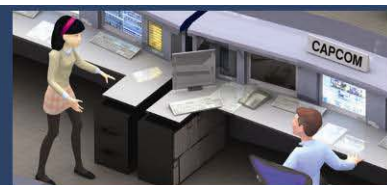




MATH AND SCIENCE @ WORK

AP* PHYSICS Educator Edition



SPACE SHUTTLE LAUNCH MOTION ANALYSIS

Instructional Objectives

Students will

- interpret and construct graphs of motion based on a space shuttle launch from ascent to orbit; and
- apply integration techniques to evaluate acceleration, speed, and distance along a flight trajectory.

Degree of Difficulty

This problem applies the standard techniques used to analyze displacement-time, speed-time, and acceleration-time graphs but begins with an acceleration-time graph (not typical). For this reason the students need to be very experienced at construction and interpretation in order to effectively analyze the graphs. In order for students to do parts C and D, they will also need to be proficient at integration and be able to evaluate and apply boundaries to the integration.

- For the average student in AP Physics C: Mechanics, this problem is at an advanced difficulty level.

Class Time Required

This problem requires 60-85 minutes.

- Introduction: 25-30 minutes (includes watching video of launch with discussion-see Appendix A)
- Student Work Time: 30-40 minutes
- Post Discussion: 5-15 minutes (could include watching Google Earth Demo of launch-see Appendix B)

Media Resources

This problem has two media files for use during the introduction and post discussion of the problem. These files will engage students in a space shuttle launch and introduce them to the different events that take place during the space shuttle's ascent into space.

- Space Shuttle Launch Video (10 minutes)
This video shows the launch of Space Shuttle Discovery STS 121 mission on July 4, 2006. To access the video, follow the link

Grade Level
11-12

Key Topic
Equations of Motion;
Graphs of Motion

Degree of Difficulty

Physics C: Mechanics:
Advanced

Teacher Prep Time
15 minutes

Class Time Required
50-70 minutes

Technology
Calculator, computer with projector, movie player software, Google Earth software (optional)

AP Course Topics
Newtonian Mechanics:
- Kinematics

NSES
Science Standards
- Physical Science
- History and Nature of Science

*AP is a trademark owned by the College Board, which was not involved in the production of, and does not endorse, this product.



provided. This should be shown to students during the introduction as it is necessary for them to answer part of the problem. It is highly recommended that the instructor become familiar with the video before showing it to students.

Appendix A provides a discussion guide that should be used with the video.

http://mfile.akamai.com/18565/wmv/etouchsyst2.download.akamai.com/18355/wm.nasa-global/sts-121/STS-121_launch.asx

- Google Earth Space Shuttle Ascent Trajectory (5-10 minutes).
The file, AscentTrajectory_Tour.kmz, shows the ascent phase of the Space Shuttle Discovery during STS 119 in Google Earth. The file is provided on the website for download with this problem. Google Earth Software must be installed on the computer to view this activity.

Appendix B provides instructions for using this file.

Background

This problem is part of a series of problems that apply Math and Science @ Work in NASA's Space Shuttle Mission Control Center.

Since its first flight in 1981, NASA has used the space shuttle for human transport, the construction of the International Space Station (ISS), and to research the effects of space on the human body. One of the keys to the success of the Space Shuttle Program is the Space Shuttle Mission Control Center (MCC). The Space Shuttle MCC at NASA Johnson Space Center uses some of the most sophisticated technology and communication equipment in the world to monitor and control the space shuttle flights.

Within the Space Shuttle MCC, teams of highly qualified engineers, scientists, doctors, and technicians, known as flight controllers, monitor the systems and activities aboard the space shuttle. They work together as a powerful team, spending many hours performing critical simulations as they prepare to support preflight, ascent, flight, and reentry of the space shuttle and the crew. The flight controllers provide the knowledge and expertise needed to support normal operations and any unexpected events.

The space shuttle follows a specific trajectory during a mission. The Flight Dynamics Officer (FDO, pronounced "Fido") in the Space Shuttle MCC is responsible for monitoring the trajectory. They must know exactly where the space shuttle is and where it is going at all times during the mission. The trajectory of the space shuttle is unique for each flight and is broken into four phases: ascent, entry, orbit, and rendezvous.

The first phase is the ascent phase which begins at lift-off and ends as the space shuttle reaches orbit. Since the space shuttle has a mass of approximately 2 million kg (weight of about 4.4 million lbs) when on the launch pad, a very powerful propulsion system is needed to launch it into orbit. The main propulsion system consists of the External Tank (ET) and the Space Shuttle Main Engines (SSMEs). This system, together with the Solid Rocket Boosters (SRBs), supplies the force (thrust) needed to accelerate to the speed of approximately 7.85 km/s that is required to attain orbit. Within 1 minute of launch the space shuttle breaks the sound barrier. The stress on the vehicle and the crew increases as the space shuttle accelerates. To ensure their safety, the acceleration of the space shuttle must be kept below 3 g (3 times the acceleration due to gravity). The space shuttle continues to accelerate along its trajectory until Main Engine Cutoff (MECO). It only takes approximately 8 minutes and 30 seconds for the space shuttle to reach MECO, which occurs around 104 km (56 miles) above the surface of the Earth. The ET is dropped away to break up and land in the ocean. Approximately 30 minutes later, the orbiter performs a final rocket engine firing to reach an orbital altitude of around 320 km (200 miles) above Earth.



Figure 1: Space Shuttle Discovery at lift-off during STS 121



Figure 2: Viewed from the Banana River Viewing Site, Space Shuttle Discovery heads toward Earth orbit and rendezvous with the International Space Station (ISS) on 28 August, 2009 during STS 128.

During the ascent phase, the FDO flight controllers work closely with the Propulsion (PROP) officer and the Booster flight controller who are both responsible for monitoring the propulsion of the space shuttle. In addition, there are several support teams for the FDO position located in backrooms of the MCC. These support teams are constantly engaged in intensive computational processing starting with the ascent phase and continuing throughout the entire mission. The FDO flight controller continues to work closely with the backroom support teams and other flight controllers in the Space Shuttle MCC throughout the mission to meet the mission objectives and to ensure crew safety.

AP Course Topics

Newtonian Mechanics

- Kinematics:
 - motion in one dimension (graphical analysis)

NSES Science Standards

Physical Science

- Motions and forces

History and Nature of Science

- Science as a human endeavor

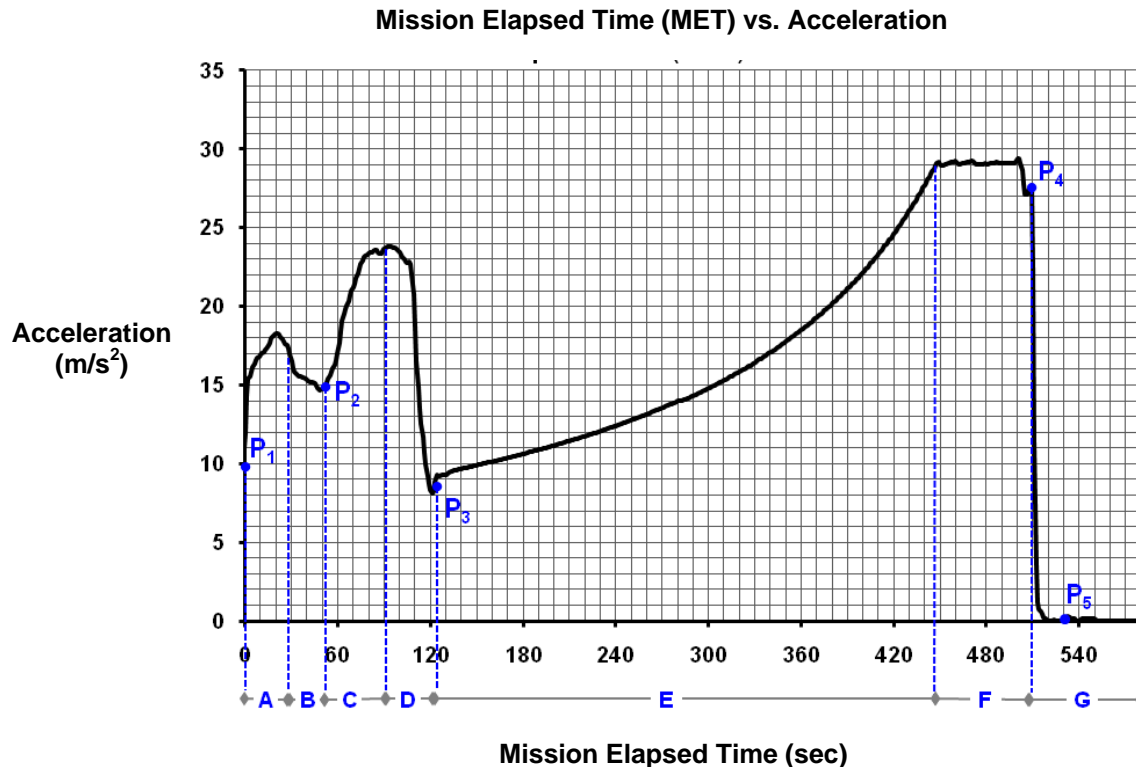
Problem

The Ascent Team, one of the backroom teams associated with FDO, has provided the acceleration data of the space shuttle during the launch of Space Transportation System (STS) 121. As the space shuttle lifts off from the launch pad and ascends into orbit, the motion along its flight path appears to be smooth. Although one might intuitively perceive the space shuttle as maintaining a constant acceleration during its climb into orbit, it actually experiences varying accelerations during this period. The final orbital speed of the space shuttle is approximately 28,000 km/hr (17,500 mph). Graph 1



provides an acceleration-time profile for the launch ascent phase of STS 121. The motion is modeled as linear motion. For purposes of this problem, centripetal or orbital motions are neglected in this model, therefore the acceleration is assumed to affect only the speed of the shuttle. The space shuttle has an initial speed of 0 m/s and distance is measured from the launch pad.

Note: Throughout this problem, when the terms “acceleration” and “displacement” are used, they are referring to the magnitude component of the vectors only.



Graph 1: Acceleration profile of STS 121 during the ascent phase.

- A. The following events take place during the launch of a space shuttle. Compare the rates of change of acceleration in Graph 1 and determine which event occurs at the marked time periods (A through G) and points (P₁ through P₅).

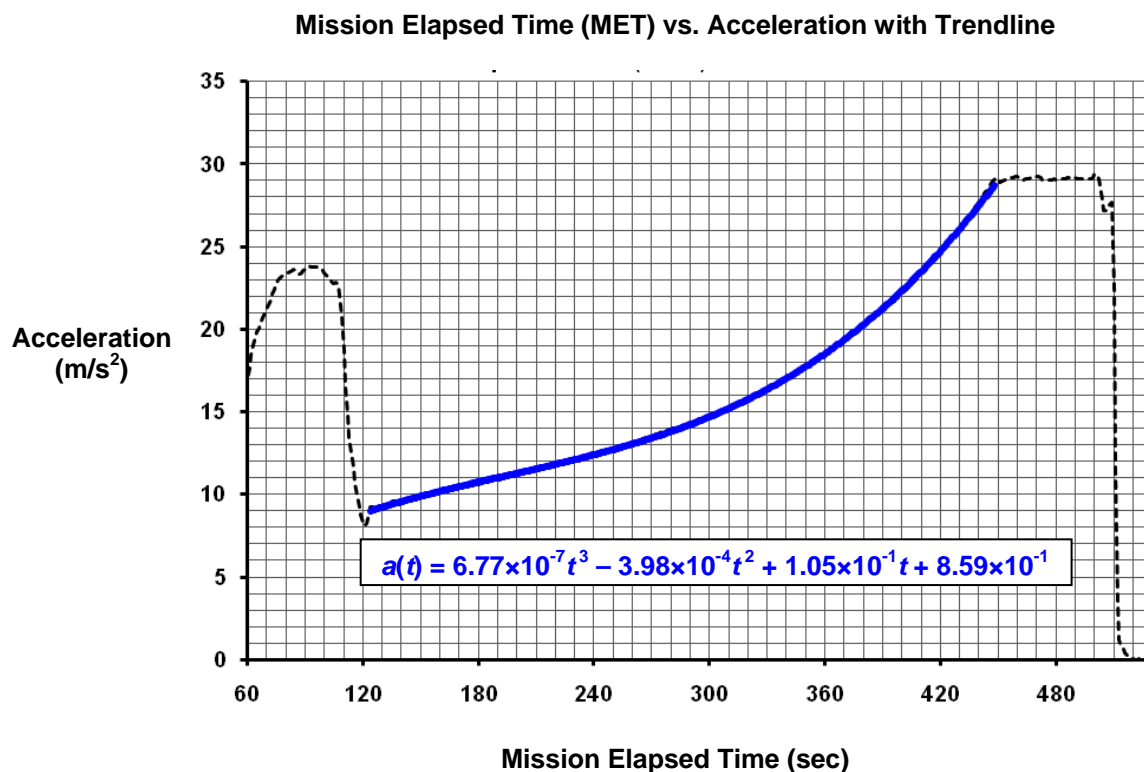
- _____ External Tank separation
- _____ Aerodynamic stress on the spacecraft in atmospheric flight is at its highest (Max Q)
- _____ Space Shuttle Main Engines provide smoothly changing acceleration
- _____ Throttle down
- _____ On orbit
- _____ Throttle up
- _____ Main Engine Cutoff
- _____ Solid Rocket Boosters burning out



- _____ Negative rate of change of acceleration (large air drag)
- _____ Constant acceleration (3 g's)
- _____ Solid Rocket Booster separation
- _____ Space shuttle lift-off

B. Identify the initial acceleration value at $t = 0$ s from Graph 1 and explain why the acceleration is not 0 m/s^2 ?

During the timeframe $t = 124$ s through $t = 446$ s the space shuttle's main engines produce a very smooth change in acceleration. Graph 2 shows how this period of changing acceleration can be best modeled by the expression: $a(t) = (6.77 \times 10^{-7})t^3 - (3.98 \times 10^{-4})t^2 + (1.05 \times 10^{-1})t + (8.59 \times 10^{-1})$.



Graph 2: Acceleration profile of STS 121 showing the trendline for the time period $t = 124$ s to $t = 446$ s.

- C. Write an expression that represents the change in speed (Δv) experienced by the space shuttle during the time period $t = 124$ s to $t = 446$ s and then evaluate the expression to determine the value.
- D. If the speed of the space shuttle at $t = 124$ s is approximately 974 m/s , write an expression that represents change in displacement along the ascent path during the time period $t = 124$ s through $t = 446$ s and evaluate the expression to determine the value.

The displacement expression derived from the acceleration expression will not yield an altitude measurement but a distance along the flight path. (This is because the data used to produce the



Mission Elapsed Time (MET) vs. Acceleration graph is a composite resultant of all three axis accelerometers on the space shuttle.)

- E. Notice that the acceleration between $t = 446$ s and $t = 500$ s remains constant at approximately $3 g$ ($g = 9.8 \text{ m/s}^2$). See Graph 2. Maintaining the acceleration at $3 g$ limits the physical stress endured by the astronauts. Make a sketch of the speed-time graph for this time period.
- F. Determine the speed of the space shuttle at Main Engine Cutoff (MECO) occurring at $t = 500$ s.

Solution Key (One Approach)

- A. The following events take place during the launch of a space shuttle. Compare the rates of change of acceleration in Graph 1 and determine which event occurs at the marked time periods (A through G) and points (P_1 through P_5).

Note to Educator: In order for students to answer this problem they will need to have watched and discussed the video of the launch.

 P_5 External Tank separation

 P_2 Aerodynamic stress on the spacecraft in atmospheric flight is at its highest (Max Q)

 E Space Shuttle Main Engines provide smoothly changing acceleration

 A Throttle down

 G On orbit

 C Throttle up

 P_4 Main Engine Cutoff

 D Solid Rocket Boosters burning out

 B Negative rate of change of acceleration (large air drag)

 F Constant acceleration ($3 g$'s)

 P_3 Solid Rocket Booster separation

 P_1 Space shuttle lift-off

- B. Identify the initial acceleration value at $t = 0$ s from Graph 1 and explain why the acceleration is not 0 km/s^2 ?

The value shown in Figure 1 appears to be just below 10 m/s^2 . This is the initial value of acceleration as the space shuttle sits on the launch pad due to Earth's gravitational pull on the space shuttle system which is 9.8 m/s^2 .

Note to Educator: While the space shuttle is not intuitively accelerating since it is sitting on the launch pad. However, an accelerometer, which is the measuring device for the displayed data, does not differentiate between acceleration due to the rocket engines and acceleration due Earth's gravitational pull.

- C. Write an expression that represents the change in speed (Δv) experienced by the space shuttle during the time period $t = 124$ s to $t = 446$ s and then evaluate the expression to determine the value.



Students should integrate $a(t)$ with respect to time and evaluate the integral for the given timeframe.

$$a(t) = (6.77 \times 10^{-7}) t^3 - (3.98 \times 10^{-4}) t^2 + (1.05 \times 10^{-1}) t + (8.59 \times 10^{-1})$$

$$\Delta v = \int_{124 \text{ s}}^{446 \text{ s}} a(t) dt = \int_{124 \text{ s}}^{446 \text{ s}} [(6.77 \times 10^{-7}) t^3 - (3.98 \times 10^{-4}) t^2 + (1.05 \times 10^{-1}) t + (8.59 \times 10^{-1}) + C] dt$$

$$\Delta v = [(1.69 \times 10^{-7}) t^4 - (1.33 \times 10^{-4}) t^3 + (5.25 \times 10^{-2}) t^2 + (8.59 \times 10^{-1}) t + C]_{124 \text{ s}}^{446 \text{ s}}$$

$$\Delta v = (5710 + C) \frac{m}{s} - (700 + C) \frac{m}{s}$$

$$\Delta v = 5010 \frac{m}{s} \text{ or } 18,036 \frac{km}{hr}$$

Note to Educator: The smooth curve during this period results from the thrust of the main engines remaining constant while the mass decreases at a smooth rate due to burning propellant.

- D. If the speed of the space shuttle at $t = 124 \text{ s}$ is approximately 974 m/s , write an expression that represents change in displacement along the ascent path during the time period $t = 124 \text{ s}$ through $t = 446 \text{ s}$ and evaluate the expression to determine the value.

The displacement expression derived from the acceleration expression will not yield an altitude measurement but a distance along the flight path. (This is because the data used to produce the Mission Elapsed Time (MET) vs. Acceleration graph is a composite resultant of all three axis accelerometers on the space shuttle.)

Students will need to find an expression for speed during the given time frame and then evaluate the integral.

$$v(t) = \int a(t) dt = \int [(6.77 \times 10^{-7}) t^3 - (3.98 \times 10^{-4}) t^2 + (1.05 \times 10^{-1}) t + (8.59 \times 10^{-1}) + C] dt$$

$$v(t) = (1.69 \times 10^{-7}) t^4 - (1.33 \times 10^{-4}) t^3 + (5.25 \times 10^{-2}) t^2 + (8.59 \times 10^{-1}) t + C$$

Solve C by substituting in $t = 124 \text{ s}$ and $v(124 \text{ s}) = 974 \text{ m/s}$

$$974 = (1.69 \times 10^{-7}) (124)^4 - (1.33 \times 10^{-4}) (124)^3 + (5.25 \times 10^{-2}) (124)^2 + (8.59 \times 10^{-1}) (124) + C$$

$$C = 274$$

$$v(t) = (1.69 \times 10^{-7}) t^4 - (1.33 \times 10^{-4}) t^3 + (5.25 \times 10^{-2}) t^2 + (8.59 \times 10^{-1}) t + 274$$

Note to teacher: C is the speed of $a(t)$ at $t=0$. Remember, however, that $a(t)$ is the approximated curve that describes only the timeframe: $t=124$ to $t=446$.

The change in displacement can now be found by evaluating the integral of $v(t)$

$$\Delta d = \int_{124 \text{ s}}^{446 \text{ s}} v(t) dt = \int_{124 \text{ s}}^{446 \text{ s}} [(1.69 \times 10^{-7}) t^4 - (1.33 \times 10^{-4}) t^3 + (5.25 \times 10^{-2}) t^2 + (8.59 \times 10^{-1}) t + 274] dt$$

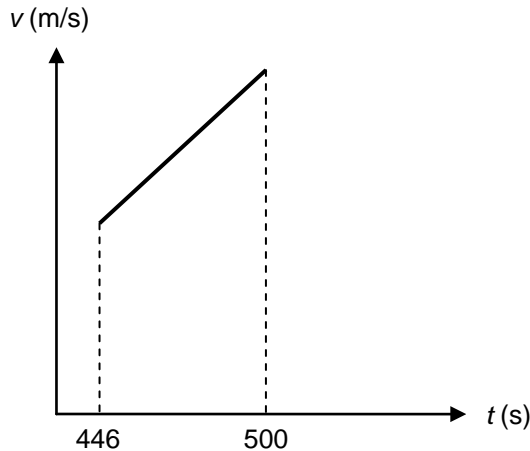
$$\Delta d = [(3.38 \times 10^{-8}) t^5 - (3.33 \times 10^{-5}) t^4 + (1.75 \times 10^{-2}) t^3 + (4.30 \times 10^{-1}) t^2 + 274t + C]_{124 \text{ s}}^{446 \text{ s}}$$



$$\Delta d = (1,039,150 + C)m - (67,072 + C)m$$

$$\Delta d = 972,000 \text{ m or } 972 \text{ km}$$

- E. Notice that the acceleration between $t = 446 \text{ s}$ and $t = 500 \text{ s}$ remains constant at approximately $3g$ ($g = 9.8 \text{ m/s}^2$). See Graph 2. Maintaining the acceleration at $3g$ limits the physical stress endured by the astronauts. Make a sketch of the speed-time graph for this time period.



- Graph is a straight line
- Positive slope ($3g$ or 29.4 m/s^2)
- Y-intercept is non-zero

- F. Determine the speed of the space shuttle at Main Engine Cutoff (MECO) occurring at $t = 500 \text{ s}$.

Students should first evaluate $v(t)$ found in question D at $t = 446 \text{ s}$.

$$v(t) = 1.69 \times 10^{-7} t^4 - 1.33 \times 10^{-4} t^3 + 5.25 \times 10^{-2} t^2 + 8.59 \times 10^{-1} t + 274$$

$$v(446) = 5990 \frac{\text{m}}{\text{s}}$$

Then find the speed at $t = 500 \text{ s}$, using $a = 3g$ or $3(9.8) = 29.4 \frac{\text{m}}{\text{s}^2}$, and $\Delta t = 54 \text{ s}$.

$$v_{f(\text{MECO})} = v_i + a\Delta t$$

$$v_{f(\text{MECO})} = 5990 \frac{\text{m}}{\text{s}} + (29.4 \frac{\text{m}}{\text{s}^2})(54 \text{ s})$$

$$v_{f(\text{MECO})} = 7580 \frac{\text{m}}{\text{s}}$$

$$v_{f(\text{MECO})} = 7580 \frac{\text{m}}{\text{s}} \text{ or } 27,288 \frac{\text{km}}{\text{hr}}$$

Note to Educator: The commonly referred to value of $28,000 \text{ km/h}$, or $17,500 \text{ mph}$ for orbital speed is ultimately achieved by an Orbital Maneuvering System (OMS) burn occurring after Main Engine Cut Off (MECO). This burn circularizes the orbit and varies for each launch. Depending on mission objectives the resulting final orbital speed is slightly different for each mission.



Scoring Guide

Suggested 15 points total to be given.

Question	Distribution of points
A <i>3 points</i>	1 point for 3-5 correct answers, 2 points for 6-8 correct answers, and 3 points for 8-12 correct answers.
B <i>2 points</i>	1 point for correct value of 9.8 m/s^2 1 point for correct explanation
C <i>3 points</i>	1 point for identifying Δv as the integral of the acceleration expression 1 point for applying the correct bounds to integral 1 point for the correct value for Δv
D <i>2 points</i>	1 point for correct integrated Δd expression 1 point for correct value for Δd
E <i>2 points</i>	1 point for a positive slope 1 point for the y-intercept being non-zero
E <i>3 points</i>	1 point for correct speed at $t = 446 \text{ s}$ 1 point for acceleration value of 29.4 m/s determined from graph 1 point for correct value of final speed at MECO

Contributors

This problem was developed by the Human Research Program Education and Outreach (HRPEO) team with the help of NASA subject matter experts and high school AP instructors.

NASA Experts

Dr. Greg Holt – FDO Flight Controller, NASA Johnson Space Center

Joseph R. Trevathan – Branch Chief, Design and Development, Systems Architecture and Integration Office, NASA Johnson Space Center

Kelly Smith – Analyst, Flight Dynamics Division, NASA Johnson Space Center

AP Physics Instructors

Doug Fackelman – Liberty High School, Academy District 20, CO



Appendix A

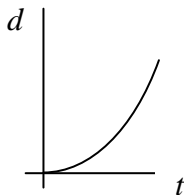
Discussion Guide for STS 121 Launch Video “The Rocket’s Red Glare”

Before playing the launch video, encourage students to pay close attention to the communication that goes on between the space shuttle crew and mission control and to strive to gain a sense of what the space shuttle is physically experiencing during the climb to orbit. After the students have watched the video discuss what they observed. The following questions and answers are just a few examples of questions that could be used in a class discussion of the video.

1. Name some of the forces that are involved in the ascent process.

Throttle down occurs as the space shuttle approaches and passes through the sound barrier where the pressure of the atmosphere produces enormous force on the space shuttle system and the crew inside the vehicle. “Accelerating more slowly” reduces these forces. Throttle up occurs after this point of maximum dynamic pressure (Max Q) has passed and allows the space shuttle to accelerate continuing its ascent.

2. After the space shuttle has cleared the tower, the video image is from the Solid Rocket Booster (SRB). You can literally see Florida “fall down” below the rising space shuttle. Sketch a d - t graph of this portion of the ascent or explain what it would look like.



The graph is concave up, does not have a constant slope and its origin is at the launch pad.

3. As the Solid Rocket Boosters (SRBs) burn out and the space shuttle approaches SRB separation, what is happening to its acceleration?

As the SRBs burn out they are no longer able to contribute significantly to accelerating the space shuttle into orbit. Therefore, the space shuttle, although still accelerating due to its own engines, slows its rate of acceleration rapidly. After SRB separation there is a large jolt as the space shuttle is able to accelerate more quickly, now that it has lost a huge amount of mass.

4. Consider the statement “from zero to 17,500 mph in 8.5 min.” Ponder the enormity of these values in comparison to the fastest car you’ve ever heard of. What are the factors that contribute to the space shuttle achieving such a magnificently huge orbital speed?

Some factors include decreasing mass (propellant use and jettison of SRBs), constant thrust, and decreasing atmospheric drag.

5. Pay special attention to the camera angle provided of the Earth as it slips past in the background. How would you describe the space shuttle’s progress (constant speed, accelerating, at rest...)?

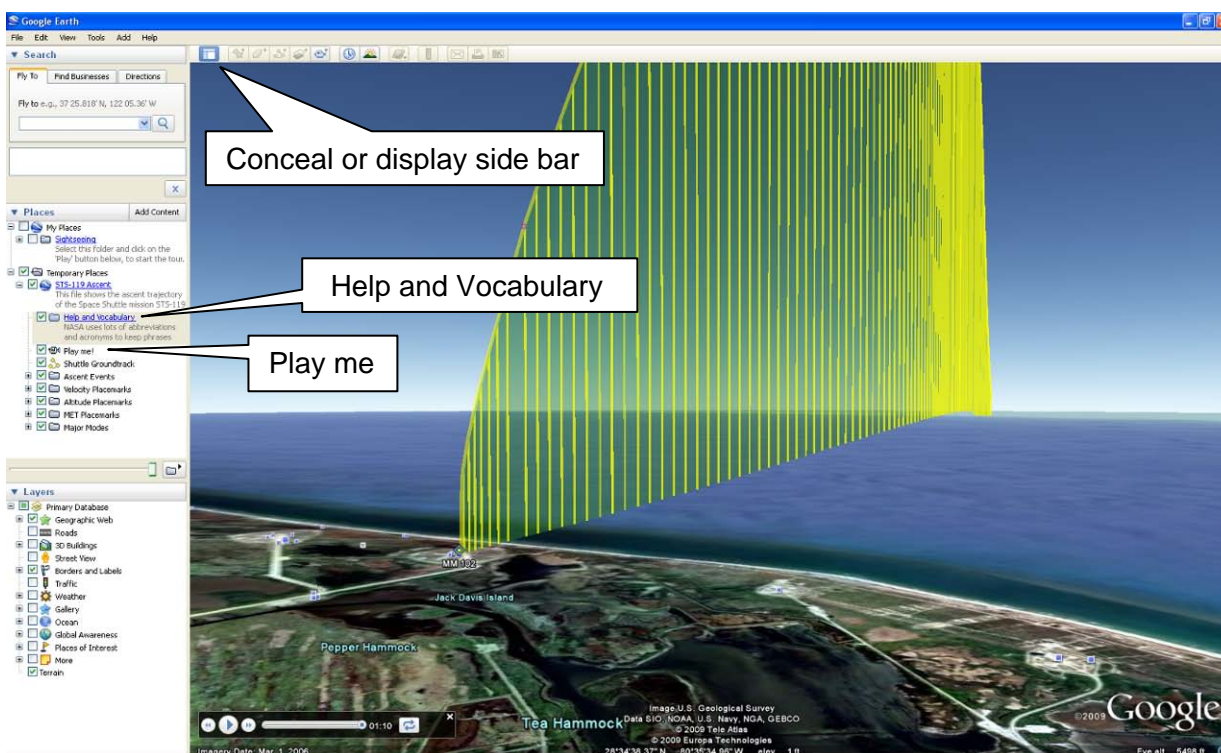
Because the space shuttle’s altitude is so great now it is nearly impossible to tell from the video that it is still accelerating. Recall, however, that the main engines are still running, propellant mass is decreasing quickly, and there is no atmospheric drag. One can conclude that the space shuttle is accelerating.



Appendix B

Instructions for Google Earth Space Shuttle Ascent Trajectory with Tour

1. Prior to presenting this file to your students, you should become familiar with its features and the features of Google Earth.
2. To access the file, you must first have Google Earth installed on the computer. To download a free version of Goggle Earth, follow the link, <http://earth.google.com>, and choose "Download Google Earth 5". The Google Earth user guide can be found under the help menu and gives a good overview of the software.
3. Download the provided file AscentTrajectory_Tour.kmz.
4. Open the file (AscentTrajectory_Tour.kmz) by selecting File > Open and select the file.



5. To learn more about this file, click on **Help and Vocabulary**, located on the left frame under **Temporary Places**.
6. An animated tour of the ascent trajectory is provided with this file and is 1:10 minutes long. To start the animation, double click on **Play Me!**
7. While watching the tour you can conceal the side bar by clicking the **Conceal** button located at the top left of the tool bar.
8. The tour begins with an overview of the entire space shuttle ascent from several different angles. This overview is 20 seconds long.
9. After the overview, the sequence will begin again. There are three 5 second pauses for inserting explanations of the ascent events: SRB Separation, MECO, and ET Separation. Each event is marked with a black and red circle.



10. There are additional features that you may want to explore with students at the conclusion of the tour. You may also want to use the following subtopics to unclutter the animation, (for example, uncheck the altitude and velocity folders).

To expand a folder in the left frame, click on the “+” next to the folder. To collapse the folder, click on the “-” next to it. For each subtopic there is a pop-up information box that can be opened by clicking on the subtopic. To close the pop-up information box, click on the “x” in the upper right corner of the box (or uncheck the small checked box to the left of the subtopic by clicking on it).

- Ascent Events Folder (expanded)
 - Click on an event and a pop-up information box will appear on the graph.
 - Notice the bullets for the events are the same as the markers (red and black circles) that appear on the graph for these events.
- Velocity Placemarks Folder (expanded)
 - Click on a velocity and a pop-up information box will appear on the graph.
 - Notice the bullets for the velocities are the same as the markers (green circles) that appear on the graph for these velocities.
- Altitude Placemarks Folder (expanded)
 - Click on an altitude and a pop-up will information box will appear on the graph.
 - Notice the bullets for the altitudes are the same as the markers (pink circles) that appear on the graph for these altitudes.
- MET (Mission Elapsed Time) Placemarks Folder (expanded)
 - Click on an MET and a pop-up information box will appear on the graph.
 - Notice the bullets for the METs are the same as the markers (orange circles) that appear on the graph for these METs.
- Major Modes (MM) Folder (expanded)
 - Click on a major mode and a pop-up information box will appear on the graph.
 - Notice the bullets for the major modes are the same as the markers (blue diamonds) that appear on the graph for these major modes.