NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

Technical Report 32-1255

Spacecraft Antenna Pointing With a Single Degree of Freedom

Gerald E. Fleischer

JET PROPULSION LABORATORY
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Abstract

A computer program which aids and evaluates the design of a single-degree-of-freedom spacecraft antenna-pointing system has been developed at the Jet Propulsion Laboratory (JPL). The program includes algorithms for determining an optimum location for the antenna's rotational axis and for fitting a function of connected line segments to the ideal antenna angular-rotation function. An on-board sequencer can then command antenna rotations according to the stored line-segment function. Specific examples of the program's application to a Jupiter flyby mission are presented, including the evaluation of major sources of pointing error. Descriptions of the program operation and its input formats are also given.

Spacecraft Antenna Pointing With a Single Degree of Freedom

I. Introduction

A natural consequence of the increasing scientific scope and sophistication of exploratory missions to the moon and the planets has been the demand for an increasing information-transmission capability. Consideration of the trade-offs between spacecraft transmitter power and weight vs spacecraft antenna gain and weight points to the use of a more directive antenna and its accompanying higher gain as the most advantageous method of improving communication system performance. However, a highly directive antenna implies a need for accurate pointing control.

This report describes the characteristics of a stored program technique of high-gain antenna pointing control using a single degree of rotational freedom. In such a system, the antenna is periodically rotated in discrete increments on command from an on-board sequencer. The sequencer is capable of generating a programmable series of pulses which update the antenna's position according to a preselected angular time function. Thus, the system is basically an open-loop controller which relies on an accurate, three-axis stabilization of the spacecraft with respect to certain celestial references. The ultimate accuracy of a stored program approach also depends heavily on the geometry of particular mission trajectories and on the geometrical effects of launch date variation and trajectory dispersions.

Of central importance to the programmed pointing system is the position of the antenna's axis of rotation. A computational method which determines an optimum location for the rotational axis is described in detail in Section III. In addition to pointing errors which result from particular trajectory geometries and the limitations of a single degree of freedom, there are errors caused by (1) spacecraft and antenna structural misalignments, (2) spacecraft attitude-control inaccuracies, and (3) antenna control system errors (including the stored pointing program). These contributions to the pointing problem are examined in detail in Section IV.

A computer algorithm is presented in Section V which approximates the optimal angular time function with a series of connected line segments. The line segment approximation is embodied in the antenna sequencer in the form of several pulse repetition frequencies (slopes) and pre-programmed times for pulse frequency changeover (breakpoints).

Finally, the report includes descriptions and listings of the several Fortran IV computer programs and subroutines developed to determine the optimum axis-of-rotation location, to fit a line segment function to the ideal angular rotation function, and to compute pointing errors under various conditions as well as to plot these errors as a function of time.

II. Antenna-Pointing Geometry

The coordinate system which provides the spacecraft's attitude reference is the quasi-inertial, sun–Canopus coordinate system, L, M, N. The right-hand set of orthogonal axes, L, M, N, with its origin at the spacecraft's center of mass will then be aligned such that +N is directed from the spacecraft toward the sun and the L–N plane contains the vector from the spacecraft to Canopus, as pictured in Fig. 1. With perfect attitude alignment, the spacecraft's body-fixed roll axis, Z, coincides with N while body-fixed axes X and Y are assumed coincident with inertial reference axes L and M as a matter of descriptive convenience. The spacecraft antenna's rotational or "hinge" axis may be positioned with respect to the body-fixed axes by angles θ and ϕ, as shown in Fig. 1. In addition, the antenna feed vector, directed along the mechanical boresight axis, is assumed to rotate about hinge axis H, at a constant angle of cant, ψ. Components of the unit feed vector **f**, in terms of θ, ϕ, ψ, and the angular position of the feed vector θ_H may be developed as follows:

i, **j**, **k** are unit vectors along X, Y, and Z.

$$\mathbf{h} = \sin\phi\cos\theta\,\mathbf{i} + \sin\phi\sin\theta\,\mathbf{j} + \cos\phi\,\mathbf{k}$$

Define

$$\mathbf{p} = \frac{\mathbf{k}\times\mathbf{h}}{|\mathbf{k}\times\mathbf{h}|} = -\sin\theta\,\mathbf{i} + \cos\theta\,\mathbf{j}$$

and

$$\mathbf{q} = \frac{\mathbf{h}\times\mathbf{p}}{|\mathbf{h}\times\mathbf{p}|} = -\cos\theta\cos\phi\,\mathbf{i} - \sin\theta\cos\phi\,\mathbf{j} + \sin\phi\,\mathbf{k}$$

Let

$$\bar{R} = \sin\psi\cos\theta_H\,\mathbf{p} + \sin\psi\sin\theta_H\,\mathbf{q}$$

Then,

$$\mathbf{f} = \cos\psi\,\mathbf{h} + \bar{R}$$

$$\begin{aligned}\mathbf{f} = {} & (\cos\theta\cos\psi\sin\phi - \sin\theta\sin\psi\cos\theta_H \\ & - \cos\theta\sin\psi\cos\phi\sin\theta_H)\,\mathbf{i} \\ & + (\sin\theta\cos\psi\sin\phi + \cos\theta\sin\psi\cos\theta_H \\ & - \sin\theta\sin\psi\cos\phi\sin\theta_H)\,\mathbf{j} \\ & + (\cos\psi\cos\phi + \sin\psi\sin\phi\sin\theta_H)\,\mathbf{k}\end{aligned}$$

$$\left.\begin{aligned}\mathbf{f} &= \alpha_F\,\mathbf{i} + \beta_F\,\mathbf{j} + \gamma_F\,\mathbf{k} \\ \mathbf{e} &= \alpha_E\,\mathbf{i} + \beta_E\,\mathbf{j} + \gamma_E\,\mathbf{k}\end{aligned}\right\}\ \text{body-fixed coordinates}$$

where

$$\mathbf{e} = \text{unit vector directed to earth}$$

Thus, the pointing-error angle θ_ϵ may be given by:

$$\theta_\epsilon = \sin^{-1}|\mathbf{f}\times\mathbf{e}|$$

$$\theta_\epsilon = \sin^{-1}[(\beta_F\gamma_E - \gamma_F\beta_E)^2 + (\gamma_F\alpha_E - \alpha_F\gamma_E)^2 + (\alpha_F\beta_E - \beta_F\alpha_E)^2]^{1/2}$$

Normally, however, the earth-pointing vector **e** is defined in terms of its components in the inertial sun–Canopus system of coordinates:

$$\mathbf{e} = \alpha'_E\,\hat{\ell} + \beta'_E\,\mathbf{m} + \gamma'_E\,\mathbf{n}$$

In general, the body-fixed coordinate axes are not coincident with the inertial axes. However, in terms of a pitch, yaw, and roll sequence of rotations about the body-fixed axes X, Y, and Z, respectively, the two systems can be related by a transformation matrix A, where:

$$\begin{Bmatrix}\alpha' \\ \beta' \\ \gamma'\end{Bmatrix}_{\text{inertial}} = [A]\begin{Bmatrix}\alpha \\ \beta \\ \gamma\end{Bmatrix}_{\text{body}}$$

and

$$[A] \cong \begin{bmatrix} 1 & \theta_R & -\theta_Y \\ -\theta_R & 1 & \theta_P \\ \theta_Y & -\theta_P & 1 \end{bmatrix} \text{for small } \theta_P, \theta_Y, \text{and } \theta_R$$

It is also useful to obtain earth-vector direction cosines in the orthogonal system defined by the unit vectors **h**, **p**, and **q**, fixed to the spacecraft body:

$$\begin{aligned} e_h &= \mathbf{h}\cdot\mathbf{e} = \sin\phi\cos\theta\alpha_E + \sin\phi\sin\theta\beta_E + \cos\phi\gamma_E \\ e_p &= \mathbf{p}\cdot\mathbf{e} = -\sin\theta\alpha_E + \cos\theta\beta_E \\ e_q &= \mathbf{q}\cdot\mathbf{e} = -\cos\theta\cos\phi\alpha_E - \sin\theta\cos\phi\beta_E + \sin\phi\gamma_E \end{aligned}$$

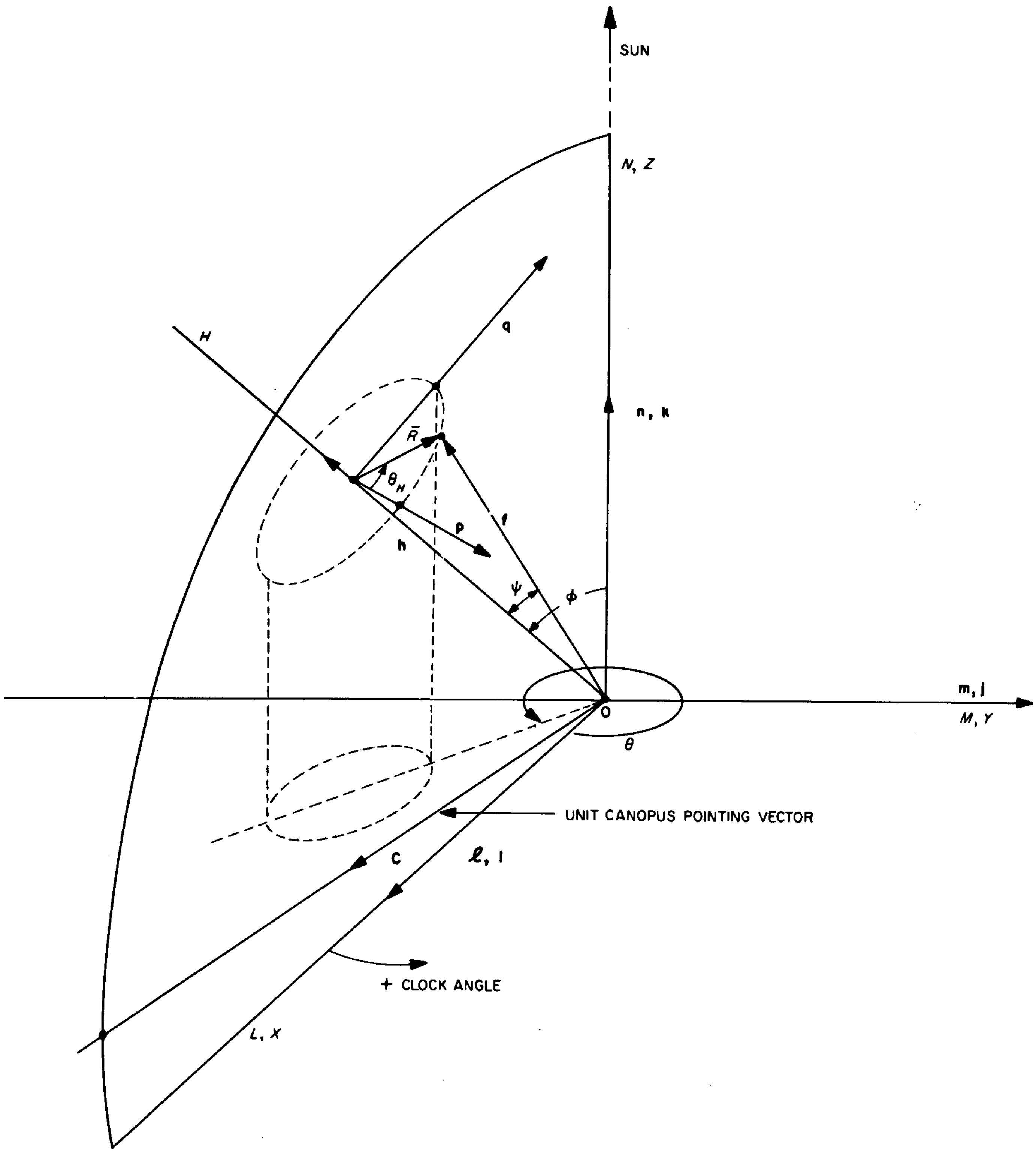


Fig. 1. Antenna-pointing geometry

Once the antenna hinge axis has been fixed to the spacecraft by specific values of θ and ϕ, the "best" angle of rotation (i.e., resulting in the least pointing error) of the feed vector about the hinge axis is defined by:

$$\theta_H^*(t) = \tan^{-1}\left(\frac{e_q}{e_p}\right)$$

where

$$e_p = f_1\,[\theta, \alpha_E(t), \beta_E(t)]$$

and

$$e_q = f_2\,[\theta, \phi, \alpha_E(t), \beta_E(t), \gamma_E(t)]$$

III. Optimum Hinge-Axis Location

The antenna's hinge axis should be fixed to the spacecraft in such a way that the pointing error, resulting solely from (a) the trajectory geometry and (b) directional

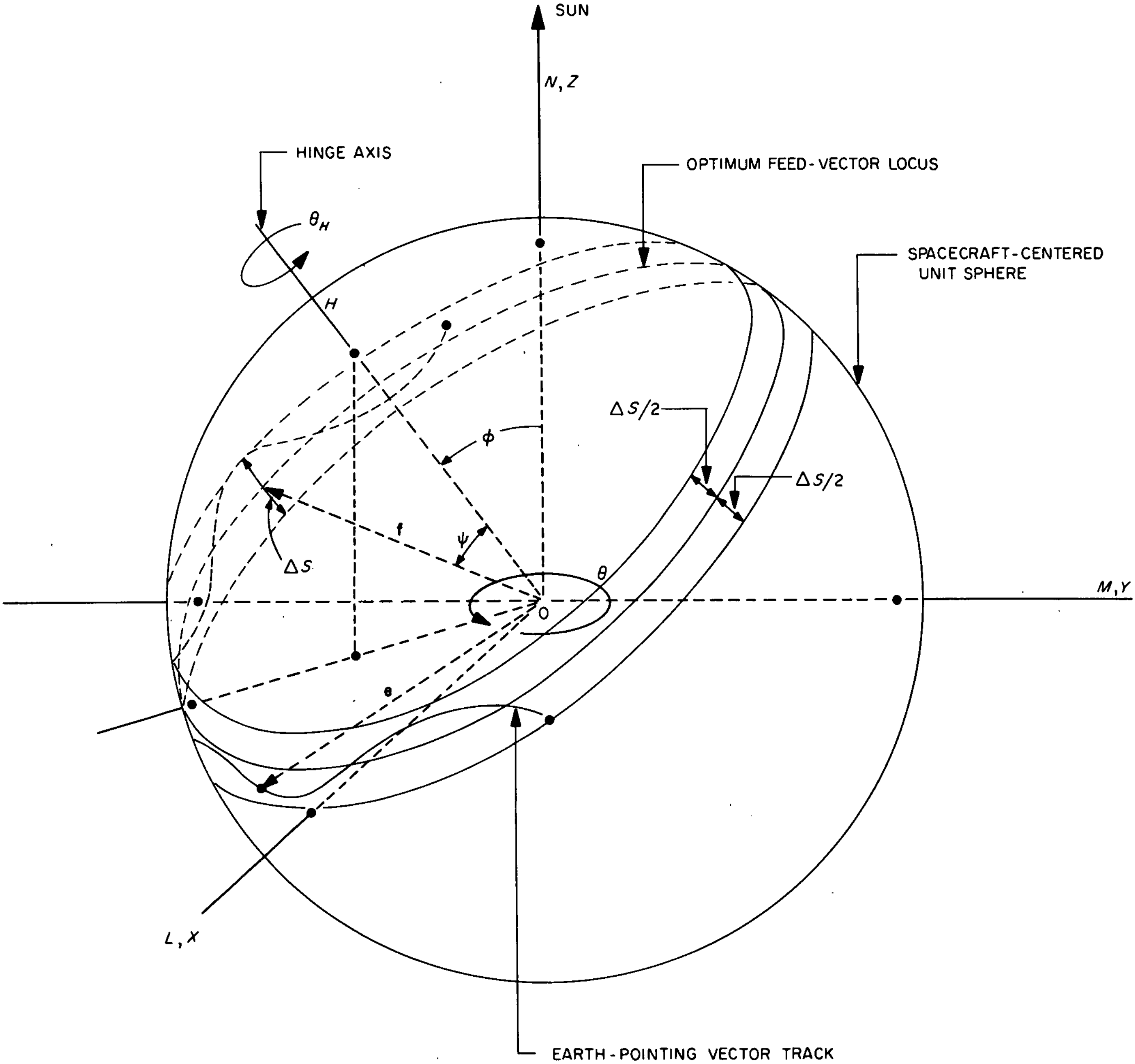


Fig. 2. Hinge-axis optimization geometry, oriented to the sun

constraints imposed by the single axis of rotation, is minimized. A useful criterion, called the minimax criterion, for optimizing the hinge-axis location is that of minimizing the maximum pointing error over the time period of interest.

Figure 2 illustrates the geometrical implications of applying the minimax criterion. The locus of earth-pointing vector intersections with a unit sphere centered at the spacecraft is shown. Body-fixed and inertial coordinate axes are assumed coincident. A rotating-antenna feed vector generates the surface of a cone with half-angle ψ, and its intersection with the unit sphere is a circle which necessarily lies in a plane perpendicular to the hinge axis. The optimization procedure then consists of enclosing the earth track within two parallel planes such that the shortest surface arc length subtended by the planes is minimized. The shortest surface arc length subtended is a portion of a great circle lying in a plane perpendicular to the two parallel planes.

Having minimized the described arc length ΔS, the locus of feed vector–sphere intersections which minimizes the maximum pointing error lies on a circle which divides ΔS in half, as shown in Fig. 2. Computationally, the arc ΔS may be examined in terms of the angle it subtends at the center of the sphere. Thus, it is required, as illustrated in Fig. 3, to minimize the expression:

$$\theta_S = \cos^{-1}(e_{h\,\min}) - \cos^{-1}(e_{h\,\max})$$

over the θ–ϕ parameter space.

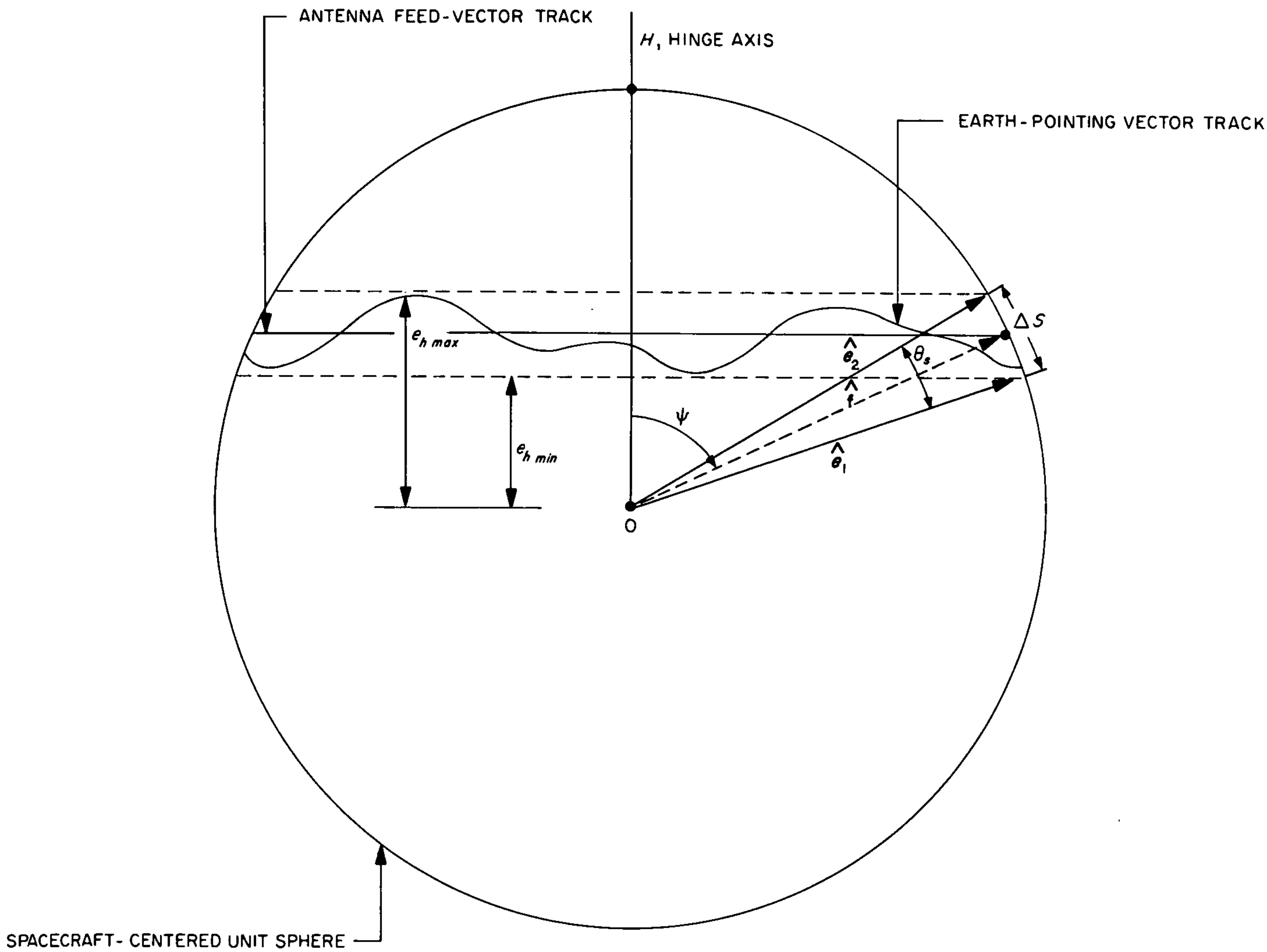


Fig. 3. Hinge-axis optimization geometry

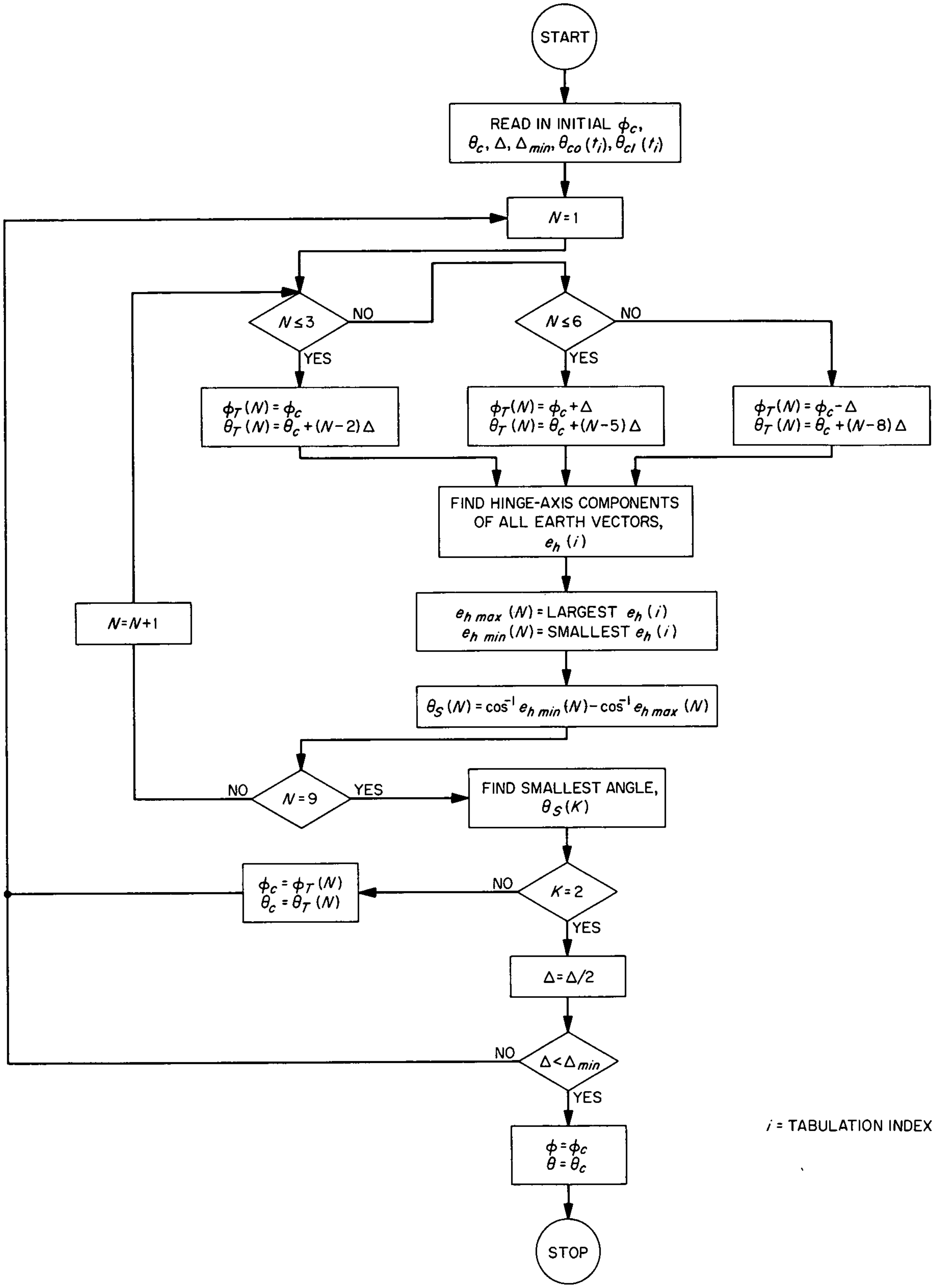


Fig. 4. Hinge-axis optimization routine

The flow diagram of a computer subroutine used to optimize the hinge-axis location is given in Fig. 4. A table of earth positions in the sun–Canopus reference system, usually at 5- to 10-day intervals, is read into the program in the form of cone and clock angles where:

$$\alpha'_E(t_i) = \sin\theta_{co}(t_i)\cos\theta_{CL}(t_i)$$

$$\beta'_E(t_i) = \sin\theta_{co}(t_i)\sin\theta_{CL}(t_i)$$

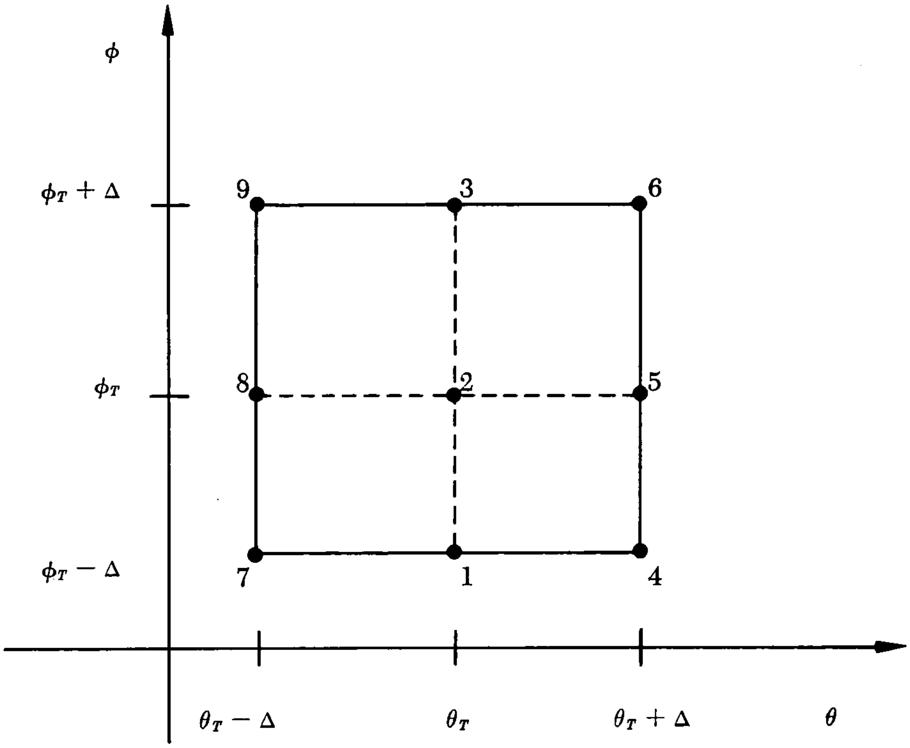
$$\gamma'_E(t_i) = \cos\theta_{co}(t_i)$$

$\theta_{co}(t_i)$ = earth's cone angle at t_i

$\theta_{CL}(t_i)$ = earth's clock angle at t_i

t_i = time of i^{th} data point, days from launch

Given trial values θ_T and ϕ_T, the immediate vicinity in the θ–ϕ plane is examined as shown below.



At each of the nine nodes, a value of θ_S is determined by scanning the entire table of earth direction cosines. That node which possesses the minimum θ_S is chosen as the new central node. If the central node results in a minimum θ_S, it remains as the central node in the next trial, but the step size Δ is halved. This process continues until Δ becomes less than some predetermined level, usually about 0.01 deg.

An example of the application of the hinge optimization subroutine, OPTLOC, to a Jupiter flyby mission is given in Figs. 5 and 6. Figure 5 is a computer printout of (a) input data in terms of earth cone and clock angles with corresponding direction cosines, (b) the computed optimal hinge-axis location, (c) a table of optimum feed-vector hinge angles, and (d) the resulting feed-vector pointing error and direction cosines at each time point. Figure 6 is a continuous plot of the antenna-pointing error vs time from launch. This error curve is ideal in the sense that it cannot be reduced, provided the antenna is mounted on a perfectly stabilized spacecraft precisely as described by θ, ϕ, and γ. Note that several identical extremums occur in the error curve (not necessarily including the endpoints) as a result of the minimax criterion.

It is also possible to optimize the antenna hinge-axis location with respect to several trajectories simultaneously. In this way, significant changes in earth-pointing geometry due to varying launch dates may be compensated by the axis-location subroutine, OPTLOC. Figure 7 indicates the result of the computations with input data representing launch dates at the beginning, center, and close of the Jupiter flyby mission launch window. Figure 8 shows resulting pointing errors for each of the trajectories.

TIME	CONE ANGLE	CLOCK ANGLE	ALPHAE	BETAE	GAMMAE
180.	23.2294049	281.5343018	0.07886450	-0.38644846	0.91893306
190.	21.9195280	282.1515617	0.07857961	-0.36493983	0.92770910
200.	20.5052531	282.6952591	0.07698211	-0.34172955	0.93664010
210.	18.9982829	283.1813393	0.07423382	-0.31696290	0.94552835
220.	17.4104259	283.6245689	0.07048239	-0.29079456	0.95418591
230.	15.7534600	284.0395889	0.06586325	-0.26338847	0.96243885
240.	14.0390500	284.4423599	0.06050150	-0.23491729	0.97013063
250.	12.2790400	284.8523903	0.05451424	-0.20556743	0.97712345
260.	10.4850740	285.2971916	0.04801071	-0.17553192	0.98330235
270.	8.6689566	285.8216515	0.04109419	-0.14501501	0.98857570
280.	6.8426313	286.5122490	0.03386271	-0.11422920	0.99287713
290.	5.0184027	287.5720482	0.02640927	-0.08339393	0.99616665
300.	3.2100890	289.6428795	0.01882382	-0.05273862	0.99843092
310.	1.4399750	296.4494095	0.01119292	-0.02249929	0.99968420
320.	0.4554972	63.0563812	0.00360218	0.00708691	0.99996840
330.	2.0605439	96.1820097	-0.00387194	0.03574643	0.99935339
340.	3.6811259	99.9914398	-0.01113937	0.06322984	0.99793682
350.	5.2251834	101.4759903	-0.01811907	0.08924962	0.99584447
360.	6.6712195	102.2888002	-0.02472591	0.11351001	0.99322913
370.	7.9990939	102.8170700	-0.03087051	0.13569008	0.99027027
380.	9.1873395	103.1998100	-0.03645864	0.15544469	0.98717158
390.	10.2123179	103.5000000	-0.04138895	0.17239760	0.98415752
400.	11.0480970	103.7511902	-0.04555226	0.18614019	0.98146667
410.	11.6663690	103.9738302	-0.04882995	0.19622822	0.97934168
420.	12.0369260	104.1822701	-0.05109428	0.20218592	0.97801341
430.	12.1285580	104.3884096	-0.05220998	0.20351558	0.97767864
440.	11.9109170	104.6042500	-0.05203950	0.19972223	0.97846968
450.	11.3572921	104.8446198	-0.05045226	0.19035399	0.98041824
460.	10.4489599	105.1315298	-0.04734128	0.17507165	0.98341686
470.	9.1806124	105.5035801	-0.04264668	0.15374182	0.98719031
480.	7.5665675	106.0406303	-0.03638508	0.12655122	0.99129254
490.	5.6469078	106.9470501	-0.02868169	0.09412469	0.99514718
500.	3.4913742	108.9649696	-0.01979131	0.05759255	0.99814398
510.	1.2113234	118.5346899	-0.01009838	0.01857209	0.99977653
512.	0.7663739	127.3009100	-0.00810547	0.01063961	0.99991055

THE OPTIMUM HINGE AXIS LOCATION

PSI	THETA	PHI
79.980512	194.374969	79.960923

TIME	HINGE ANGLE	ERROR	ALPHAF	BETAF	GAMMAF
180.	66.39507	0.3174133	0.08431446	-0.38545652	0.91886578
190.	67.73217	0.1161652	0.08057253	-0.36456801	0.92768433
200.	69.17422	0.0347505	0.07638649	-0.34184371	0.93664719
210.	70.70914	0.1443203	0.07176229	-0.31744812	0.94555636
220.	72.32495	0.2204824	0.06671013	-0.29155397	0.95422535
230.	74.00984	0.2702023	0.06124496	-0.26434207	0.96248236
240.	75.75221	0.2995610	0.05538682	-0.23600063	0.97017317
250.	77.54025	0.3138047	0.04916234	-0.20673032	0.97716203
260.	79.36242	0.3174135	0.04260359	-0.17673712	0.98333560
270.	81.20701	0.3141402	0.03574936	-0.14623685	0.98860344
280.	83.06223	0.3070732	0.02864474	-0.11545224	0.99289991
290.	84.91625	0.2986559	0.02134095	-0.08461139	0.99618545
300.	86.75679	0.2907500	0.01389624	-0.05395092	0.99844687
310.	88.57131	0.2846571	0.00637511	-0.02371235	0.99969848
320.	90.34710	0.2813299	-0.00115287	0.00586272	0.99998213
330.	92.06934	0.2804117	-0.00860512	0.03450181	0.99936757
340.	93.72406	0.2823633	-0.01589923	0.06195300	0.99795241
350.	95.29491	0.2863174	-0.02293945	0.08793229	0.99586228
360.	96.76449	0.2912431	-0.02962331	0.11214862	0.99324977
370.	98.11345	0.2957172	-0.03583756	0.13428790	0.99029410
380.	99.32039	0.2979956	-0.04145887	0.15401386	0.98719849
390.	100.36157	0.2960759	-0.04635257	0.17096072	0.98418690
400.	101.21080	0.2877889	-0.05037342	0.18473141	0.98149722
410.	101.83946	0.2709390	-0.05336632	0.19489340	0.97937152
420.	102.21693	0.2435060	-0.05516994	0.20098152	0.97804023
430.	102.31154	0.2039213	-0.05562274	0.20250574	0.97770011
440.	102.09237	0.1514321	-0.05457422	0.19897364	0.97848409
450.	101.53208	0.0865308	-0.05190127	0.18992836	0.98042513
460.	100.61134	0.0114113	-0.04753245	0.17501581	0.98341756
470.	99.32426	0.0697154	-0.04147696	0.15407688	0.98718790
480.	97.68481	0.1505929	-0.03385495	0.12726301	0.99129105
490.	95.73286	0.2234311	-0.02492170	0.09515881	0.99515008
500.	93.53723	0.2800920	-0.01506927	0.05885743	0.99815264
510.	91.19388	0.3137557	-0.00479876	0.01995090	0.99978943
512.	90.71748	0.3173447	-0.00274319	0.01202632	0.99992391

AVERAGE POINTING ERROR = 0.2414658 DEGREES

Fig. 5. Optimum hinge-axis location and resulting pointing errors for a 512-day mission to Jupiter

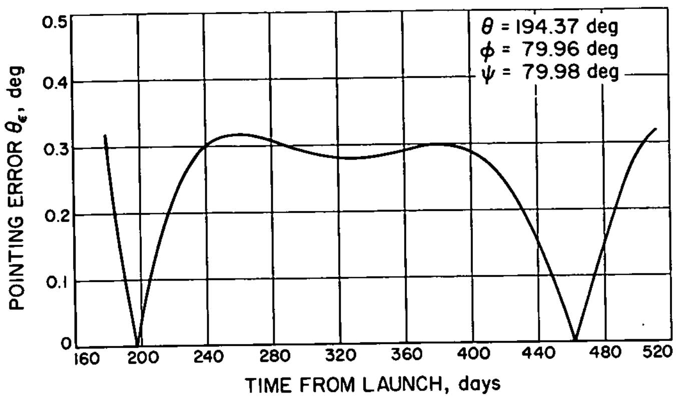


Fig. 6. Pointing error vs time for a 512-day Jupiter flyby mission (launch date: May 18, 1974)

IV. Other Sources of Pointing Error

A. Effects of Spacecraft Attitude Errors

It has been pointed out that the antenna pointing error generated, so far, reflects only the unavoidable geometrical limitations inherent in the single degree of rotational freedom. Methods for minimizing this error under various criteria can be devised. Section III described a scheme which minimizes the maximum error due to the constraint of a single-axis rotation. However, present and past designs for interplanetary spacecraft attitude-control systems have allowed errors about each control axis of approximately 0.25–0.50 deg. Thus, an additional and rather significant source of pointing error is introduced by spacecraft attitude misalignments.

Pitch, yaw, and roll rotations about the spacecraft body-fixed axes may be used to relate vector components in the inertial system to body-referenced components.

$$\begin{Bmatrix} \alpha' \\ \beta' \\ \gamma' \end{Bmatrix}_{\text{inertial}} = [A]^{-1} \begin{Bmatrix} \alpha \\ \beta \\ \gamma \end{Bmatrix}_{\text{body}}$$

where

$$[A]^{-1} = \begin{bmatrix} 1 & -\theta_R & \theta_Y \\ \theta_R & 1 & -\theta_P \\ -\theta_Y & \theta_P & 1 \end{bmatrix} \text{for small } \theta_P, \theta_Y, \text{and } \theta_R$$

Thus, the antenna feed-vector body components α_F, β_F, γ_F are transformed to the inertial system, and the total pointing error is obtained by a cross-product with the earth-pointing vector (in inertial coordinates).

$$\begin{Bmatrix} \alpha'_F \\ \beta'_F \\ \gamma'_F \end{Bmatrix} = \begin{bmatrix} 1 & -\theta_R & \theta_Y \\ \theta_R & 1 & -\theta_P \\ -\theta_Y & \theta_P & 1 \end{bmatrix} \begin{Bmatrix} \alpha_F(\phi, \theta, \psi, \theta^*_H) \\ \beta_F(\phi, \theta, \psi, \theta^*_H) \\ \gamma_F(\phi, \theta, \psi, \theta^*_H) \end{Bmatrix}$$

$$\theta_\epsilon = \sin^{-1}|\mathbf{f}' \times \mathbf{e}'|$$

$$\theta_\epsilon = \sin^{-1}[(\beta'_F\gamma'_E - \gamma'_F\beta'_E)^2 + (\gamma'_F\alpha'_E - \alpha'_F\gamma'_E)^2 + (\alpha'_F\beta'_E - \beta'_F\alpha'_E)^2]^{1/2}$$

Spacecraft attitude drift rates are on the order of a few μrad/s in the typical *bang-bang* system with a deadband. However, the possibility that, in the space of a few days, the worst attitude (from the standpoint of causing antenna-pointing error) could be reached must be taken into consideration. The maximum value of θ_ϵ must occur at some combination of extreme values of θ_P, θ_Y, and θ_R, i.e., values corresponding to either end of the pitch, yaw, and roll deadbands. A computer search of the eight possible deadband edge combinations can easily be made to determine the worst pointing error resulting from combined spacecraft attitude drifts and hinge-axis location. The effects of ±¼-deg deadbands (in each of the three control axes) on the Jupiter mission's antenna-pointing error are illustrated in Fig. 9. In general, the angular separation of vectors **f** and **f**′ (rotated **f**) is given by

$$\theta_{\epsilon f} = \sin^{-1}|\mathbf{f}' \times \mathbf{f}|$$

and

$$\theta_{\epsilon f} \cong [(\alpha_F\theta_R - \gamma_F\theta_P)^2 + (\beta_F\theta_R - \gamma_F\theta_Y)^2 + (\beta_F\theta_P - \alpha_F\theta_Y)^2]^{1/2}$$

for small angles.

If for example, $\gamma_F = 0$, and $\alpha_F = \beta_F = \frac{\sqrt{2}}{2}$,

then

$$\theta_{\epsilon f} = \left[\theta_R^2 + \left(\frac{\sqrt{2}}{2}\theta_P - \frac{\sqrt{2}}{2}\theta_Y\right)^2\right]^{1/2}$$

for,

$$\theta_R = \theta_{DB} = \theta_P = -\theta_Y$$

$$\theta_{\epsilon f} = (3\theta_{DB}^2)^{1/2} = \sqrt{3}\theta_{DB}$$

LAUNCH DATE	TIME	CONE ANGLE	CLOCK ANGLE	ALPHAE	BETAE	GAMMAE
5/18/74	180.	23.2294049	281.5343018	0.07886450	-0.38644846	0.91893306
	190.	21.9195280	282.1515617	0.07857961	-0.36493983	0.92770910
	200.	20.5052531	282.6952591	0.07698211	-0.34172955	0.93664010
	210.	18.9982829	283.1813393	0.07423382	-0.31696290	0.94552835
	220.	17.4104259	283.6245689	0.07048239	-0.29079456	0.95418591
	230.	15.7534600	284.0395889	0.06586325	-0.26338847	0.96243885
	240.	14.0390500	284.4423599	0.06050150	-0.23491729	0.97013063
	250.	12.2790400	284.8523903	0.05451424	-0.20556743	0.97712345
	260.	10.4850740	285.2971916	0.04801071	-0.17553192	0.98330235
	270.	8.6689566	285.8216515	0.04109419	-0.14501501	0.98857570
	280.	6.8426313	286.5122490	0.03386271	-0.11422920	0.99287713
	290.	5.0184027	287.5720482	0.02640927	-0.08339393	0.99616665
	300.	3.2100890	289.6428795	0.01882382	-0.05273862	0.99843092
	310.	1.4399750	296.4494095	0.01119292	-0.02249929	0.99968420
	320.	0.4554972	63.0563812	0.00360218	0.00708691	0.99996840
	330.	2.0605439	96.1820097	-0.00387194	0.03574643	0.99935339
	340.	3.6811259	99.9914398	-0.01113937	0.06322984	0.99793682
	350.	5.2251834	101.4759903	-0.01811907	0.08924962	0.99584447
	360.	6.6712195	102.2888002	-0.02472591	0.11351001	0.99322913
	370.	7.9990939	102.8170700	-0.03087051	0.13569008	0.99027027
	380.	9.1873395	103.1998100	-0.03645864	0.15544469	0.98717158
	390.	10.2123179	103.5000000	-0.04138895	0.17239760	0.98415752
	400.	11.0480970	103.7511902	-0.04555226	0.18614019	0.98146667
	410.	11.6663690	103.9738302	-0.04882995	0.19622822	0.97934168
	420.	12.0369260	104.1822701	-0.05109428	0.20218592	0.97801341
	430.	12.1285580	104.3884096	-0.05220998	0.20351558	0.97767864
	450.	11.3572921	104.8446198	-0.05045226	0.19035399	0.98041824
	460.	10.4489599	105.1315298	-0.04734128	0.17507165	0.98341686
	470.	9.1806124	105.5035801	-0.04264668	0.15374182	0.98719031
	480.	7.5665675	106.0406303	-0.03638508	0.12655122	0.99129254
	490.	5.6469078	106.9470501	-0.02868169	0.09412469	0.99514718
	500.	3.4913742	108.9649696	-0.01979131	0.05759255	0.99814398
	510.	1.2113234	118.5346899	-0.01009838	0.01857209	0.99977653
	512.	0.7663739	127.3009100	-0.00810547	0.01063961	0.99991055
5/8/74	190.	22.7842009	281.4000511	0.07654511	-0.37962109	0.92196999
	200.	21.5677891	282.0775185	0.07691484	-0.35946508	0.92998332
	210.	20.2302711	282.6695518	0.07584196	-0.33737434	0.93831048
	220.	18.7864349	283.1952286	0.07351213	-0.31353900	0.94672555
	230.	17.2504520	283.6717606	0.07009190	-0.28814662	0.95501762
	240.	15.6359980	284.1156807	0.06573165	-0.26138669	0.96299343
	250.	13.9561690	284.5446587	0.06056833	-0.23345029	0.97048052
	260.	12.2240990	284.9798584	0.05472923	-0.20454042	0.97732693
	270.	10.4523970	285.4507294	0.04833151	-0.17486207	0.98340598
	280.	8.6537158	286.0049591	0.04148542	-0.14463002	0.98861576
	290.	6.8406449	286.7339096	0.03429446	-0.11406437	0.99288127
	300.	5.0262868	287.8513107	0.02685744	-0.08339469	0.99615461
	310.	3.2250165	290.0290413	0.01926793	-0.05285495	0.99841630
	320.	1.4605823	297.1129417	0.01161659	-0.02268820	0.99967510
	330.	0.4547759	59.8071599	0.00399174	0.00686047	0.99996850
	340.	2.0454200	95.6677103	-0.00352487	0.03551724	0.99936285
	350.	3.6664310	99.7618999	-0.01084257	0.06302171	0.99795326
	360.	5.2131029	101.3473597	-0.01787735	0.08908421	0.99586365
	370.	6.6627965	102.2111597	-0.02454119	0.11340070	0.99324620
	380.	7.9949766	102.7697802	-0.03074278	0.13564612	0.99028027
	390.	9.1877944	103.1722898	-0.03638576	0.15546982	0.98717031
	400.	10.2171770	103.4860897	-0.04136656	0.17248881	0.98414248
	410.	11.0568060	103.7469797	-0.04557403	0.18628845	0.98143753
	420.	11.6779360	103.9766598	-0.04888739	0.19641766	0.97930083
	430.	12.0498840	104.1902599	-0.05117669	0.20239322	0.97796622
	440.	12.1409780	104.4001904	-0.05230453	0.20371011	0.97763307
	460.	11.3610760	104.8612804	-0.05052422	0.19040190	0.98040523
	470.	10.4440440	105.1499596	-0.04737554	0.17497496	0.98343242
	480.	9.1640637	105.5240402	-0.04262527	0.15345185	0.98723635
	490.	7.5359566	106.0649300	-0.03629219	0.12602684	0.99136276
	500.	5.6007009	106.9826899	-0.02850581	0.09333924	0.99522620
	510.	3.4294651	109.0530996	-0.01952779	0.05654258	0.99820920
	520.	1.1372189	119.4306202	-0.00975216	0.01728569	0.99980303
5/28/74	170.	23.6464369	281.6071091	0.08069911	-0.39288939	0.91603798
	180.	22.2396491	282.1873817	0.07990075	-0.36995135	0.92560892
	190.	20.7464581	282.7011414	0.07788345	-0.34556521	0.93515713
	200.	19.1754050	283.1623802	0.07479418	-0.31983211	0.94451747
	210.	17.5357990	283.5844002	0.07076874	-0.29287272	0.95352890
	220.	15.8375590	283.9805717	0.06593314	-0.26482671	0.96203931
	230.	14.0906500	284.3658104	0.06040433	-0.23584419	0.96991177
	240.	12.3057990	284.7584190	0.05429325	-0.20609783	0.97702402
	250.	10.4936190	285.1844597	0.04770366	-0.17576757	0.98327520
	260.	8.6649756	285.6866798	0.04073389	-0.14504528	0.98858617
	270.	6.8310937	286.3474693	0.03347777	-0.11413426	0.99290111
	280.	5.0036862	287.3606911	0.02602511	-0.08324656	0.99618909
	290.	3.1957613	289.3398209	0.01846192	-0.05260186	0.99844489
	300.	1.4281811	295.8606987	0.01087140	-0.02242794	0.99968936
	310.	0.4463971	64.6492672	0.00333579	0.00704077	0.99996965
	320.	2.0504737	96.5279703	-0.00406774	0.03554789	0.99935970
	330.	3.6604108	100.1546402	-0.01125582	0.06284270	0.99795997
	340.	5.1917839	101.5700197	-0.01814908	0.08865104	0.99589738
	350.	6.6238067	102.3465405	-0.02466454	0.11268209	0.99332493
	360.	7.9369709	102.8524799	-0.03071551	0.13462410	0.99042057
	370.	9.1104349	103.2201004	-0.03621063	0.15414173	0.98738499
	380.	10.1212350	103.5094099	-0.04105174	0.17086936	0.98443812
	390.	10.9441630	103.7523699	-0.04513278	0.18440962	0.98181267
	400.	11.5517581	103.9685297	-0.04833881	0.19433126	0.97974421
	410.	11.9147320	104.1716805	-0.05054613	0.20017258	0.97845594
	420.	12.0030020	104.3732996	-0.05162435	0.20145347	0.97813671
	440.	11.2419950	104.8213100	-0.04987003	0.18846685	0.98081253
	450.	10.3492870	105.1036100	-0.04681012	0.17344275	0.98373087
	460.	9.1043134	105.4694500	-0.04220442	0.15250007	0.98740190
	470.	7.5210483	105.9960403	-0.03606955	0.12582241	0.99139684
	480.	5.6383295	106.8791199	-0.02852681	0.09401605	0.99516190
	490.	3.5232307	108.8159904	-0.01982048	0.05816912	0.99810997
	500.	1.2816636	117.4798403	-0.01032112	0.01984375	0.99974982

THE OPTIMUM HINGE AXIS LOCATION

PSI	THETA	PHI
79.219141	194.374969	79.179675

Fig. 7. Optimum hinge-axis location and resulting errors for three Jupiter flyby trajectories

TIME	HINGE ANGLE	ERROR	ALPHAF	BETAF	GAMMAF
180.	66.33592	0.2717629	0.08353495	-0.38562382	0.91886678
190.	67.67725	0.0776391	0.07991279	-0.36469804	0.92769028
200.	69.12344	0.0660367	0.07584922	-0.34194100	0.93665534
210.	70.66244	0.1684108	0.07134731	-0.31751636	0.94556485
220.	72.28234	0.2375717	0.06641451	-0.29159629	0.95423304
230.	73.97137	0.2806231	0.06106326	-0.26436106	0.96248870
240.	75.71792	0.3037710	0.05531136	-0.23599864	0.97017796
250.	77.51023	0.3123707	0.04918346	-0.20670944	0.97716537
260.	79.33674	0.3109938	0.04271003	-0.17669946	0.98333775
270.	81.18573	0.3034662	0.03592854	-0.14618459	0.98860466
280.	83.04540	0.2929295	0.02888317	-0.11538771	0.99290051
290.	84.90389	0.2818587	0.02162462	-0.08453712	0.99618564
300.	86.74890	0.2721265	0.01421100	-0.05386971	0.99844683
310.	88.56783	0.2650229	0.00670704	-0.02362717	0.99969833
320.	90.34795	0.2614665	-0.00081704	0.00594878	0.99998195
330.	92.07439	0.2610448	-0.00827764	0.03458558	0.99936745
340.	93.73314	0.2641393	-0.01559095	0.06203143	0.99795240
350.	95.30779	0.2697815	-0.02265948	0.08800263	0.99586247
360.	96.78093	0.2768170	-0.02937863	0.11220855	0.99325027
370.	98.13313	0.2836781	-0.03563274	0.13433582	0.99029499
380.	99.34298	0.2884564	-0.04129573	0.15404900	0.98719985
390.	100.38665	0.2889690	-0.04622997	0.17098338	0.98418872
400.	101.23793	0.2828540	-0.05028706	0.18474307	0.98149946
410.	101.86813	0.2677208	-0.05330874	0.19489681	0.97937398
420.	102.24659	0.2413631	-0.05513062	0.20098054	0.97804265
430.	102.34154	0.2020534	-0.05558856	0.20250522	0.97770216
450.	101.56055	0.0824627	-0.05183204	0.18994463	0.98042565
460.	100.63775	0.0049265	-0.04742371	0.17504720	0.98341723
470.	99.34769	0.0792451	-0.04131792	0.15412565	0.98718696
480.	97.70430	0.1634364	-0.03364060	0.12732875	0.99128990
490.	95.74753	0.2393914	-0.02465464	0.09523821	0.99514914
500.	93.54634	0.2984776	-0.01476046	0.05894474	0.99815211
510.	91.19697	0.3334633	-0.00446630	0.02003912	0.99978921
512.	90.71933	0.3371610	-0.00240860	0.01211390	0.99992371
190.	66.78791	0.3366919	0.08233030	-0.37859269	0.92189439
200.	68.03426	0.1170730	0.07892482	-0.35909849	0.92995652
210.	69.40274	0.0464214	0.07504570	-0.33752366	0.93832079
220.	70.87798	0.1639521	0.07070238	-0.31407965	0.94676033
230.	72.44554	0.2444504	0.06590666	-0.28897374	0.95506568
240.	74.09167	0.2957042	0.06067412	-0.26241362	0.96304606
250.	75.80332	0.3244923	0.05502452	-0.23460710	0.97053170
260.	77.56740	0.3366544	0.04898432	-0.20577249	0.97737311
270.	79.37139	0.3371614	0.04258502	-0.17612871	0.98344555
280.	81.20275	0.3301871	0.03586504	-0.14590307	0.98864856
290.	83.04908	0.3191623	0.02886904	-0.11532680	0.99290799
300.	84.89775	0.3068269	0.02164895	-0.08463913	0.99617644
310.	86.73613	0.2952563	0.01426299	-0.05408200	0.99843462
320.	88.55113	0.2859280	0.00677680	-0.02390479	0.99969126
330.	90.32951	0.2798468	-0.00073822	0.00564264	0.99998379
340.	92.05631	0.2769280	-0.00819871	0.03428607	0.99937842
350.	93.71690	0.2774580	-0.01551869	0.06176318	0.99797016
360.	95.29472	0.2806235	-0.02260029	0.08778735	0.99588282
370.	96.77184	0.2853187	-0.02933686	0.11205946	0.99326834
380.	98.12859	0.2900259	-0.03561160	0.13426165	0.99030581
390.	99.34321	0.2928680	-0.04129684	0.15405280	0.98719921
400.	100.39146	0.2916709	-0.04625283	0.17106131	0.98417411
410.	101.24674	0.2840726	-0.05032919	0.18488519	0.98147054
420.	101.87991	0.2676746	-0.05336536	0.19508632	0.97933316
430.	102.25983	0.2402556	-0.05519446	0.20119320	0.97799532
440.	102.35427	0.2000734	-0.05564995	0.20270950	0.97765634
460.	101.56450	0.0792939	-0.05185097	0.19000822	0.98041233
470.	100.63282	0.0014799	-0.04740025	0.17496746	0.98343255
480.	99.33088	0.0826859	-0.04123880	0.15385228	0.98723290
490.	97.67311	0.1665101	-0.03349601	0.12681871	0.99136018
500.	95.70038	0.2416696	-0.02444027	0.09446276	0.99522833
510.	93.48303	0.2995022	-0.01447938	0.05789835	0.99821746
520.	91.11917	0.3328355	-0.00413030	0.01874852	0.99981569
170.	65.91216	0.2298249	0.08464957	-0.39219653	0.91597833
180.	67.35202	0.0525997	0.08080411	-0.36978840	0.92559560
190.	68.87824	0.0777370	0.07654966	-0.34581314	0.93517559
200.	70.48213	0.1697677	0.07188412	-0.32038844	0.94455487
210.	72.15439	0.2311043	0.06681128	-0.29365101	0.95357502
220.	73.88517	0.2684316	0.06134140	-0.26575569	0.96208686
230.	75.66461	0.2876079	0.05549014	-0.23686701	0.96995610
240.	77.48207	0.2936705	0.04928147	-0.20717087	0.97706271
250.	79.32702	0.2909197	0.04274505	-0.17685944	0.98330747
260.	81.18868	0.2829463	0.03591751	-0.14613570	0.98861229
270.	83.05596	0.2726743	0.02884256	-0.11521271	0.99292201
280.	84.91750	0.2624078	0.02157069	-0.08431102	0.99620597
290.	86.76168	0.2538509	0.01415892	-0.05365714	0.99845901
300.	88.57643	0.2481527	0.00667109	-0.02348412	0.99970194
310.	90.34921	0.2460262	-0.00082246	0.00596981	0.99998183
320.	92.06610	0.2472975	-0.00824145	0.03444826	0.99937248
330.	93.71307	0.2519278	-0.01550166	0.06169992	0.99797434
340.	95.27448	0.2590322	-0.02250871	0.08745416	0.99591421
350.	96.73318	0.2674072	-0.02915928	0.11142551	0.99334489
360.	98.07030	0.2754804	-0.03534037	0.13330984	0.99044410
370.	99.26503	0.2813576	-0.04092901	0.15278149	0.98741207
380.	100.29421	0.2828800	-0.04579114	0.16948628	0.98446815
390.	101.13235	0.2777359	-0.04978235	0.18303929	0.98184434
400.	101.75163	0.2635923	-0.05274900	0.19302195	0.97977550
410.	102.12232	0.2383041	-0.05453178	0.19898403	0.97848435
420.	102.21379	0.2002097	-0.05497250	0.20045367	0.97815966
440.	101.44312	0.0836151	-0.05126925	0.18805229	0.98081996
450.	100.53620	0.0077971	-0.04694064	0.17340424	0.98373143
460.	99.26994	0.0746785	-0.04095213	0.15286143	0.98739874
470.	97.65796	0.1574305	-0.03342579	0.12657094	0.99139421
480.	95.73897	0.2324975	-0.02461569	0.09509736	0.99516357
490.	93.57935	0.2915169	-0.01490709	0.05949029	0.99811756
500.	91.27212	0.3274387	-0.00479115	0.02128565	0.99976194

AVERAGE POINTING ERROR = 0.2387016 DEGREES

Fig. 7 (contd)

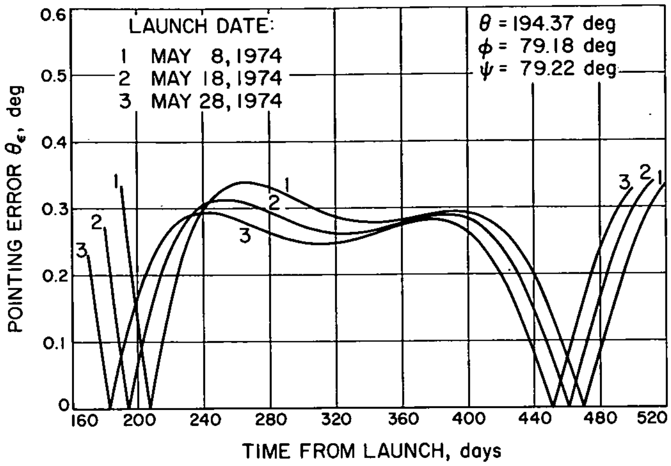


Fig. 8. Pointing error vs time for three Jupiter flyby trajectories

This is the maximum angular perturbation of the antenna feed vector due to symmetrical deadbands of identical width, $2\theta_{DB}$.

B. Effects of Structural Misalignments

Additional sources of significant antenna pointing errors are the various structural misalignments which occur, particularly those associated with actually placing the antenna hinge axis in the computed optimal position with respect to optical sensor null directions. Antenna electrical–mechanical boresight misalignments are of interest, as are the effects of in-flight thermal gradient distortions of both the antenna and spacecraft structure.

This type of error is not, in general, easy to predict. However, assuming that at least upper bounds can be estimated, it is convenient for purposes of this study to lump structural misalignments into specifications on the values of θ, ϕ, and ψ. Thus, for example, the computed optimum values of θ and ϕ may actually vary by as much as $\pm 1/2$ deg, while ψ–optimum may be held to within $\pm 1/4$ deg. Figure 10 compares the Jupiter mission pointing-error level, including the misalignment estimates specified above, and attitude-control error against the ideal pointing errors due to geometry only. Like the computations involved in reaching a worst-case attitude effect on pointing, the effect of θ, ϕ, and ψ misalignment is conservatively taken as that combination of the eight possible combinations of θ, ϕ, and ψ extremes, which results in a maximum pointing error at each instant of time.

C. Hinge-Angle Control System Errors

Pointing-error sources are also present in the hinge-angle control system in the form of (1) feedback potentiometer linearity and null offsets (if a potentiometer is used), (2) drive-train hysteresis, (3) built-in deadzones, and (4) hinge-angle stored-program approximations. These may result in a total deviation on the order of ½ to 1 deg from the optimum hinge-angle value.

Evaluation of the effects of hinge-angle errors due to control-system components and hinge-program approximations can be made by an examination of the pointing geometry. Figure 11 pictures an antenna-beam cross-section superimposed on the track of a unit earth-pointing vector. In general, the antenna beam is elliptically shaped. Particularly, in the case of single-degree-of-freedom pointing, it is desirable from a gain standpoint to widen the beam in the direction of no positional control and to narrow it along the direction of hinge rotation.

The antenna's feed vector is pictured in Fig. 11 as rotating with the hinge angle along a path which reflects the presence of geometric errors, attitude perturbations, and mechanical misalignments. The angle θ_0, describing the relative position of the antenna-beam's major axis and the earth-pointing vector, is a parameter of interest for measuring possible losses in antenna gain as the earth moves off the beam's major axis.

A unit vector directed along the beam's minor axis may be developed as

$$\mathbf{v}_{MI} = \frac{\mathbf{h} \times \mathbf{f}}{|\mathbf{h} \times \mathbf{f}|} = \frac{1}{\sin\psi}[(\gamma_F \sin\phi \sin\theta - \beta_F \cos\phi)\,\mathbf{i} + (\alpha_F \cos\phi - \gamma_F \sin\phi \cos\theta)\,\mathbf{j} + (\beta_F \sin\phi \cos\theta - \alpha_F \sin\phi \sin\theta)\,\mathbf{k}]$$

A unit vector along the major axis is then

$$\mathbf{v}_{MA} = \mathbf{v}_{MI} \times \mathbf{f} = (\beta_{MI}\gamma_F - \beta_F\gamma_{MI})\,\mathbf{i} + (\gamma_{MI}\alpha_F - \alpha_{MI}\gamma_F)\,\mathbf{j} + (\alpha_{MI}\beta_F - \beta_{MI}\alpha_F)\,\mathbf{k}$$

where

$$\mathbf{v}_{MI} = \alpha_{MI}\,\mathbf{i} + \beta_{MI}\,\mathbf{j} + \gamma_{MI}\,\mathbf{k}$$

Thus,

$$\tan\theta_0 = \frac{|\mathbf{e}\cdot\mathbf{v}_{MI}|}{|\mathbf{e}\cdot\mathbf{v}_{MA}|}$$

The vector $\mathbf{e}$ equals the unit earth vector (in body-fixed coordinates).

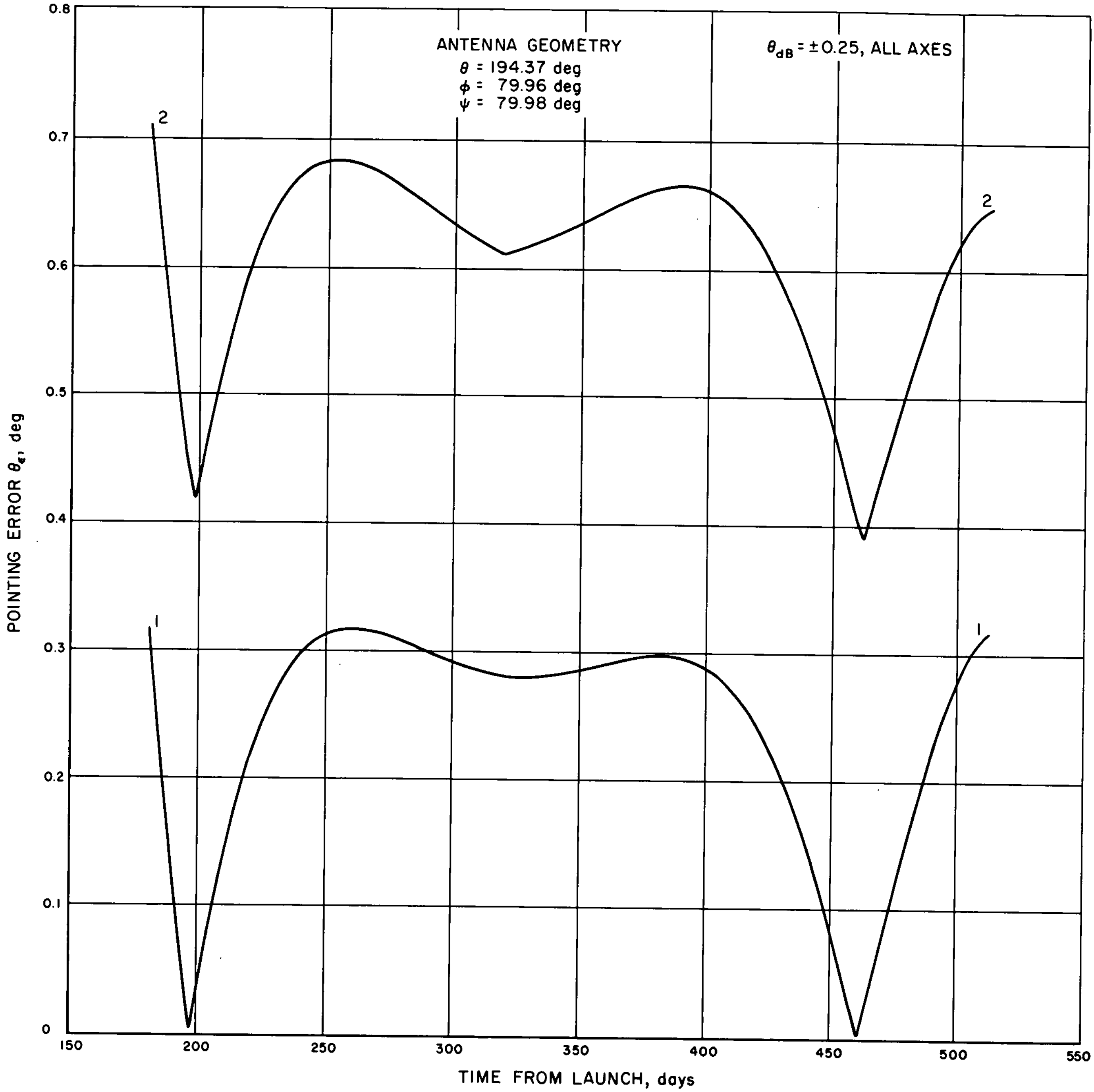


Fig. 9. Antenna-pointing error vs time: (1) ideal error for given antenna and trajectory geometry; (2) effect of worst-case attitude-control system errors

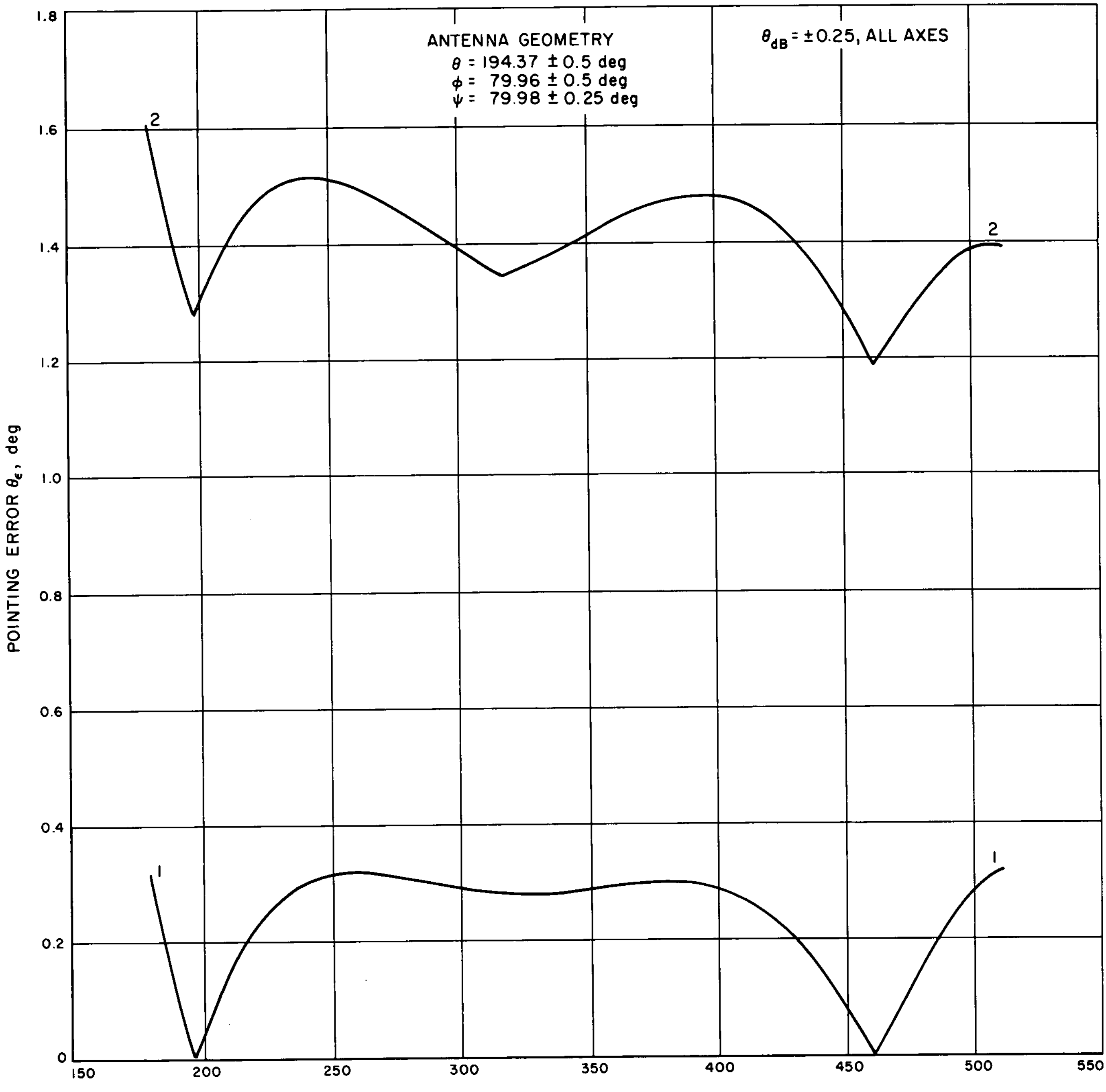


Fig. 10. Antenna-pointing error vs time: (1) ideal error for given antenna and trajectory geometry; (2) effect of worst-case attitude-control and mechanical misalignment errors

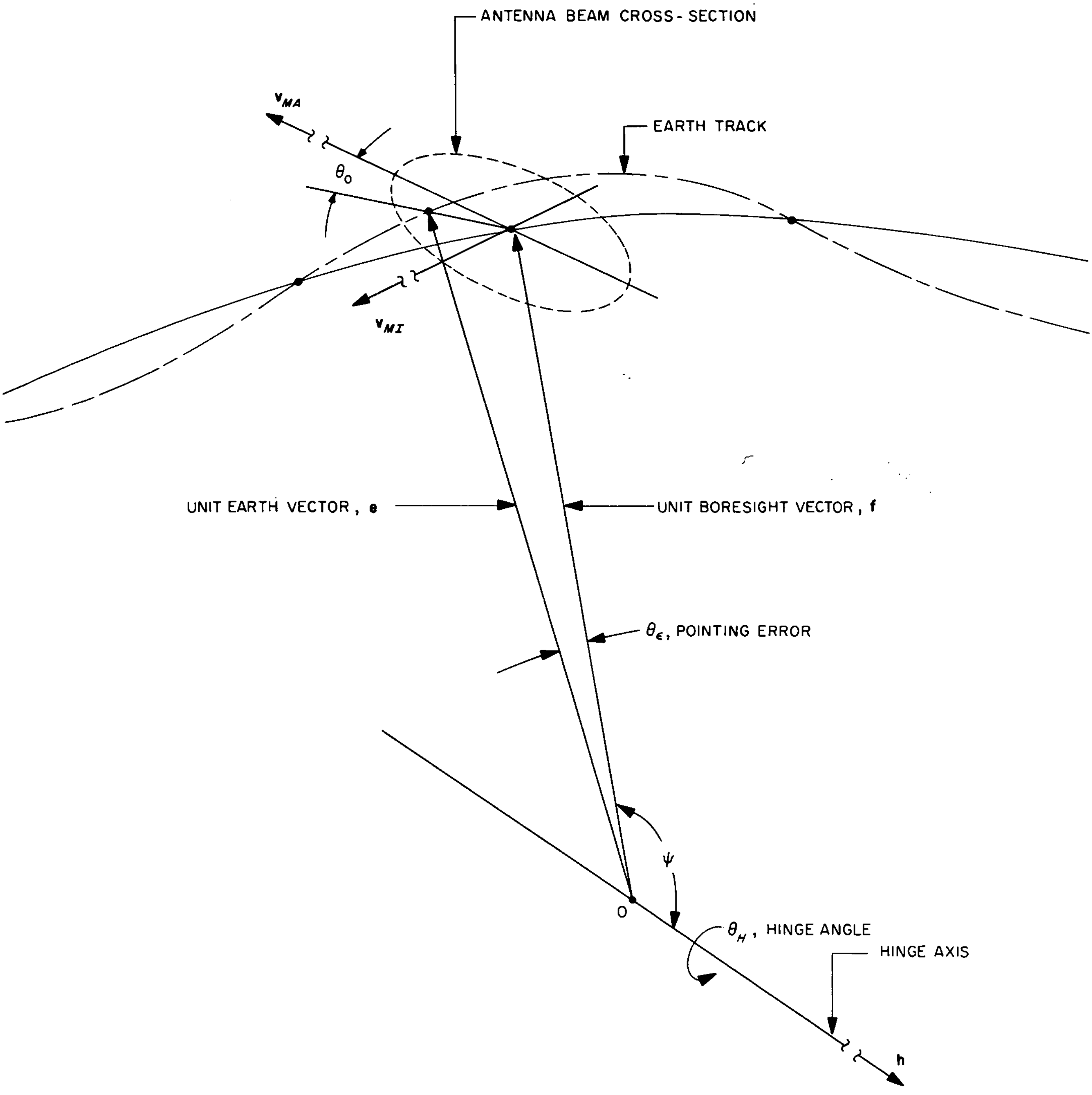


Fig. 11. Pointing vector and antenna beam geometry

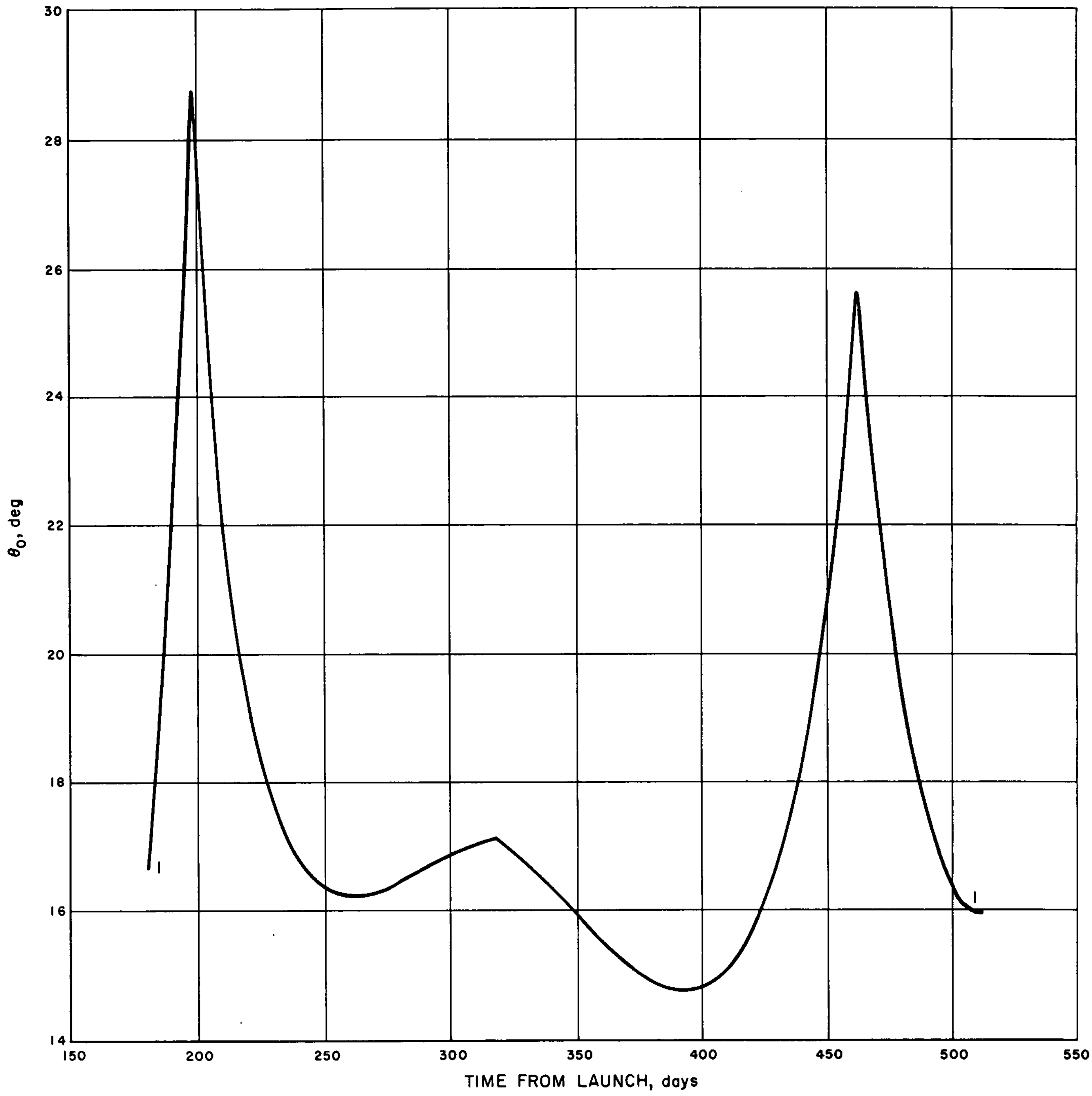


Fig. 12. Angular deviation of earth vector from antenna beam's major axis (corresponding to the worst-case pointing error)

In general, body-fixed components of e are a function of the attitude errors θ_P, θ_Y, θ_R and the inertial components of e:

$$\begin{Bmatrix} \alpha_E \\ \beta_E \\ \gamma_E \end{Bmatrix}_{\text{body}} \cong \begin{bmatrix} 1 & \theta_R & -\theta_Y \\ -\theta_R & 1 & \theta_P \\ \theta_Y & -\theta_P & 1 \end{bmatrix} \begin{Bmatrix} \alpha'_E \\ \beta'_E \\ \gamma'_E \end{Bmatrix}_{\text{inertial}} = [A] \begin{Bmatrix} \sin\theta_{co} \cdot \cos\theta_{CL} \\ \sin\theta_{co} \cdot \sin\theta_{CL} \\ \cos\theta_{co} \end{Bmatrix}$$

$$\mathbf{e} = \alpha_E \mathbf{i} + \beta_E \mathbf{j} + \gamma_E \mathbf{k} = \alpha'_E \hat{\mathbf{l}} + \beta'_E \mathbf{m} + \gamma'_E \mathbf{n}$$

For the Jupiter mission, under worst-case conditions of attitude perturbation and mechanical misalignments, a time history of θ_0 was calculated from the above relations and is plotted in Fig. 12. This computation of $\theta_0(t)$ was performed assuming that $\theta_H = \theta^*_H(t)$, i.e., the hinge-angle function was the ideal one associated with the optimum hinge-axis placement. Another $\theta_0(t)$ could be calculated based on a $\theta_H(t)$ which is a line segment fit to $\theta^*_H(t)$ (see Sect. V). Of course, $\theta^*_H(t)$ is no longer an ideal function in the sense of a minimum pointing error when attitude and misalignment errors have shifted the hinge-axis location.

The sensitivity of a pointing error to θ_H changes can be described by the relation

$$\theta_\epsilon = (\theta'^2_\epsilon + \Delta\theta^2_H)^{1/2}$$

where

θ_ϵ = total pointing error

θ'_ϵ = pointing error when $\theta_H = \theta^*_H$

$\Delta\theta_H$ = deviation of hinge angle from the optimum value

For example, for

$$\theta'_\epsilon = 2.0 \text{ deg}, \qquad \Delta\theta_H = 0.5 \text{ deg}$$

and

$$\theta_\epsilon = \sqrt{4.25} = 2.06 \text{ deg}$$

an increase of only 0.06 deg. The orthogonality of θ'_ϵ and $\Delta\theta_H$ therefore minimizes the effect of hinge-angle error when the earth vector is well off the path of feed-vector rotation, i.e., θ'_ϵ is large. For single-degree-of-freedom pointing, θ'_ϵ tends to be substantially larger than possible hinge-angle perturbations, except at those few points where the earth track and the feed-vector rotational track happen to cross.

In general, the use of an elliptical antenna beam to minimize the effects of feed-vector perturbations perpendicular to the nominal rotational track tends to restrict allowable hinge-rotation errors, since the beam is narrowed in that direction. A circular beam, on the other hand, which is capable of handling feed-axis movements and geometric errors which predominate in a direction perpendicular to hinge rotation, tends to allow greater hinge-angle error and, thus, a more coarse approximation to the ideal hinge function, $\theta^*_H(t)$. The result is a trade-off between possible savings in circuitry to generate a hinge-angle function vs the increased power necessary to provide a circular beam of diameter equal to that of the major axis of an elliptical beam.

V. Hinge-Angle Approximation

The optimum hinge-angle function $\theta^*_H(t)$, corresponding to the optimum placement of the antenna hinge axis, must be stored on board the spacecraft in suitable form for use as the input to the antenna's control system. A function approximation technique, which is particularly well suited for implementation by a spacecraft central computer and sequencer (CC&S) that is timer-oriented, takes the form of a series of connected line segments, as shown in Fig. 13. The time function is then represented by a series of "breakpoint" times and their associated slope values. An n-stage binary counter can be preprogrammed to divide a basic clock frequency into the required pulse frequency (slope) at the appropriate time

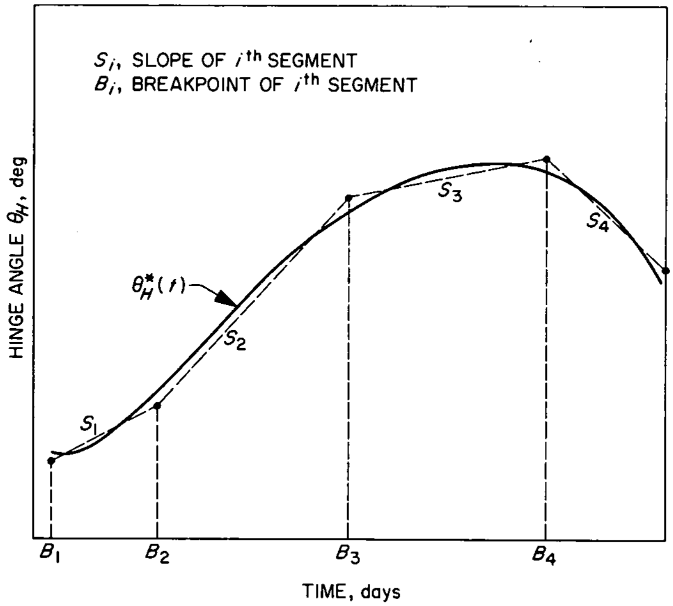


Fig. 13. Line segment approximation of an optimum hinge-angle function

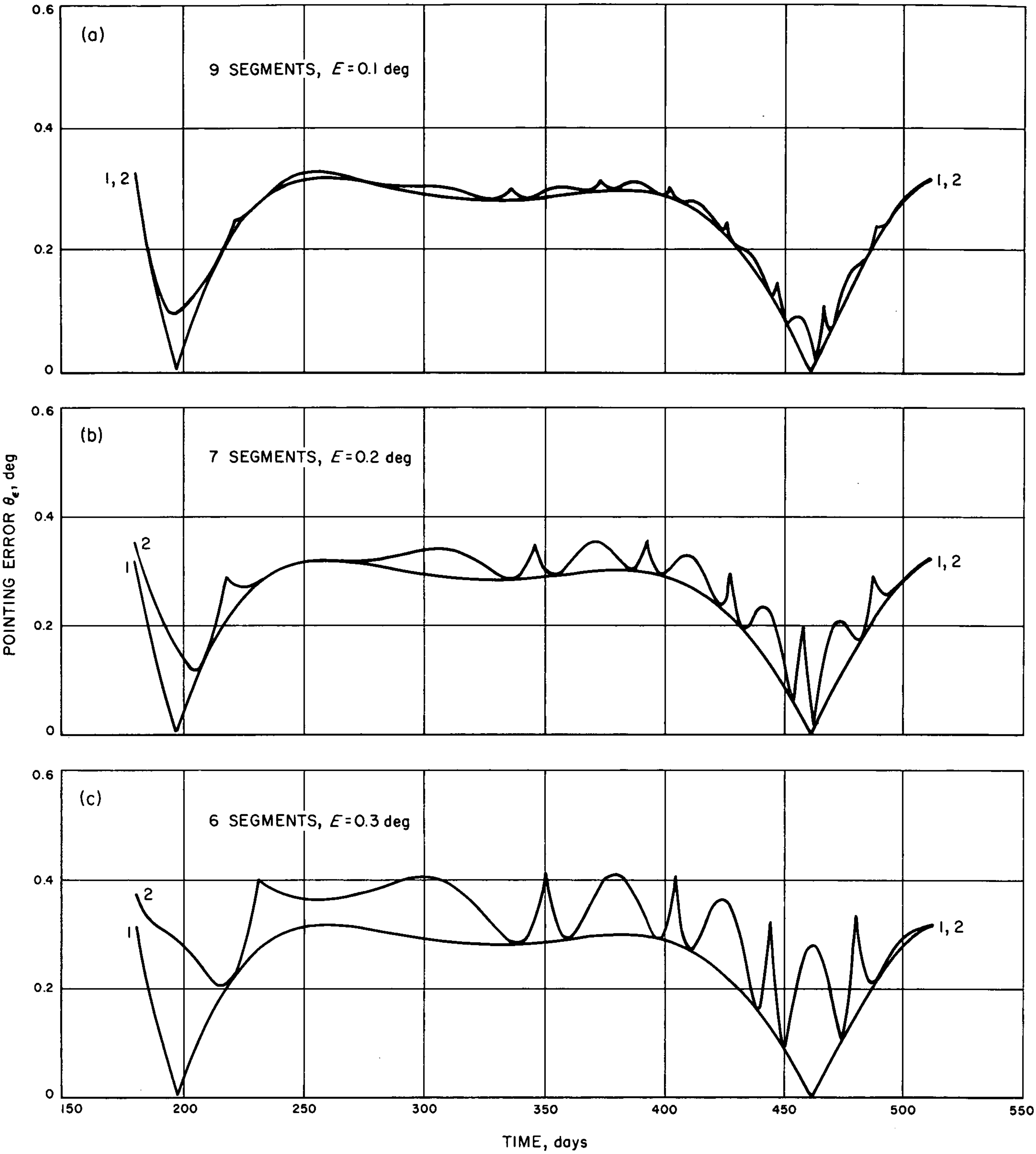


Fig. 14. Pointing error vs time: (1) ideal error; (2) effect of line-segment hinge-angle approximation

The algorithm used to obtain a "best" fit to $\theta^*_H(t)$ embodies the following basic steps:

(1) Beginning at t_1, the start of the time period of interest, the starting ordinate of the first line segment is taken as $\theta^*_H(t) - E$, where E is the allowable deviation from the $\theta^*_H(t)$ function.

(2) The endpoint of the first segment is placed at $\theta^*_H(t_i) \pm E$ such that t_i is as far from t_1 as possible. However, no portion of the line must fall outside $\theta^*_H \pm E$ at any intermediate time point. The endpoint of the first segment then becomes the starting point of the second segment.

(3) Step 2 is repeated for each successive line segment up to the last segment whose endpoint is arbitrarily placed at $\theta^*_H(t_n)$, t_n being the last day of interest.

(4) Using the slope and starting time for each segment, the approximating hinge-angle function is reconstructed, knowing that the first segment begins at $\theta^*_H(t_1) - E$. The pointing error is then computed at each interval of the approximate curve, and the maximum error is recorded.

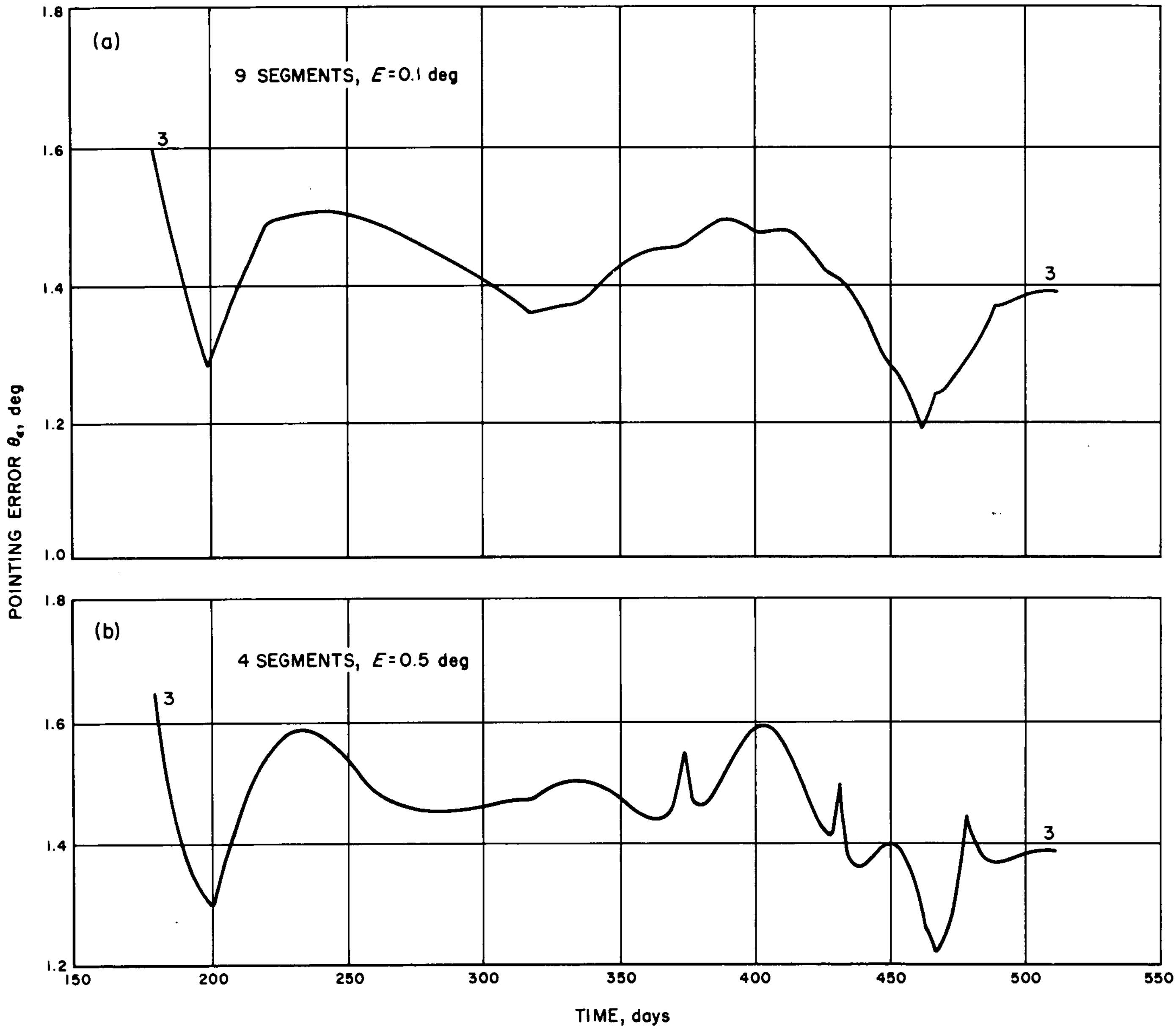


Fig. 15. Worst-case pointing error resulting from two hinge-angle programs

(breakpoint). The derived pulses may be applied directly to the antenna's "stepper" motor, or they may be accumulated in a "command position" register for comparison with a true-position feedback signal, the error being used to drive the antenna motor.

The problem of determining a "best" line segment fit to $\theta_H^*(t)$ requires some attention. Obviously, it is desirable that the number of line segments needed to approximate $\theta_H^*(t)$ should be kept to a minimum. Assuming one specifies a maximum allowable deviation of the approximating curve from $\theta_H^*(t)$, an infinite number of line segment-type curves may be fitted within these boundaries. From among those curves with the least number of line segments, it is further desirable that *the* curve which provides the smallest maximum pointing error be chosen for use in the pointing system. An alternate criterion for choosing a best approximation might emphasize the minimization of the average pointing error rather than the maximum.

To simplify the process of fitting the connected line segments to $\theta_H^*(t)$, segments were required to begin and end on the maximum deviation boundary line, except for the start of the first segment and end of the last segment. Further, since the data used are available only at discrete time intervals, each line segment is required to span a time interval of an integral number of days.

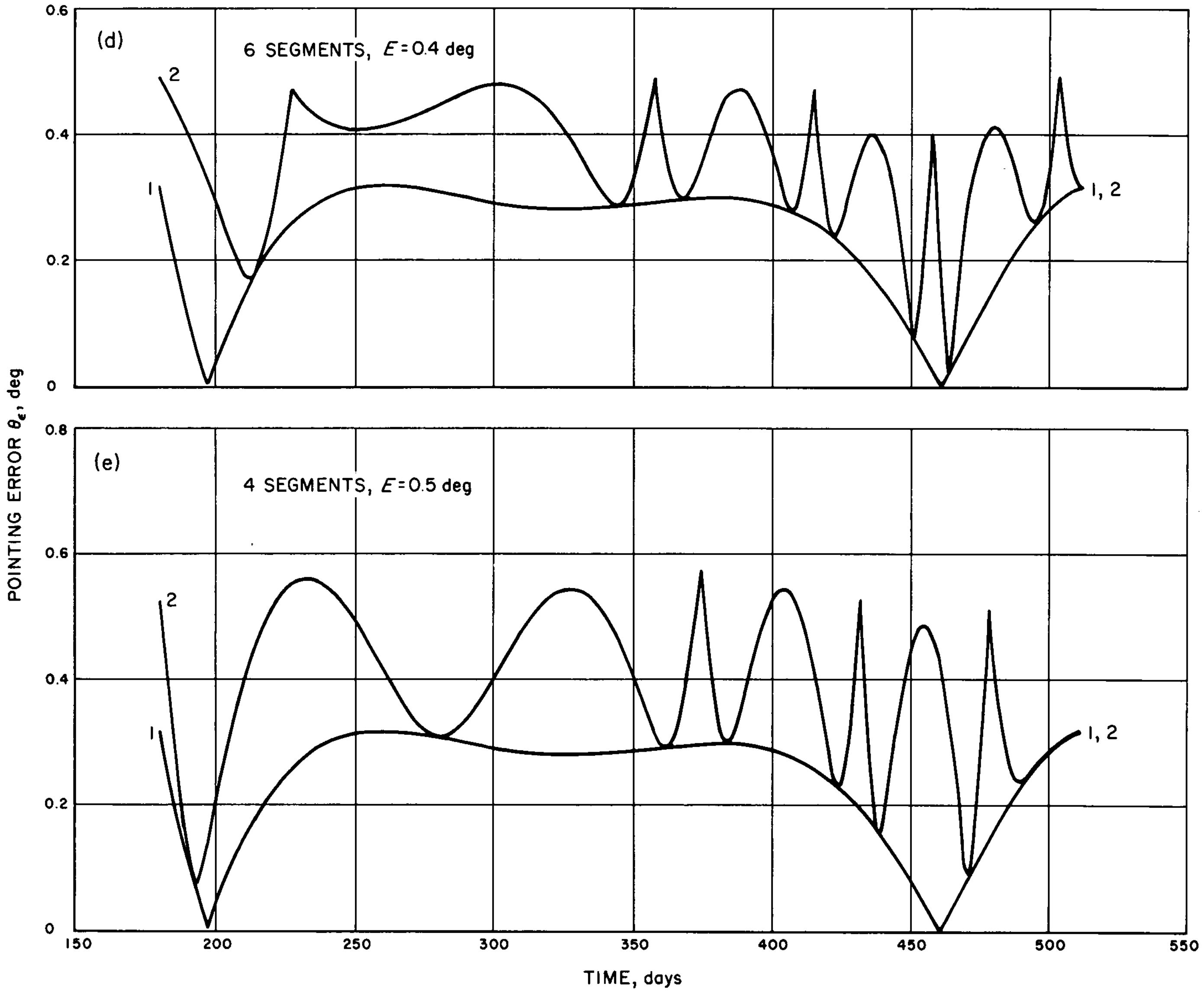


Fig. 14 (contd)

(5) A new approximating curve is then generated by beginning the first line segment at $\theta^*_H(t_1) - E + \Delta\theta$, where $\Delta\theta$ is a positive, predetermined angle increment. Steps 2, 3, and 4 are repeated for the new curve, and the resulting maximum pointing error is compared to that of the previous curve(s), as well as the number of segments required.

(6) Step 5 is repeated until the first segment's starting value:

$$\theta^*_H(t_1) - E + n\,\Delta\theta > \theta^*_H(t_1) + E$$

or

$$n\,\Delta\theta > 2E \cdot \left(\frac{2E}{\Delta\theta} + 1 = \text{number of iterations}\right)$$

(7) Of all the curves generated by changing the starting value (within the maximum deviation boundary, $\theta^*_H \pm E$), the curve which, out of the group of curves using the least number of segments, results in the smallest maximum pointing error is saved as the "best."

(8) Finally, the value of deviation, E, may be increased or reduced to see the effect on a pointing error of coarser or finer fits to θ^*_H.

Figure 14 pictures the pointing error resulting from line-segment approximations to the optimal hinge-angle function for the Jupiter mission. The effect of using a four-segment pointing program as opposed to a nine-segment program can be observed in comparing the worst-case pointing errors for the two programs in Fig. 15. The four-segment fit apparently raises the total error less than 0.1 deg above that of the nine-segment fit. It will be remembered that the total worst-case error reflects the addition of worst combinations of attitude drifts and hinge-axis misalignments. It is clear that, in this case, a rather coarse hinge-angle approximation can be used without seriously affecting the overall pointing-system capability.

Appendix A
Antenna-Pointing Subroutine Descriptions

I. ANTENA

ANTENA is the main program for single-degree-of-freedom antenna-pointing computations. It reads the various input data, depending on the desired options, and computes the direction cosines, ALPHA, BETA, and GAMMA of successive earth-pointing vectors, storing these computations in the labeled common region EARTH for use by other subroutines.

ANTENA calls the major subroutine SLOPP which in turn calls other subroutines.

A. Input:

1. ***NOFIT, NOPT, MULTRJ***

   Format (3I1)

   NOFIT —If NOFIT = 1, no hinge-angle curve fitting will be performed and pointing-error plots will be based on the ideal hinge-angle function, θ^*_H.

   NOPT —If NOPT = 1, no determination of an optimum hinge-axis location will be made. Values for ϕ, θ, and ψ must be supplied (See Appendix B).

   MULTRJ—If MULTRJ = 1, data from several trajectories may be input at once (up to 100 data points). An optimum hinge-axis location will be determined. However, no curve fits or error plots will be generated.

2. ***JT, (T (I), THETCO (I), THETCL (I), I = 1, JT)***

   Format (I3/(3F20.16))

   JT —Number of time points for which data are given

   T —Time, days

   THETCO—Earth-pointing vector cone angle, deg

   THETCL—Earth-pointing vector clock angle, deg

3. ***DP, DY, DR, DPHI, DTHETA, DPSI***

   Format (3F20.16)

   DP, DY, DR—Pitch, yaw, and roll *half* deadband size in the typical "bang-bang" type attitude-control system, deg

   DPHI, DTHETA, DPSI—Misalignment specifications for the hinge-axis placement based on ϕ, θ, and ψ, deg

4. ***E, EMAX, DELE, DELY (Input only when NOFIT $\neq$ 1)***

   Format (3F20.16)

   E —Starting value of maximum deviation allowed in line-segment fitting, deg

   EMAX—Maximum value of E to be used, deg

   DELE—Incremental value of E, deg

   DELY—Increment for changing starting value of first line segment, deg

5. ***DELTA1, DELTA2 (Input only when NOPT $\neq$ 1)***

   Format (3F20.16)

   DELTA1—Initial incremental value for hinge-axis optimization, deg

   DELTA2—Smallest angular increment for hinge-axis optimization routine, deg

6. ***PHI, THETA, PSI (input only when NOPT = 1)***

   Format (3F20.16)

   PHI, THETA, PSI—Antenna hinge-axis location, deg

B. Output:

1. ***Table of earth cone and clock angles and direction cosines vs time.***

2. ***The optimum (or given) axis location.***

3. ***Table of best hinge angle, pointing error, and antenna feed-vector direction cosines vs time.***

4. ***Tables of hinge-angle breakpoints and slopes for a line-segment hinge program at each value of E (NOFIT $\neq$ 1).***

5. ***Plots of antenna pointing error based on (1) an ideal hinge-angle function, (2) a line-segment approximation (NOFIT $\neq$ 1), and (3) a total worst case, including spacecraft attitude drifts and hinge-axis misalignments.***

6. *A plot of θ_0, angular deviation of earth from the antenna beam's major lobe axis, under worst-case pointing-error conditions (for each value of E).*

II. SUBROUTINE SLOPP (JT, FJ, DELTA1, DELTA2)

JT—Number of given data points.

FJ—Floating point value of JT.

DELTA1, DELTA2—Defined in ANTENA and used by SUBROUTINE OPTLOC, rad

SLOPP is the major subroutine of the program. It calls SUBROUTINES: INIT, OPTLOC, PTERR, INTRPO, SLPFIT, and ERPLTI.

III. SUBROUTINE INIT (L)

L—Dimension of the given data.

The routine INIT is a preliminary step to optimum location of the antenna hinge axis. Trial locations of the hinge axis are made at 5-deg intervals throughout the unit sphere. The entire table of earth-pointing vectors is projected on each trial position of the hinge axis in a search for a minimum "spread." The resulting position is used as the starting point for SUBROUTINE OPTLOC.

IV. SUBROUTINE OPTLOC (THET1, THET2, DELTA, DELLIM, JT, THET3)

THET1—Clock angle of the hinge axis, rad

THET2—Cone angle of the hinge axis, rad

DELTA—Initial increment for THET1 and THET2, rad

DELLIM—Smallest value used for DELTA. (The angular resolution of the optimum hinge-axis location), rad

JT—Dimension of the data

THET3—Optimum value of angle between antenna feed vector and the hinge axis, rad

OPTLOC, as explained in Sect. III, systematically searches the THET1–THET2 plane for a best location of the antenna's rotational or hinge axis. The iterative process begins with a search step size of DELTA which is successively halved until it becomes less than DELLIM.

V. SUBROUTINE SLPFIT (JT, EE, DELTAH, YSB)

JT—Dimension of the given data

EE—Maximum deviation of the fitted curve from $\theta^*_H(t)$, deg

DELTAH—Increment for changing initial value of the line-segment hinge-angle function, deg

YSB—Initial value of the line-segment approximation, deg

SLPFIT fits a curve consisting of connected line segments to the ideal hinge-angle function, $\theta^*_H(t)$, which is stored in the labeled common HINGE2. (See Sect. V for description of algorithm.) The best fit is stored in the labeled common HTRIAL.

VI. SUBROUTINE INTRPO (IXX, JT, Y, YY)

IXX—Interval at which data is desired, days

JT —Dimension of input data.

Y —Given function

YY —Desired function

INTRPO uses a 4-point Lagrange interpolation formula to obtain input data values at intervals of IXX days. Typically, input earth cone and clock angles are supplied at 5- or 10-day intervals.

VII. SUBROUTINE PTERR (XTHETA, XALPHA, XBETA, XGAMMA, X AVGERR, J, FJ, YMAX, L)

XTHETA—Hinge angle, deg

XALPHA, XBETA, XGAMMA—Direction cosines of the earth-pointing errors

X—Pointing error, deg

AVGERR—Average pointing error, deg

XMAX—Maximum pointing error, deg

J, FJ—Fixed and floating point dimension of input data

L—If L = 0, the program writes a table of XTHETA, X, and feed-vector direction cosines at each time.

If L = 1, no table is written.

As described in Sect. II, PTERR determines pointing error due to constraints imposed by the single degree of rotational freedom. Based on a hinge-axis location determined by ϕ, θ, and ψ, feed-vector direction cosines are computed knowing the hinge-angle function. The earth-vector–feed-vector cross-product is obtained, and the pointing-error angle is given by the inverse sine of this cross-product.

VIII. SUBROUTINE WORSTE (THETH, JT)

THETH—Hinge angle, deg

JT —Dimension of the given data

This subroutine calculates antenna-pointing error due not only to implicit single degree-of-freedom constraints, but to vehicle attitude drifts, antenna hinge-axis misalignments, and approximations used in stored hinge-angle programs. The worst combination of attitude drifts and hinge-axis misalignment is obtained at each time point. The resulting error is stored in the labeled common area WCASE, along with the corresponding angular deviation of the earth direction from the antenna beam's major axis, θ_0.

IX. SUBROUTINE ERPLTI (JT)

ERPLTI is a routine which appropriately collects and prepares data which are to be plotted by an SC-4020 plotter using a graphical output routine called KDPLOT. If a line-segment fit to the ideal hinge-angle function $\theta_H^*(t)$ has been called for (NOFIT $\neq 1$), ERPLTI generates the approximate hinge program, calls PTERR to calculate the corresponding pointing error (no additional error sources), and calls WORSTE for the worst-case table of pointing errors along with $\theta_0(t)$. KDPLOT is called to plot three superimposed pointing-error curves corresponding to (1) the ideal hinge-angle function, (2) the approximated hinge function, and (3) the total worst-case error. $\theta_0(t)$ for worst-case pointing is plotted separately. If no curve fitting is desired (NOFIT = 1), ERPLTI is called only once and the pointing error curve (2) is eliminated. Otherwise, ERPLTI is called for each discrete value of allowable hinge-angle deviation E.

Appendix B
Antenna-Pointing-Program Data-Input Information

As described in Appendix A, the main program, ANTENA, reads the input data supplied by the user according to the values specified for the three "option indicators," NOFIT, NOPT, MULTRJ. Specifically, the sequence of data to be supplied to the program must be as follows.

I. For MULTRJ = 0 (A single trajectory is to be examined):

Case (a) NOFIT = 0

NOPT = 0

Data:

Card 1–NOFIT, NOPT, MULTRJ–Format (3I1)

Card 2–JT (JT ≤ 100) –Format (I3)

Card 3–T(1), THETCO(1), THETCL(1)–Format (3F20.16)

" " " " "

" " " " "

" " " " "

Card JT + 2–T(JT), THETCO(JT), THETCL(JT)–Format (3F20.16)

Card JT + 3–DP, DY, DR –Format (3F20.16)

Card JT + 4–DPHI, DTHETA, DPSI –Format (3F20.16)

Card JT + 5–E, EMAX, DELE –Format (3F20.16)

Card JT + 6–DELH –Format (3F20.16)

Card JT + 7–DELTA1, DELTA2 –Format (3F20.16)

Case (b) NOFIT = 0

NOPT = 1

Data:

Card 1–NOFIT, NOPT, MULTRJ–Format (3I1)

Card 2–JT –Format (I3)

Card 3–T(1), THETCO(1), THETCL(1)–Format (3F20.16)

" " " " "

" " " " "

" " " " "

Card JT + 2–T(JT), THETCO(JT), THETCL(JT)–Format (3F20.16)

Card JT + 3–DP, DY, DR –Format (3F20.16)

Card JT + 4–DPHI, DTHETA, DPSI –Format (3F20.16)

Card JT + 5–E, EMAX, DELE –Format (3F20.16)

Card JT + 6–DELH –Format (3F20.16)

Card JT + 7–PHI, THETA, PSI –Format (3F20.16)

Case (c) NOFIT = 1

NOPT = 0

Data: Card 1—NOFIT, NOPT, MULTRJ—Format (3I1)

Card 2—JT —Format (I3)

Card 3—T(1), THETCO(1), THETCL(1)—Format (3F20.16)

" " " " "

" " " " "

Card JT + 2—T(JT), THETCO(JT), THETCL(JT)—Format (3F20.16)

Card JT + 3—DP, DY, DR —Format (3F20.16)

Card JT + 4—DPHI, DTHETA, DPSI —Format (3F20.16)

Card JT + 5—DELTA1, DELTA2 —Format (3F20.16)

Case (d) NOFIT = 1

NOPT = 1

Data: Card 1—NOFIT, NOPT, MULTRJ—Format (3I1)

Card 2—JT —Format (I3)

Card 3—T(1), THETCO(1), THETCL(1)—Format (3F20.16)

" " " " "

" " " " "

Card JT + 2—T(JT), THETCO(JT), THETCL(JT)—Format (3F20.16)

Card JT + 3—DP, DY, DR —Format (3F20.16)

Card JT + 4—DPHI, DTHETA, DPSI —Format (3F20.16)

Card JT + 5—PHI, THETA, PSI —Format (3F20.16)

II. For MULTRJ = 1 (Hinge-axis location is to be optimized for several trajectories):

Data: Card 1—NOFIT, NOPT, MULTRJ—Format (3I1)

Card 2—JT ($JT \leq 100$) —Format (I3)

Card 3—T(1), THETCO(1), THETCL(1)—Format (3F20.16)

" " " " "

" " " " "

Card JT + 2—T(JT), THETCO(JT), THETCL(JT)—Format (3F20.16)

Card JT + 3—DELTA1, DELTA2 —Format (3F20.16)

Appendix C
Fortran IV Program Listing

```
                    JPL,J265000,44808-0,28572,AF IC      IBJOB V13MJO   7
         ANTENA      -  EFN    SCURCE STATEMENT  -  IFN(S)  -

C
C        ANTENNA POINTING PROGRAM
C
1        CIMENSICN THETCO(100),THETCL(100)                              ANTEN
2        COMMCN/EARTH/ALPHA(1CO),BETA(100),GAMMA(100)                    ANTEN
3        COMMON/TIME/T(1CO)                                             ANTEN
4        COMMON/CONST/E,EMAX,CELE,DELY,NCFIT,NOPT,MULTRJ                ANTEN
401      COMMCN/HINGAX/PHI,THETA,PSI                                    ANTEN
402      COMMON/DRIFT/DP,DY,DR,DPHI,DTHETA,DPSI                         ANTEN
5        FORMAT(I3/(3F20.16))                                           ANTEN
501      FORMAT (3I1)                                                   ANTEN
502      FORMAT(////10X,93HERROR - NUMBER OF LINE SEGMENT SEARCH INCREMENTSANTEN
503     1 EXCEEDS 1C1 (DECREASE EMAX OR INCREASE DELY))                 ANTEN
6        FORMAT (1H1, 8X,4HTIME,13X,10HCONE ANGLE,9X,11HCLOCK ANGLE,12X, ANTEN
7       16HALPHAE,15X,5HBETAE,14X,6HGAMMAE//(7X,F5.0,11X,F12.7,7X,      ANTEN
8       2F13.7, 9X,F11.8, 9X,F11.8, 9X,F11.8))                          ANTEN
9        FORMAT (3F20.16)                                               ANTEN
901      READ (5,501) NOFIT,NCPT,MULTRJ                                 ANTEN
10       READ (5,5) JT,(T(I),THETCO(I),THETCL(I),I=1,JT)                ANTEN
11       CO 15 I=1,JT                                                   ANTEN
C
C        COMPUTE COMPONENTS OF EARTH POINTING VECTOR
C
12       ALPHA(I) =SIN(THETCO(I)*.01745329)*COS(THETCL(I)*.01745329)    ANTEN
13       BETA(I) =SIN(THETCO(I)*.01745329)*SIN(THETCL(I)*.01745329)     ANTEN
14       GAMMA(I) =COS(THETCO(I)*.01745329)                             ANTEN
15       CONTINUE                                                       ANTEN
16       WRITE (6,6) (T(I),THETCO(I),THETCL(I),ALPHA(I),BETA(I),GAMMA(I), ANTEN
17      1I=1,JT)                                                        ANTEN
170      IF(MULTRJ.EQ.1) GO TC 1802                                     ANTEN
1001     READ (5,9) DP,DY,DR,CPHI,DTHETA,DPSI                           ANTEN
1002     DP = DP*.C1745329                                              ANTEN
1003     DY = DY*.C1745329                                              ANTEN
1004     DR = DR*.01745329                                              ANTEN
1005     DPHI = DPHI*.01745329                                          ANTEN
1006     DTHETA = DTHETA*.01745329                                      ANTEN
1007     DPSI = DPSI*.01745329                                          ANTEN
1701     IF(NOFIT.EQ.1) GO TO 18                                        ANTEN
1702     READ (5,9) E,EMAX,DELE,DELY                                    ANTEN
1703     IF(((2.*EMAX/DELY)+1.).GT.101.) GO TC 2002                     ANTEN
18       IF(NOPT.EQ.1) GO TO 1806                                       ANTEN
1802     READ (5,9) DELTA1,DELTA2                                       ANTEN
1803     DELTA1 = DELTA1*.01745329                                      ANTEN
1804     DELTA2 = DELTA2*.01745329                                      ANTEN
1805     GO TO 19                                                       ANTEN
1806     READ (5,9) PHI,THETA,PSI                                       ANTEN
1807     PHI = PHI*.01745329                                            ANTEN
1808     THETA = THETA*.C1745329                                        ANTEN
1809     PSI = PSI*.01745329                                            ANTEN
19       FJ=JT                                                          ANTEN
20       CALL SLOPP(JT,FJ,DELTA1,DELTA2)                                ANTEN
2001     GO TO 21                                                       ANTEN
2002     WRITE (6,502)                                                  ANTEN
21       STOP                                                           ANTEN
         END                                                            ANTEN
```

```
          SLP1        -  EFN   SOURCE STATEMENT  -  IFN(S)  -

 1        SUBROUTINE SLOPP(JT,AJ,DEL1,DEL2)                              SLP1
 C
 C        THIS IS A MAJOR SUBRCUTINE
 C
 2        COMMON/HNGLOC/A1,A2,A3,B1,B2,B3,G1,G2                          SLP1
 3        COMMON/START/THECL,THECO                                       SLP1
 4        COMMON/CONST/EE,EEMAX,DELTAE,DELTAH,NOFIT,NOPT,MULTRJ          SLP1
 5        COMMON/EARTH/ALPHAE(100),BETAE(100),GAMMAE(100)                SLP1
 6        COMMON/TIME/T(100)                                             SLP1
 7        COMMON/HINGE1/THETAH(100)                                      SLP1
 8        COMMON/HINGE2/YH(450)                                          SLP1
 9        COMMON/ERROR1/TRUERR(100)                                      SLP1
 10       COMMON/ERROR2/TR(450)                                          SLP1
 11       COMMON/EARTH2/YALPHA(450),YBETA(450),YGAMMA(450)               SLP1
 12       COMMON/HINGAX/PHI,THETA,PSI                                    SLP1
 13       COMMON/INDEX2/LMIN                                             SLP1
 15       COMMON/SMALL/YMXMN                                             SLP1
 16       COMMON/BEST/JA,AVGDEV                                          SLP1
 18       FORMAT (/////8X,31HTHE OPTIMUM HINGE AXIS LOCATION //18X,3HPSI,  SLP1
 19      136X,5HTHETA,36X,3HPHI)                                         SLP1
 20       FORMAT (/13X,F12.6,27X,F12.6,29X,F12.6)                        SLP1
 201      IF(NOPT.EQ.1) GO TO 28                                         SLP1
 C
 C        CALL INIT TO OBTAIN INITIAL ANGLES TO BE USED BY OPTLOC
 C
 21       CALL INIT(JT)                                                  SLP1
 22       THET1 = THECL                                                  SLP1
 23       THET2 = THECO                                                  SLP1
 C
 C        CALL OPTLOC TO OBTAIN OPTIMUM LOCATION OF HINGE AXIS
 C
 24       CALL OPTLOC(THET1,THET2,DEL1,DEL2,JT,THET3)                    SLP1
 C
 C        COMPUTE COMPUTATIONAL CONSTANTS
 C
 25       PHI = THET2                                                    SLP1
 26       THETA = THET1                                                  SLP1
 27       PSI = THET3                                                    SLP1
 28       PHIS = SIN(PHI)                                                SLP1
 29       PHIC = COS(PHI)                                                SLP1
 30       THETAS = SIN(THETA)                                            SLP1
 31       THETAC = COS(THETA)                                            SLP1
 32       PSIS = SIN(PSI)                                                SLP1
 33       PSIC = COS(PSI)                                                SLP1
 38       A1 = THETAC*PSIC*PHIS                                          SLP1
 39       A2 = THETAS*PSIS                                               SLP1
 40       A3 = THETAC*PSIS*PHIC                                          SLP1
 41       B1 = THETAS*PSIC*PHIS                                          SLP1
 42       B2 = THETAC*PSIS                                               SLP1
 43       B3 = THETAS*PSIS*PHIC                                          SLP1
 44       G1 = PSIC*PHIC                                                 SLP1
 45       G2 = PSIS*PHIS                                                 SLP1
 46       PSID = PSI*57.2957795                                          SLP1
 47       THETAD = THETA*57.2957795                                      SLP1
 48       PHID = PHI*57.2957795                                          SLP1
```

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        SLP1          -  EFN   SOURCE STATEMENT   -  IFN(S)  -

 49     WRITE (6, 18)                                                      SLP1
 50     WRITE (6, 20) PSID,THETAD,PHID                                     SLP1
 C
 C      COMPUTE HINGE ANGLE
 C
 51     DO 55 I=1,JT                                                       SLP1
 52     ALPHAH = -THETAS*ALPHAE(I) + THETAC*BETAE(I)                       SLP1
 53     BETAH = -THETAC*PHIC*ALPHAE(I)-THETAS*PHIC*BETAE(I)+PHIS*GAMMAE(I)SLP1
 54     THETAH(I)=57.2957795*ATAN2(BETAH,ALPHAH)                           SLP1
 55     CONTINUE                                                           SLP1
 58     NT = T(JT) - T(1) + 1.                                             SLP1
 59     IXX = NT/451 + 1                                                   SLP1
 5902   MIDT = (NT - (1/IXX))/(NT/451 + 1) + 1                             SLP1
 C
 C      CALL PTERR WITH L=0 TO WRITE HINGE ANGLE,ERROR AND COMPONENTS OF
 C      ANTENNA FEED VECTORS
 C
 60     CALL PTERR(THETAH,ALPHAE,BETAE,GAMMAE,TRUERR,AVGDEV,JT,AJ,YMAX,0)  SLP1
 C
 601    IF(MULTRJ.EQ.1) GO TO 76                                           SLP1
 C      INTERPOLATE HINGE ANGLE AND EARTH POINTING VECTORS
 C
 61     CALL INTRPO(IXX,JT,THETAH,YH)                                      SLP1
 63     CALL INTRPO(IXX,JT,ALPHAE,YALPHA)                                  SLP1
 64     CALL INTRPO(IXX,JT,BETAE,YBETA)                                    SLP1
 65     CALL INTRPO(IXX,JT,GAMMAE,YGAMMA)                                  SLP1
 66     FMIDT = MIDT                                                       SLP1
 C
 C      CALL PTERR TO CALCULATE POINTING ERROR AT ONE DAY INTERVALS
 C
 67     CALL PTERR(YH,YALPHA,YBETA,YGAMMA,TR,AVG,MIDT,FMIDT,YMAX,1)        SLP1
 6701   IF(NOFIT.EQ.1) GO TO 75                                            SLP1
 68     YSB = YH(1) - EE                                                   SLP1
 C
 C      CALL SLPFIT TO START LINE SEGMENT FIT
 C
 69     CALL SLPFIT(JT,EE,DELTAH,YSB)                                      SLP1
 70     CALL ERPLT1(JT)                                                    SLP1
 C
 C      INCREASE EE AND TRY AGAIN IF EE.LE.EEMAX
 C
 71     EE = EE + DELTAE                                                   SLP1
 72     IF(EE.GT.EEMAX) GO TO 76                                           SLP1
 73     YSB = YH(1) - EE                                                   SLP1
 74     GO TO 69                                                           SLP1
 75     CALL ERPLT1(JT)                                                    SLP1
 76     RETURN                                                             SLP1
        END                                                                SLP1
```

SLP2 - EFN SOURCE STATEMENT - IFN(S) -

```
1       SUBROUTINE SLPFIT(JT,EE,DELTAH,YSB)                                 SLP2
C
C       SLPFIT   4 ARGUMENTS
C           INPUT ARGUMENTS
C                  JT  DIMENSION OF DATA
C                  EE  MAX. DEVIATION IN LINE SEGMENT FIT
C              DELTAH  INCREMENTAL VALUE FOR YSB
C                 YSB  INITIAL VALUE OF FIRST SEGMENT
C
2       DIMENSION ERROR(450)                                                SLP2
3       COMMON/HINGE2/YN(450)                                               SLP2
4       COMMON/TIME/T(100)                                                  SLP2
5       INTEGER A                                                           SLP2
6       DIMENSION A(101)                                                    SLP2
7       COMMON/INDEX2/LMIN                                                  SLP2
9       COMMON/INDEX1/TIM(101,24),SLOPE(101,24),HANGLE(101,24)              SLP2
10      COMMON/HTRIAL/HTHETA(450)                                           SLP2
11      COMMON/HNGLOC/A1,A2,A3,B1,B2,B3,G1,G2                               SLP2
12      COMMON/EARTH2/YALPHA(450),YBETA(450),YGAMMA(450)                    SLP2
13      COMMON/BEST/JA,AVGDEV                                               SLP2
14      COMMON/SMALL/YMXMN                                                  SLP2
15      FORMAT (1H1,4X,55HTHE LINE SEGMENT FIT AS A FUNCTION OF MAXIMUM DESLP2
16     1VIATION)                                                            SLP2
17      FORMAT (///8X,24HTHE MAXIMUM DEVIATION = ,F5.2,1X,7HDEGREES)        SLP2
18      FORMAT (//11X,I3,1X,16HLINE SEGMENT FIT)                            SLP2
19      FORMAT (//30X,4HTIME,17X,11HHINGE ANGLE,17X,5HSLOPE//(29X,F5.1,F28SLP2
20     1.5,F22.6))                                                          SLP2
21      FORMAT (/25X,24HAVERAGE POINTING ERROR =,F11.7,1X,7HDEGREES)        SLP2
2101    FORMAT (25X,24HMAXIMUM POINTING ERROR = ,F11.7,1X,7HDEGREES///)
2102    NT = T(JT) - T(1) + 1.                                              SLP2
22      KL = (NT - (1/(NT/451 + 1)))/(NT/451 + 1) + 1                       SLP2
23      FKL=KL                                                              SLP2
2301    TX = NT/451 + 1                                                     SLP2
24      WRITE (6, 15)                                                       SLP2
25      WRITE (6, 17) EE                                                    SLP2
26      Z = (2.*EE)/DELTAH                                                  SLP2
C
C       M = NUMBER OF TIME YSB IS CHANGED FOR GIVEN EE
C
27      M = Z + 1.                                                          SLP2
28      LMIN = 1000                                                         SLP2
29      YMXMN = 1000.                                                       SLP2
31      DO 97 I=1,M                                                         SLP2
33      SLHL = 1000.                                                        SLP2
34      SLLH = -1000.                                                       SLP2
35      YS = YSB                                                            SLP2
36      TM = -1.                                                            SLP2
37      IF (YS.GE.YN(1)) TM=-TM                                             SLP2
38      LL = 1                                                              SLP2
39      KF = 2                                                              SLP2
40      DO 55 J=KF,KL                                                       SLP2
41      FJ = J                                                              SLP2
42      FKF = KF                                                            SLP2
43      IF (J.EQ.KL) GO TO 68                                               SLP2
44      SLH = (YN(J)+EE-YS)/((FJ-FKF+1.)*TX)                                SLP2
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          SLP2         -  EFN    SOURCE STATEMENT  -  IFN(S)  -

 45    SLL = (YN(J)-EE-YS)/((FJ-FKF+1.)*TX)                             SLP2
 46    IF (SLL.GT.SLHL.OR.SLH.LT.SLLH.OR.SLH.GT.SLHL.AND.SLL.LT.SLLH) GO SLP2
 47   1TO 52                                                             SLP2
 C
 C     SLH AND SLL ARE BOTH GOOD.  IF(TM=1),CHOOSE SLL  IF(TM=-1),SLH
 C
 48    IF (SLH.LE.SLHL.AND.SLL.GE.SLLH) YB=YN(J)-TM*EE                  SLP2
 C
 C     ONLY SLH IS GOOD
 C
 49    IF (SLH.LE.SLHL.AND.SLL.LT.SLLH) YB=YN(J)+EE                     SLP2
 C
 C     ONLY SLL IS GOOD
 C
 50    IF (SLH.GT.SLHL.AND.SLL.GE.SLLH) YB=YN(J)-EE                     SLP2
 51    FFJ = J                                                          SLP2
 52    IF (SLH.LE.SLHL) SLHL=SLH                                        SLP2
 53    IF (SLL.GE.SLLH) SLLH=SLL                                        SLP2
 C
 C     STOP IF AND ONLY IF(SLHL.LT.SLLH)
 C
 54    IF (SLHL.LT.SLLH) GO TO 56                                       SLP2
 55    CONTINUE                                                         SLP2
 56    SLOPE(I,LL) = (YB-YS)/((FFJ-FKF+1.)*TX)                          SLP2
 57    TIM(I,LL) = T(1) + (FKF-2.)*TX                                   SLP2
 58    KTIM = FFJ                                                       SLP2
 59    HANGLE(I,LL) = YS                                                SLP2
 60    KF = FFJ + 1.                                                    SLP2
 61    LL = LL + 1                                                      SLP2
 62    SLHL = 1000.                                                     SLP2
 63    SLLH = -1000.                                                    SLP2
 64    YS = YB                                                          SLP2
 65    TM = -1.                                                         SLP2
 66    IF (YS.GE.YN(KTIM)) TM=-TM                                       SLP2
 67    GO TO 40                                                         SLP2
 C
 C     FSLOPE IS THE LAST LINE SEGMENT
 C
 68    FSLOPE = (YN(KL)-YS)/(T(JT)-T(1)-(FKF-2.)*TX)                    SLP2
 69    IF (.NOT.(FSLOPE.LE.SLHL.AND.FSLOPE.GE.SLLH)) GO TO 56           SLP2
 70    SLOPE(I,LL) = FSLOPE                                             SLP2
 71    TIM(I,LL) = T(1) + (FKF-2.)*TX                                   SLP2
 72    HANGLE(I,LL) = YS                                                SLP2
 C
 C     A(I) IS THE NO. OF SEGMENTS USED
 C
 73    A(I) = LL                                                        SLP2
 74    LM = LL                                                          SLP2
 75    TIM(I,LL+1) = T(JT)                                              SLP2
 7501  IF(LM.GT.LMIN) GO TO 96                                          SLP2
 7502  IF(LM.EQ.LMIN) GO TO 7504                                        SLP2
 7503  IF(LM.LT.LMIN) YMXMN = 1000.                                     SLP2
 7504  LMIN = LM                                                        SLP2
 C
 C     COMPUTE NEW HINGE ANGLE THAT RESULTS FROM LINEARIZATION
 C
```

```
            SLP2          -  EFN   SOURCE STATEMENT   -  IFN(S)  -

  76     L = 1                                                          SLP2
  77     DO 86 N=1,LM                                                   SLP2
  78     NN = N + 1                                                     SLP2
  79     DO 84 K=L,KL                                                   SLP2
  80     FK = K                                                         SLP2
  81     FL = L                                                         SLP2
  82     IF(K.EQ.KL) GO TO 87                                           SLP2
  83     IF((T(1)+(FK-1.)*TX).EQ.TIM(I,NN))GO TO 85                     SLP2
  84     HTHETA(K) = HANGLE(I,N) + SLOPE(I,N)*(FK-FL)*TX                SLP2
  85     LK = (TIM(I,NN)-T(1))/TX                                       SLP2
  86     L = 1 + LK                                                     SLP2
  87     HTHETA(KL) = HANGLE(I,LM) + SLOPE(I,LM)*(T(JT)-TIM(I,LM))      SLP2
 C
 C       CALL PTERR TO CALCULATE AVERAGE POINTING ERROR AND MAXIMUM
 C       POINTING ERROR
 C
  89     CALL PTERR(HTHETA,YALPHA,YBETA,YGAMMA,ERROR,AVGERR,KL,FKL,YMAX,1) SLP2
  90     IF(YMAX.LE.YMXMN) GO TO 92                                     SLP2
  91     GO TO 96                                                       SLP2
  92     YMXMN = YMAX                                                   SLP2
  93     JA = I                                                         SLP2
  94     KA = LMIN                                                      SLP2
  95     AVGDEV = AVGERR                                                SLP2
  96     YSB = YSB + DELTAH                                             SLP2
  97     CONTINUE                                                       SLP2
  98     WRITE (6,18) KA                                                SLP2
  99     WRITE (6,19) (TIM(JA,N),HANGLE(JA,N),SLOPE(JA,N),N=1,KA)       SLP2
 100     WRITE (6,21) AVGDEV                                            SLP2
 101     WRITE (6,2101) YMXMN                                           SLP2
 102     RETURN                                                         SLP2
         END                                                            SLP2
```

```
                              JPL,J265000,44808-0,28572,AF IC      IBJOB V13MJO  7
        SLP3          -  EFN   SOURCE STATEMENT   -  IFN(S)  -

1         SUBROUTINE ERPLT1(JT)                                                 SLP3
C
C         ERPLT1 PLOTS POINTING ERROR VS. TIME CORRESPONDING TO THE IDEAL
C              HINGE ANGLE FUNCTION (GEOMETRIC ERROR ONLY) AND WORST CASE
C              PTG. ERROR (GEOM. ERROR + ATTITUDE DRIFTS + STRUCTURAL
C              MISALIGNMENTS).   IF A LINE SEGMENT APPROXIMATION TO THE IDEAL
C              HINGE ANGLE FCT. IS REQUIRED, A PTG. ERROR PLOT FOR EACH LINE
C              SEGMENT FIT (EE = CONST.) IS ALSC PLOTTED.
C
C         ERPLT1   1 ARGUMENT
C
C              INPUT ARGUMENTS
C                    JT   DIMENSICN OF GIVEN DATA
C
2         COMMON/SMALL/YMXMN                                                    SLP3
3         COMMON/BEST/JA,AVGDEV                                                 SLP3
4         COMMON/HINGE2/YN(450)                                                 SLP3
5         COMMON/TIME/T(100)                                                    SLP3
6         COMMON/EARTH2/EALPHA(450),EBETA(450),EGAMMA(450)                      SLP3
7         COMMON/ERROR2/TR(450)                                                 SLP3
8         COMMON/INDEX1/TIM(101,24),SLOPE(101,24),HANGLE(101,24)                SLP3
9         COMMON/HTRIAL/HTHETA(450)                                             SLP3
10        COMMON/INDEX2/LMIN                                                    SLP3
1001      COMMON/WCASE/ERROR(450),THETO(450)                                    SLP3
11        COMMON/CONST/EE,EEMAX,DELTAE,DELTAH,NOFIT,NOPT,MULTRJ                 SLP3
12        DIMENSION NP(3),IX(3),IY(3),INTERP(3),SYMBOL(3),XY(4,450),YM(10),     SLP3
13       1TITLE1(14),TITLE2(14),XNAME(14),YNAME(10),YY(450),TITLE3(14)          SLP3
14        DATA SYMBOL(1)/18H1      2      3      /                              SLP3
15        DATA TITLE1(1)/84HPOINTING ERROR - (1)BEST GEOMETRIC ERROR (2)BESTSLP3
16       1 HINGE ANGLE PROGRAM (3)WORST CASE   /                                SLP3
1601      DATA TITLE2(1)/84HPOINTING ERROR - (1)BEST GEOMETRIC ERROR (2)WORSSLP3
1602     1T CASE ERROR (USING CPT.HINGE ANGLE)/                                 SLP3
1603      DATA TITLE3(1)/84HANGULAR DEVIATION OF EARTH POSITION FROM ANTENNASLP3
1604     1 BEAM MAJOR AXIS                          /                           SLP3
1605      DATA YM(1)/60H                       THETO   (DEGREES)                SLP3
1606     1      /                                                               SLP3
17        DATA XNAME(1)/84H                                 TIME FROM LAUNCH SLP3
18       1  (DAYS)                             /                                SLP3
19        DATA YNAME(1)/60H                    POINTING ERROR    (DEGREES)      SLP3
20       1          /                                                           SLP3
21        NT = T(JT) - T(1) + 1.                                                SLP3
2101      KL = (NT - (1/(NT/451 + 1)))/(NT/451 + 1) + 1                         SLP3
2102      IXX = NT/451 + 1                                                      SLP3
2103      FXX = IXX                                                             SLP3
22        FKL = KL                                                              SLP3
2201      IF(NOFIT.EQ.1) GO TO 62                                               SLP3
23        I = JA                                                                SLP3
24        FI = I                                                                SLP3
25        YSB = YN(1)-EE+(FI-1.)*DELTAH                                         SLP3
26        LM = LMIN                                                             SLP3
C
C         COMPUTE HTHETA
C
31        L = 1                                                                 SLP3
32        DO 41 N=1,LM                                                          SLP3
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      SLP3          -   EFN   SOURCE STATEMENT   -   IFN(S)   -

 33       NN = N + 1                                                        SLP3
 34       DO 39 K=L,KL                                                      SLP3
 35       FK = K                                                            SLP3
 36       FL = L                                                            SLP3
 37       IF(K.EQ.KL) GO TO 42                                              SLP3
 38       IF((T(1)+(FK-1.)*FXX).EQ.TIM(I,NN)) GO TO 40                      SLP3
 39       HTHETA(K) = HANGLE(I,N) + SLOPE(I,N)*(FK-FL)*FXX                  SLP3
 40       LK = (TIM(I,NN)-T(1))/FXX                                         SLP3
 41       L = 1 + LK                                                        SLP3
 42       HTHETA(KL) = HANGLE(I,LM) + SLOPE(I,LM)*(T(JT)-TIM(I,LM))         SLP3
C
C         COMPUTE AND PLOT ERROR VS. TIME
C
 44       CALL PTERR(HTHETA,EALPHA,EBETA,EGAMMA,YY,AVG,KL,FKL,YMAX,1)       SLP3
 45       CALL WORSTE(HTHETA,JT)                                            SLP3
 46       DO 51 K=1,KL                                                      SLP3
 47       FK = K                                                            SLP3
 48       XY(1,K) = TR(K)                                                   SLP3
 49       XY(2,K) = YY(K)                                                   SLP3
 50       XY(3,K) = ERROR(K)                                                SLP3
 51       XY(4,K) = T(1) + (FK-1.)*FXX                                      SLP3
 52       DO 56 I=1,3                                                       SLP3
 53       IX(I) = 4                                                         SLP3
 54       IY(I) = I                                                         SLP3
 55       NP(I) = KL                                                        SLP3
 56       INTERP(I) = 1                                                     SLP3
 57       CALL KDPLOT(XY,4,3,IX,IY,NP,INTERP,SYMBOL,TITLE1,XNAME,YNAME,1)   SLP3
 58       DO 59 K=1,KL                                                      SLP3
 59       XY(1,K) = THETO(K)                                                SLP3
 60       CALL KDPLOT(XY,4,1,IX,IY,NP,INTERP,SYMBOL,TITLE3,XNAME,YM,1)      SLP3
 61       GO TO 77                                                          SLP3
 62       CALL WORSTE(YN,JT)                                                SLP3
 63       DO 67 K=1,KL                                                      SLP3
 64       FK = K                                                            SLP3
 65       XY(1,K) = TR(K)                                                   SLP3
 66       XY(2,K) = ERROR(K)                                                SLP3
 67       XY(3,K) = T(1) + (FK-1.)*FXX                                      SLP3
 68       DO 72 I=1,2                                                       SLP3
 69       IX(I) = 3                                                         SLP3
 70       IY(I) = I                                                         SLP3
 71       NP(I) = KL                                                        SLP3
 72       INTERP(I) = 1                                                     SLP3
 73       CALL KDPLOT(XY,4,2,IX,IY,NP,INTERP,SYMBOL,TITLE2,XNAME,YNAME,1)   SLP3
 74       DO 75 K=1,KL                                                      SLP3
 75       XY(1,K) = THETO(K)                                                SLP3
 76       CALL KDPLOT(XY,4,1,IX,IY,NP,INTERP,SYMBOL,TITLE3,XNAME,YM,1)      SLP3
 77       RETURN                                                            SLP3
          END                                                               SLP3
```

```
        WORS          -  EFN    SOURCE STATEMENT  -  IFN(S)  -

1       SUBROUTINE WORSTE(THETH,JT)                                    WORS
C
C       WORSTE   2 ARGUMENTS
C
C           INPUT ARGUMENTS
C                THETH  HINGE ANGLE FUNCTION
C                JT     DIMENSION OF GIVEN DATA
C
2       DIMENSION THETH(450)                                           WORS
2001    INTEGER O                                                      WORS
3       COMMON/TIME/T(100)                                             WORS
3001    COMMON/WCASE/ERROR(450),THETO(450)                             WORS
4       COMMON/EARTH2/ALPHAE(450),BETAE(450),GAMMAE(450)               WORS
5       COMMON/HINGAX/PHI,THETA,PSI                                    WORS
6       COMMON/DRIFT/DP,DY,DR,DPHI,DTHETA,DPSI                         WORS
601     NT = T(JT) - T(1) + 1.                                         WORS
7       KL = (NT - (1/(NT/451 + 1)))/(NT/451 + 1) + 1                  WORS
8       DO 61 I=1,KL                                                   WORS
8001    ERM = 0.                                                       WORS
9       DO 61 J=1,3,2                                                  WORS
10      RJ = J                                                         WORS
11      PHIT = PHI + (RJ-2.)*DPHI                                      WORS
12      PHIS = SIN(PHIT)                                               WORS
13      PHIC = COS(PHIT)                                               WORS
14      DO 61 K=1,3,2                                                  WORS
15      RK = K                                                         WORS
16      THETAT = THETA + (RK-2.)*DTHETA                                WORS
17      THETAS = SIN(THETAT)                                           WORS
18      THETAC = COS(THETAT)                                           WORS
19      DO 61 L=1,3,2                                                  WORS
20      RL = L                                                         WORS
21      PSIT = PSI + (RL-2.)*DPSI                                      WORS
22      PSIS = SIN(PSIT)                                               WORS
23      PSIC = COS(PSIT)                                               WORS
24      A1 = THETAC*PSIC*PHIS                                          WORS
25      A2 = THETAS*PSIS                                               WORS
26      A3 = THETAC*PSIS*PHIC                                          WORS
27      B1 = THETAS*PSIC*PHIS                                          WORS
28      B2 = THETAC*PSIS                                               WORS
29      B3 = THETAS*PSIS*PHIC                                          WORS
30      G1 = PSIC*PHIC                                                 WORS
31      G2 = PSIS*PHIS                                                 WORS
32      TS = SIN(THETH(I)*.01745329)                                   WORS
33      TC = COS(THETH(I)*.01745329)                                   WORS
34      ALPHFN = A1 - A2*TC - A3*TS                                    WORS
35      BETFN = B1 + B2*TC - B3*TS                                     WORS
36      GAMMFN = G1 + G2*TS                                            WORS
37      DO 61 M=1,3,2                                                  WORS
38      DO 61 N=1,3,2                                                  WORS
39      DO 61 O=1,3,2                                                  WORS
40      RM = M                                                         WORS
41      RN = N                                                         WORS
42      RO = O                                                         WORS
43      ALPHAF = ALPHFN - BETFN*(RO-2.)*DR + GAMMFN*(RN-2.)*DY         WORS
44      BETAF = BETFN + ALPHFN*(RO-2.)*DR - GAMMFN*(RM-2.)*DP          WORS
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        WORS          -   EFN    SOURCE STATEMENT   -   IFN(S)   -

 45     GAMMAF = GAMMFN - ALPHFN*(RN-2.)*DY + BETFN*(RM-2.)*DP              WORS
 54     ZZ = ((BETAF*GAMMAE(I) - GAMMAF*BETAE(I))**2                        WORS
 55    1          + (GAMMAF*ALPHAE(I) - ALPHAF*GAMMAE(I))**2                WORS
 56    2          + (ALPHAF*BETAE(I) - BETAF*ALPHAE(I))**2)                 WORS
 57     IF(ZZ.LE.ERM) GO TO 61                                              WORS
 58     ERM = ZZ                                                            WORS
 59     ERROR(I) = 57.2957795*ARSIN(SQRT(ZZ))                               WORS
 46     AMI = (GAMMAF*PHIS*THETAS - BETAF*PHIC)/PSIS                        WORS
 47     BMI = (ALPHAF*PHIC - GAMMAF*PHIS*THETAC)/PSIS                       WORS
 48     GMI = (BETAF*PHIS*THETAC - ALPHAF*PHIS*THETAS)/PSIS                 WORS
 49     AMA = BMI*GAMMAF - GMI*BETAF                                        WORS
 50     BMA = GMI*ALPHAF - AMI*GAMMAF                                       WORS
 51     GMA = AMI*BETAF - BMI*ALPHAF                                        WORS
 52     THETO(I) = 57.2957795*ATAN(ABS(ALPHAE(I)*AMI + BETAE(I)*BMI +       WORS
 53    1GAMMAE(I)*GMI)/ABS(ALPHAE(I)*AMA + BETAE(I)*BMA + GAMMAE(I)*GMA))   WORS
 61     CONTINUE                                                            WORS
 62     RETURN                                                              WORS
        END                                                                 WORS
```

TRPO - EFN SOURCE STATEMENT - IFN(S) -

```
C       INTERPOLATION ROUTINE                                        TRPO
C
C           INPUT ARGUMENTS
C             IXX   INTERVAL AT WHICH YY IS WANTED
C             Y     GIVEN FUNCTION
C           OUTPUT ARGUMENTS
C             YY    WANTED FUNCTION
C
        SUBROUTINE INTRPO(IXX,JT,Y,YY)                               TRPO
        COMMON/TIME/T(100)                                           TRPO
        DIMENSION Y(1),YY(1)                                         TRPO
        NT = T(JT) - T(1) + 1.                                       TRPO
        KL = (NT - (1/IXX))/IXX + 1                                  TRPO
        FXX = IXX                                                    TRPO
        DO 24 I=1,KL                                                 TRPO
        FI = I                                                       TRPO
        X = T(1) + (FI-1.)*FXX                                       TRPO
        IF(I.EQ.KL) X = T(JT)                                        TRPO
        DO 15 J=1,JT                                                 TRPO
        IF(X-T(J)) 13,15,15                                          TRPO
        N = J - 1                                                    TRPO
        GO TO 16                                                     TRPO
        CONTINUE                                                     TRPO
        IF(X.GE.T(JT-2)) N = JT-2                                    TRPO
        IF(X.LT.T(2)) N = 2                                          TRPO
        YY(I) = Y(N-1)*(X-T(N))*(X-T(N+1))*(X-T(N+2))/((T(N-1)-T(N))*(T(N-TRPO
       11)-T(N+1))*(T(N-1)-T(N+2))) + Y(N)*(X-T(N-1))*(X-T(N+1))*(X-T(N+2)TRPO
       2)/((T(N)-T(N-1))*(T(N)-T(N+1))*(T(N)-T(N+2))) + Y(N+1)*(X-T(N-1))*TRPO
       3(X-T(N))*(X-T(N+2))/((T(N+1)-T(N-1))*(T(N+1)-T(N))*(T(N+1)-T(N+2))TRPO
       4) + Y(N+2)*(X-T(N-1))*(X-T(N))*(X-T(N+1))/((T(N+2)-T(N-1))*(T(N+2)TRPO
       5-T(N))*(T(N+2)-T(N+1)))                                     TRPO
        CONTINUE                                                     TRPO
        RETURN                                                       TRPO
        END                                                          TRPO
```

```
C      OPTIMUM HINGE AXIS ROUTINE                                        OPT
1      SUBROUTINE OPTLOC(THET1,THET2,DELTA,DELLIM,JT,THET3)              OPT
C
C      OPTLOC HAS 7 ARGUMENTS
C          INPUT ARGUMENTS
C            THET1 = INITIAL CLOCK ANGLE
C            THET2 = INITIAL CONE ANGLE
C            DELTA = INITIAL VALUE FOR INCREMENTING THET1 AND THET2
C           DELLIM = SMALLEST VALUE FOR INCREMENTING THET1 AND THET2
C               JT = DIMENSION OF GIVEN DATA
C          OUTPUT ARGUMENTS
C            THET1 = OPTIMUM HINGE AXIS CLOCK ANGLE
C            THET2 = OPTIMUM HINGE AXIS CONE ANGLE
C            THET3 = OPTIMUM CONE ANGLE OF FEED VECTOR ABOUT HINGE AXIS
C
2      DIMENSION THETA1(9),THETA2(9),GAMMAL(9),GAMMAS(9)                 OPT
3      DIMENSION PPSIL(9),PPSIS(9),DELPSI(9)                             OPT
4      COMMON/EARTH/ALPHAE(100),BETAE(100),GAMMAE(100)                   OPT
5      DO 32 N=1,9                                                       OPT
6      DN = N                                                            OPT
7      IF (N.LE.3) GO TO 10                                              OPT
8      IF (N.LE.6) GO TO 13                                              OPT
9      GO TO 16                                                          OPT
C
C      (THETA1(2),THETA2(2)) IS THE CENTER OF SQUARE
C
10     THETA1(N) = THET1                                                 OPT
11     THETA2(N) = THET2 + (DN-2.)*DELTA                                 OPT
12     GO TO 18                                                          OPT
13     THETA1(N) = THET1 + DELTA                                         OPT
14     THETA2(N) = THET2 + (DN-5.)*DELTA                                 OPT
15     GO TO 18                                                          OPT
16     THETA1(N) = THET1 - DELTA                                         OPT
17     THETA2(N) = THET2 + (DN-8.)*DELTA                                 OPT
18     S1 = SIN(THETA1(N))                                               OPT
19     C1 = COS(THETA1(N))                                               OPT
20     S2 = SIN(THETA2(N))                                               OPT
21     C2 = COS(THETA2(N))                                               OPT
22     GAMMAL(N)=-1.                                                     OPT
23     GAMMAS(N)= 1.                                                     OPT
24     DO 28 I=1,JT                                                      OPT
25     GAMMAR = S2*C1*ALPHAE(I) + S2*S1*BETAE(I) + C2*GAMMAE(I)          OPT
26     IF (GAMMAR.GT.GAMMAL(N)) GAMMAL(N) = GAMMAR                       OPT
27     IF (GAMMAR.LT.GAMMAS(N)) GAMMAS(N) = GAMMAR                       OPT
28     CONTINUE                                                          OPT
29     PPSIL(N) = ARCOS(GAMMAS(N))                                       OPT
30     PPSIS(N) = ARCOS(GAMMAL(N))                                       OPT
31     DELPSI(N) = PPSIL(N) - PPSIS (N)                                  OPT
32     CONTINUE                                                          OPT
33     DELMIN = 3.14159265                                               OPT
C
C      FIND N FOR WHICH DELPSI(N) IS MINIMUM
C
34     DO 40 N=1,9                                                       OPT
35     IF (DELPSI(N).GE.DELMIN) GO TO 40                                 OPT
```

```
       OPT            -  EFN    SOURCE STATEMENT  -  IFN(S)  -

 36      DELMIN = DELPSI(N)                                         OPT
 37      PPSI = PPSIS(N)                                            OPT
 38      IN = N                                                     OPT
 39      K = IN                                                     OPT
 40      CONTINUE                                                   OPT
 C
 C       IF.NOT.(N=2), SHIFT SQUARE SO THAT (THETA1(N),THETA2(N))
 C       BECOMES A CENTER
 C       IF N=2, REDUCE INCREMENTAL VALUE DELTA
 C
 41      IF (K.EQ.2) GO TO 45                                       OPT
 42      THET1 = THETA1(K)                                          OPT
 43      THET2 = THETA2(K)                                          OPT
 44      GO TO 5                                                    OPT
 45      DELTA = DELTA*.5                                           OPT
 46      IF (DELTA.GE.DELLIM) GO TO 5                               OPT
 460     THET3 = PPSI + DELMIN*.5                                   OPT
 47      RETURN                                                     OPT
         END                                                        OPT
```

```
                         JPL,J265000,44808-0,28572,AF IC      IBJOB V13MJO  7
      PTER        -  EFN   SOURCE STATEMENT  -  IFN(S)  -

C       POINTING ERROR ROUTINE                                              PTER
1       SUBROUTINE PTERR(XTHETA,XALPHA,XBETA,XGAMMA,X,AVGERR,J,FJ,YMAX,L)   PTER
C
C       PTERR    IC ARGUMENTS
C           INPUT ARGUMENTS
C             XTHETA  HINGE ANGLE
C             XALPHA  COMPONENT OF EARTH POINTING VECTOR
C             XBETA   COMPONENT OF EARTH POINTING VECTOR
C             XGAMMA  COMPONENT OF EARTH POINTING VECTOR
C             J,FJ    DIMENSION OF ABOVE ARGUMENTS
C             L       INDEX    L=0  WRITE OUT T,XTHETA,X,FEEDING VECTOR
C                              L=1  NO WRITE OUT
C           OUTPUT ARGUMENTS
C             X       POINTING ERROR
C             AVGERR  AVERAGE POINTING ERROR
C             YMAX    MAXIMUM X
C
2       DIMENSION XTHETA(J),XALPHA(J),XBETA(J),XGAMMA(J),X(J)               PTER
3       COMMON/HNGLOC/A,B,C,D,E,F,G,H                                       PTER
4       COMMON/TIME/T(100)                                                  PTER
5       FORMAT (1H1,8X,4HTIME,12X,11HHINGE ANGLE,13X,5HERROR,13X,6HALPHAF,PTER
6      115X,5HBETAF,14X,6HGAMMAF //)                                        PTER
7       FORMAT (7X,F5.0,12X,F11.5,8X,F11.7, 9X,F11.8, 9X,F11.8, 9X,F11.8) PTER
8       FORMAT (//10X,25HAVERAGE POINTING ERROR = ,F11.7,2X,7HDEGREES)      PTER
9       SUMERR = 0.                                                         PTER
9001    YMAX = 0.                                                           PTER
10      IQ = 0                                                              PTER
11      DO 25 I=1,J                                                         PTER
C
C       ALPHAF,BETAF,AND GAMMAF ARE ANTENNA FEED VECTOR COMPONENTS
C
12      ALPHAF=A-B*COS(XTHETA(I)*0.01745329)-C*SIN(XTHETA(I)*0.01745329)    PTER
13      BETAF=D+E*COS(XTHETA(I)*0.01745329)-F*SIN(XTHETA(I)*0.01745329)     PTER
14      GAMMAF=G+H*SIN(XTHETA(I)*0.01745329)                                PTER
15      Z = SQRT((BETAF*XGAMMA(I)-GAMMAF*XBETA(I))**2                       PTER
16     1 + (GAMMAF*XALPHA(I)-ALPHAF*XGAMMA(I))**2                           PTER
17     2 + (ALPHAF*XBETA(I)-BETAF*XALPHA(I))**2)                            PTER
18      X(I) = 57.2957795*ARSIN(Z)                                          PTER
19      IF (X(I).GE.YMAX) YMAX=X(I)                                         PTER
20      SUMERR = X(I) + SUMERR                                              PTER
21      IF (L.EQ.1) GO TO 25                                                PTER
22      IF (IQ.EQ.0) WRITE (6, 5)                                           PTER
23      WRITE (6, 7) T(I),XTHETA(I),X(I),ALPHAF,BETAF,GAMMAF                PTER
24      IQ = 1                                                              PTER
25      CONTINUE                                                            PTER
26      AVGERR = SUMERR/FJ                                                  PTER
27      IF (L.EQ.0) WRITE (6, 8) AVGERR                                     PTER
28      RETURN                                                              PTER
        END                                                                 PTER
```

PREL - EFN SOURCE STATEMENT - IFN(S) -

```
C      INITIALIZATION ROUTINE                                                PREL
1      SUBROUTINE INIT(L)                                                    PREL
C
C      INIT HAS ONLY ONE ARGUMENT,L = DIMENSION OF TRAJ. DATA
C
C      INIT COMPUTES BEST HINGE AXIS LOCATION USING A ROUGH GRID WITH A
C            SPACING OF 5 DEGREES
C
2      COMMON/START/CLOCK,CONE                                               PREL
C
C      LABELED COMMON REGION EARTH CONTAINS EARTH POINTING VECTOR
C
3      COMMON/EARTH/A(100),B(100),G(100)                                     PREL
4      DELMIN = 3.14159265                                                   PREL
5      DO 26 I=1,18                                                          PREL
6      DO 25 J=1,72                                                          PREL
7      FI = I*5                                                              PREL
8      FJ = J*5                                                              PREL
9      F = FI*.01745329                                                      PREL
10     H = FJ*.01745329                                                      PREL
1001   SF = SIN(F)                                                           PREL
1002   CF = COS(F)                                                           PREL
1003   SH = SIN(H)                                                           PREL
1004   CH = COS(H)                                                           PREL
11     GAMMAL = -1.                                                          PREL
12     GAMMAS = 1.                                                           PREL
13     DO 17 K=1,L                                                           PREL
14     GAMMAR = SF*CH*A(K) + SF*SH*B(K) + CF*G(K)                            PREL
15     IF (GAMMAR.GT.GAMMAL) GAMMAL=GAMMAR                                   PREL
16     IF (GAMMAR.LT.GAMMAS) GAMMAS=GAMMAR                                   PREL
17     CONTINUE                                                              PREL
18     PPSIL = ARCOS(GAMMAS)                                                 PREL
19     PPSIS = ARCOS(GAMMAL)                                                 PREL
20     DELPSI = PPSIL - PPSIS                                                PREL
C
C      WANT F AND H THAT MINIMIZE DELPSI
C           F = CONE ANGLE
C           H = CLOCK ANGLE
C
21     IF (DELPSI.GT.DELMIN) GO TO 25                                        PREL
22     DELMIN = DELPSI                                                       PREL
23     CLOCK = H                                                             PREL
24     CONE = F                                                              PREL
25     CONTINUE                                                              PREL
26     CONTINUE                                                              PREL
27     RETURN                                                                PREL
       END                                                                   PREL
```

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