $j = 2k-1, 2k$, add node N^j to the candidate list. $k = k+1$. Go to step 2.
4. $x_{\bar{UB}}$ is an optimal solution for (SPLCCAP) with $f^* = UB_f$. Terminate.

At each node of the branch and bound tree we generate a new LP, each of which is a knapsack problem (KP) of size Q. Using a greedy approach based on each suppliers linearized unit cost, we can optimally solve each KP by inspection. Further, each KP solution is feasible to our original NLP. By inputting our test problems, this branch and bound algorithm validates the optimal solutions obtained using LINGO's Global Solver. Table 4-8 provides a summary of the number of subproblems solved for each test problem.

Table 4–8: Number of Subproblems Solved for each Test Problem

Test Problem	1	2	3	4	5	6	7	8	9	10
Sub Problems	3	7	5	3	3	11	1	3	7	5
Test Problem	11	12	13	14	15	16	17	18	19	20
Sub Problems	1	7	5	5	3	5	3	3	1	5
Test Problem	21	22	23	24	25	26	27	28	29	30
Sub Problems	3	5	3	3	1	7	1	5	1	5

4.6 Conclusions

The analysis of alternate supplier base pricing schemes in this chapter provides guidance for a buying firm's optimal sourcing strategy. For the constant cost case, if the lowest cost supplier has enough capacity to satisfy all of the buying firm's product requirements, single sourcing is optimal, otherwise multiple sourcing is best. For sourcing scenarios where supplier bases offer quantity discount pricing schemes, if all suppliers in the base possess enough capacity to individually provide Q units, then it is optimal to single source from the least cost supplier evaluated at Q units. However, if even one supplier in the base is incapable of providing Q units, then single sourcing may not be the optimal sourcing strategy.

In cases where quantity discount quoting suppliers' capacity is individually inadequate for a buying firm's product requirements, a firm's sourcing problem can become extremely complex. Therefore, identifying an optimal solution may be resource (time) prohibitive for supply chain sourcing professionals. In such cases, the heuristics developed in this chapter can be expected to efficiently provide good quality solutions. In sum, the heuristic solutions to randomly generated test problems arrived at the optimal solution in 62% of the instances. Furthermore, in the 38% of non-optimal solution instances, a respectable .64% average optimality gap exists.

CHAPTER 5
STRATEGIC SOURCING WITH DIVERSIFICATION CONSIDERATION

5.1 Introduction

To be successful in the long term, a firm's sourcing strategy today must be consistent with the vision of the firm for tomorrow. This vision is at the core of a firm's corporate strategy. Corporate strategy is not only a company's pattern of purposes and goals that defines the type of company that it is, it is also the policies that direct goal achievement (Andrews, 1980). The intent of the general model developed in this chapter is to provide a strategic sourcing framework to capture subjective elements of a firm's corporate strategy that directly influence sourcing decisions. The general approach in this chapter for examining supplier selection and quantity allocation decisions is two fold. First, an integrated model is developed to address both of these decisions. Given uncertain product demand, we simultaneously consider supplier costs, supplier reliabilities, supplier capacities, manufacturer inventory costs, and manufacturer diversification benefits in making these integrated decisions. Second, using our model, we characterize conditions under which single-sourcing and multiple-sourcing strategies are optimal. A key feature of the model facilitating this analysis is an explicit treatment of supply diversification benefits. These executive level valuations of alternate supply base sizes are assumed to be consistent with the firm's competitive strategy. This strategic direction may be presently inconvenient, in that it may indicate that current profits should be foregone in lieu of potential future gains.

This work is most closely related to those of Pan (1989) and Parlar and Wang (1993). Pan (1989) proposes a linear programming model to optimally identify the number of suppliers and their respective quantity allocations to meet pre-specified product requirements. Other constraints incorporated are related to aggregate incoming quality, lead times, and service level. The overall objective is to minimize the price per unit as a weighted average of selected suppliers' prices. It is assumed that product requirements are deterministic and supply is reliable and unlimited. Parlar and Wang (1993) compare the costs of single versus dual-sourcing for a firm assuming that the overall objective is to minimize purchasing and inventory related costs. In their approach, they assume that actual incoming quantities are a function of a random variable representing the yield. Separately using an EOQ and newsboy based ordering policy, they are able to show that in certain cases dual-sourcing dominates single-sourcing. Both of these studies ignore the supplier capacity issue in making supplier selection and quantity allocation decisions. Further, Parlar and Wang (1993) note that supplier yields and demand uncertainty play a critical role in the analysis.

We build upon both of these studies by analyzing the simultaneous supplier selection and quantity allocation decisions for a single firm facing supply unreliability and demand uncertainty. Further, we incorporate an explicit benefit related to requirements diversification among the supplier base in identifying optimal sourcing strategies. In the next section, the integrated supplier selection and quantity allocation model which forms the basis of our analysis is described. In Section 5.3, we characterize the optimal sourcing strategies under various scenarios. In Section 5.4, we proceed to discuss extensions to this modeling effort and this is followed by an extensive numerical analysis in Section 5.5. Finally, conclusions and managerial implications are discussed in Section 5.6.

5.2 Integrated Selection/Allocation Model

5.2.1 Preliminaries

Our examination of the supplier selection and quantity allocation decisions focuses on a single-period analysis of a two stage supply chain consisting of N

suppliers ($i = 1, \ldots, N$) and one buying firm. All N suppliers are assumed to have been pre-screened by the firm and are thus, included in the supplier base. The firm faces an uncertain single-period demand w (with $f(w)$ and $F(w)$ representing density and distribution functions, respectively) for the product requirements which it satisfies through procurement from the N suppliers. We assume that this product is being supplied to the next stage of the supply chain at a unit price p. Excess inventory of the product is disposed of at the end of the single period by the firm which receives a price of s per unit while unsatisfied demand "costs" the firm u per unit. For each supplier i we assume that the firm has information on (a) the cost per unit c_i; (b) the capacity (in units) y_i; and (c) the reliability index r_i representing the historical percentage of "good" units (i.e., $0 \leq r_i \leq 1 \; \forall i$) received from the supplier[1] . In line with the assumptions in single-period inventory models, we assume that $p > c_i > s$ for all i (Silver, Pyke, and Peterson, 1998). We assume zero fixed order placement costs for the firm since in current industrial settings where orders are issued online (through, for example, B2B exchanges), these costs are negligible (Nahmias, 2001).

A final component of our model characterizes the diversification benefit function $d(\cdot)$. The motivation for incorporating this function stems from observed industry practices. For example, consider HP's sophisticated Procurement Risk Management program which was initially aimed at better managing the procurement risk of critical memory components. Incorporated in this program is a portfolio approach to assess and mitigate pricing and availability exposure to insure

[1] In contrast to Parlar and Wang (1993), we assume that the firm has exact knowledge on the reliability of each supplier. This is primarily due to the fact that the uncertainty in this parameter required Parlar and Wang (1993) to impose some very restrictive conditions on other parameters in order to show that dual-sourcing is preferred to single-sourcing. We do, however, analyze the impact of changes in this parameter on the resulting selection/allocation policies.

margin stability. HP expects the revenue contribution of this program to approach \$1 billion as it expands its use throughout the company. This financial benefit stems from reduced unit costs and avoiding costs that arise when product delivery is delayed due to sourcing allocation. Additionally, as a result of this program, HP has learned that by segmenting its expected requirements it can target contracts to take advantage of a supplier's particular strength(s). This results in more efficient supply chain practices that create shared savings for HP and its suppliers (Shah, 2002). However, these supply chain savings are sometimes difficult to quantify.

The issue of positive diversification benefits is well documented through anecdotal and/or case examples. In addition to HP, Unifine Richardson's decision to change from a single honey supplier to multiple honey suppliers to meet more stringent regulation requirements; and Wendy's decision to find a second high-volume supplier of chicken given the increasing demand for its chicken products illustrates the importance of these diversification benefits (Prahinski, 2002; Lambert and Knemeyer, 2004). Toyota and Honda also have a policy of sourcing all components from a minimum of two or three suppliers (Liker and Choi, 2004).

The diversification function $d(\cdot)$ reflects buyer specific supply chain efficiency savings and strategic positioning benefits. In general, this function captures the net benefits of choosing to source product requirements from multiple suppliers and is analogous to the risk-averse expected utility function maximized in the portfolio selection problem (Gerchak and Parlar, 1990). By choosing multiple suppliers the firm can reduce the risk associated with selecting a single supplier (Ramasesh et al., 1991). Consequently, the diversification function is essentially insurance against supply disruptions attributable to the size of the supply base for a specified part.

We also recognize that the there is a potential decline in diversification benefits if the number of selected suppliers is too large due to excessive individual order related costs. We incorporate this by assuming that that $d(X)$ is strictly concave in X where X represents the number of suppliers selected by the firm. To support this specific functional form, consider the findings of Agarwal and Nahmias (1997). These authors show that expected firm profits are concave in the number of suppliers selected in a setting characterized by stochastic supplier quantity reliability. Further, the notion of such benefits being piece-wise concave is documented in prior work (e.g., Gerchak and Parlar, 1990; and Ramasesh et al., 1991). Thus, our $d(X)$ function could be regarded as a proxy for diversification benefits associated with uncertainty in the quantities delivered by each supplier. While the diversification function may be difficult to quantify, a value/utility function procedure outlined below can be used to gauge diversification valuations of a firm's decision maker(s).

1. To start with assume that diversification benefits are not included. Then using the result of Theorem 5.1, determine the total quantity ordered by the firm $MQ_{nc} = q_{[1]}$ and the corresponding expected firm level profits be MZ_{nc}.
2. Set $j = 2$
3. Pose the following question to the firm level sourcing manager:

   Selecting $j - 1$ suppliers, you will order MQ_{nc} units and expect profits of MZ_{nc}. Assuming that you decide to select j suppliers for ordering MQ_{nc} units, your expected profits will decline for certain by \$ (c_j- c_{j-1}) per unit ordered from supplier j . What \$ amount would compensate you for the decline in expected profits assuming that you decide to: (a) source exactly MQ_{nc} units from all j suppliers?; and (b) order at least 1 unit from supplier j.

   Record this \$ amount as $d(j)$.
4. If $j = N$ stop else set $j = j + 1$ and repeat Step (3).

The process outlined is based on established work in utility theory and decision making under uncertainty, and is an adaptation of the reference gamble procedure. Furthermore, it utilizes results from Theorem 5.1 where no diversification benefits are realized. The outcome of this process is to determine a "value" for $d(i)$ for $i = 2, \ldots, n)$ ($d(1) = 0$ since no diversification benefits are realized by the firm with a single supplier). However, elicitation of these values from procurement policy setters or decision makers is not he focus of our analysis.

The key decision variables for our selection/allocation model are both the number of suppliers and the order quantity for each supplier. We define the binary decision variable x_i to be 1 if we choose to source from supplier i, and 0 otherwise; and the related allocation quantity q_i (in units) procured from supplier i.

5.2.2 Model Development

Let us start by expressing the profit function without diversification benefits as[2] :

$$\Pi = \begin{cases} pw - \sum_{i=1}^{N} c_i r_i q_i + s[\sum_{i=1}^{N} r_i q_i - w] & \text{if } w < \sum_{i=1}^{N} r_i q_i \\ p\sum_{i=1}^{N} r_i q_i - \sum_{i=1}^{N} c_i r_i q_i - u[w - \sum_{i=1}^{N} r_i q_i] & \text{if } w \geq \sum_{i=1}^{N} r_i q_i \end{cases}$$

Since this profit is uncertain and depends on the exact realization of demand w, we use the traditional newsboy analysis for this profit function (Silver, Pyke, and Peterson, 1998), and determine the expected profits as:

$$E(\Pi) = (p-s)\mu - \sum_{i=1}^{N} c_i q_i r_i + s\sum_{i=1}^{N} r_i q_i - (p-s+u)ES$$

[2] Similar to Agarwal and Nahmias (1997), this profit realization characterizes a situation where the firm compensates suppliers for only the "good" units supplied (i.e., $r_i q_i$). Later in the chapter, we consider the impact of the firm compensating the supplier for the complete order quantity (i.e., q_i).

where

$$
\begin{aligned}
\mu &= \text{mean demand} \\
ES &= \text{expected number of units short} \\
&= \int_{(\sum_{i=1}^{N} q_i r_i)}^{\infty} [w - (\sum_{i=1}^{N} q_i r_i)] f(w) \mathrm{d}w
\end{aligned}
$$

and depending upon the distribution of demand, we can specify the expected shortage ES[3] .

Based on this, the firm's expected profit (including the diversification benefit) maximization sourcing model can be defined as follows:

Maximize $Z_{q_i,x_i} = E(\Pi) + d(X)$

$$
= (p-s)\mu - \sum_{i=1}^{N} c_i q_i r_i + s \sum_{i=1}^{N} q_i r_i - (p-s+u)ES + d(X) \tag{5.1}
$$

subject to:

$$
\begin{aligned}
q_i &\leq y_i x_i & \forall i \quad & (5.2) \\
X &= \sum_{i=1}^{N} x_i & & (5.3) \\
q_i &\geq 0 & \forall i \quad & (5.4) \\
x_i &= \{0, 1\} & \forall i \quad & (5.5)
\end{aligned}
$$

where constraint set (5.2) integrates capacity limitations when $x_i = 1$ (or supplier i is selected) or forces the quantity allocation decision q_i to be 0, if $x_i = 0$ (or when supplier i is not selected), constraint (5.3) determines the total number of suppliers chosen for sourcing total product requirements, and equations (5.4) and (5.5)

[3] For example, if demand is normally distributed with mean μ and standard deviation σ, then $ES = \sigma G(Z)$ where $G(\cdot)$ is the loss function and Z is the standard normal deviate (i.e., $Z = \frac{(\sum_{i=1}^{N} q_i r_i - \mu)}{\sigma}$).

are the non-negativity and binary restrictions on the decision variables q_i and x_i, respectively. In the next section, we proceed to analyze this model and characterize the optimal solutions and sourcing strategies for several cases.

5.3 Analysis

The focus of our paper is on investigating sourcing strategies for the supply chain. Primarily we are interested in identifying when it is optimal for the manufacturer to use multiple suppliers versus a single supplier. We start our analysis by first assuming a zero diversification benefit. These results will serve as a base case in our analysis.

5.3.1 No Diversification Benefit

Assuming that the firm does not obtain any explicit diversification benefit for choosing to source from more than one supplier, let us examine the structure of the optimal sourcing policies. When the diversification benefit $d(X) = 0$, our integrated sourcing model can be formulated as follows.

$$Maximize Z_{q_i} = (p-s)\mu - \sum_{i=1}^{N} c_i q_i r_i + s\sum_{i=1}^{N} q_i r_i - (p-s+u)ES \tag{5.6}$$

subject to:

$$q_i \leq y_i \qquad \forall i \tag{5.7}$$

$$q_i \geq 0 \qquad \forall i \tag{5.8}$$

Let us first analyze this problem assuming that supplier capacity is not a significant issue. Note that this scenario may be relevant to smaller manufacturing firms with larger suppliers. In this case, it is relatively easy to show that the firm commits all its requirements to a single supplier and this supplier is, as would be expected, the one which offers the lowest per unit cost to the firm. The theorem given below formalizes this result.

<u>Theorem 5.1</u>: [4] When the suppliers are uncapacitated and there are no diversification benefits and there is a unique least cost supplier, then it is optimal for the firm to order its total requirements from the least cost supplier. Under this scenario, the total usable quantity ordered from the least cost supplier (i.e., $q_{[1]}r_{[1]}$) is determined such that:

$$F(q_{[1]}r_{[1]}) = \frac{p - c_{[1]} + u}{p - s + u}$$

where $c_{[1]}$ is the cost per unit charged by the lowest cost supplier.

<u>Proof</u>: See APPENDIX C.

One surprising result of this single sourcing strategy is that supplier reliabilities do not impact the supplier choice (i.e., the supplier choice is based strictly on cost considerations regardless of the quantity reliability parameter r_i). On further investigation, we find that this result is solely due to the fact that the manufacturer only incurs the purchasing cost for "good" units (i.e., incurs cost c_i per unit for $r_i q_i$ units). In certain situations, the cost of defective units in a delivery may need to be absorbed by the manufacturer. To reflect this scenario, the uncapacitated supplier model without diversification benefits can be reformulated as:

---

[4] While this result is analytically trivial, it is driven by the fact that for all suppliers $j = 1, ..., N$, $c_j r_j$ is a constant. However, it points to the fact that when the firm uses historical information of supplier reliabilities and only compensates a supplier for the "good" units received, then only cost considerations play a role in determining which supplier will be chosen to received the complete order. This result also forms the basis of how the firm's single supplier selection decision is moderated should it decide to compensate suppliers for all units ordered (i.e., the firm absorbs the complete costs of defective units which could occur in situations where suppliers hold more "power" in the channel). In this case, we show that ratio of costs to reliability drives the choice of the single supplier who will receive the entire order from the firm.

Maximize $E(\Pi)^e$

$$= (p-s)\mu - \sum_{i=1}^{N} c_i q_i + s\sum_{i=1}^{N} q_i r_i - (p-s+u)\int_{(\sum_{i=1}^{N} q_i r_i)}^{\infty} [w - (\sum_{i=1}^{N} q_i r_i)] f(w) dw \quad (5.9)$$

subject to:

$$q_i \geq 0 \qquad \forall i \qquad (5.10)$$

It is easy to show that $E(\Pi)^e$ is strictly concave in q_i, and thus, the FOC are necessary and sufficient to identify a global optimal solution to this model. Essentially, the manufacturer determines the total "good" quantity received from all suppliers (i.e., $\sum_{i=1}^{N} q_i r_i$) such that:

$$F(\sum_{i=1}^{N} q_i r_i) = \frac{p - (c_i/r_i) + u}{p - s + u} \qquad (5.11)$$

The issue, of course, is which supplier's reliability adjusted unit cost $(i.e., c_i/r_i)$ is relevant in determining this total quantity. The optimal policy, which can be easily verified, is for the manufacturer to place an order for the total quantity from a single supplier with the lowest cost/reliability ratio. Hence, if suppliers are indexed in order of decreasing cost/reliability ratio such that:

$$\frac{c_{[1]}}{r_{[1]}} \leq \frac{c_{[2]}}{r_{[2]}} \leq \ldots \leq \frac{c_{[N]}}{r_{[N]}} \qquad (5.12)$$

then the manufacturer determines the quantity to purchase from supplier [1] such that:

$$F(q_{[1]} r_{[1]}) = \frac{p - (c_{[1]}/r_{[1]}) + u}{p - s + u} \qquad (5.13)$$

and orders zero units from all other suppliers. Thus, even in this case, it is optimal for the manufacturer to adopt a single sourcing strategy except that the choice of the supplier is based on the lowest cost/reliable unit.

Given that eliminating capacity constraints results in single sourcing, let us now proceed to examine how this solution changes if supplier capacity constraints do not necessarily permit the firm to place orders for all requirements with the least cost supplier[5] . The theorem below characterizes the optimal supplier selection and quantity allocation policy with capacitated suppliers.

Theorem 5.2: When suppliers are capacitated and there are no diversification benefits, then the optimal number of suppliers selected and the corresponding quantity allocated to each supplier can be determined as follows.

Step 1: Index all suppliers in increasing order of cost per unit (i.e., $c_{[1]} \leq c_{[2]} \leq c_{[3]} \ldots \leq c_{[N]}$).

Step 2: For each supplier $[i]$ $(i = 1, \ldots, N)$, determine $Q_{[i]}$ such that:

$F(Q_{[i]}) = \frac{p - c_{[i]} + u}{p - s + u}$

and based on this determine:

For $i = 1$, $t_{[i]} = Q_{[i]}$. Otherwise, let $t_{[i]} = Q_{[i]} - \sum_{j=1}^{i-1} y_{[j]} r_{[j]}$

Step 3: The optimal number of suppliers selected (k) is $\max\{1 \leq k \leq N | t_{[k]} \geq 0\}$.

Step 4: The quantities allocated to supplier $j = 1, \ldots, k-1$ are determined such that $q_{[j]} = y_{[j]}$, and the quantity allocated to supplier k is $q_{[k]} = \min\{t_{[k]}, y_{[k]}\}$. The total quantity ordered by the firm from all suppliers can be determined as $\min\{Q_{[k]}, \sum_{j=1}^{k} y_{[j]}\}$.

Proof: See APPENDIX C.

An interesting observation based on these results is that the firm's optimal total order quantity when suppliers are capacity constrained is always lower than the optimal total order quantity when the lowest cost supplier's capacity is not binding. This leads to the general result that expected profits for the firm dealing

[5] All the analysis in the remainder of this chapter reflects the original assumption that suppliers are compensated by the firm for only the "good" units received by the firm rather than all the units supplied.

with capacitated suppliers are never higher than the profits realized by a firm dealing with uncapacitated suppliers. This serves as a rationale for the observed industry practice of firms expending resources to encourage lower cost suppliers to increase capacity.

5.3.2 Diversification Benefit

In this section, we analyze our complete model which includes explicit benefits derived from the size of the selected supplier pool. To start with assume that suppliers are uncapacitated. In this case, our sourcing model is:

$$\begin{aligned}\text{Maximize } Z_{q_i,x_i} &= (p-s)\mu - \sum_{i=1}^{N} c_i q_i r_i + s\sum_{i=1}^{N} q_i r_i - \\ &\quad -(p-s+u)ES + d(X) \qquad (5.14)\end{aligned}$$

subject to:

$$X = \sum_{i=1}^{N} x_i \qquad (5.15)$$

$$q_i \geq 0 \qquad \forall i \qquad (5.16)$$

$$x_i = \{0,1\} \qquad \forall i \qquad (5.17)$$

The optimal solution to this problem is characterized in Theorem 5.3 below.

Theorem 5.3: If the diversification benefits are positive and suppliers are uncapacitated, then the optimal number of suppliers to source from is v^*, where $1 \leq v^* \leq n$, and is determined such that v^* maximizes the diversification benefit function $d(X)$.

The proof of this theorem is a direct extension of Theorem 5.1. Note that based on Theorem 5.1, we choose to source the entire quantity from the lowest cost supplier. If the diversification benefit function is maximized when we choose v^* suppliers where $1 < v^* \leq n$,[6] then we order the total requirements from the lowest cost supplier and simply include the others (i.e., suppliers $2, \ldots, v^*$; indexed in order of increasing unit costs) in the supplier selection set.

In this setting the diversification benefits drive the choice of the optimal number of suppliers. In particular, if it is relatively costless to source from additional suppliers at a negligible level, then the firm can reap the v^* selected supply pool diversification benefits without incurring additional costs. When the marginal benefits of sourcing from an additional supplier are positive, we can always find an order quantity that is sufficiently small enough to warrant selecting that supplier. Qualitatively, the supplementary suppliers $(2, \ldots, v^*)$ provide the firm with those suppliers to consider for emergency supply.

For the capacitated supplier case, the structural insights into the optimal sourcing strategy are characterized in the following theorem. The proof follows directly from Theorem 5.3 and is omitted.

Theorem 5.4: If the suppliers are capacitated and diversification benefits are positive, identify the number of suppliers v^* where $1 \leq v^* \leq n$, such that v^* maximizes the diversification benefit function $d(X)$. Using the results of Theorem 5.2, identify the optimal number of suppliers k^* and optimal order quantities for the capacitated suppliers problem without diversification benefits. If $v^* \geq k^* \geq 1$, then allocate a minimal quantity $\epsilon > 0$ to suppliers $k^* + 1, \ldots, v^*$ (assuming that suppliers are indexed in increasing order of unit costs).

[6] Obviously if $v^* = 1$, then we would simply source the entire quantity from a single lowest cost supplier

The implications of Theorem 5.3 and 5.4 for decision making concerning an appropriate supplier base are clear. Recall from the introduction that there are three interrelated decisions with regards to a firms sourcing strategy (Burke and Vakharia, 2002) (a) criteria for establishing a supplier base; (b) criteria for selecting suppliers (a subset of the base) who will receive an order from the firm; and (c) the quantity of goods to order from each supplier selected. Theorem 5.3 offers direct managerial guidance concerning the second decision, or the number of suppliers who will receive an order from the firm. Furthermore, as a consequence of the positive diversification benefits, the total order quantity will slightly exceed that of the original solution for the capacitated suppliers. Finally, there may also be benefits beyond those directly captured by the model for ordering at a negligible level from some of the suppliers. Depending upon the particular contracts negotiated with these suppliers concerning upside order flexibility, the manufacturing firm could potentially place larger orders with these suppliers in the event that lower cost suppliers cannot deliver good units.

Note that if $1 \leq v^* < k^*$, then the optimal v^* that maximizes the diversification benefit function $d(X)$ is actually smaller than the number of suppliers needed to satisfy total requirements due to capacity limitations. In this case the firm foregoes profit from product sales for a greater strategic benefit of a smaller selected supplier pool. Analytically, the optimal strategy would be to source from any number of suppliers m^* such that $v^* \leq m^* \leq k^*$. We now turn to describing an extension to our modeling effort.

5.4 Model Extensions

In this section, we extend our model to examine the impact of including a minimal order quantity when sourcing from a supplier. In certain situations, firms may have to commit to ordering some minimal order quantity from each supplier in order to reap the benefits of diversification. Suppose that we add an additional constraint which reflects the minimum order quantity for each supplier as shown below.

$$q_i \geq z_i x_i \qquad \forall i \tag{5.18}$$

While the complete solution algorithm for this model is fairly complicated, we can easily obtain boundaries for the optimal number of suppliers. As compared to Theorems 5.3 and 5.4 in the previous section, we now have non-negligible costs associated with including additional suppliers in our supplier base. In particular, the costs are associated with shifting enough units from a lower cost supplier to a higher cost supplier to meet that higher cost supplier's minimum order quantity. The intuition developed in Theorems 5.3 and 5.4 as to the appropriate number of suppliers is still relevant. In particular, we would want to consider all candidate solutions for the number of suppliers where the marginal diversification benefits are non-negative. Therefore, its likely that the optimal number of suppliers will be less than or equal to the number determined in Theorem 5.3.

While these diversification benefits may be difficult to quantify precisely, the firm can analyze the marginal costs associated with a re-allocation strategy to evaluate the diversification benefits. To illustrate, if the order quantity for the $n + 1$th supplier is fairly low, and the cost per unit of the $n + 1$th supplier is only slightly higher than for the nth supplier (i.e., when regret is more likely), then it may be worthwhile to source from $n + 1$ suppliers. Recall the Toyota dilemma discussed in the introduction where Toyota was forced to shut down an assembly plant because of a problem with its sole supplier (Nishiguchi and Beaudet, 1998). In this case, a secondary supplier hedges against the potential costs incurred from problems associated with a single supplier strategy. The marginal costs of including a secondary supplier at a minimum required level can be utilized as a proxy for the "insurance" premium necessary to reap the benefits of a larger pool of suppliers. Likewise, the implicit risk premium for single sourcing can be determined by comparing procurement costs for single versus multiple supplier allocation strategies.

If we assume that the firm has already determined the subset of suppliers that will receive orders, then Theorem 5.5 below specifies the structure of a simple algorithm which can be used to determine optimal order quantities.

Theorem 5.5: When each supplier has both maximum and minimum limitations placed on the size of the order, then the optimal quantity allocated to each supplier can be determined as follows.

Step 1: Index all chosen suppliers in increasing order of cost per unit (i.e., $c_{[1]} \leq c_{[2]} \leq c_{[3]} \ldots \leq c_{[X]}$).

Step 2: For each supplier $[i]$ $(i = 1, \ldots, X)$, determine $Q_{[i]}$ such that:

$F(Q_{[i]}) = \frac{p-c_{[i]}+u}{p-s+u}$

and based on this determine:

$t_{[i]} = Q_{[i]} - \sum_{j=1}^{i-1} y_{[j]}r_{[j]} - \sum_{j=i+1}^{X} z_{[j]}r_{[j]}$

Step 3: The quantity allocated to supplier i is $q_{[i]} = \min\{\max\{\ t_{[i]}, z_{[i]}r_{[i]}\}, y_{[i]}r_{[i]}\}$ and the total quantity ordered by the firm from all suppliers can be determined as $\sum_{i=1}^{X} q_{[i]}$.

Proof: See APPENDIX C.

From Theorem 5.5, we know that at most one of the chosen suppliers will be unconstrained. Suppose that supplier $[i]$ is unconstrained (i.e., $q_{[i]} = t_{[i]}$). Then, the optimal order quantity for the lower cost suppliers $(j = 1, \ldots, i-1)$ is determined by the capacity constraint for each supplier. Similarly, the optimal order quantity for the higher cost suppliers $(j = i+1, \ldots, X)$ is determined by the minimum order quantity dictated by each supplier. Interestingly, the total order quantity in this situation (ie. sum of all orders placed to the subset of suppliers) is determined by the cost of the unconstrained supplier $[i]$ and is such that $F(Q_{[i]}) = \frac{p-c_{[i]}+u}{p-s+u}$.

However, because the generalized structure of this problem has a wide range of supplier options corresponding to alternate cost levels with differing combinations of minimum and maximum order quantities, a simple solution algorithm cannot be easily derived. This leads us to the problem of determining the optimal subset of potential suppliers that will receive an order from the firm, in addition to allocating appropriate order quantities. Branch and bound methodologies can be utilized based on Theorem 5.5 to enumerate all possible subsets of suppliers for an optimal solution. In addition to the optimal mathematical solution, firms may want to consider different qualitative evaluation measures in determining an appropriate subset of suppliers to source from. For example, in an international sourcing context, firms may wish to pick a subset of suppliers in a variety of countries, thereby hedging against country specific risks such as changing political climate and/or exchange rates.

We now turn to an extensive numerical analysis in order to illustrate some of our results and explore the sensitivity of these results for key parameters in our analysis.

5.5 Numerical Analysis

Analytic results have been presented offering insights concerning the optimal choice of suppliers and appropriate order quantities for a manufacturer. In this section, we present the results of a numerical study to illustrate several key cases for which the analytical insights cannot be obtained. We also examine the sensitivity of our results based on changes in the key input parameters. Our intention is to show an overview of these examples which offer insights concerning the relative impact of these factors on a firm's sourcing strategy.

5.5.1 Experimental Design

The parameters and functions were chosen to capture the underlying assumptions outlined in Section 3. The explicit numerical parameters selected for the base case example reflect those shown in Jucker and Rosenblatt (1985). For the manufacturer: (a) price/unit (p) = \$ 19; (b) salvage value (s) = \$ 2/unit; (c) lost sales cost (u) = \$ 6/unit; and (d) demand is assumed to be uniformly distributed with parameters [300,700]. Our supplier base consists of 5 suppliers (i.e., $i = 1, \ldots, 5$) with identical reliabilities ($r_i = 0.9$), minimum order quantities ($z_i = 200\ \forall i$), and capacities ($y_i = 300\ \forall i$). Suppliers are assumed to be heterogeneous with respect to costs (i.e, c_i) and these parameter settings are $c_1 = 6.5$; $c_2 = 7$; $c_3 = 8$; $c_4 = 9$; and $c_5 = 10$. While the diversification benefit function is discrete, we assume that it is roughly quadratic in the number of suppliers (X) and when $X = 1, \ldots, 5$, this function is defined as $d(X) = d_1 - d_2(d_3 - X)^2$ and when $X = 0$, $d(0) = 0$. This functional form of $d(X)$ was chosen since the single parameter d_3 represents the optimal number of suppliers which maximizes this function. All the results discussed next were obtained using LINGO optimization software.

5.5.2 Results

Table 5.1 summarize the results of a set of numerical examples which show sensitivity of the optimal supplier strategy to changes

in parameter values. Model A represents the case where both diversification benefits and supplier minimum order quantities are included. For the remaining examples in Table 5.1, the parameter changes are specified in the variable range column. Not surprisingly, models B and C in Table 1 show that an increase in the price or salvage value of the items increases the total quantity ordered, the total number of suppliers sourced from and the total profit. Similarly, an increase in the underage costs increases the total quantity ordered, the total number of suppliers, but decreases the total profit earned.

Model E is intended to illustrate the impact of changes in the diversification benefit function on the optimal sourcing strategy. In this model, the peak in the magnitude of diversification benefits earned (i.e., d_1) is varied between \$250 and

\$2000 (it is set to \$1000 for the base case example). In response to an increase in the peak value of the diversification function, the total order quantity and the optimal number of suppliers remains the same, while the profit increases. This would indicate that the optimal sourcing policy is fairly robust in that it is not sensitive to large increases in the peak diversification value for this example.

Models F and G illustrate how the optimal sourcing policy changes with alterations in the first supplier's cost and reliability. While small increases in the first supplier's cost does not change the optimal number of suppliers, it does decrease the total quantity ordered and the total profit. Similarly, an increase in the first supplier's reliability decreases the total number of units ordered and increases profit. In general, the firm simply compensates for small changes in reliability by ordering proportionately more items since it does not pay for the bad units.

Table 5–1: Sensitivity Analysis of the Key Parameters

Model	Parameter Range	Firm Profit (\$)	n*	Q*	Nonzero Allocations		
					q_1	q_2	q_3
A	NA	6103.59	3	691	291	200	200
B	p=[10,25]	[1737,9065]	[2,3]	[600,701]	[300,300]	[300,201]	[0,200]
C	s=[-6,6]	[5375,6703]	[2,3]	[591,704]	[300,300]	[291,254]	[0,200]
D	u=[0,12]	[6166,6065]	[3,3]	[660,701]	[260,300]	[200,201]	[200,200]
E	$d_1 = [250, 2000]$	[5354, 7103]	[3,3]	[691,691]	[291,291]	[200,200]	[200,200]
F	$c_1 = [6.25, 6.75]$	[6170,6039]	[3,3]	[667,658]	[296,286]	[200,200]	[200,200]
G	$r_1 = [0.5, 1]$	[6045,6104]	[3,3]	[700,662]	[300,262]	[200,200]	[200,200]

Next we show the impact of the minimum order quantity constraints on the optimal sourcing strategy (see model extensions described in Section 5.5). In this case, we choose three scenarios to illustrate our results. In all three scenarios, the results are generated assuming capacitated suppliers. Table 5.2 contains the results for these three scenarios in the following manner: Model A is the same as in Table 5.1; model H represents the case where diversification benefits are included without supplier minimum order quantities; and model I represents the case where supplier minimum order quantities are incorporated in the absence of diversification benefits. For models A and H, the diversification benefit function parameters were $d_1 = 1000$, $d_2 = 62.5$ and $d_3 = 4$.

Comparing models A and H, the optimal number of suppliers is reduced in the presence of the minimum order quantity constraints. In this case, the marginal benefits of diversification from including an additional supplier do not outweigh the marginal costs of placing a minimal order of 200 units from the 4th supplier. In model H, note that the optimal number of suppliers is 4 with the fourth supplier actually receiving an order of zero. This occurs due to the absence of a minimum order quantity, and reflects the situation where the buying firm would optimally qualify the fourth supplier. Ideally, a contractual arrangement would be negotiated with this supplier facilitating an agreement whereby the buying firm could place an actual order in an emergency situation. Comparing models A and I, the optimal number of suppliers is reduced when the diversification benefits are equal to zero. More specifically, when no diversification benefits exist, then the marginal cost of placing an order with the third supplier at the minimal level of 200 units is not economical great.

Table 5–2: Impact of Minimum Order Quantity on the Sourcing Strategy

Model	Firm Profit ($)			n*	Q*	Quantity Allocations				
						q_1	q_2	q_3	q_4	q_5
	Sales	Diversification	Total							
A	5166.09	937.50	6103.59	3	691	291	200	200	0	0
H	5288.04	1000.00	6288.04	4	662	300	300	62	0	0
I	5199.00	0.00	5199.00	2	600	300	300	0	0	0

A third set of numerical examples shown in Table 5.3 illustrates further interesting interactions that can occur between the minimum order quantities and the reliability factors for the suppliers. Diversification benefits, minimum order quantities, and capacity constraints are included for both of these models. In both of these examples, the supplier costs are $c_1 = 6.5$; $c_2 = 7$; $c_3 = 8$; $c_4 = 9$; and $c_5 = 10$. In model A, the reliabilities of the individual suppliers are homogenous and equal to 0.90. In model J, the reliability factor of the 4th supplier is set to equal half of that of other suppliers, (i.e., 0.45). The impact of the lower reliability factor on the optimal solution is that supplier 4 receives an order while supplier 3 does not. Moreover, the buying firm orders more units and makes more profit. This result is somewhat counter-intuitive, in that lower reliability leads to higher profit and total order quantities. Moreover, profit increases by sourcing from a higher cost supplier. Recall first that in the original model from Section 5.3.2, we assume that the buying firm pays only for good units delivered. The net effect of this assumption is that the minimum order quantity in terms of the good units delivered is much lower for the lower reliability supplier. Also, note that the total quantity delivered is actually lower for model J than for model A due to the lower reliability factor. Therefore, there may be situations where it is optimal to source more units from a lower reliability yet higher cost supplier.

Table 5–3: Interactions between Minimum Order Quantities and Reliabilities

Model	Firm Profit ($)			n*	Q*	Quantity Allocations				
	Sales	Diversification	Total			q_1	q_2	q_3	q_4	q_5
A	5166.09	937.50	6103.59	3	691	291	200	200	0	0
J	5142.39	937.50	6079.89	3	781	300	281	0	200	0

Finally, based on model A and adjusted capacities of 700 for each supplier, we examine cases where the three lowest cost suppliers also have differing reliabilities and minimum order quantities. Each of the three lowest cost suppliers is ranked as best(B), middle(M), or worst(W) for cost, reliability and minimum order quantity. Notationally, instance BWM signifies that the lowest cost supplier is worst in regard to reliability and middle in regard to minimum order quantity. Table 5.4 provides a summary of parameter values used for this experiment for each rank order.

Table 5–4: Parameter Values for Ranked Supplier Characteristics

	Cost	Reliability	Minimum Order Quantity
(B)est	6.50	.9	100
(M)iddle	7.00	.8	150
(W)orst	8.00	.7	200

Table 5.5 summarizes the optimal sourcing strategies for selected instances. A general insight from this experiment is that the total usable quantity is determined by the lowest cost supplier who is not optimally allocated an amount equal to its minimum order quantity or its capacity (confirming the intuitive result developed in Theorem 5.5). Additionally, it is typically preferable to have higher cost suppliers with lower reliabilities. This situation effectively lowers the minimum order quantities of higher cost suppliers and requires shifting fewer units from lower cost suppliers to gain incremental diversification benefits.

Table 5–5: Selected Results for Capacity Adjusted Model A

Supplier Characteristic Vector (c,r,z)			Firm Profit ($)			Optimal Quantity Allocations			Good Units
Supp. 1	Supp. 2	Supp. 3	Sales	Divers.	Total	q_1	q_2	q_3	Q*
BBB	MMM	WWW	5466.09	750.00	6216.09	557	150	0	622
BBB	MMW	WWM	5288.59	937.50	6226.09	396	200	150	622
BBW	MMM	WWB	5361.09	937.50	6298.59	480	150	100	622
BMB	MBW	WWM	5278.59	937.50	6216.09	421	200	150	622
BMB	MWW	WBM	5456.09	750.00	6206.09	602	200	0	622
BWB	MBW	WMM	5256.09	937.50	6193.59	460	200	150	622
BWB	MMM	WBW	5462.39	750.00	6212.39	700	154	0	613
BWM	MBB	WMW	5462.39	750.00	6212.39	700	137	0	613
BWM	MBW	WMB	5316.09	937.50	6253.59	517	200	100	622

5.6 Conclusions and Implications

Analytic and numerical analysis of our model provide several managerial insights. To start with, consider the situation where supplier minimum order quantities are not considered and when the firm does not obtain any explicit benefits by diversifying its supplier base. First, the industry practice of single sourcing is only optimal when supplier capacities are relatively large as compared to product demand. In such a case, the firm's optimal choice is to source all its requirements from the least cost supplier. Interestingly, supplier reliabilities do not moderate the choice of the supplier unless the firm is required to compensate suppliers for all units ordered rather than simply the "good" units received. In the latter case, the ratio of costs to reliabilities is relevant in determining the supplier from which all demand is sourced. Second, we show that when supplier capacities are relevant, the optimal strategy for the firm is to source from multiple suppliers. Under this scenario we find that the firm's total order quantity (across all suppliers) and expected profits are both lower than that compared to the scenario where suppliers are uncapacitated. The difference in profits could be regarded as the value to the firm which could be realized if the lowest cost supplier could be motivated to increase his/her capacity.

When positive net diversification benefits are incorporated (without supplier minimum order quantities), the key results are as follows. If suppliers are uncapacitated, then multiple supplier sourcing strategies are always optimal where the number of suppliers is determined by the diversification benefit function. Managerially, this implies that the firm should determine the total order quantity based on the least cost supplier. However, in placing orders, it should order the required amount from the least cost supplier and order marginal quantities from all the other selected suppliers. When suppliers are capacitated, a similar simple decision rule can be used by the firm when the number of suppliers which optimizes the diversification benefits is larger than the number which are selected without such a benefit.

Through an extensive numerical analysis we also examine the robustness of our results when supplier minimum order quantities are relevant in making a firm's sourcing decisions. A counter-intuitive insight we obtain for this case is that there is an interaction between reliabilities, costs, and minimum order quantities. For example, we show that in certain cases, it may be optimal to source from a higher cost, lower reliability supplier as compared to a lower cost, higher reliability supplier. This is generally the case when a lower effective minimum order quantity is economically preferable. The insight here is that the flexibility of a supplier may have greater bearing on selection than unit cost.

The general model in this chapter incorporates strategic diversification considerations within the traditional newsvendor framework to determine the optimal number of suppliers to place an order with and the corresponding quantities of those orders. Through the introduction of the diversification benefit function, we explicitly account for buyer specific supply base management benefits based on the size of a selected supplier pool. In essence, this function could be construed as capturing the monetized utility of insurance naturally provided by the size of the selected supplier pool. This consideration highlights the need for a purchasing manager to execute sourcing strategies consistently with the firm's operations and broader corporate strategy.

CHAPTER 6
SUMMARY

The strategic importance of sourcing activities is inherent in their vitality to a firm's means of value creation. Sourcing activities link the firm to its chain of supply. A firm's sourcing strategy is therefore a key driver of an effective supply chain strategy. A firm's sourcing strategy can be characterized by three key decision: (a) criteria for establishing a supply base; (b) criteria for selecting suppliers from the approved base to receive orders; and (c) the quantity of goods to order from each supplier selected. Decisions (b) and (c) are decisions of selection and allocation respectively. In this dissertation the selection and allocation decisions of a firm's sourcing strategy are studied under various operating conditions commonly encountered in practice. We are especially focused on determining when it is optimal for a firm to choose a specialized (single sourcing) strategy versus a generalized (multiple sourcing) strategy. In particular, the supply chain we examine consists of a single intermediary buying firm serviced by multiple upstream suppliers. By separately modeling and examining the influence of uncertainty in product supply and demand, alternate quantity pricing schemes quoted by suppliers, and explicit benefits of a diversified sourcing strategy, we provide managerial insights for this buying firm's optimal sourcing strategy. The remainder of this summary chapter reviews the key results of our analysis and numerical experiments in each of chapters 3, 4 and 5, and discusses various avenues for future research related to this dissertation.

6.1 Key Results and Directions for Future Research from Chapter 3

Business operations are often faced with uncertainties in product supply and demand. Chapter 3 provides structural and numerical results for determining an appropriate supplier sourcing strategy in the presence of upstream and downstream uncertainty. In the context of the supplier selection decision, our results are in line with observed practice. For example, Verma and Pullman (1996) find that while supply managers recognize the importance of quality, cost primarily drives their supplier selection decisions. In our model, a supplier's cost (and not its reliability) is the key factor which comes into play when a firm is deciding whether or not to place an order with that supplier. Consequently, the lowest cost supplier in the pre-qualified pool will always receive a positive order. An exception to this rule shown through numerical examples is when the lowest cost supplier has a restrictively high minimum order quantity. It follows that if all pre-qualified suppliers have similar costs, then it's optimal to place an order with all suppliers in the pool.

Also, for convenient evaluation of the firm's supplier selection decision, we derive a simple ratio to analytically determine whether or not a single supplier strategy is appropriate. This ratio reflects a trade-off between the first supplier's reliability and its cost advantage relative to other suppliers. Essentially, if the lowest cost supplier has a reliability distribution with a high mean and a low standard deviation, and has a large cost advantage, then a single supplier strategy is warranted.

Another key factor influencing the firm's supplier selection decision is the firm's anticipated demand. Both analytic and numerical results confirm that a single supplier strategy is favorable when the mean demand is low. However, if a firm anticipates a significant increase in demand, it should consider enlarging its supplier base even when the low cost supplier could provide the full order quantity. Surprisingly, an increase in the variability in demand also favors a single sourcing strategy. In this case, the firm limits its financial risk by sourcing only with the single lowest cost supplier when the firm anticipates great uncertainty in demand.

The firm's order quantity allocation decision concerns how much to order from each selected supplier. Analytic expressions are developed which determine the optimal order quantity for each selected supplier under a variety of circumstances. The most general case is shown in Theorem 3.1 which addresses the situation where all suppliers have different costs and reliability functions. In this case, each supplier will receive an order amount based on its unit cost, mean reliability, and variance in reliability. Note that while the lowest cost supplier is guaranteed to receive a positive order, he/she won't necessarily receive the largest order. In contrast, we also analyze the situation where the suppliers are homogeneous in their costs and reliability functions in Theorem 3.4. In this situation, all pre-qualified suppliers receive an equivalent order quantity.

There are several areas of research related to chapter 3 which warrant further investigation. First, the model may be extended to encompass multiple periods and multiple products. Also, a more detailed model could be developed which addresses the fixed and potentially cumbersome costs of the initial supplier qualification process. Another modification of the model for future consideration is to incorporate not only quality reliability, but also timeliness of delivery in a multiple period model. Further, we assume that demand is uniformly distributed to facilitate the development of simplified expressions. Other types of demand distributions could be explored to enhance the generalizability of chapter 3's results. Another modification of the model for future consideration is to incorporate not only quality reliability, but also timeliness of delivery in a multiple period model. Lastly, the focus of this model is on decision making at the buying firm; future research could incorporate the supplier's decision making process as well.

6.2 Key Results and Directions for Future Research from Chapter 4

Evaluating alternative pricing schedules offered as quantity discounts is a quantitative problem commonly presented to sourcing decision makers. In chapter 4, we model a central purchasing organization's supplier selection and order quantity allocation decisions with supplier quoted quantity discount pricing. We examine these decisions for environments where suppliers offer constant, linear discount, incremental discount, and all-unit discount pricing schemes.

Our analysis of alternate supplier base pricing schemes provides guidance for a buying firm's optimal sourcing strategy. For the constant cost case, if the lowest cost supplier has enough capacity to satisfy all of the buying firm's product requirements, single sourcing is optimal, otherwise multiple sourcing is best. For sourcing scenarios where supplier bases offer quantity discount pricing schemes, if all suppliers in the base possess enough capacity to individually provide Q units, then it is optimal to single source from the least cost supplier evaluated at Q units. However, if even one supplier in the base is incapable of providing Q units, then single sourcing may not be the optimal sourcing strategy.

In cases where quantity discount quoting suppliers' capacity is individually inadequate for a buying firm's product requirements, a firm's sourcing problem can become extremely complex. Therefore, identifying an optimal solution may be resource (time) prohibitive for supply chain sourcing professionals. In such cases, the heuristics developed in this chapter can be expected to efficiently provide good quality solutions. In sum, the heuristic solutions to randomly generated test problems arrived at the optimal solution in 62% of the instances. Furthermore, in the 38% of non-optimal solution instances, a respectable .64% average optimality gap exists.

Leveraging Result 4.1, we have developed efficient and well-performing heuristics to generate supplier quantity allocations where the selected supplier(s) capacity binds. Additionally, we validate LINGO's Global Solver optimal solutions for our incremental discounts test problems by developing a branch and bound algorithm.

An immediate extension of the models developed in chapter 4 is to consider multiple products required by the buying firm and supplied by multiple suppliers.

Additionally, quantity discounts can be applied based on total purchase value instead of number of units. Another noteworthy modification is for each suppliers aggregate capacity to depend on the product mix or bundle ordered by the firm. Finally, incorporating downstream price dependency or demand uncertainty with upstream quantity discount pricing warrants further examination.

6.3 Key Results and Directions for Future Research from Chapter 5

Chapter five models a buying firms source selection and order quantity allocation decisions in an operating environment characterized by unreliable upstream supply and uncertain downstream demand. Analytic and numerical analysis of this model provide several managerial insights. First, if supplier minimum order quantities are not considered and the firm obtains no explicit benefits by diversifying its supplier base, then single sourcing is optimal when the lowest cost suppliers capacity is relatively large as compared to product demand. Interestingly, supplier reliabilities do not moderate the choice of the supplier unless the firm is required to compensate suppliers for all units ordered rather than simply the “good” units received. In the latter case, the ratio of costs to reliabilities is relevant in determining the supplier from which all demand is sourced.

Second, we show that when supplier capacities are relevant, the optimal strategy for the firm is to source from multiple suppliers. Under this scenario we find that the firm’s total order quantity (across all suppliers) and expected profits are both lower than that compared to the scenario where suppliers are uncapacitated. The difference in profits could be regarded as the value to the firm which could be realized if the lowest cost supplier could be motivated to increase his/her capacity.

When positive net diversification benefits are incorporated (without supplier minimum order quantities), the key results are as follows. If suppliers are uncapacitated, then multiple supplier sourcing strategies are always optimal where the number of suppliers is determined by the diversification benefit function. Managerially, this implies that the firm should determine the total order quantity based on the least cost supplier. However, in placing orders, it should order the required amount from the least cost supplier and order marginal quantities from all the other selected suppliers. When suppliers are capacitated, a similar simple decision rule can be used by the firm when the number of suppliers which optimizes the diversification benefits is larger than the number selected without such a benefit.

Through extensive numerical analysis with supplier minimum order quantity constraints, an insight we obtain is that there is a counter-intuitive interaction between reliabilities, costs, and minimum order quantities. For example, we show that in certain cases, it may be optimal to source from a higher cost, lower reliability supplier as compared to a lower cost, higher reliability supplier. This is generally the case when a lower effective minimum order quantity is economically preferable. The insight here is that the flexibility of a supplier may have greater bearing on selection than unit cost. This is also similar to a total cost of ownership approach to supplier selection.

The explicit diversification benefits model developed in chapter five can be extended for future research by examining multiple product sourcing scenarios as well as multiple period problems. An interesting and growing area of practical application is in combinatorial procurement auctions. Also, operationalizing the diversification benefit function is an area of empirical research interest.

APPENDIX A
PROOFS FOR CHAPTER 3

A.1 Proof of Corollary 3.1

Corollary 3.1: The expected profit function shown in Equation (3.1) is concave in the order quantities q_i for N suppliers when demand is uniformly distributed between $[a, b]$.

Proof: Given that demand is uniformly distributed, the objective function in Equation (3.1) can be simplified:

$$E[\pi] = \tfrac{-u(a+b)}{2} - \tfrac{(p+u-s)a^2}{2(b-a)} + \sum_{i=1}^{N}(p+u-c_i)\bar{r}_i q_i + \tfrac{(p+u-s)a}{(b-a)}\sum_{i=1}^{N}\bar{r}_i q_i$$
$$-\tfrac{(p+u-s)}{2(b-a)}\left(\sum_{i=1}^{N} V_i q_i^2 + 2\sum_{i=1}^{N-1}\sum_{j=i}^{N}\bar{r}_i\bar{r}_j q_i q_j\right)$$

The Hessian for the objective function with N suppliers is as follows:

$$H_N = L'H' = L'\begin{pmatrix} -V_1 & -\bar{r}_1\bar{r}_2 & . & -\bar{r}_1\bar{r}_i & . & . & -\bar{r}_1\bar{r}_N \\ & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ -\bar{r}_i\bar{r}_1 & -\bar{r}_i\bar{r}_2 & . & -V_i & . & . & -\bar{r}_i\bar{r}_N \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ -\bar{r}_N\bar{r}_1 & -\bar{r}_N\bar{r}_2 & . & -\bar{r}_N\bar{r}_i & . & . & -V_N \end{pmatrix}$$

where $L' = \tfrac{(p+u-s)}{(b-a)}$ and $V_i = \sigma_i^2 + \bar{r}_i^2$.

Then, the determinant of the Hessian for N suppliers is:

$$|H_N| = (-L')^N\left[\sum_{i=1}^{N}\bar{r}_i^2\prod_{j\neq i}\sigma_j^2 + \prod_{i=1}^{N}\sigma_i^2\right]$$

Since the mean and variance for all reliability distributions are positive, then the sign of the determinant is determined by $(-L')^N$. Also, utilizing standard assumptions for newsboy problems, we have $p > c_i \quad > s$. Since b>a for the uniform distribution, then $L' > 0$and the sign of the determinant of the Hessian matrix is simply $(-1)^N$. In order to establish concavity of the objective, all of the k principal minors for the Hessian matrix must have the same sign as $(-1)^k$. By substituting k for N in the expression for the determinant of the Hessian, it follows that the principal minors have the appropriate sign such that the objective is negative definite and consequently concave for any N.

A.2 Proof of Theorem 3.1

Theorem 3.1: When suppliers are heterogeneous with respect to costs and reliability parameters, then the optimal sourcing quantity for each supplier i is:

$$q_i^* = \frac{\frac{\bar{r}_i}{\sigma_i^2}\left[\sum_{j=1}^{N}(b_i - b_j)\left(\frac{\bar{r}_j}{\sigma_j}\right)^2 + b_i\right]}{\sum_{j=1}^{N}\left(\frac{\bar{r}_j}{\sigma_j}\right)^2 + 1} \tag{A.1}$$

where $b_i = \ K_i(b-a) + a$, and $K_i = \frac{p+u-c_i}{p+u-s}$.

Proof: The proof follows directly from the first order conditions of optimality. Consider first the optimal order quantity for the first supplier when N=1-5:

$q_1* = \frac{\bar{r}_1 b_1}{\bar{r}_1^2 + \sigma_1^2}$; for N=1

$q_1* = \frac{\bar{r}_1 [\bar{r}_2^2(b_1-b_2)+b_1\sigma_2^2]}{D}$ where $D = \sigma_1^2\sigma_2^2 + \bar{r}_2^2\sigma_1^2 + \bar{r}_1^2\sigma_2^2$; for N=2

$q_1 = \frac{\bar{r}_1\left[(b_1-b_2)\bar{r}_2^2\sigma_3^2+(b_1-b_3)\bar{r}_3^2\sigma_2^2+b_1\sigma_2^2\sigma_3^2\right]}{\bar{r}_1^2\sigma_2^2\sigma_3^2+\bar{r}_2^2\sigma_1^2\sigma_3^2+\bar{r}_3^2\sigma_1^2\sigma_2^2+\sigma_1^2\sigma_2^2\sigma_3^2}$ for N=3

$q_1 = \frac{\bar{r}_1\left[(b_1-b_2)\bar{r}_2^2\sigma_3^2\sigma_4^2+(b_1-b_3)\bar{r}_3^2\sigma_2^2\sigma_4^2+(b_1-b_4)\bar{r}_4^2\sigma_2^2\sigma_3^2+b_1\sigma_2^2\sigma_3^2\sigma_4^2\right]}{\bar{r}_1^2\sigma_2^2\sigma_3^2\sigma_4^2+\bar{r}_2^2\sigma_1^2\sigma_3^2\sigma_4^2+\bar{r}_3^2\sigma_1^2\sigma_2^2\sigma_4^2+\bar{r}_4^2\sigma_1^2\sigma_2^2\sigma_3^2+\sigma_1^2\sigma_2^2\sigma_3^2\sigma_4^2}$ for N=4

$q_1 = \frac{\bar{r}_1\left[(b_1-b_2)\bar{r}_2^2\sigma_3^2\sigma_4^2\sigma_5^2+(b_1-b_3)\bar{r}_3^2\sigma_2^2\sigma_4^2\sigma_5^2+(b_1-b_4)\bar{r}_4^2\sigma_2^2\sigma_3^2\sigma_5^2+(b_1-b_5)\bar{r}_5^2\sigma_2^2\sigma_3^2\sigma_4^2+b_1\sigma_2^2\sigma_3^2\sigma_4^2\sigma_5^2\right]}{\bar{r}_1^2\sigma_2^2\sigma_3^2\sigma_4^2\sigma_5^2+\bar{r}_2^2\sigma_1^2\sigma_3^2\sigma_4^2\sigma_5^2+\bar{r}_3^2\sigma_1^2\sigma_2^2\sigma_4^2\sigma_5^2+\bar{r}_4^2\sigma_1^2\sigma_2^2\sigma_3^2\sigma_5^2+\bar{r}_5^2\sigma_1^2\sigma_2^2\sigma_3^2\sigma_4^2+\sigma_1^2\sigma_2^2\sigma_3^2\sigma_4^2\sigma_5^2}$ for N=5

It follows that for $N > 2$:

$$q_i^* = \frac{\bar{r}_i \left[\sum\limits_{\substack{j=1 \\ j \neq i}}^{N} (b_i - b_j)\bar{r}_j^2 \prod\limits_{\substack{k=1 \\ k \neq i \\ k \neq j}}^{N} \sigma_k^2 + b_i \prod\limits_{\substack{j=1 \\ j \neq i}}^{N} \sigma_j^2 \right]}{\sum\limits_{i=1}^{N} \bar{r}_i^2 \prod\limits_{\substack{j=1 \\ j \neq i}}^{N} \sigma_j^2 + \prod\limits_{i=1}^{N} \sigma_i^2} \text{ or } q_i^* = \frac{\frac{\bar{r}_i}{\sigma_i^2}\left[\sum\limits_{j=1}^{N} (b_i - b_j)\left(\frac{\bar{r}_j}{\sigma_j}\right)^2 + b_i\right]}{\sum\limits_{j=1}^{N} \left(\frac{\bar{r}_j}{\sigma_j}\right)^2 + 1}.$$

A.3 Proof of Corollary 3.2

Corollary 3.2: The firm will always order a positive quantity from the lowest cost supplier.

Proof: Suppose that $c_i < c_j$ for all $j \in [1, N]$ and $j \neq i$. It follows that $K_i > K_j$, and $b_i > b_k$. From Equation (A.1), it is clear that $q_i* > 0$.

A.4 Proof for Corollary 3.3

Corollary 3.3: The firm will source all its requirements from the lowest cost supplier (i.e., use a single sourcing strategy) if and only if:

$$\left(\frac{\sigma_1}{\bar{r}_1}\right)^2 < \frac{b_1 - b_j}{b_j} = \frac{(c_j - c_1)(b - a)}{(p + u - c_j)b + (c_j - s)a} \; for\ j = 2, ..N \tag{A.2}$$

Proof: Consider the solution as shown in Equation (A.1). The proof follows directly from Corollary 3.2 and by setting $q_j \leq 0$ for $j = 2, .., N$.

A.5 Proof of Corollary 3.4

Corollary 3.4: When a multiple supplier sourcing strategy is optimal, the lowest cost supplier will not necessarily receive the highest order quantity.

Proof: Consider the case where a multiple supplier sourcing strategy is optimal where n^* suppliers ($n* > 1$) receive an order. Then, if the following Equation holds, supplier j $(j = 2, .., n*)$ will receive a higher quantity order than supplier 1:

$$(b_1 - b_j) = \frac{(b-a)(c_j - c_1)}{(p+u-s)} \leq \frac{\bar{r}_j b_j \sigma_1^2 - \bar{r}_1 b_1 \sigma_j^2}{\bar{r}_1 \bar{r}_j (\bar{r}_1 - \bar{r}_j)}$$

A.6 Proof of Corollary 3.5

Corollary 3.5: A higher cost supplier will never receive a positive order when a lower cost supplier's order quantity is equal to zero.

Proof: Consider a three supplier example, and order the suppliers such that $c_1 < c_2 < c_3$. Let the variables λ_1, λ_2 and λ_3 represent the Lagrangian multipliers associated with the non-negativity constraints for q_1,q_2 and q_3 respectively. From Corollary 3.2, we know that λ_1 =0 in any optimal solution. Suppose that $\lambda_2 > 0$ (i.e., q_2= 0) and $\lambda_3 = 0$ (i.e., $q_3 > 0$). From $\lambda_2 \quad > 0$ and solving for c_1, we have:

$$\lambda_2 = \frac{r_2 \left[(c_2 - c_3) r_3^2 \sigma_1^2 + (c_2 - c_1) r_1^2 \sigma_3^2 + (c_2 - ah - p - u) \sigma_1^2 \sigma_3^2 \right]}{r_3^2 \sigma_1^2 + r_1^2 \sigma_3^2 + \sigma_1^2 \sigma_3^2} > 0$$

$$c_1 < K_1 = \frac{(c_2 - c_3) r_3^2 \sigma_1^2 + c_2 r_1^2 \sigma_3^2 + (c_2 - ah - p - u) \sigma_1^2 \sigma_3^2}{r_1^2 \sigma_3^2}$$

From $q_3 > 0$ and solving for c_1, we have:

$$q_3 = c_1 r_1^2 - c_3 r_1^2 - c_3 \sigma_1^2 + \sigma_1^2 (ah + p + u) > 0$$

$$c_1 > K_2 = \frac{c_3 r_1^2 + c_3 \sigma_1^2 - (ah + p + u) \sigma_1^2}{r_1^2}$$

Since $K_1 < K_2$, the proof follows by contradiction.

A.7 Proof of Corollary 3.6

Corollary 3.6: The optimal subset of suppliers (n^*) receiving a positive order quantity is the lowest cost subset of suppliers such that the following relationships hold:

$$b_{n*} > \frac{\sum_{j=1}^{n*-1} b_j \left(\frac{\bar{r}_j}{\sigma_j} \right)^2}{1 + \sum_{j=1}^{n*-1} \left(\frac{\bar{r}_j}{\sigma_j} \right)^2} \text{ and } b_{n*+1} \leq \frac{\sum_{j=1}^{n*} b_j \left(\frac{\bar{r}_j}{\sigma_j} \right)^2}{1 + \sum_{j=1}^{n*} \left(\frac{\bar{r}_j}{\sigma_j} \right)^2}$$

Proof: The proof follows directly from the first order conditions of optimality shown in Equation (A.2) assuming that $q_{n*} > 0$ and $q_{n*+1} \leq 0$.

A.8 Proof of Theorem 3.2

Theorem 3.2: When suppliers (a) have *heterogeneous* cost structures and (b) have *identical* reliability distributions, then there is no one dominant supplier sourcing strategy. The optimal order quantity for each supplier i is:

$$q_i^* = \frac{\bar{r}\left[\bar{r}^2 \sum_{j=1}^{N} (b_i - b_j) + b_i \sigma^2\right]}{N\bar{r}^2\sigma^2 + \sigma^4} \quad \forall i.$$

Proof:Follows directly from Equation (A.1).

A.9 Proof of Theorem 3.3

Theorem 3.3: When suppliers (a) have *identical* cost structures and (b) have *unique* reliability distributions, then it is optimal to order from all suppliers. The optimal order quantity for each supplier i is:

$$q_i^* = \frac{\frac{\bar{r}_i}{\sigma_i^2} b'}{\sum_{j=1}^{N} \left(\frac{\bar{r}_j}{\sigma_j}\right)^2 + 1} \quad \forall i$$

where $b' = K(b-a) + a$, and $K = \frac{p+u-c}{p+u-s}$.

Proof: Follows directly from Equation (A.1).

A.10 Proof of Theorem 3.4

Theorem 3.4: When suppliers (a) have *identical* cost structures and (b) have *identical* reliability distributions, then it is optimal to order the same amount from all suppliers. The optimal order quantity for each supplier is:

$$q_i* = \frac{\bar{r}b'}{\sigma^2 + N\bar{r}^2} \quad \forall i. \tag{A.3}$$

Proof: Let $c_i = c$, $r_i = r$, $g_i = g$, $G_i = G$, and $\sigma_i = \sigma$ in Equation (A.1). Then, the optimal order quantity for all suppliers is:

$$q_i* = \frac{\bar{r}\left[0 + b'(\sigma^2)^{N-1}\right]}{(\sigma^2)^N + N\bar{r}^2(\sigma^2)^{N-1}} \quad \forall i \quad \text{or} \quad q_i* = \frac{\bar{r}b'}{\sigma^2 + N\bar{r}^2} \quad \forall i.$$

A.11 Proof of Corollary 3.7

Corollary 3.7: Suppose m of the N suppliers (with $m < N$) have identical cost and reliability distributions. Then, the same order quantity should be placed with all m suppliers.

Proof: The proof follows directly from Equation (A.1). It may also be the case where the m homogeneous suppliers do not receive a positive order quantity (i.e., q_i = 0.)

APPENDIX B
PROOFS FOR CHAPTER 4

B.1 Proof of Theorem 4.1

Theorem 4.1:Under constant supplier prices, the optimal sourcing policy for the firm is:

- Index suppliers in non-decreasing order of prices (i.e., $c_1 \leq c_2 \leq \ldots \leq c_n$).
- If $y_1 \geq Q$, then source the complete requirement Q from supplier, i.e., $q_i^* = Q$ and $q_j = 0 \; \forall \; j = 2, \ldots, n$.
- If $y_1 < Q$, then the following algorithm determines the optimal sourcing strategy for the firm:
  1. Set i=1.
  2. Order $q_i = \min\{Q, y_i\}$ units from supplier i. If $q_i = 0$, then set $q_j = 0$ $\forall j = i+1, \ldots, n$ and Stop, else goto 3.
  3. Set $Q = Q - q_i$, $i = i + 1$ and repeat 2.

Proof: Given a suppplier set indexed in non-decreasing order of prices (i.e., $c_1 \leq c_2 \leq \ldots \leq c_n$), and Supplier 1's capacity, $y_1 \geq Q$, our total cost for single sourcing requirements, Q, from Supplier 1 is:

$$TC_1 = c_1 Q$$

Now consider shifting any ϵ order quantity from Supplier 1 to any other Supplier $j > 1$, such that $c_j \geq c_1$. In this case our total cost of not single sourcing from Supplier 1 is:

$$TC_2 = c_1(Q - \epsilon) + c_j\epsilon = c_1 Q - c_1\epsilon + c_j\epsilon = c_1 Q + (c_j - c_1)\epsilon$$

And since $c_j \geq c_1$, $TC_2 - TC_1 \geq 0$. This conclude our proof.

B.2 Proof of Result 4.1

Result 4.1: There exists at least one optimal solution to our sourcing model such that $q_i = 0$ or $q_i = y_i$ for all i suppliers except that there may be at most one supplier j for which $0 < q_j < y_j$.

The cost minimization problem is:

$$MinZ = \sum_{i=1}^{n} f_i(q_i)$$
$$s.t.$$
$$\sum_{i=1}^{n} q_i = Q$$
$$0 \le q_i \le y_i \quad \forall i$$

where $f_i(q_i) = a_i q_i - (biqi^2)$

Proof:We have a concave cost minimization problem for supply of single product. As such, the proof of this result is quite similar to that of Chauhan and Proth (2003, p.375-376).

Let $S^1 = \{q_1^1, q_2^1, ..., q_n^1\}$ be a feasible solution to the above problem. Assume that there exists $i, j \in \{1, 2, ..., n\}$ such that:

$0 < q_i^1 < y_j$ and $0 < q_j^1 < y_j$

and that:

$$\frac{\partial f_i}{\partial q_i}(q_i^1) \le \frac{\partial f_j}{\partial q_j}(q_j^1) \tag{B.1}$$

choose $\delta = Min\{y_i - q_i^1, q_j^1\}$

Set:

$$q_i^2 = q_i^1 + \delta \tag{B.2}$$

$$q_j^2 = q_j^1 - \delta \tag{B.3}$$

Obtain a new feasible solution,S^2, by replacing q_i^1 with q_i^2 and q_j^1 with q_j^2.

Adding (B.2) and (B.3):

$q_i^2 + q_j^2 = q_i^1 + q_j^1$

This together with the definition of δ, insure that S^2 is feasible.

Now, since relation (B.1) holds for all feasible solutions we know that:

$$\frac{\partial f_i}{\partial q_i}(q_i^*) \leq \frac{\partial f_j}{\partial q_j}(q_j^*) \tag{B.4}$$

and for any $q_{i^*} \geq q_i^1$ and $q_{j^*} \leq q_j^1$,

we know that:

$f_i(q_i^*) - f_i(q_i^1) \leq f_j(q_j^1) - f_j(q_j^*)$.

So

$f_i(q_i^2) - f_i(q_i^1) \leq f_j(q_j^1) - f_j(q_j^2)$,

and therefore, $\sum_{i=1}^n f_i(q_i^2) \leq \sum_{i=1}^n f_i(q_i^1)$.

Repeating this process will lead to a solution S^2 that verifies Result 3.1's conditions.

B.3 Linear Discount Pricing Test Problems Data

Table B–1: Linear Discount Pricing Test Problems Data (1-12)

	1			2			3			4		
Supplier	y	a	b	y	a	b	y	a	b	y	a	b
1	945	71	0.02	164	121	0.55	12	15	0.33	138	108	0.54
2	91	109	0.71	83	57	0.49	126	56	0.26	130	27	0.11
3	250	165	0.5	860	198	0.07	296	116	0.05	201	197	0.64
4	35	21	0.47	21	56	0.83	45	83	0.79	72	162	0.68
5	367	142	0.13	219	116	0.02	468	88	0.06	746	170	0.14
6	777	175	0.16	249	108	0.38	7	22	0.13	18	17	0.84
7	978	165	0.01	42	30	0.33	74	189	0.2	193	184	0.7
8	50	171	0.13	74	70	0.58	766	95	0.05	981	87	0.05
9	295	91	0.29	762	17	0.02	80	176	0.97	77	154	0.81
10	35	125	0.38	12	8	0.4	143	103	0.54	128	81	0.57
	5			6			7			8		
Supplier	y	a	b	y	a	b	y	a	b	y	a	b
1	968	185	0.08	78	166	0.99	190	67	0.19	139	125	0.6
2	674	195	0.23	240	47	0.09	956	158	0.13	58	177	0.81
3	89	195	0.42	468	169	0.29	4	7	0.68	597	180	0.29
4	15	16	0.43	810	138	0.03	49	36	0.09	129	156	0.4
5	156	120	0.68	9	21	0.69	169	99	0.53	57	84	0.54
6	239	38	0.1	17	75	0.76	46	125	0.86	87	147	0.35
7	74	60	0.71	179	196	0.75	479	167	0.33	901	133	0.08
8	70	62	0.71	28	102	0.92	439	137	0.15	259	27	0.02
9	29	149	0.98	290	189	0.15	278	159	0.35	25	180	0.41
10	62	103	0.92	133	95	0.67	91	135	0.69	264	199	0.37
	9			10			11			12		
Supplier	y	a	b	y	a	b	y	a	b	y	a	b
1	139	125	0.6	120	110	0.27	139	125	0.6	422	75	0.17
2	58	177	0.81	169	181	0.85	58	177	0.81	26	36	0.39
3	597	180	0.29	70	56	0.77	597	180	0.29	268	106	0.23
4	129	156	0.4	125	109	0.36	129	156	0.4	124	99	0.59
5	57	84	0.54	93	93	0.78	57	84	0.54	274	136	0.33
6	87	147	0.35	525	90	0.13	87	147	0.35	817	53	0.05
7	901	133	0.08	397	24	0.02	901	133	0.08	22	103	0.7
8	259	27	0.02	83	180	0.9	259	27	0.02	53	73	0.87
9	25	180	0.41	855	177	0.04	25	180	0.41	3	65	0.45
10	264	199	0.37	47	178	0.25	264	199	0.37	35	35	0.01

Table B–2: Linear Discount Pricing Test Problems Data (13-24)

	13			14			15			16		
Supplier	y	a	b	y	a	b	y	a	b	y	a	b
1	140	153	0.96	542	110	0.09	6	4	0.54	251	150	0.55
2	8	4	0.46	574	175	0.06	65	167	0.68	8	4	0.48
3	32	160	0.56	90	152	0.76	157	165	0.6	763	158	0.11
4	24	162	0.65	130	140	0.92	612	199	0.06	134	21	0.12
5	247	172	0.17	0	10	0.99	42	69	0.96	67	143	0.44
6	543	190	0.19	6	31	0.09	53	144	0.93	15	66	0.73
7	577	185	0.28	62	83	0.37	241	132	0.35	160	135	0.2
8	520	171	0.23	308	96	0.18	476	182	0.06	151	177	0.55
9	68	44	0.61	166	62	0.09	150	184	0.99	556	188	0.07
10	37	137	0.85	704	158	0.22	545	91	0.13	12	33	0.52
	17			18			19			20		
Supplier	y	a	b	y	a	b	y	a	b	y	a	b
1	32	98	0.64	168	79	0.36	17	4	0.09	252	193	0.21
2	329	84	0.14	732	135	0.09	179	127	0.37	54	21	0.39
3	382	98	0.01	12	17	0.24	703	182	0.24	39	102	0.74
4	73	65	0.15	129	177	0.82	31	127	0.81	407	152	0.14
5	754	186	0.17	22	115	0.43	321	152	0.21	185	178	0.3
6	28	43	0.65	88	13	0.11	296	155	0.4	252	40	0.07
7	287	86	0.11	907	105	0.06	41	20	0.01	15	18	0.93
8	10	93	0.93	258	189	0.62	428	197	0.27	160	71	0.03
9	106	79	0.3	197	98	0.32	45	155	0.68	652	133	0.06
10	95	157	0.17	28	53	0.75	7	5	0.06	12	48	0.82
	21			22			23			24		
Supplier	y	a	b	y	a	b	y	a	b	y	a	b
1	78	57	0.25	831	190	0.07	249	188	0.33	578	170	0.21
2	233	176	0.6	93	26	0.23	10	28	0.67	940	189	0.06
3	57	83	0.79	873	141	0.15	112	97	0.16	121	145	0.71
4	64	67	0.61	206	67	0.3	159	182	0.94	250	172	0.39
5	2	173	0.69	120	189	0.05	204	174	0.85	162	172	0.72
6	25	54	0.17	235	94	0.35	95	119	0.57	291	141	0.45
7	38	174	0.85	93	112	0.74	985	139	0.11	207	119	0.17
8	47	100	0.97	957	160	0.13	36	107	0.99	134	129	0.56
9	486	41	0.01	3	2	0.62	67	144	0.95	21	161	0.44
10	998	141	0.14	94	27	0.22	130	111	0.45	219	90	0.11

Table B–3: Linear Discount Pricing Test Problems Data (25-30)

	25			26			27			28		
Supplier	y	a	b	y	a	b	y	a	b	y	a	b
1	379	142	0.21	34	73	0.9	60	155	0.75	196	135	0.43
2	165	156	0.8	350	67	0.08	163	176	0.77	16	28	0.4
3	697	116	0.09	600	161	0.22	125	94	0.72	171	68	0.16
4	159	91	0.28	40	91	0.82	283	160	0.29	176	82	0.38
5	637	121	0.18	161	92	0.44	910	199	0.14	809	154	0.06
6	132	105	0.69	40	11	0.12	303	100	0.33	306	147	0.46
7	47	29	0.15	63	152	0.77	120	34	0.15	139	141	0.97
8	28	153	0.91	652	179	0.18	241	149	0.06	106	56	0.24
9	18	20	0.55	170	141	0.65	483	137	0.11	95	121	0.02
10	77	68	0.58	14	47	0.61	966	152	0.14	23	68	0.71
	29			30								
Supplier	y	a	b	y	a	b						
1	18	193	0.45	185	73	0.35						
2	19	60	0.99	45	50	0.84						
3	200	49	0.01	471	34	0.01						
4	55	138	0.59	841	181	0.01						
5	154	78	0.35	10	92	0.89						
6	36	86	0.94	112	140	0.14						
7	373	91	0.13	10	24	0.84						
8	373	108	0.26	112	165	0.82						
9	837	72	0.06	150	166	0.19						
10	93	176	0.81	133	41	0.2						

B.4 Incremental and All-Unit Discount Pricing Test Problems Data

Table B–4: Incremental and All-Units Pricing Test Problems Data (1-3)

Dataset 1				Dataset 2				Dataset 3			
LB	UB	Cost	Incost	LB	UB	Cost	Incost	LB	UB	Cost	Incost
0	65	1.66	0	0	96	1.96	0	0	58	1.59	0
0	47	1.48	0	97	167	1.91	188.16	59	101	1.54	92.22
48	83	1.43	69.56	0	12	1.12	0	102	141	1.49	158.44
84	87	1.38	121.04	13	22	1.07	13.44	0	80	1.81	0
88	148	1.33	126.56	23	80	1.02	24.14	81	139	1.76	144.8
149	190	1.28	207.69	81	165	0.97	83.3	140	154	1.71	248.64
191	202	1.23	261.45	166	214	0.92	165.75	155	210	1.66	274.29
0	92	1.93	0	215	244	0.87	210.83	211	262	1.61	367.25
93	160	1.88	177.56	245	255	0.82	236.93	0	86	1.86	0
161	230	1.83	305.4	256	339	0.77	245.95	87	150	1.81	159.96
231	327	1.78	433.5	0	36	1.37	0	151	238	1.76	275.8
328	383	1.73	606.16	37	63	1.32	49.32	239	243	1.71	430.68
384	440	1.68	703.04	64	123	1.27	84.96	0	79	1.79	0
441	520	1.63	798.8	124	150	1.22	161.16	80	137	1.74	141.41
521	590	1.58	929.2	151	199	1.17	194.1	138	236	1.69	242.33
591	682	1.53	1039.8	0	39	1.39	0	237	323	1.64	409.64
0	82	1.83	0	40	68	1.34	54.21	324	412	1.59	552.32
83	143	1.78	150.06	69	163	1.29	93.07	413	441	1.54	693.83
144	188	1.73	258.64	164	183	1.24	215.62	0	85	1.86	0
189	252	1.68	336.49	184	224	1.19	240.42	86	148	1.81	158.1
253	339	1.63	444.01	225	317	1.14	289.21	149	228	1.76	272.13

Table B-4. Continued

Dataset 1				Dataset 2				Dataset 3			
LB	UB	Cost	Incost	LB	UB	Cost	Incost	LB	UB	Cost	Incost
340	422	1.58	585.82	318	413	1.09	395.23	229	299	1.71	412.93
423	439	1.53	716.96	0	50	1.5	0	0	7	1.07	0
0	75	1.76	0	51	87	1.45	75	0	5	1.05	0
76	131	1.71	132	88	120	1.4	128.65	6	9	1	5.25
132	184	1.66	227.76	0	61	1.62	0	10	76	0.95	9.25
185	187	1.61	315.74	62	107	1.57	98.82	77	127	0.9	72.9
188	230	1.56	320.57	108	184	1.52	171.04	128	202	0.85	118.8
231	255	1.51	387.65	185	200	1.47	288.08	203	277	0.8	182.55
256	275	1.46	425.4	201	269	1.42	311.6	278	355	0.75	242.55
0	38	1.39	0	270	344	1.37	409.58	0	33	1.33	0
39	67	1.34	52.82	345	424	1.32	512.33	34	58	1.28	43.89
68	157	1.29	91.68	425	489	1.27	617.93	59	76	1.23	75.89
158	191	1.24	207.78	490	499	1.22	700.48	77	78	1.18	98.03
192	264	1.19	249.94	0	71	1.72	0	0	33	1.33	0
265	305	1.14	336.81	72	124	1.67	122.12	34	58	1.28	43.89
0	1	1.02	0	125	129	1.62	210.63	59	75	1.23	75.89
2	3	0.97	1.02	130	215	1.57	218.73	76	153	1.18	96.8
4	27	0.92	2.96	216	291	1.52	353.75	154	195	1.13	188.84
28	35	0.87	25.04	292	297	1.47	469.27	0	8	1.08	0
36	73	0.82	32	298	389	1.42	478.09	9	15	1.03	8.64
0	45	1.45	0	390	465	1.37	608.73	16	21	0.98	15.85
46	79	1.4	65.25	0	99	1.99	0	22	24	0.93	21.73
80	148	1.35	112.85	0	72	1.73	0	25	63	0.88	24.52
149	228	1.3	206	0	53	1.53	0				

Table B-4. Continued											
Dataset 1				Dataset 2				Dataset 3			
LB	UB	Cost	Incost	LB	UB	Cost	Incost	LB	UB	Cost	Incost
0	69	1.7	0								
70	120	1.65	117.3								
121	198	1.6	201.45								
199	227	1.55	326.25								
228	309	1.5	371.2								
310	318	1.45	494.2								
319	371	1.4	507.25								
0	1	1.01	0								
2	3	0.96	1.01								
4	16	0.91	2.93								
17	24	0.86	14.76								
25	56	0.81	21.64								

Table B–5: Incremental and All-Units Pricing Test Problems Data (4-6)

Dataset 4				Dataset 5				Dataset 6			
LB	UB	Cost	Incost	LB	UB	Cost	Incost	LB	UB	Cost	Incost
0	14	1.14	0	0	13	1.14	0	0	66	1.66	0
15	25	1.09	15.96	14	24	1.09	14.82	67	115	1.61	109.56
26	103	1.04	27.95	25	97	1.04	26.81	116	148	1.56	188.45
104	142	0.99	109.07	98	143	0.99	102.73	149	216	1.51	239.93
143	201	0.94	147.68	144	231	0.94	148.27	217	274	1.46	342.61
202	209	0.89	203.14	232	233	0.89	230.99	275	359	1.41	427.29
210	242	0.84	210.26	234	258	0.84	232.77	360	422	1.36	547.14
243	302	0.79	237.98	259	274	0.79	253.77	423	496	1.31	632.82
303	330	0.74	285.38	0	73	1.74	0	0	61	1.61	0
0	78	1.78	0	74	128	1.69	127.02	62	106	1.56	98.21
79	136	1.73	138.84	129	159	1.64	219.97	107	174	1.51	168.41
137	218	1.68	239.18	160	191	1.59	270.81	175	242	1.46	271.09
219	229	1.63	376.94	0	34	1.35	0	243	298	1.41	370.37
230	307	1.58	394.87	35	60	1.3	45.9	299	361	1.36	449.33
308	340	1.53	518.11	0	39	1.39	0	362	441	1.31	535.01
0	55	1.55	0	40	68	1.34	54.21	0	71	1.71	0
56	96	1.5	85.25	69	161	1.29	93.07	72	123	1.66	121.41
97	194	1.45	146.75	162	245	1.24	213.04	124	218	1.61	207.73
0	75	1.75	0	246	277	1.19	317.2	219	240	1.56	360.68
76	131	1.7	131.25	278	317	1.14	355.28	241	311	1.51	395
0	46	1.47	0	318	404	1.09	400.88	312	328	1.46	502.21
47	81	1.42	67.62	0	86	1.87	0	329	414	1.41	527.03

Table B-5. Continued											
Dataset 4				Dataset 5				Dataset 6			
LB	UB	Cost	Incost	LB	UB	Cost	Incost	LB	UB	Cost	Incost
82	172	1.37	117.32	87	150	1.82	160.82	415	482	1.36	648.29
173	238	1.32	241.99	151	241	1.77	277.3	483	551	1.31	740.77
239	268	1.27	329.11	242	297	1.72	438.37	0	83	1.84	0
269	282	1.22	367.21	298	359	1.67	534.69	84	145	1.79	152.72
0	27	1.28	0	0	52	1.53	0	0	51	1.52	0
28	48	1.23	34.56	53	91	1.48	79.56	0	37	1.38	0
49	100	1.18	60.39	0	64	1.65	0	0	27	1.28	0
0	93	1.94	0	65	112	1.6	105.6	28	48	1.23	34.56
94	162	1.89	180.42	113	130	1.55	182.4	49	97	1.18	60.39
163	244	1.84	310.83	131	138	1.5	210.3	98	139	1.13	118.21
245	340	1.79	461.71	139	171	1.45	222.3	0	10	1.1	0
341	390	1.74	633.55	172	223	1.4	270.15	11	18	1.05	11
0	57	1.58	0	224	308	1.35	342.95	19	47	1	19.4
58	100	1.53	90.06	309	355	1.3	457.7	48	138	0.95	48.4
101	126	1.48	155.85	0	95	1.96	0	139	204	0.9	134.85
127	173	1.43	194.33	0	70	1.7	0	205	226	0.85	194.25
174	271	1.38	261.54	71	122	1.65	119	0	64	1.65	0
272	348	1.33	396.78	123	204	1.6	204.8	65	112	1.6	105.6
0	21	1.22	0	205	214	1.55	336	113	129	1.55	182.4
0	15	1.16	0	215	285	1.5	351.5	130	205	1.5	208.75
16	27	1.11	17.4	286	292	1.45	458	0	3	1.03	0
28	29	1.06	30.72	293	304	1.4	468.15	4	6	0.98	3.09
30	52	1.01	32.84	0	1	1.01	0	7	45	0.93	6.03
53	145	0.96	56.07	2	3	0.96	1.01	46	102	0.88	42.3

Table B-5. Continued

Dataset 4				Dataset 5				Dataset 6			
LB	UB	Cost	Incost	LB	UB	Cost	Incost	LB	UB	Cost	Incost
146	239	0.91	145.35	4	14	0.91	2.93	103	170	0.83	92.46
				15	100	0.86	12.94	171	236	0.78	148.9
				101	175	0.81	86.9	237	256	0.73	200.38
				176	249	0.76	147.65	257	299	0.68	214.98
				250	323	0.71	203.89	300	329	0.63	244.22
				324	383	0.66	256.43				
				384	404	0.61	296.03				

Table B–6: Incremental and All-Units Pricing Test Problems Data (7-9)

Dataset 7				Dataset 8				Dataset 9			
LB	UB	Cost	Incost	LB	UB	Cost	Incost	LB	UB	Cost	Incost
0	52	1.53	0	0	81	1.81	0	0	40	1.4	0
53	91	1.48	79.56	82	141	1.76	146.61	41	70	1.35	56
92	155	1.43	137.28	142	167	1.71	252.21	71	77	1.3	96.5
0	99	1.99	0	168	199	1.66	296.67	78	91	1.25	105.6
100	172	1.94	197.01	200	245	1.61	349.79	92	121	1.2	123.1
173	225	1.89	338.63	246	321	1.56	423.85	122	123	1.15	159.1
226	325	1.84	438.8	322	419	1.51	542.41	124	146	1.1	161.4
0	5	1.06	0	0	75	1.76	0	147	237	1.05	186.7
6	10	1.01	5.3	0	55	1.55	0	0	56	1.56	0
11	80	0.96	10.35	56	96	1.5	85.25	57	98	1.51	87.36
0	94	1.94	0	97	192	1.45	146.75	99	105	1.46	150.78
0	68	1.69	0	193	234	1.4	285.95	106	113	1.41	161
69	119	1.64	114.92	235	245	1.35	344.75	114	143	1.36	172.28
120	185	1.59	198.56	246	322	1.3	359.6	144	158	1.31	213.08
186	217	1.54	303.5	323	330	1.25	459.7	159	203	1.26	232.73
218	262	1.49	352.78	0	37	1.37	0	204	268	1.21	289.43
0	72	1.72	0	0	27	1.27	0	0	2	1.03	0
73	125	1.67	123.84	28	48	1.22	34.29	3	4	0.98	2.06
126	137	1.62	212.35	49	93	1.17	59.91	5	36	0.93	4.02
138	235	1.57	231.79	94	155	1.12	112.56	0	41	1.42	0
0	79	1.79	0	156	212	1.07	182	42	72	1.37	58.22
80	137	1.74	141.41	213	285	1.02	242.99	73	98	1.32	100.69

Table B-6. Continued

Dataset 7				Dataset 8				Dataset 9			
LB	UB	Cost	Incost	LB	UB	Cost	Incost	LB	UB	Cost	Incost
138	235	1.69	242.33	0	46	1.47	0	99	145	1.27	135.01
0	82	1.83	0	47	81	1.42	67.62	146	239	1.22	194.7
83	143	1.78	150.06	82	170	1.37	117.32	240	249	1.17	309.38
144	182	1.73	258.64	171	197	1.32	239.25	250	315	1.12	321.08
183	252	1.68	326.11	198	251	1.27	274.89	316	335	1.07	395
253	345	1.63	443.71	252	274	1.22	343.47	336	377	1.02	416.4
346	432	1.58	595.3	0	81	1.82	0	0	14	1.15	0
433	523	1.53	732.76	82	141	1.77	147.42	15	25	1.1	16.1
524	577	1.48	871.99	142	171	1.72	253.62	26	114	1.05	28.2
0	21	1.22	0	172	177	1.67	305.22	115	130	1	121.65
22	37	1.17	25.62	0	95	1.96	0	131	205	0.95	137.65
0	73	1.74	0	0	69	1.7	0	0	86	1.86	0
				70	121	1.65	117.3	0	62	1.63	0
				122	201	1.6	203.1	63	109	1.58	101.06
				202	275	1.55	331.1	110	202	1.53	175.32
				276	347	1.5	445.8	203	296	1.48	317.61
				348	385	1.45	553.8	297	311	1.43	456.73
				386	427	1.4	608.9	0	45	1.46	0
				0	16	1.16	0	46	79	1.41	65.7
								80	157	1.36	113.64
								158	191	1.31	219.72
								192	265	1.26	264.26
								0	70	1.71	0
								71	122	1.66	113.64

Table B-6. Continued

Dataset 7				Dataset 8				Dataset 9			
LB	UB	Cost	Incost	LB	UB	Cost	Incost	LB	UB	Cost	Incost
								123	214	1.61	185.02
								0	77	1.78	0
								78	134	1.73	137.06
								135	211	1.68	235.67
								212	232	1.63	365.03
								233	285	1.58	399.26
								286	381	1.53	483
								382	420	1.48	629.88
								421	487	1.43	687.60
								488	527	1.38	783.41

Table B–7: Incremental and All-Units Pricing Test Problems Data (10-12)

Dataset 10				Dataset 11				Dataset 12			
LB	UB	Cost	Incost	LB	UB	Cost	Incost	LB	UB	Cost	Incost
0	84	1.85	0	0	21	1.22	0	0	30	1.3	0
85	146	1.8	155.4	22	37	1.17	25.62	31	52	1.25	39
147	213	1.75	267	38	111	1.12	44.34	53	131	1.2	66.5
214	255	1.7	384.25	112	174	1.07	127.22	132	183	1.15	161.3
256	275	1.65	455.65	175	255	1.02	194.63	184	263	1.1	221.1
0	41	1.41	0	256	344	0.97	277.25	0	67	1.67	0
42	72	1.36	57.81	345	374	0.92	363.58	68	117	1.62	111.89
73	93	1.31	99.97	0	5	1.06	0	118	165	1.57	192.89
94	140	1.26	127.48	6	10	1.01	5.3	166	184	1.52	268.25
141	148	1.21	186.7	11	84	0.96	10.35	185	208	1.47	297.13
149	182	1.16	196.38	85	142	0.91	81.39	0	11	1.12	0
183	252	1.11	235.82	0	96	1.97	0	0	8	1.08	0
253	254	1.06	313.52	0	70	1.71	0	9	15	1.03	8.64
255	269	1.01	315.64	71	122	1.66	119.7	16	22	0.98	15.85
0	46	1.46	0	0	92	1.93	0	23	36	0.93	22.71
47	80	1.41	67.16	93	160	1.88	177.56	37	70	0.88	35.73
0	83	1.83	0	161	229	1.83	305.4	71	144	0.83	65.65
84	144	1.78	151.89	230	301	1.78	431.67	145	205	0.78	127.07
145	194	1.73	260.47	302	333	1.73	559.83	206	239	0.73	174.65
195	243	1.68	346.97	334	378	1.68	615.19	0	73	1.74	0
0	31	1.32	0	379	444	1.63	690.79	74	127	1.69	127.02
32	55	1.27	40.92	445	475	1.58	798.37	128	153	1.64	218.28

Table B-7. Continued

Dataset 10				Dataset 11				Dataset 12			
LB	UB	Cost	Incost	LB	UB	Cost	Incost	LB	UB	Cost	Incost
56	154	1.22	71.4	0	21	1.22	0	154	197	1.59	260.92
155	250	1.17	192.18	0	15	1.16	0	198	252	1.54	330.88
251	301	1.12	304.5	16	27	1.11	17.4	253	295	1.49	415.58
302	365	1.07	361.62	28	127	1.06	30.72	296	319	1.44	479.65
366	458	1.02	430.1	0	7	1.08	0	320	417	1.39	514.21
459	460	0.97	524.96	8	13	1.03	7.56	0	74	1.74	0
0	1	1.01	0	14	110	0.98	13.74	75	129	1.69	128.76
2	3	0.96	1.01	0	59	1.59	0	130	164	1.64	221.71
4	19	0.91	2.93	0	43	1.43	0	165	166	1.59	279.11
20	78	0.86	17.49	44	75	1.38	61.49	167	171	1.54	282.29
79	80	0.81	68.23	76	122	1.33	105.65	172	255	1.49	289.99
81	113	0.76	69.85	123	215	1.28	168.16	256	293	1.44	415.15
114	179	0.71	94.93	216	314	1.23	287.2	0	47	1.48	0
0	34	1.34	0	315	405	1.18	408.97	48	82	1.43	69.56
35	60	1.29	45.56	406	457	1.13	516.35	83	84	1.38	119.61
61	93	1.24	79.1	458	537	1.08	575.11	85	97	1.33	122.37
0	54	1.55	0					98	114	1.28	139.66
0	40	1.4	0					115	202	1.23	161.42
41	70	1.35	56					203	301	1.18	269.66
0	4	1.05	0					302	393	1.13	386.48
5	8	1	4.2					394	476	1.08	490.44
9	71	0.95	8.2					0	18	1.19	0
72	146	0.9	68.05					19	32	1.14	21.42
147	224	0.85	135.55					33	70	1.09	37.38

Table B-7. Continued

Dataset 10				Dataset 11				Dataset 12			
LB	UB	Cost	Incost	LB	UB	Cost	Incost	LB	UB	Cost	Incost
225	260	0.8	201.85					71	110	1.04	78.8
261	279	0.75	230.65					111	185	0.99	120.4
								186	260	0.94	194.65
								261	350	0.89	265.15
								351	391	0.84	345.25
								392	394	0.79	379.69
								0	48	1.49	0
								49	84	1.44	71.52
								85	101	1.39	123.36
								102	178	1.34	146.99
								179	200	1.29	250.17
								201	275	1.24	278.55
								276	354	1.19	371.55
								0	52	1.53	0

Table B–8: Incremental and All-Units Pricing Test Problems Data (13-15)

Dataset 13				Dataset 14				Dataset 15			
LB	UB	Cost	Incost	LB	UB	Cost	Incost	LB	UB	Cost	Incost
0	88	1.89	0	0	13	1.14	0	0	57	1.58	0
89	154	1.84	166.32	14	24	1.09	14.82	0	42	1.42	0
155	173	1.79	287.76	25	98	1.04	26.81	43	73	1.37	59.64
174	195	1.74	321.77	99	155	0.99	103.77	74	105	1.32	102.11
196	268	1.69	360.05	156	226	0.94	160.2	106	154	1.27	144.35
269	310	1.64	483.42	227	236	0.89	226.94	155	191	1.22	206.58
0	19	1.19	0	237	298	0.84	235.84	192	221	1.17	251.72
20	34	1.14	22.61	0	64	1.65	0	222	230	1.12	286.82
35	77	1.09	39.71	65	112	1.6	105.6	0	50	1.5	0
78	105	1.04	86.58	0	15	1.16	0	51	87	1.45	75
106	182	0.99	115.7	16	27	1.11	17.4	88	121	1.4	128.65
183	193	0.94	191.93	28	125	1.06	30.72	122	190	1.35	176.25
0	82	1.83	0	126	208	1.01	134.6	191	267	1.3	269.4
83	143	1.78	150.06	209	226	0.96	218.43	268	275	1.25	369.5
144	182	1.73	258.64	0	98	1.98	0	276	310	1.2	379.5
183	250	1.68	326.11	0	71	1.72	0	311	400	1.15	421.5
251	319	1.63	440.35	72	124	1.67	122.12	401	441	1.1	525
320	405	1.58	552.82	125	129	1.62	210.63	0	93	1.93	0
0	78	1.79	0	0	73	1.74	0	94	162	1.88	179.49
79	136	1.74	139.62	74	127	1.69	127.02	163	236	1.83	309.21
137	225	1.69	240.54	128	157	1.64	218.28	237	297	1.78	444.63

Table B-8. Continued

Dataset 13				Dataset 14				Dataset 15			
LB	UB	Cost	Incost	LB	UB	Cost	Incost	LB	UB	Cost	Incost
226	257	1.64	390.95	158	159	1.59	267.48	298	343	1.73	553.21
258	299	1.59	443.43	160	167	1.54	270.66	344	430	1.68	632.79
0	9	1.1	0	168	200	1.49	282.98	431	519	1.63	778.95
10	17	1.05	9.9	201	266	1.44	332.15	0	28	1.29	0
18	39	1	18.3	267	293	1.39	427.19	29	49	1.24	36.12
40	106	0.95	40.3	0	63	1.64	0	50	110	1.19	62.16
0	44	1.44	0	64	110	1.59	103.32	111	154	1.14	134.75
0	32	1.32	0	111	114	1.54	178.05	155	205	1.09	184.91
0	23	1.24	0	115	170	1.49	184.21	206	272	1.04	240.5
24	41	1.19	28.52	171	224	1.44	267.65	273	317	0.99	310.18
42	138	1.14	49.94	225	244	1.39	345.41	318	385	0.94	354.73
0	63	1.63	0	245	273	1.34	373.21	386	453	0.89	418.65
64	110	1.58	102.69	274	372	1.29	412.07	0	58	1.59	0
111	210	1.53	176.95	0	98	1.99	0	59	101	1.54	92.22
211	215	1.48	329.95	99	171	1.94	195.02	102	139	1.49	158.44
216	301	1.43	337.35	172	216	1.89	336.64	140	189	1.44	215.06
302	375	1.38	460.33	217	274	1.84	421.69	0	60	1.6	0
376	440	1.33	562.45	275	361	1.79	528.41	61	105	1.55	96
441	443	1.28	648.9	0	95	1.95	0	106	163	1.5	165.75
0	48	1.48	0	96	165	1.9	185.25	164	260	1.45	252.75
				166	263	1.85	318.25	0	64	1.65	0
				0	84	1.85	0	65	112	1.6	105.6
				85	146	1.8	155.4	113	129	1.55	182.4
				147	213	1.75	267	130	220	1.5	208.75

Table B-8. Continued

Dataset 13				Dataset 14				Dataset 15			
LB	UB	Cost	Incost	LB	UB	Cost	Incost	LB	UB	Cost	Incost
				214	259	1.7	384.25	221	275	1.45	345.25
				260	337	1.65	462.45	0	40	1.4	0
								41	70	1.35	56
								71	77	1.3	96.5
								78	93	1.25	105.6
								94	169	1.20	125.6
								170	262	1.15	216.8
								263	346	1.10	323.75
								347	391	1.05	416.15
								0	62	1.63	0

Table B–9: Incremental and All-Units Pricing Test Problems Data (16-18)

Dataset 16				Dataset 17				Dataset 18			
LB	UB	Cost	Incost	LB	UB	Cost	Incost	LB	UB	Cost	Incost
0	60	1.6	0	0	46	1.47	0	0	70	1.71	0
61	105	1.55	96	0	34	1.34	0	0	51	1.52	0
106	166	1.5	165.75	35	59	1.29	45.56	52	89	1.47	77.52
167	207	1.45	257.25	60	89	1.24	77.81	90	141	1.42	133.38
208	296	1.4	316.7	90	96	1.19	115.01	142	232	1.37	207.22
297	327	1.35	441.3	97	104	1.14	123.34	233	290	1.32	331.89
328	350	1.3	483.15	105	139	1.09	132.46	0	87	1.87	0
351	431	1.25	513.05	0	96	1.96	0	88	151	1.82	162.69
0	79	1.79	0	97	167	1.91	188.16	152	250	1.77	279.17
80	138	1.74	141.41	168	177	1.86	323.77	251	341	1.72	454.4
139	140	1.69	244.07	178	240	1.81	342.37	0	65	1.65	0
141	151	1.64	247.45	241	313	1.76	456.4	66	113	1.6	107.25
152	241	1.59	265.49	314	354	1.71	584.88	114	136	1.55	184.05
0	35	1.35	0	355	359	1.66	654.99	137	217	1.5	219.7
36	61	1.3	47.25	360	445	1.61	663.29	218	309	1.45	341.2
62	105	1.25	81.05	446	520	1.56	801.75	310	381	1.4	474.6
106	161	1.2	136.05	0	84	1.84	0	382	415	1.35	575.4
162	211	1.15	203.25	85	146	1.79	154.56	0	72	1.73	0
0	48	1.49	0	147	204	1.74	265.54	0	53	1.53	0
49	84	1.44	71.52	0	96	1.96	0	54	93	1.48	81.09
85	100	1.39	123.36	97	167	1.91	188.16	94	165	1.43	140.29
101	161	1.34	145.6	168	177	1.86	323.77	166	194	1.38	243.25

Table B-9. Continued

Dataset 16				Dataset 17				Dataset 18			
LB	UB	Cost	Incost	LB	UB	Cost	Incost	LB	UB	Cost	Incost
162	208	1.29	227.34	178	240	1.81	342.37	195	287	1.33	283.27
209	215	1.24	287.97	0	72	1.72	0	0	97	1.97	0
216	236	1.19	296.65	73	125	1.67	123.84	98	169	1.92	191.09
237	283	1.14	321.64	126	133	1.62	212.35	0	24	1.24	0
284	289	1.09	375.22	134	167	1.57	225.31	0	17	1.18	0
0	90	1.91	0	168	236	1.52	278.69	18	30	1.13	20.06
91	157	1.86	171.9	237	313	1.47	383.57	31	53	1.08	34.75
158	197	1.81	296.52	314	337	1.42	496.76	0	92	1.93	0
198	276	1.76	368.92	338	435	1.37	530.84	93	160	1.88	177.56
277	325	1.71	507.96	436	508	1.32	665.1	161	230	1.83	305.4
326	362	1.66	591.75	0	50	1.5	0	231	323	1.78	433.5
363	383	1.61	653.17	51	87	1.45	75	324	422	1.73	599.04
384	437	1.56	686.98	88	119	1.4	128.65				
0	23	1.24	0	120	159	1.35	173.45				
24	41	1.19	28.52	160	232	1.3	227.45				
42	137	1.14	49.94	0	41	1.41	0				
138	186	1.09	159.38	42	72	1.36	57.81				
187	227	1.04	212.79	73	89	1.31	99.97				
228	231	0.99	255.43	90	170	1.26	122.24				
0	61	1.62	0	171	254	1.21	224.3				
62	107	1.57	98.82	255	284	1.16	325.94				
108	187	1.52	171.04	285	289	1.11	360.74				
0	62	1.63	0	0	69	1.69	0				
63	108	1.58	101.06	70	120	1.64	116.61				

Table B-9. Continued

Dataset 16				Dataset 17				Dataset 18			
LB	UB	Cost	Incost	LB	UB	Cost	Incost	LB	UB	Cost	Incost
109	200	1.53	173.74	121	194	1.59	200.25				
201	269	1.48	314.5	0	68	1.68	0				
270	348	1.43	416.62								
349	408	1.38	529.59								
0	19	1.2	0								
20	34	1.15	22.8								
35	80	1.1	40.05								
81	166	1.05	90.65								
167	239	1	180.95								
240	289	0.95	253.95								
290	335	0.9	301.45								
336	414	0.85	342.85								
415	462	0.8	410								
0	22	1.23	0								
23	39	1.18	27.06								
40	125	1.13	47.12								
126	196	1.08	144.3								
197	209	1.03	220.98								
210	228	0.98	234.37								
229	254	0.93	252.99								

Table B–10: Incremental and All-Units Pricing Test Problems Data (19-21)

Dataset 19				Dataset 20				Dataset 21			
LB	UB	Cost	Incost	LB	UB	Cost	Incost	LB	UB	Cost	Incost
0	43	1.43	0	0	36	1.37	0	0	67	1.67	0
44	75	1.38	61.49	37	64	1.32	49.32	68	117	1.62	111.89
76	121	1.33	105.65	65	129	1.27	86.28	118	165	1.57	192.89
0	85	1.86	0	130	145	1.22	168.83	166	181	1.52	268.25
86	148	1.81	158.1	146	220	1.17	188.35	182	248	1.47	292.57
149	226	1.76	272.13	221	297	1.12	276.1	249	296	1.42	391.06
227	252	1.71	409.41	0	12	1.12	0	297	311	1.37	459.22
253	292	1.66	453.87	13	22	1.07	13.44	312	353	1.32	479.77
293	378	1.61	520.27	23	78	1.02	24.14	354	375	1.27	535.21
379	447	1.56	658.73	79	135	0.97	81.26	0	67	1.67	0
0	82	1.83	0	136	214	0.92	136.55	68	117	1.62	111.89
0	60	1.6	0	215	262	0.87	209.23	118	161	1.57	192.89
61	105	1.55	96	263	277	0.82	250.99	162	209	1.52	261.97
106	166	1.5	165.75	278	329	0.77	263.29	210	229	1.47	334.93
0	39	1.39	0	0	79	1.8	0	230	262	1.42	364.33
40	68	1.34	54.21	80	138	1.75	142.2	263	324	1.37	411.19
69	164	1.29	93.07	139	142	1.7	245.45	325	380	1.32	496.13
165	210	1.24	216.91	143	201	1.65	252.25	381	438	1.27	570.05
211	297	1.19	273.95	202	215	1.6	349.6	0	96	1.97	0
298	392	1.14	377.48	0	38	1.39	0	97	167	1.92	189.12
393	420	1.09	485.78	39	67	1.34	52.82	168	182	1.87	325.44
0	67	1.68	0	68	154	1.29	91.68	183	237	1.82	353.49

Table B-10. Continued											
Dataset 19				Dataset 20				Dataset 21			
LB	UB	Cost	Incost	LB	UB	Cost	Incost	LB	UB	Cost	Incost
0	49	1.49	0	155	237	1.24	203.91	238	281	1.77	453.59
50	86	1.44	73.01	238	260	1.19	306.83	282	329	1.72	531.47
87	109	1.39	126.29	261	344	1.14	334.2	330	343	1.67	614.03
110	193	1.34	158.26	0	36	1.37	0	344	384	1.62	637.41
194	228	1.29	270.82	37	64	1.32	49.32	385	386	1.57	703.83
229	314	1.24	315.97	65	130	1.27	86.28	0	22	1.23	0
315	389	1.19	422.61	131	156	1.22	170.1	0	16	1.17	0
0	81	1.81	0	157	191	1.17	201.82	0	12	1.12	0
82	141	1.76	146.61	192	286	1.12	242.77	13	21	1.07	13.44
142	163	1.71	252.21	287	320	1.07	349.17	22	76	1.02	23.07
164	231	1.66	289.83	0	73	1.74	0	77	111	0.97	79.17
232	298	1.61	402.71	74	128	1.69	127.02	112	200	0.92	113.12
299	339	1.56	510.58	129	158	1.64	219.97	201	230	0.87	195
0	91	1.91	0	0	7	1.07	0	231	246	0.82	221.1
92	158	1.86	173.81	8	13	1.02	7.49	0	66	1.66	0
159	206	1.81	298.43	14	103	0.97	13.61	67	115	1.61	109.56
207	225	1.76	385.31	104	142	0.92	100.91	116	151	1.56	188.45
226	249	1.71	418.75	0	56	1.57	0	152	161	1.51	244.61
0	8	1.09	0	57	98	1.52	87.92	162	219	1.46	259.71
				99	114	1.47	151.76	220	317	1.41	344.39
				115	174	1.42	175.28	318	401	1.36	482.57
				175	206	1.37	260.48	402	444	1.31	596.81
				207	249	1.32	304.32	445	471	1.26	653.14
				0	35	1.35	0	0	62	1.62	0

Table B-10. Continued

Dataset 19				Dataset 20				Dataset 21			
LB	UB	Cost	Incost	LB	UB	Cost	Incost	LB	UB	Cost	Incost
				36	61	1.3	47.25	0	45	1.46	0
				62	104	1.25	81.05	0	33	1.33	0
				105	138	1.2	134.8	34	58	1.28	43.89
				0	83	1.83	0	59	78	1.23	75.89
				84	144	1.78	151.89	79	116	1.18	100.49
				145	193	1.73	260.47				
				194	221	1.68	345.24				
				222	298	1.63	392.28				
				299	310	1.58	517.79				

Table B–11: Incremental and All-Units Pricing Test Problems Data (22-24)

Dataset 22				Dataset 23				Dataset 24			
LB	UB	Cost	Incost	LB	UB	Cost	Incost	LB	UB	Cost	Incost
0	69	1.7	0	0	91	1.91	0	0	12	1.13	0
70	121	1.65	117.3	92	158	1.86	173.81	13	22	1.08	13.56
122	201	1.6	203.1	159	208	1.81	298.43	23	84	1.03	24.36
202	264	1.55	331.1	209	253	1.76	388.93	85	148	0.98	88.22
265	342	1.5	428.75	254	324	1.71	468.13	149	239	0.93	150.94
343	377	1.45	545.75	325	336	1.66	589.54	240	298	0.88	235.57
378	379	1.4	596.5	337	432	1.61	609.46	299	397	0.83	287.49
380	395	1.35	599.3	433	483	1.56	764.02	398	492	0.78	369.66
0	63	1.63	0	484	553	1.51	843.58	0	25	1.26	0
64	110	1.58	102.69	0	96	1.97	0	26	44	1.21	31.5
111	207	1.53	176.95	0	70	1.71	0	45	71	1.16	54.49
208	271	1.48	325.36	71	122	1.66	119.7	72	125	1.11	85.81
272	356	1.43	420.08	123	214	1.61	206.02	126	153	1.06	145.75
357	406	1.38	541.63	0	70	1.7	0	154	225	1.01	175.43
407	465	1.33	610.63	71	122	1.65	119	226	258	0.96	248.15
466	479	1.28	689.1	123	207	1.6	204.8	259	321	0.91	279.83
480	518	1.23	707.02	208	265	1.55	340.8	0	69	1.69	0
0	58	1.59	0	266	360	1.5	430.7	0	50	1.51	0
59	101	1.54	92.22	361	392	1.45	573.2	51	88	1.46	75.5
102	140	1.49	158.44	393	431	1.4	619.6	89	126	1.41	130.98
141	202	1.44	216.55	0	60	1.6	0	127	165	1.36	184.56
203	258	1.39	305.83	0	44	1.44	0	166	235	1.31	237.6

Table B-11. Continued

Dataset 22				Dataset 23				Dataset 24			
LB	UB	Cost	Incost	LB	UB	Cost	Incost	LB	UB	Cost	Incost
259	309	1.34	383.67	0	32	1.32	0	236	327	1.26	329.3
310	380	1.29	452.01	0	23	1.24	0	328	406	1.21	445.22
381	386	1.24	543.6	24	41	1.19	28.52	407	460	1.16	540.81
0	88	1.88	0	42	139	1.14	49.94	461	475	1.11	603.45
89	153	1.83	165.44	140	214	1.09	161.66	0	54	1.55	0
154	162	1.78	284.39	215	302	1.04	243.41	55	95	1.5	83.7
163	202	1.73	300.41	303	308	0.99	334.93	96	187	1.45	145.2
203	289	1.68	369.61	0	92	1.93	0	188	256	1.4	278.6
290	383	1.63	515.77	93	160	1.88	177.56	0	83	1.83	0
384	392	1.58	668.99	161	225	1.83	305.4	0	60	1.61	0
393	440	1.53	683.21	226	236	1.78	424.35	61	105	1.56	96.6
441	457	1.48	756.65	0	78	1.78	0	0	64	1.64	0
0	80	1.81	0	79	136	1.73	138.84	65	112	1.59	104.96
81	140	1.76	144.8					113	121	1.54	181.28
141	160	1.71	250.4					122	175	1.49	195.14
161	194	1.66	284.6					176	203	1.44	275.6
195	265	1.61	341.04					204	283	1.39	315.92
0	14	1.14	0					284	356	1.34	427.12
15	25	1.09	15.96					357	403	1.29	524.94
26	102	1.04	27.95					0	94	1.94	0
103	127	0.99	108.03					95	163	1.89	182.36
128	145	0.94	132.78					164	247	1.84	312.77
146	149	0.89	149.7					248	283	1.79	467.33
150	207	0.84	153.26					0	4	1.05	0

Table B-11. Continued

Dataset 22				Dataset 23				Dataset 24			
LB	UB	Cost	Incost	LB	UB	Cost	Incost	LB	UB	Cost	Incost
0	80	1.81	0					5	8	1	4.2
0	59	1.59	0								
0	43	1.43	0								
44	75	1.38	61.49								
76	120	1.33	105.65								
121	193	1.28	165.5								
0	43	1.43	0								
44	75	1.38	61.49								
76	122	1.33	105.65								
123	129	1.28	168.16								
130	148	1.23	177.12								
149	165	1.18	200.49								

Table B–12: Incremental and All-Units Pricing Test Problems Data (25-27)

Dataset 25				Dataset 26				Dataset 27			
LB	UB	Cost	Incost	LB	UB	Cost	Incost	LB	UB	Cost	Incost
0	47	1.48	0	0	69	1.7	0	0	49	1.5	0
48	82	1.43	69.56	70	121	1.65	117.3	50	86	1.45	73.5
83	84	1.38	119.61	0	78	1.79	0	87	113	1.4	127.15
85	92	1.33	122.37	79	136	1.74	139.62	114	169	1.35	164.95
93	129	1.28	133.01	137	231	1.69	240.54	0	49	1.5	0
130	152	1.23	180.37	232	253	1.64	401.09	50	86	1.45	73.5
0	79	1.8	0	254	331	1.59	437.17	87	115	1.4	127.15
0	58	1.58	0	332	373	1.54	561.19	116	203	1.35	167.75
59	101	1.53	91.64	374	391	1.49	625.87	204	303	1.3	286.55
102	137	1.48	157.43	392	396	1.44	652.69	304	319	1.25	416.55
0	18	1.18	0	0	79	1.8	0	0	72	1.72	0
19	32	1.13	21.24	0	58	1.58	0	73	125	1.67	123.84
33	60	1.08	37.06	0	42	1.43	0	126	136	1.62	212.35
61	127	1.03	67.3	43	74	1.38	60.06	137	216	1.57	230.17
128	165	0.98	136.31	75	110	1.33	104.22	217	287	1.52	355.77
166	216	0.93	173.55	111	120	1.28	152.1	0	12	1.12	0
217	281	0.88	220.98	121	190	1.23	164.9	13	22	1.07	13.44
282	284	0.83	278.18	191	285	1.18	251	23	78	1.02	24.14
0	35	1.36	0	286	318	1.13	363.1	79	129	0.97	81.26
36	62	1.31	47.6	0	64	1.64	0	130	205	0.92	130.73
63	112	1.26	82.97	65	111	1.59	104.96	206	210	0.87	200.65
113	164	1.21	145.97	112	120	1.54	179.69	211	284	0.82	205

Table B-12. Continued

Dataset 25				Dataset 26				Dataset 27			
LB	UB	Cost	Incost	LB	UB	Cost	Incost	LB	UB	Cost	Incost
165	242	1.16	208.89	121	162	1.49	193.55	0	70	1.71	0
243	283	1.11	299.37	163	184	1.44	256.13	0	51	1.52	0
0	92	1.93	0	0	70	1.7	0	52	89	1.47	77.52
93	160	1.88	177.56	71	122	1.65	119	90	140	1.42	133.38
161	229	1.83	305.4	123	209	1.6	204.8	141	210	1.37	205.8
230	307	1.78	431.67	210	295	1.55	344	211	213	1.32	301.7
308	342	1.73	570.51	296	360	1.5	477.3	214	258	1.27	305.66
343	441	1.68	631.06	361	362	1.45	574.8	259	329	1.22	362.81
442	443	1.63	797.38	363	391	1.4	577.7	330	349	1.17	449.43
444	457	1.58	800.64	392	486	1.35	618.3	0	28	1.28	0
458	496	1.53	822.76	487	516	1.3	746.55	29	49	1.23	35.84
0	56	1.56	0	0	1	1.01	0	50	107	1.18	61.67
57	98	1.51	87.36	2	3	0.96	1.01	0	87	1.87	0
99	109	1.46	150.78	4	12	0.91	2.93	0	63	1.64	0
0	84	1.84	0	13	56	0.86	11.12	64	110	1.59	103.32
85	146	1.79	154.56	57	98	0.81	48.96	111	112	1.54	178.05
147	205	1.74	265.54	99	108	0.76	82.98	113	133	1.49	181.13
206	212	1.69	368.2	109	166	0.71	90.58	134	180	1.44	212.42
0	4	1.04	0	0	94	1.94	0	181	278	1.39	280.1
5	8	0.99	4.16	95	164	1.89	182.36	279	365	1.34	416.32
9	64	0.94	8.12	0	89	1.89	0	0	82	1.83	0
65	118	0.89	60.76	90	155	1.84	168.21				
119	144	0.84	108.82	156	179	1.79	289.65				
0	32	1.32	0	180	188	1.74	332.61				

Table B-12. Continued

Dataset 25				Dataset 26				Dataset 27			
LB	UB	Cost	Incost	LB	UB	Cost	Incost	LB	UB	Cost	Incost
33	56	1.27	42.24	189	240	1.69	348.27				
57	61	1.22	72.72	241	329	1.64	436.15				
62	140	1.17	78.82	330	357	1.59	582.11				

Table B–13: Incremental and All-Units Pricing Test Problems Data (28-30)

Dataset 28				Dataset 29				Dataset 30			
LB	UB	Cost	Incost	LB	UB	Cost	Incost	LB	UB	Cost	Incost
0	95	1.96	0	0	27	1.27	0	0	53	1.53	0
96	165	1.91	186.2	28	48	1.22	34.29	54	93	1.48	81.09
166	168	1.86	319.9	49	94	1.17	59.91	94	165	1.43	140.29
169	210	1.81	325.48	0	90	1.91	0	166	191	1.38	243.25
211	219	1.76	401.5	0	66	1.66	0	192	238	1.33	279.13
220	262	1.71	417.34	67	115	1.61	109.56	0	7	1.07	0
263	288	1.66	490.87	116	150	1.56	188.45	8	13	1.02	7.49
289	329	1.61	534.03	151	250	1.51	243.05	14	104	0.97	13.61
330	422	1.56	600.04	251	269	1.46	394.05	105	162	0.92	101.88
0	99	1.99	0	270	286	1.41	421.79	163	251	0.87	155.24
100	172	1.94	197.01	0	75	1.75	0	252	282	0.82	232.67
173	221	1.89	338.63	0	55	1.55	0	283	312	0.77	258.09
222	248	1.84	431.24	56	96	1.5	85.25	313	316	0.72	281.19
249	305	1.79	480.92	97	191	1.45	146.75	317	385	0.67	284.07
0	78	1.78	0	192	210	1.4	284.5	0	78	1.78	0
79	136	1.73	138.84	0	20	1.21	0	79	136	1.73	138.84
137	224	1.68	239.18	21	36	1.16	24.2	137	224	1.68	239.18
225	324	1.63	387.02	37	99	1.11	42.76	225	237	1.63	387.02
325	337	1.58	550.02	100	172	1.06	112.69	238	258	1.58	408.21
338	349	1.53	570.56	173	219	1.01	190.07	259	310	1.53	441.39
350	441	1.48	588.92	220	227	0.96	237.54	0	87	1.88	0
442	509	1.43	725.08	228	256	0.91	245.22	88	152	1.83	163.56

Table B-13. Continued

Dataset 28				Dataset 29				Dataset 30			
LB	UB	Cost	Incost	LB	UB	Cost	Incost	LB	UB	Cost	Incost
510	565	1.38	822.32	257	350	0.86	271.61	153	157	1.78	282.51
0	51	1.52	0	0	15	1.16	0	158	238	1.73	291.41
0	37	1.38	0	16	27	1.11	17.4	239	326	1.68	431.54
38	65	1.33	51.06	28	126	1.06	30.72	327	339	1.63	579.38
66	143	1.28	88.3	127	217	1.01	135.66	0	16	1.16	0
144	178	1.23	188.14	0	54	1.54	0	17	28	1.11	18.56
179	180	1.18	231.19	55	94	1.49	83.16	29	33	1.06	31.88
181	189	1.13	233.55	95	180	1.44	142.76	34	112	1.01	37.18
190	238	1.08	243.72	181	249	1.39	266.6	113	168	0.96	116.97
239	278	1.03	296.64	250	335	1.34	362.51	0	46	1.47	0
279	362	0.98	337.84	336	414	1.29	477.75	47	81	1.42	67.62
0	39	1.4	0	415	464	1.24	579.66	82	171	1.37	117.32
40	69	1.35	54.6	0	50	1.5	0	172	205	1.32	240.62
70	72	1.3	95.1	51	87	1.45	75	206	274	1.27	285.5
0	39	1.39	0	88	121	1.4	128.65	0	71	1.72	0
40	68	1.34	54.21	0	69	1.69	0	72	124	1.67	122.12
69	165	1.29	93.07	70	120	1.64	116.61	125	126	1.62	210.63
166	224	1.24	218.2	121	189	1.59	200.25	127	145	1.57	213.87
225	229	1.19	291.36					146	165	1.52	243.7
230	307	1.14	297.31					166	205	1.47	274.1
308	341	1.09	386.23					206	284	1.42	332.9
0	74	1.74	0					285	327	1.37	445.08
75	129	1.69	128.76					0	39	1.4	0
130	162	1.64	221.71					0	28	1.29	0

Table B-13. Continued

Dataset 28				Dataset 29				Dataset 30			
LB	UB	Cost	Incost	LB	UB	Cost	Incost	LB	UB	Cost	Incost
163	222	1.59	275.83					29	50	1.24	36.12
223	238	1.54	371.23					51	114	1.19	63.4
239	304	1.49	395.87					0	98	1.98	0
305	335	1.44	494.21					99	170	1.93	194.04
336	361	1.39	538.85					171	206	1.88	333
0	44	1.45	0					207	223	1.83	400.68
45	77	1.4	63.8					224	303	1.78	431.79
78	143	1.35	110					304	373	1.73	574.19
0	24	1.24	0					374	472	1.68	695.29
25	42	1.19	29.76								
43	49	1.14	51.18								
50	63	1.09	59.16								

APPENDIX C
PROOFS FOR CHAPTER 5

C.1 Proof of Theorem 5.1

Theorem 5.1: When the suppliers are uncapacitated and there are no diversification benefits and there is a unique least cost supplier, then the firm will choose to order its total requirements from the least cost supplier. Under this scenario, the total usable quantity ordered from the least cost supplier is determined such that:

$$F(r_{[1]}q_{[1]}) = \frac{p - c_{[1]} + u}{p - s + u}$$

where $c_{[1]}$ is the cost per unit charged by the lowest cost supplier. If multiple suppliers have the same lowest cost, then the total order will be split amongst all of the lowest cost suppliers such that the total usable quantity ordered still satisfies the above critical ratio.

Proof: Before proving the result in this theorem, we first characterize the optimal solution to the uncapacitated suppliers problem with no diversification benefit. This problem can be formalized as follows:

$$\text{Maximize } Z_{q_i \geq 0} = (p - s)\mu - \sum_{i=1}^{N} c_i r_i q_i + s\sum_{i=1}^{N} r_i q_i - (p - s + u)ES \qquad \text{(C.1)}$$

Adding the non-negativity constraints to this objective, the corresponding KKT conditions are:

$$-c_i r_i + s r_i + (p - s + u) r_i (1 - F(\sum_{i=1}^{N} r_i q_i)) + \lambda_i = 0 \quad \forall i \qquad \text{(C.2)}$$

$$q_i \lambda_i = 0 \quad \forall i \qquad \text{(C.3)}$$

$$\lambda_i \geq 0 \quad \forall i \qquad \text{(C.4)}$$

We also note that $\frac{\partial^2 Z^U}{\partial {q_i}^2} = -(p-s+u)r_i\frac{\partial F}{\partial q_i} < 0$ for all i. Thus the objective is concave (i.e., the Hessian is negative semidefinite), and the KKT conditions are necessary and sufficient to obtain a global optimum solution to equation (C.1). Assuming that $q_i > 0$ for any supplier i, the FOC results in the following relationship:

$$F(\sum_{i=1}^{N} r_i q_i) = \frac{p-c_i+u}{p-s+u} \tag{C.5}$$

If the costs per unit (c_i) are not equal among suppliers, then this relationship can only hold for any one supplier i. Further, for all other suppliers j ($j \neq i$), it is obvious that $q_j = 0$. Thus, equation (C.5) provides the following result for supplier i:

$$F(r_i q_i) = \frac{p-c_i+u}{p-s+u} \tag{C.6}$$

Using this result, suppose that $q_i > 0$ for some supplier i, and q_j=0 $\forall j \neq i$. Then, for all suppliers j we have the following relationships:

$$F(r_i q_i) = \frac{(p-c_j+u)}{(p-s+u)} + \frac{\lambda_j}{r_j(p-s+u)} \tag{C.7}$$

$$\lambda_j = r_j(p-s+u)F(r_i q_i) - r_j(p-c_j+u) \tag{C.8}$$

$$\lambda_j = r_j(c_j - c_i) \tag{C.9}$$

In order to satisfy the KKT conditions, then λ_j must be non-negative $\forall j \neq i$. This can only occur when the ith supplier is the lowest cost supplier such that $c_j > c_{[1]}$, where $c_{[1]}$ is the cost per unit charged by the lowest cost supplier. For the situation where more than one supplier has the lowest cost, then it is obvious from the KKT conditions of optimality that the total usable quantity ordered must still satisfy the above critical ratio in equation (C.2) . This concludes our proof.

C.2 Proof of Theorem 5.2

Theorem 5.2: When suppliers are capacitated and there are no diversification benefits, then the optimal number of suppliers selected and the corresponding quantity allocated to each supplier can be determined as follows.

Step 1: Index all suppliers in increasing order of cost per unit (i.e., $c_{[1]} \leq c_{[2]} \leq c_{[3]} \ldots \leq c_{[N]}$).

Step 2: For each supplier $[i]$ $(i = 1, \ldots, N)$, compute $Q_{[i]}$ such that:

$F(Q_{[i]}) = \frac{p-c_{[i]}+u}{p-s+u}$

and based on this determine:

$t_{[i]} = Q_{[i]} - \sum_{j=1}^{i-1} y_{[j]} r_{[j]}$

Step 3: The optimal number of suppliers selected (k) can be identified as $\max\{1 \leq k \leq N | t_{[k]} \geq 0\}$.

Step 4: The quantities allocated to supplier $j = 1, \ldots, k-1$ are $q_{[j]} = y_{[j]}$, the quantity allocated to supplier k is $q_{[k]} = \max\{t_{[k]}, y_{[k]}\}$, and the total quantity ordered by the firm from all suppliers can be determined as $\min\{Q_{[k]}, \sum_{j=1}^{k} y_{[j]}\}$.

Proof: Recall the capacitated suppliers problem with no diversification benefit is as follows:

$$\text{Maximize } Z_{q_i} = (p-s)\mu - \sum_{i=1}^{N} c_i r_i q_i + s \sum_{i=1}^{N} r_i q_i - (p-s+u)ES$$

subject to:

$$q_i \leq y_i \qquad \forall i \tag{C.10}$$

$$q_i \geq 0 \qquad \forall i \tag{C.11}$$

Based on the proof of Theorem 5.1, we know that Z is strictly concave in q_i and the constraints are all linear. Thus, by noting the concavity of the lagrangean function L $(L = Z + \sum_{i=1}^{n} \lambda_i (y_i - q_i))$, we know that Karush-Kuhn-Tucker (KKT) conditions are necessary and sufficient to identify the optimal solution to this problem. The KKT conditions for the lagrangean are:

$$q_i[\frac{\partial L}{\partial q_i}] = r_i q_i[-c_i + s - (p - s + u)(1 - F(\sum_{i=1}^{N} r_i q_i) - \lambda_i] \ \ \forall i \tag{C.12}$$

$$\lambda_i[\frac{\partial L}{\partial \lambda_i}] = \lambda_i[y_i - q_i] \qquad \forall i \tag{C.13}$$

To start with assume that there is some amount Q which will be sourced from all the suppliers. Then it is obvious that using the underlying logic of Theorem 5.1, we would choose to source the maximum amount possible from the lowest cost suppliers. Thus, our optimal algorithm (given Q) has the following structure:

1. Index suppliers in increasing order of the costs c_i such that $c_{[1]} \leq c_{[2]} \leq \ldots \leq c_{[N]}$. Set $s = 0$ and $q_i = 0\ \forall i$.
2. $s = s + 1$. Determine $t_{[s]} = \max\{\ 0, Q - \sum_{i=1}^{s-1} r_{[i]} q_{[i]}\ \}$ and based on this, $q_{[s]} = \min\ \{y_{[s]}, t_{[s]}\}$.
3. If $s = n$ stop, else repeat 2.

Before proceeding to determine the optimal quantity Q, note that for each supplier where $q_i = y_i > 0$, the lagrange multiplier λ_i shows the marginal profit which could be obtained by increasing the capacity of supplier i and this can be determined from equation (C.12).

To determine the optimal quantity Q which should be sourced from all the suppliers, we note that for any one supplier k, $-c_k + s + (p - s + u)(1 - F(Q)) = 0$ must hold. Obviously, since profits are maximized by ordering first from the lower cost suppliers, determining k iteratively as in our Theorem must hold. This concludes our proof.

C.3 Proof of Theorem 5.5

Theorem 5.5: When each chosen supplier has both maximum and minimimum limitations placed on the size of the order, then the optimal quantity allocated to each supplier can be determined as follows.

Step 1: Index all chosen suppliers in increasing order of cost per unit (i.e., $c_{[1]} \leq c_{[2]} \leq c_{[3]} \ldots \leq c_{[X]}$).

Step 2: For each supplier $[i]$ $(i = 1, \ldots, X)$, determine $Q_{[i]}$ such that:

$F(Q_{[i]}) = \frac{(p-c_{[i]}+u)}{(p-s+u)}$

and based on this determine:

$t_{[i]} = Q_{[i]} - \sum_{j=1}^{i-1} y_{[j]} - \sum_{j=i+1}^{X} z_{[j]}$

Step 3: The quantity allocated to supplier i is $q_{[i]} = \min\{\max\{ t_{[i]}, z_{[i]}\}, y_{[i]}\}$ and the total quantity ordered by the firm from all suppliers can be determined as $\sum_{i=1}^{X} q_{[i]}$.

Proof: Recall the capacitated suppliers problem with no diversification benefit is as follows:

$$\text{Maximize } Z_{q_i} = \frac{(p-s)(b+a)}{2} - \sum_{i=1}^{N} c_i r_i q_i + s\sum_{i=1}^{N} r_i q_i - (p-s+u)ES$$

subject to:

$$q_i \leq y_i \qquad \forall i \qquad \text{(C.14)}$$

$$q_i \geq 0 \qquad \forall i \qquad \text{(C.15)}$$

$$q_i \geq z_i \qquad \forall i \qquad \text{(C.16)}$$

Since Z is strictly concave in q_i and the constraints are all linear, the KKT conditions for the lagrangean function ($L = Z + \sum_{i=1}^{N} \lambda_i(y_i - q_i) + \sum_{i=1}^{N} \theta_i(q_i - z_i)$), are necessary and sufficient to identify the optimal solution to this problem. These conditions are:

$$q_i[\frac{\partial L}{\partial q_i}] = r_i[-c_i + s + (p - s + u)(1 - F(Q))] - \lambda_i + \theta_i \qquad \forall i \tag{C.17}$$

$$\lambda_i[\frac{\partial L}{\partial \lambda_i}] = \lambda_i[y_i - q_i] \qquad \forall i \tag{C.18}$$

$$\theta_i[\frac{\partial L}{\partial \theta_i}] = \theta_i[q_i - z_i] \qquad \forall i \tag{C.19}$$

To start with assume that there is some amount Q which will be sourced from all the suppliers. Then, similar to the underlying logic of Theorems 5.1 and 5.2, we know that at most one supplier can be unconstrained in the optimal solution.

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BIOGRAPHICAL SKETCH

In 1967, Gerard Joseph Burke Jr. was born in Jacksonville Beach, Florida. He graduated from the University of Florida with a Bachelor of Science degree in business administration in 1991. From 1991 through 1999, Gerard worked as a restaurant manager for casual dining companies. During this time he gained valuable experience in the field as a general manager of production and service operations. In 1999, he was awarded a graduate assistantship from the University of Florida's MBA program and became a Poe Ethics Fellow. After completing the MBA program in 2001 with a specialization in supply chain management, Gerard decided to pursue his newfound calling in academics and remained at the University of Florida in the Department of Decision and Information Sciences' doctoral program for operations management. During his doctoral studies from 2001 to 2005, he was awarded a University of Florida Alumni Graduate Fellowship and became a member of the decision sciences honorary society, Alpha Iota Delta. He is also a member of the Institute for Operations Research and the Management Sciences (INFORMS), Decision Sciences Institute (DSI), and Production and Operations Management Society (POMS). He has presented, session chaired, or refereed for annual meetings of INFORMS and DSI. Upon completion of his doctorate at the University of Florida, Gerard will be on faculty at Georgia Southern University's College of Business Administration.