# Spherical Pressure Vessels

Pressure vessels are closed structures containing liquids or gases under pressure. Examples include tanks, pipes, pressurized cabins, etc.

Shell structures : When pressure vessels have walls that are thin in comparison to their radii and length. In the case of thin walled pressure vessels of spherical shape the ratio of radius **r** to wall thickness **t** is greater than **10**. A sphere is the theoretical ideal shape for a vessel that resists internal pressure.

$$\frac{r}{t} > 10$$

To determine the stresses in an spherical vessel let us cut through the sphere on a vertical diameter plane and isolate half of the shell and its fluid contents as a single free body. Acting on this free body are the tensile stress σ in the wall of the vessel and the fluid pressure **p** .
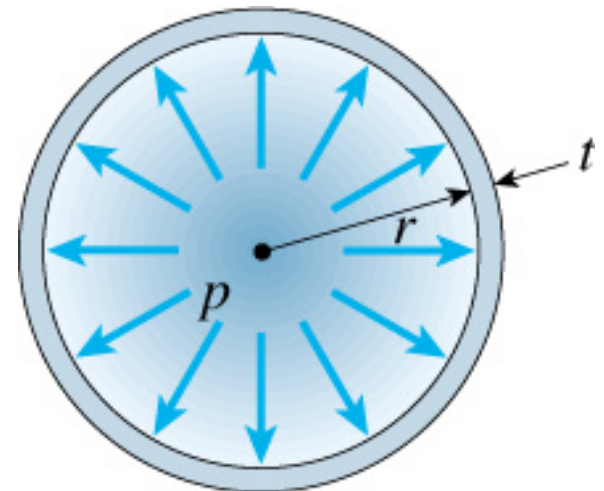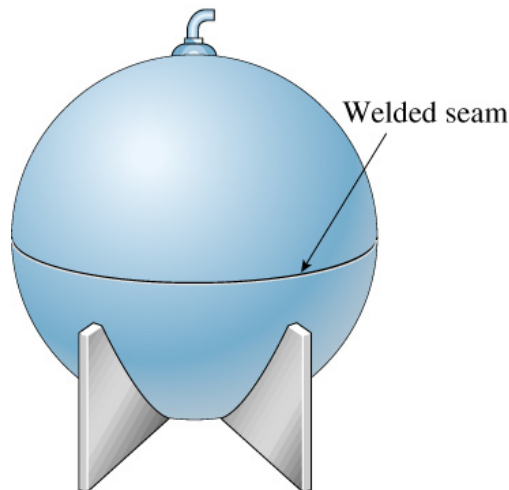
The pressure that acts horizontally against the plane circular area is uniform and gives a resultant pressure force of :

$$P = p\pi r^2$$

Where **p** is the gage or internal pressure (above the pressure acting in the outside of the vessel).

The stress is uniform around the circumference and it is uniformly distributed across the thickness **t** (because the wall is thin). The resultant horizontal force is :
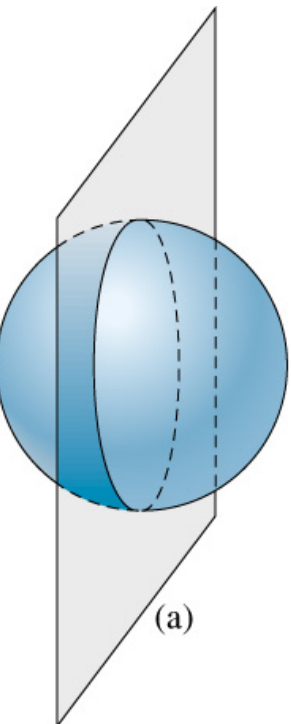
$$\text{Horizontal Force} = \sigma(2\pi r_m)t$$
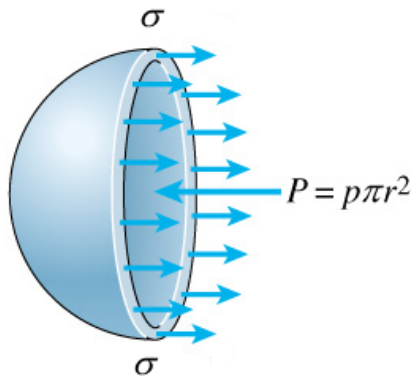
$$r_m = r + \frac{t}{2}$$

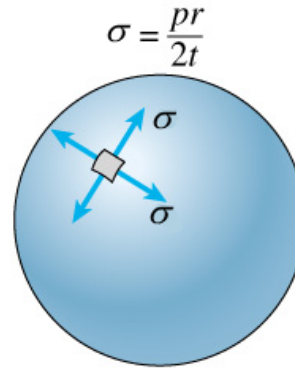Equilibrium of forces in the horizontal direction:

$$\sigma(2\pi r_m)t = p\pi r^2$$

$$\sigma = \frac{pr}{2t}$$

σ

$P = p\pi r^2$

σ

$\sigma = \frac{pr}{2t}$

σ

σ

(a)   (b)   (c)

$r_m \sim r$ for thin walls. Therefore the formula to calculate the stress in a thin walled spherical vessels is

As is evident from the symmetry of a spherical shell that we will obtain the same equation regardless of the direction of the cut through the center.

The wall of a pressurized spherical vessel is subjected to uniform tensile stresses σ in all directions.

Stresses that act tangentially to the curved surface of a shell are known as *membrane stresses*.

Limitations of the thin-shell theory:
1. The wall thickness must be small (r/t > 10)
2. The internal pressure must exceed the external pressure.
3. The analysis is based only on the effects of internal pressure.
4. The formulas derived are valid throughout the wall of the vessel except near points of stress concentration.
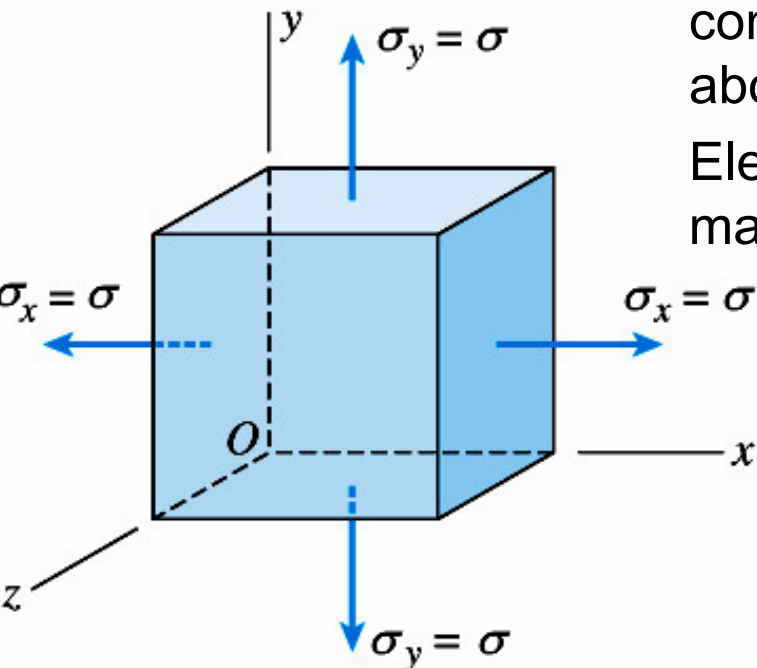
Stresses at the Outer Surfaces.

The element below has the *x* and *y* axes tangential to the surface of the sphere and the *z* axis is perpendicular to the surface.

Thus, the normal stresses σ_x and σ_y are equal to the membrane stress σ and the normal stress σ_z is zero.

The principal stresses are

$$\sigma_1 = \sigma_2 = \sigma = \frac{pr}{2t}$$

and $\sigma_3 = 0$. Any rotation element about the *z* axis will have a shear stress equals to **zero**.

To obtain the maximum shear stresses, we must consider out of plane rotations, that is, rotations about the *x* and *y* axis.

Elements oriented at *45°* of the *x* or *y* axis have maximum shear stresses equal to *σ/2* or

$$\tau_{Max} = \frac{\sigma}{2} = \frac{pr}{4t}$$

Stresses at the Inner Surface

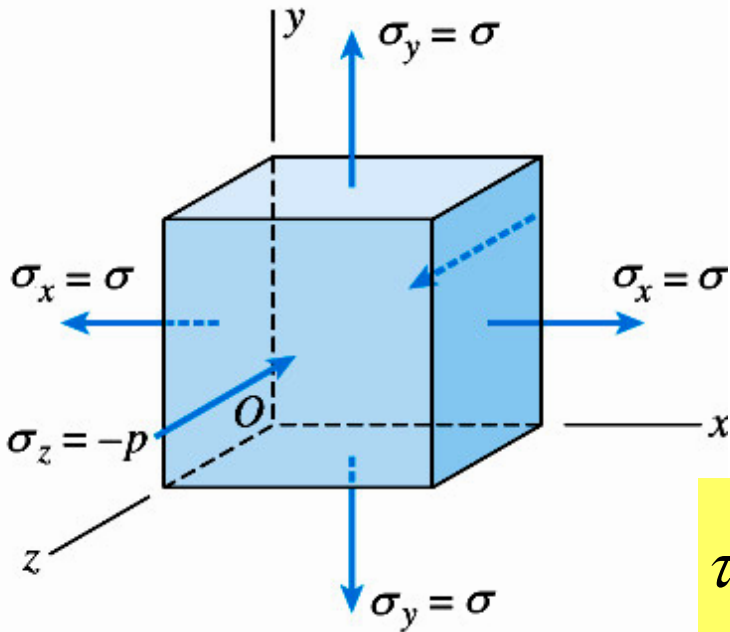At the inner wall the stresses in the *x* and *y* direction are equal to the membrane stress $\sigma_x = \sigma_y = \sigma$, but the stress in the *z* direction is not *zero*, and it is equal to the pressure *p* in compression.

This compressive stress decreases from *p* at the inner surface to *zero* at the outer surface.

The element is in *triaxial* stress

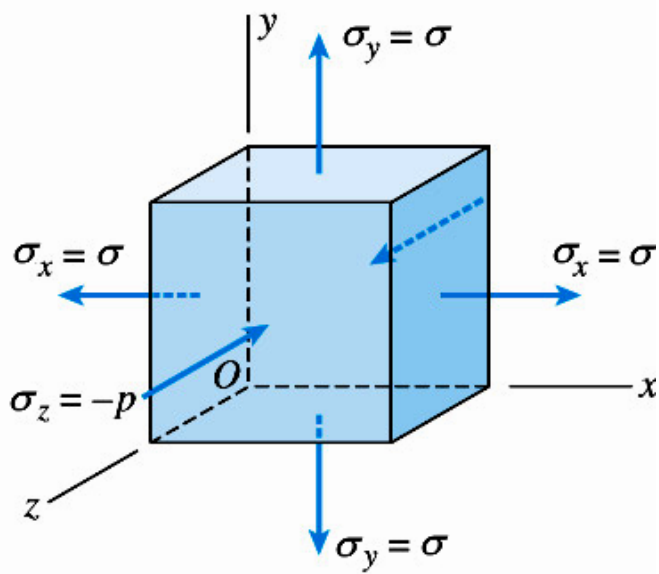$$\sigma_1 = \sigma_2 = \sigma = \frac{pr}{2t}$$

$$\sigma_3 = -p$$

The **in-plane shear stress are zero**, but the maximum out-of-plane shear stress (obtained at *45º* rotation about either the *x* or *y* axis) is

$$\tau_{Max} = \frac{(\sigma + p)}{2} = \frac{pr}{4t} + \frac{p}{2} \qquad \text{or}$$
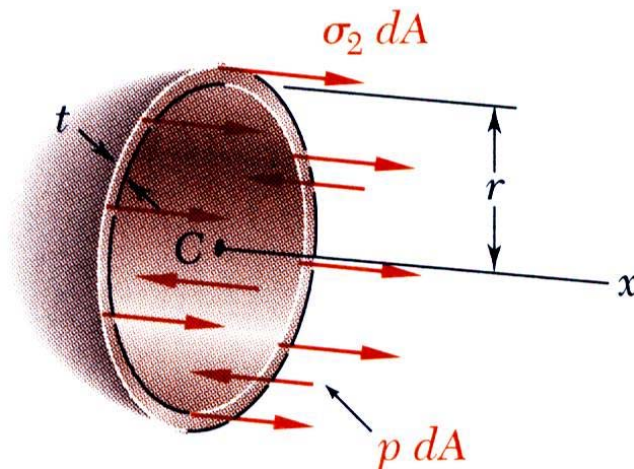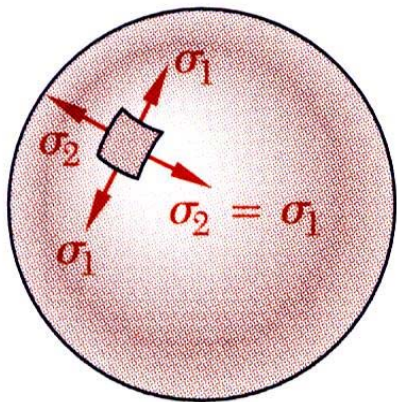
$$\tau_{Max} = \frac{p}{2}\left(1 + \frac{r}{2t}\right)$$

When the vessels is thin walled and the ratio *r/t* is large, we can disregard the number *1* and
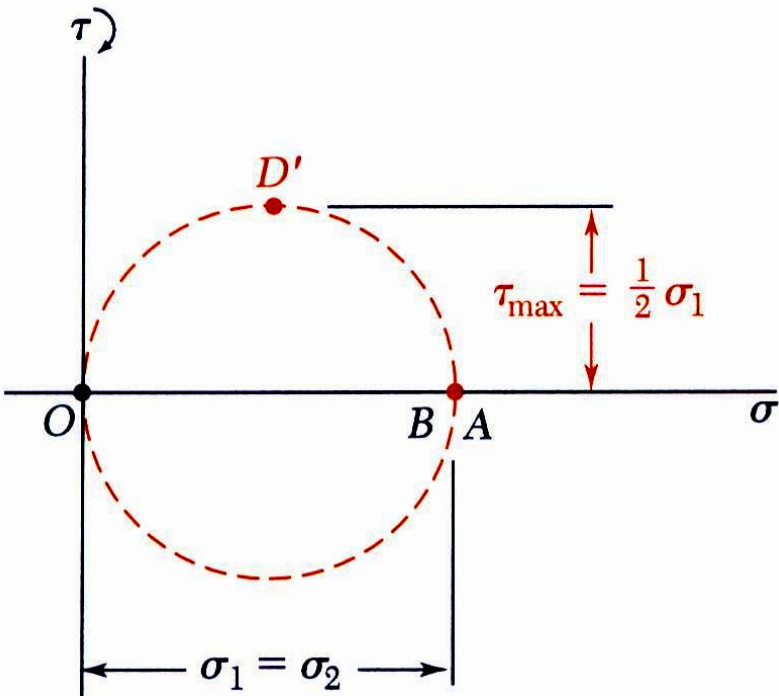
$$\tau_{Max} = \frac{pr}{4t}$$

Consequently, we can consider the stress state at the inner surface to be the same as the outer surface.

Summary for Spherical pressure vessel with r/t large:

$$\sigma_1 = \sigma_2 = \frac{pr}{2t}$$

As the two stresses are equal, Mohr's circle for in-plane transformations reduces to a point

$$\sigma = \sigma_1 = \sigma_2 = \text{constant}$$

$$\tau_{\text{max(in-plane)}} = 0$$

Maximum out-of-plane shearing stress

$$\tau_{\text{max}} = \frac{1}{2}\sigma_1 = \frac{pr}{4t}$$

A compressed air tank having an inner diameter of **18 inches** and a wall thickness of ¼ **inch** is formed by welding two steel hemispheres (see figure).

(a) If the allowable tensile stress in the steel is **14000psi**, what is the maximum permissible air pressure p_a in the tank?.

(b) If the allowable shear stress in the steel is **6000psi**, what is the maximum permissible pressure p_b?.

(c) If the normal strain in the outer surface of the tank is not to exceed **0.0003**, what is the maximum permissible pressure p_c? (Assume Hooke's law is obeyed $E = 29x10^6 psi$ and Poisson's ratio is $\nu = 0.28$)
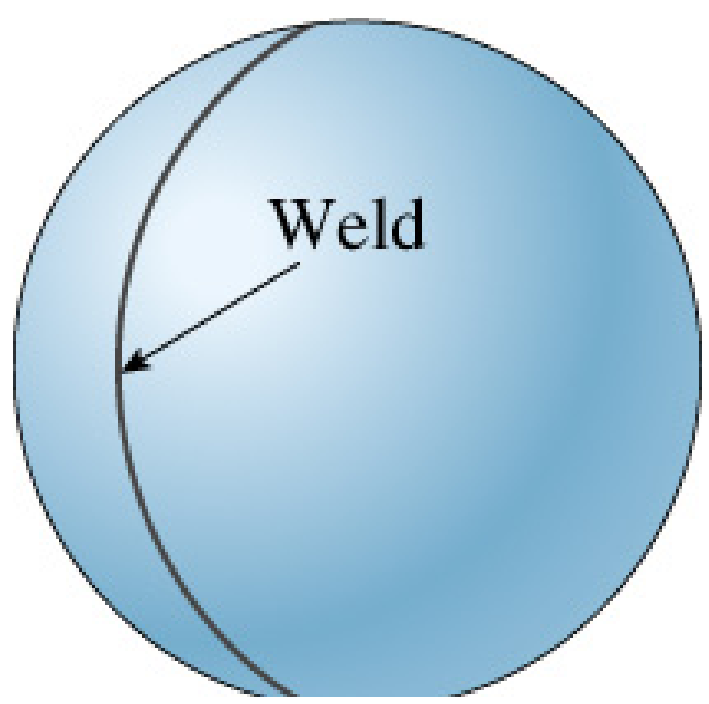
(d) Tests on the welded seam show that failure occurs when the tensile **load** on the welds exceeds **8.1kips per inch** of weld. If the required factor of safety against failure of the weld is **2.5**, what is the maximum permissible pressure p_d?

(e) Considering the four preceding factors, what is the allowable pressure p_{allow} in the tank?

Weld

Solution

(a)Allowable pressure based on the tensile stress in the steel. .We will use the equation

$$\sigma_{allowed} = \frac{p_a r}{2t} \quad \text{then}$$

$$p_a = \frac{2t\sigma_{allowed}}{r} = \frac{2(0.25inch)(14000\,psi)}{9.0inch} = 777.8\,psi$$

(b) Allowable pressure based upon the shear stress of the steel. We will use

$$\tau_{allowed} = \frac{\sigma}{2} = \frac{p_b r}{4t} \quad \text{then}$$

$$p_b = \frac{4t\tau_{allowed}}{r} = 4\frac{(0.25inch)(6000\,psi)}{9.0inch} = 666.7\,psi$$

(c) Allowable pressure based upon the normal strain in the steel. For biaxial stress

$$\varepsilon_X = \frac{(\sigma_X - \upsilon\sigma_Y)}{E} \quad substituting \quad \sigma_X = \sigma_Y = \sigma = \frac{pr}{2t}$$

$$then \quad \varepsilon_X = \frac{(1-\upsilon)\sigma}{E} = \frac{(1-\upsilon)pr}{2tE}$$

this equation can be solved for pressure p_c

$$p_c = \frac{2tE\,\varepsilon_{allowed}}{r(1-\upsilon)} = \frac{2(0.25\,inch\,)(29\,x10^6\,psi\,)(0.0003\,)}{(9.0inch\,)(1-0.28\,)} = 671.3\,psi$$

(d) Allowable pressure based upon the tension in the welded seam.

The allowable tensile load on the welded seam is equal to the failure load divided by the factor of safety

$$T_{allowed} = \frac{T_{failure}}{n} = \frac{8.1 kips/inch}{2.5}$$

$$T_{allowed} = 3.24 kips/inch = 3240 lb/inch$$

The corresponding allowable tensile stress is equal to the allowable load on 1inch length of weld divided by the cross-sectional area of a 1inch length of weld:

$$\sigma_{allowed} = \frac{T_{allowed}(1.0 inch)}{(1.0 inch)(t)} = \frac{(3240\ lb/inch)(1.0 inch)}{(1.0 inch)(0.25\ inch)} = 12960\ psi$$

$$p_d = \frac{2t\sigma_{allowed}}{r} = \frac{(2)(0.25\ inch)(12960\ psi)}{9.0 inch} = 720.0\ psi$$

(e) Allowable pressure

Comparing the preceding results for p_a, p_b, p_c and p_d, we see that the shear stress in the wall governs and the allowable pressure in the tank is $p_{allow} = 666 psi.$

Cylindrical Pressure Vessel

Examples: Compressed air tanks, rocket motors, fire extinguishers, spray cans, propane tanks, grain silos, pressurized pipes, etc.

We will consider the normal stresses in a thin walled circular tank *AB* subjected to internal pressure *p*.

σ_1 and σ_2 are the membrane stresses in the wall. No shear stresses act on these elements because of the symmetry of the vessel and its loading, therefore σ_1 and σ_2 are the principal stresses.
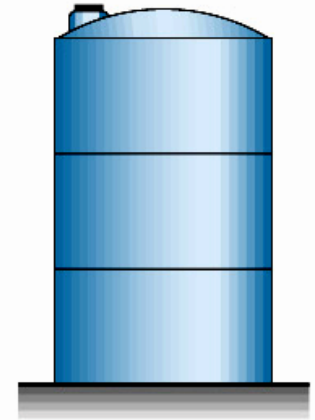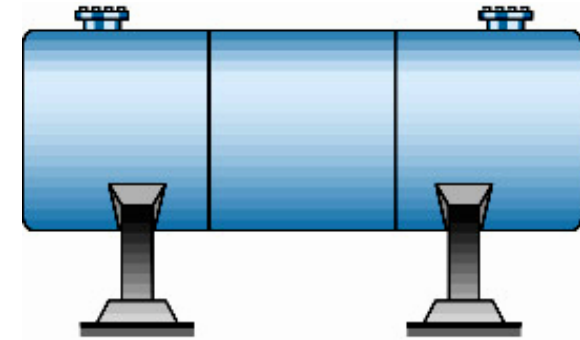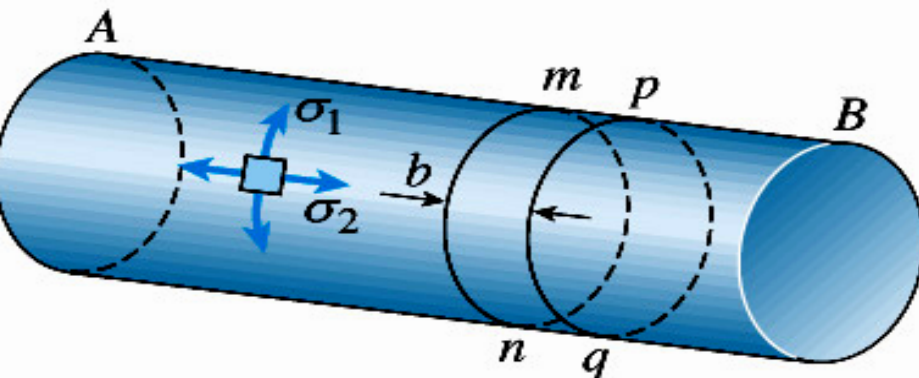
Because of their directions, the stress σ_1 is called *circumferential stress* or the *hoop stress*, and the stress σ_2 is called the *longitudinal stress* or the *axial stress*.

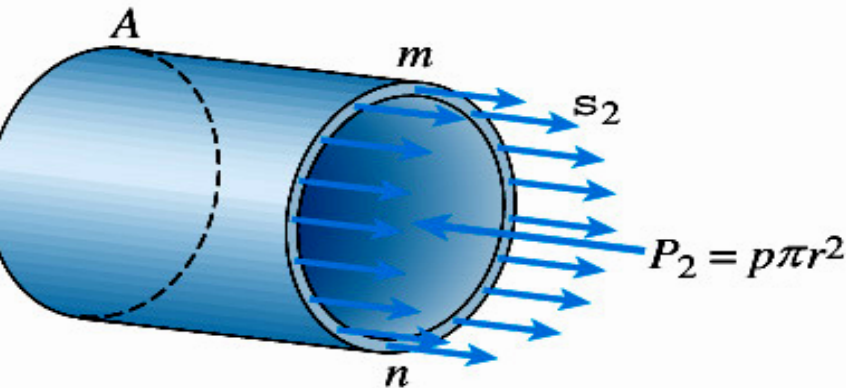Equilibrium of forces to find the circumferential stress:

$$\sigma_1(2b)t = p(2b)r$$

$$\sigma_1 = \frac{pr}{t}$$

(b)

$P_1 = 2pbr$
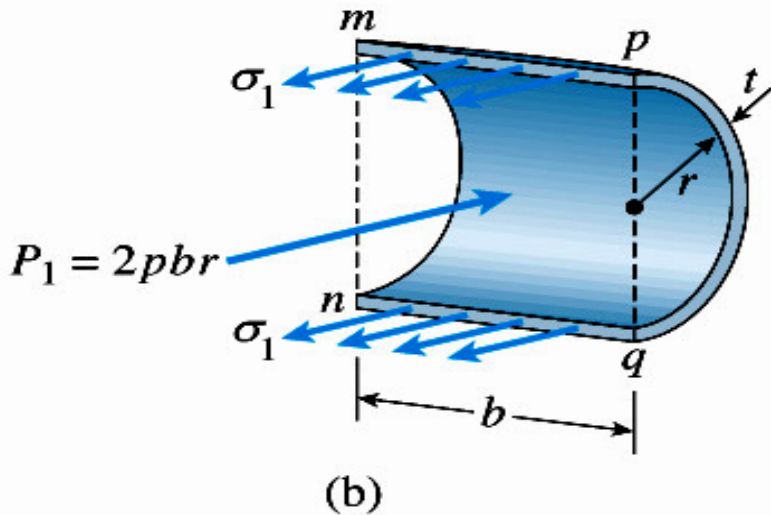
Equilibrium of forces to find the longitudinal stress:

$$\sigma_2(2\pi r)t = p(\pi r^2)$$

$$\sigma_2 = \frac{pr}{2t}$$

$P_2 = p\pi r^2$

This longitudinal stress is equal to the membrane stress in a spherical vessel. Then:

$$\sigma_1 = 2\sigma_2$$

We note that the longitudinal welded seam in a pressure tank must be twice as strong as the circumferential seam.

Stresses at the Outer Surface

The principal stresses σ_1 and σ_2 at the outer surface of a cylindrical vessel are shown below. Since σ_3 is zero, the element is in biaxial stress.

The maximum in plane shear stress occurs on planes that are rotate 45° about the *z-axis*

$$\left(\tau_{Max}\right)_z = \frac{\left(\sigma_1 - \sigma_2\right)}{2} = \frac{pr}{4t}$$
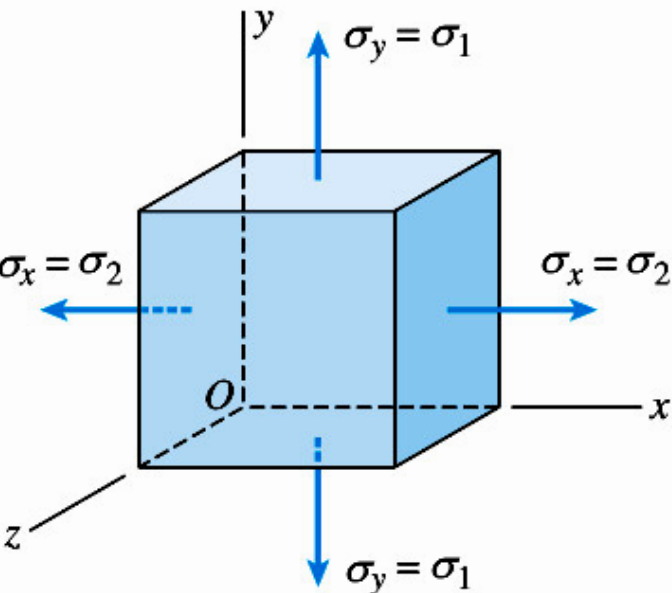
The maximum out of plane shear stresses are obtain by 45° rotations about the *x* and *y* axes respectively.

$$\tau_{Max,x} = \frac{\sigma_1}{2} = \frac{pr}{2t}$$

$$\tau_{Max,y} = \frac{\sigma_2}{2} = \frac{pr}{4t}$$

(a)

Then, the absolute maximum shear stress is $\tau_{max} = pr/2t$, which occurs on a plane that has been rotated 45° about the *x-axis*.
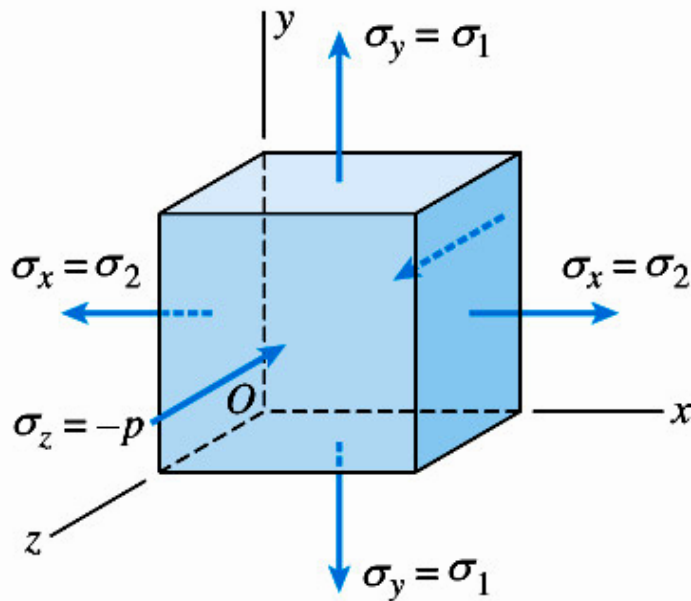
Stresses at the Inner Surface

The principal stresses are

$$\sigma_1 = \frac{pr}{t} \qquad \sigma_2 = \frac{pr}{2t} \qquad \text{and} \qquad \sigma_3 = -p$$

The three maximum shear stresses, obtained by *45°* rotations about the *x*, *y* and *z* axes are

$$(\tau_{MAX})_x = \frac{(\sigma_1 - \sigma_3)}{2} = \frac{pr}{2t} + \frac{p}{2}$$

$$(\tau_{Max})_y = \frac{(\sigma_2 - \sigma_3)}{2} = \frac{pr}{4t} + \frac{p}{2}$$

$$(\tau_{Max})_z = \frac{(\sigma_1 - \sigma_2)}{2} = \frac{pr}{4t}$$

(b)

The first of these three stresses is the largest. However, when **r/t** is very large (thin walled), the term **p/2** can be disregarded, and the equations are the same as the stresses at the outer surface.

Summary for Cylindrical vessels with r/t large

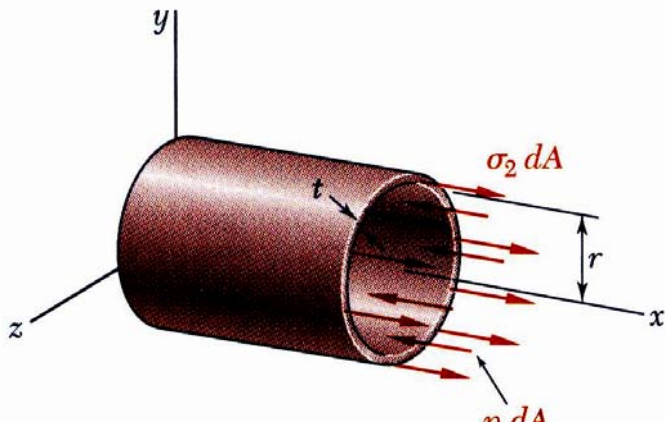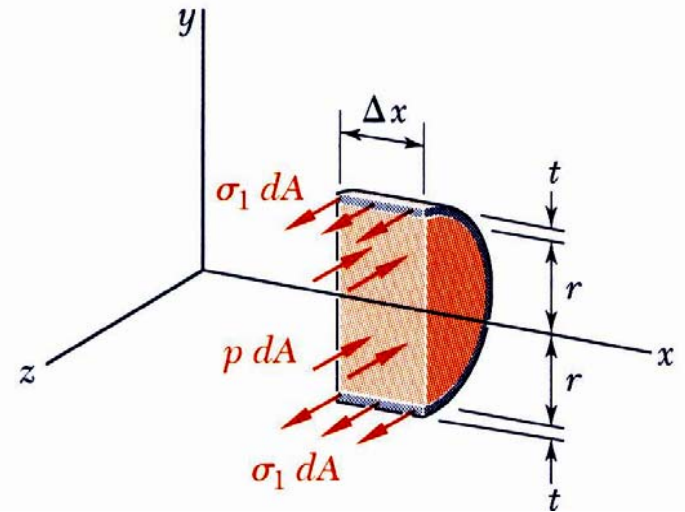Cylindrical vessel with principal stresses

σ_1 = hoop stress

σ_2 = longitudinal stress
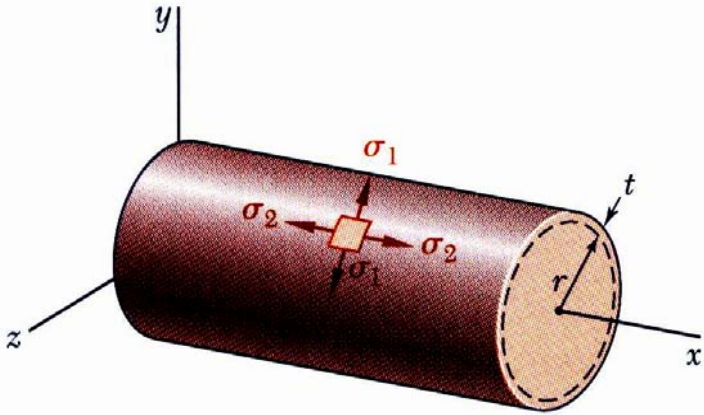
Hoop stress:

$$\sum F_z = 0 = \sigma_1(2t\,\Delta x) - p(2r\,\Delta x)$$
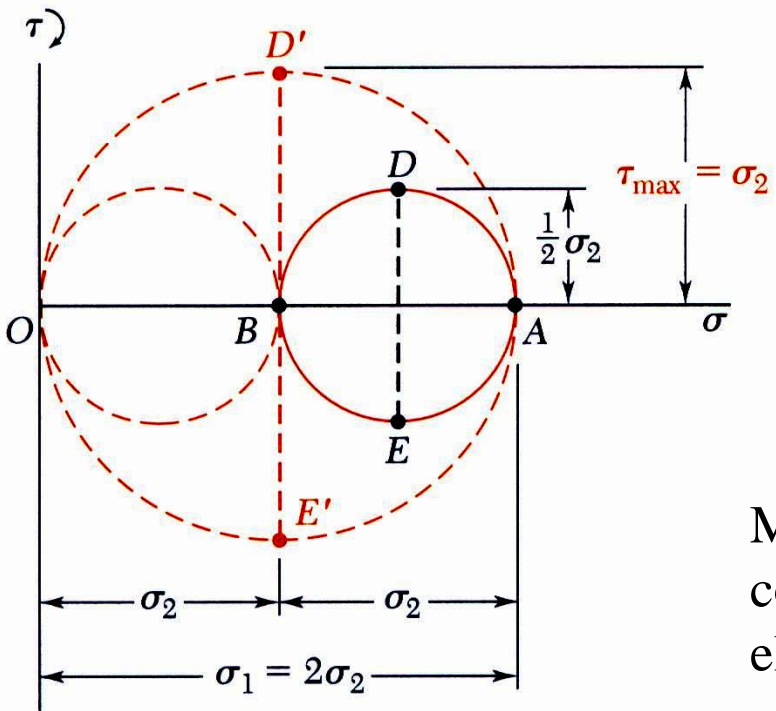
$$\sigma_1 = \frac{pr}{t}$$

Longitudinal stress:

$$\sum F_x = 0 = \sigma_2(2\pi rt) - p(\pi r^2)$$

$$\sigma_2 = \frac{pr}{2t}$$

$$\sigma_1 = 2\sigma_2$$

Points A and B correspond to hoop stress, σ_1, and longitudinal stress, σ_2

Maximum in-plane shearing stress:

$$\tau_{\text{max(in-plane)}} = \frac{1}{2}\sigma_2 = \frac{pr}{4t}$$

Maximum out-of-plane shearing stress corresponds to a 45° rotation of the plane stress element around a longitudinal axis

$$\tau_{\text{max}} = \sigma_2 = \frac{pr}{2t}$$

A cylindrical pressure vessel is constructed from a long, narrow steel plate by wrapping the plate around a mandrel and then welding along the edges of the plate to make an helical joint (see figure below). The helical weld makes an angle $\alpha = 55^o$ with the longitudinal axis. The vessel has an inner radius $r=1.8m$ and a wall thickness $t = 20mm$. The material is steel with a modulus $E=200GPa$ and a Poisson's ratio $\nu=0.30$. The internal pressure p is $800kPa$

Calculate the following quantities for the cylindrical part of the vessel:

The circumferential and longitudinal stresses σ_1 and σ_2 respectively;

The maximum in-plane and out-of-plane shear stresses

The circumferential and longitudinal strains ε_1 and ε_2 respectively, and
The normal stress σ_w and shear stress τ_w acting perpendicular and parallel, respectively, to the welded seam.
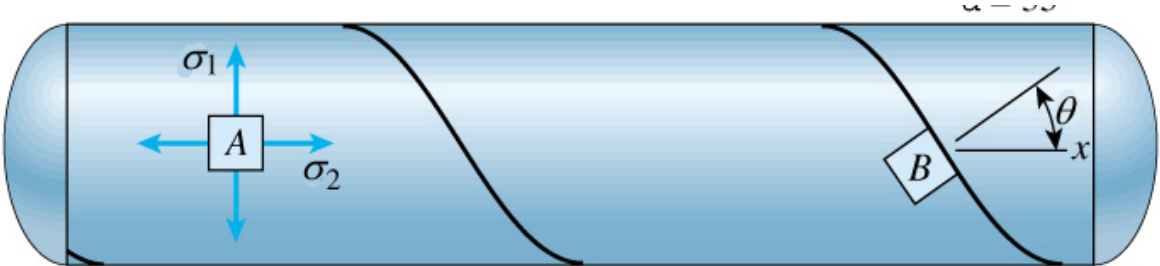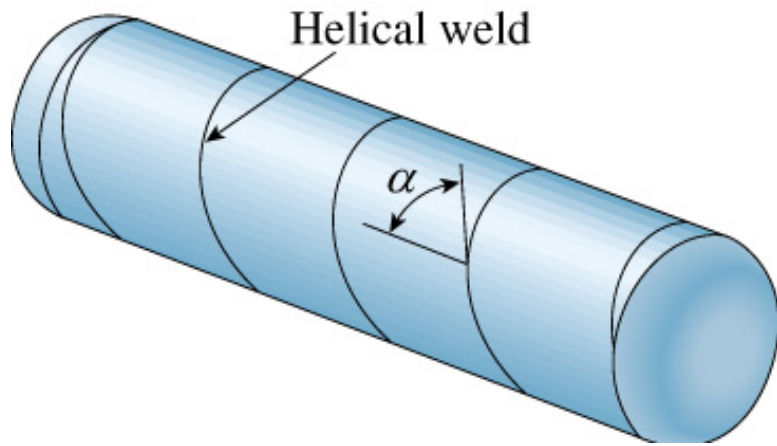
Solution

Circumferential and longitudinal stresses:

$$\sigma_1 = \frac{pr}{t} = \frac{(800\text{kPa})(1.8m)}{0.02m} = 72 MPa$$

$$\sigma_2 = \frac{pr}{2t} = \frac{(800\text{kPa})(1.8m)}{2(0.02m)} = 36 MPa$$

(b) Maximum Shear Stress

The largest in-plane shear stress is obtained from the equation

$$\left(\tau_{Max}\right)_{in-plane} = \frac{(\sigma_1 - \sigma_2)}{2} = \frac{pr}{4t} = 18 MPa$$

The largest out-of-plane shear stress is obtained from the equation:

$$\left(\tau_{Max}\right)_{out-of-plane} = \frac{(\sigma_1)}{2} = \frac{pr}{t} = 36 MPa$$

This last stress is the absolute maximum shear stress in the wall of the vessel.

(c) Circumferential and longitudinal strains.

Assume the Hook's law applies to the wall of the vessel. Using the equations

$$\varepsilon_x = \frac{\sigma_x}{E} - \upsilon \frac{\sigma_y}{E}$$

$$\varepsilon_y = -\upsilon \frac{\sigma_x}{E} + \frac{\sigma_y}{E}$$

We note that $\varepsilon_x = \varepsilon_2$ and $\varepsilon_y = \varepsilon_1$ and also $\sigma_x = \sigma_2$ and $\sigma_y = \sigma_1$. Therefore the above equations can be written in the following forms:
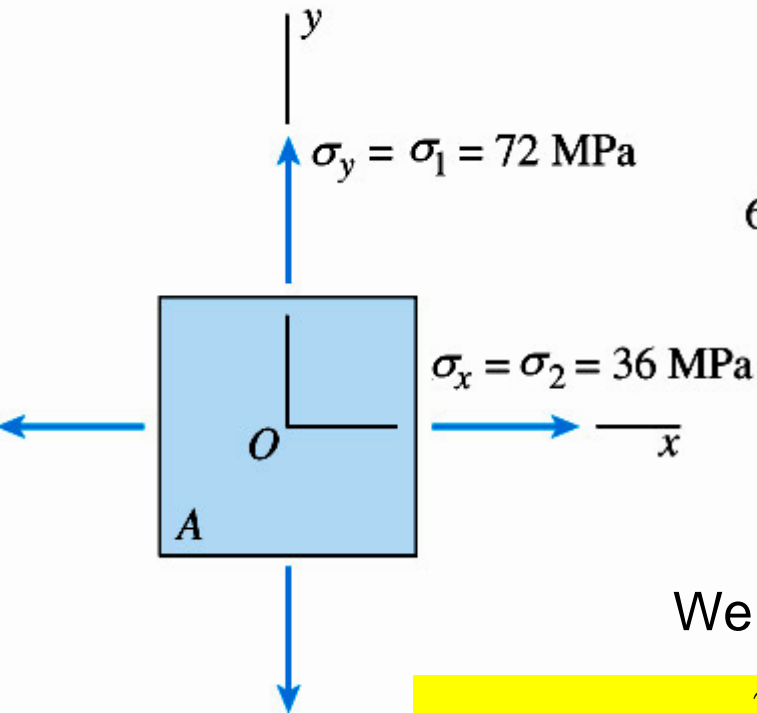
$$\varepsilon_2 = \frac{(1-2\upsilon)pr}{2tE} = \frac{(1-2\upsilon)\sigma_2}{E} = \frac{(1-2(0.30))(36\,MPa)}{200\,GPa} = 72\,x10^{-6}$$

$$\varepsilon_1 = \frac{(2-\upsilon)pr}{2tE} = \frac{(2-\upsilon)\sigma_1}{2E} = \frac{(2-0.30)(72\,MPa)}{(2)(200\,GPa)} = 306\,x10^{-6}$$

$\sigma_y = \sigma_1 = 72$ MPa

$\sigma_x = \sigma_2 = 36$ MPa

6

(d) Normal and shear stresses acting on the weld seam

The angle θ for the stress element at point **B** in the wall of the cylinder with sides parallel and perpendicular to the weld is

$$\theta = 90^o - \alpha = 35^o$$

We will use the stress transformation equations:

$$\sigma_{x1} = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy}\sin\theta\cos\theta$$

$$\tau_{x1y1} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$\sigma_{x1} + \sigma_{y1} = \sigma_x + \sigma_y$$

$$\sigma_{x1} = 36(\cos(35))^2 + 72(\sin(35))^2 = 47.8 MPa$$

$$\tau_{x1y1} = -36 \cdot \sin(35) \cdot \cos(35) + 72 \cdot \sin(35) \cdot \cos(35) = 16.9 MPa$$

$$47.8 + \sigma_{y1} = 36 + 72 \Rightarrow \sigma_{y1} = 60.2 MPa$$

Mohr's circle

Coordinates of point A (for θ = 0) σ_2 = 36MPa and shear stress = 0

Coordinates of point B (for θ = 90) σ_1 = 72MPa and shear stress = 0
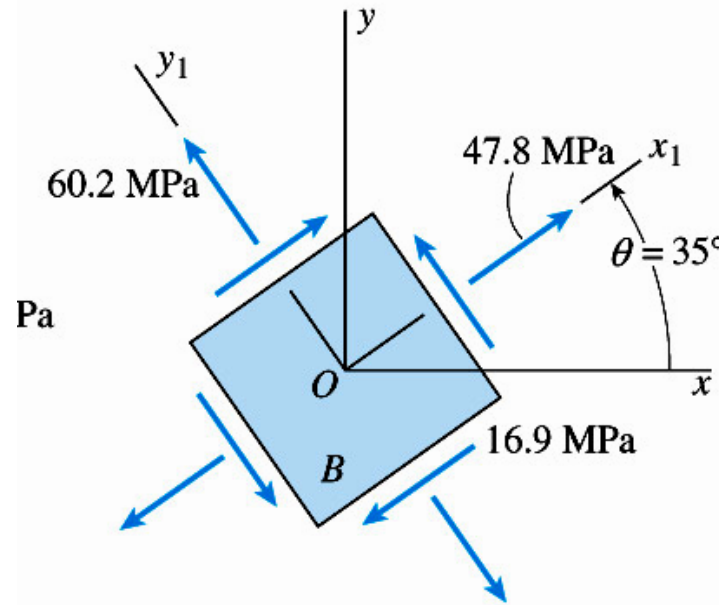
Center (point C) = (σ_1 + σ_2) / 2 = 54MPa

Radius = (σ_1- σ_2) / 2 =18MPa
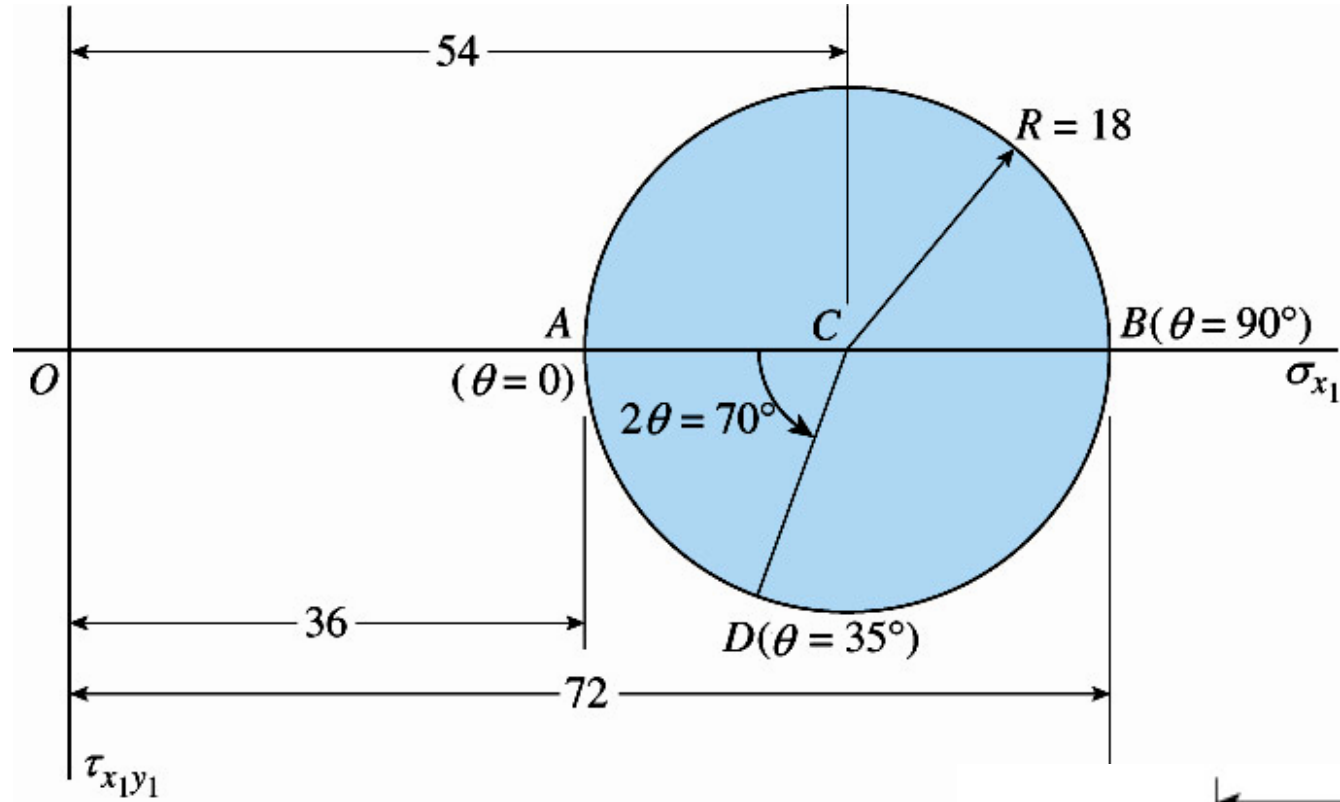

A counterclockwise angle *2θ = 70º* (measured on the circle from point *A*) locates point *D*, which corresponds to the stresses on the *x₁* face (*θ = 35º*) of the element.

The coordinates of point *D*

σ_{x1} = 54 – R cos 70º = 54MPa – (18MPa)(cos 70º) = 47.8MPa

τ_{x1y1} = R sin 70º = (18MPa)(sin 70º) = 16.9MPa

Figure labels (Mohr's circle diagram):
- 54
- $R = 18$
- A $(\theta = 0)$
- C
- $B(\theta = 90°)$, σ_{x_1}
- $2\theta = 70°$
- O
- 36
- $D(\theta = 35°)$
- 72
- $\tau_{x_1 y_1}$

Note:

When seen in a side view, a **helix** follows the shape of a sine curve (see Figure below). The pitch of the helix is $p = \pi d \tan \theta$, where d is the diameter of the circular cylinder and θ is the angle between a normal to the helix and a longitudinal line. The width of the flat plate that wraps into the cylinder shape is $w = \pi d \sin \theta$.

Figure labels: p, θ, d
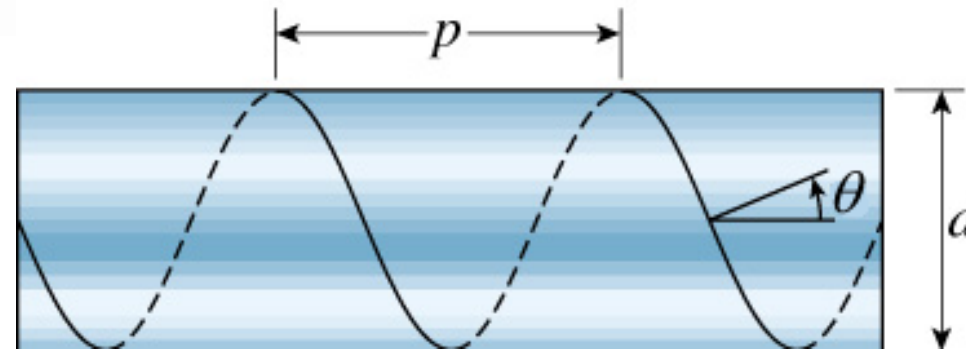
For practical reasons, the angle θ is usually in the range from **20° to 35°**.

Quiz

A close-end pressure vessel has an inside diameter of 1600 mm an a wall thickness of 40mm. It is pressurized to an internal pressure of 6.5MPa an it has a centric compressive force of 750kN applied as shown. The tank is welded together along the helix making an angle of 25° to the horizontal. Determine the normal and shear stresses along the helix.

750kN

25°
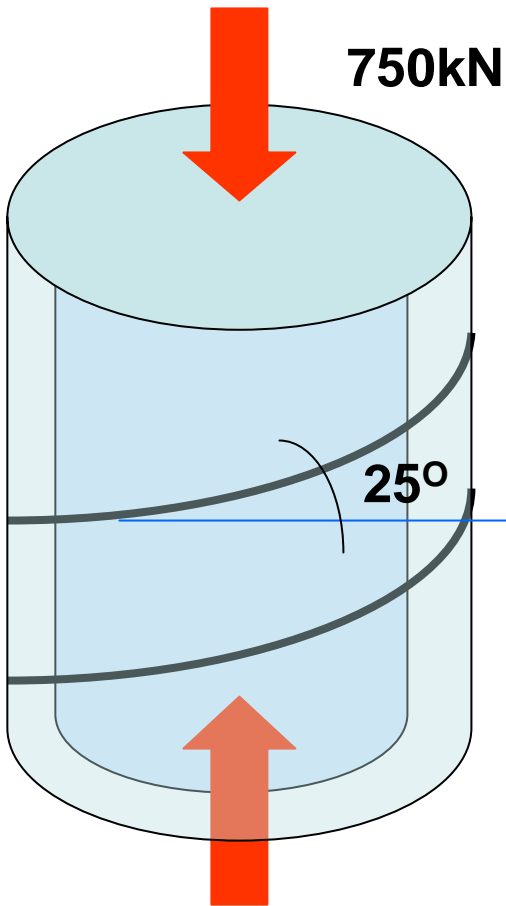
The material is steel with a modulus **E = 200GPa** and a Poisson's ratio $v = 0.30$

Calculate the following quantities for the cylindrical part of the vessel:

(a) The circumferential and longitudinal stresses σ_1 and σ_2 respectively;

(b) The maximum in-plane and out-of-plane shear stresses

(c) The circumferential and longitudinal strains ε_1 and ε_2 respectively, and

(d) The normal stress σ_w and shear stress τ_w acting perpendicular and parallel, respectively, to the welded seam.

General State of Stresses

The most general state of stress at a point may be represented by 6 components
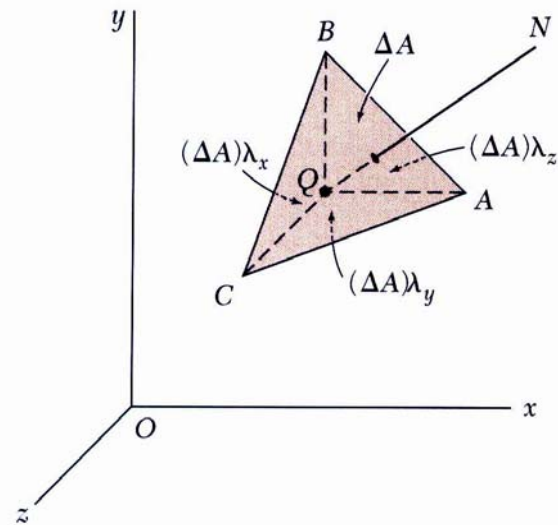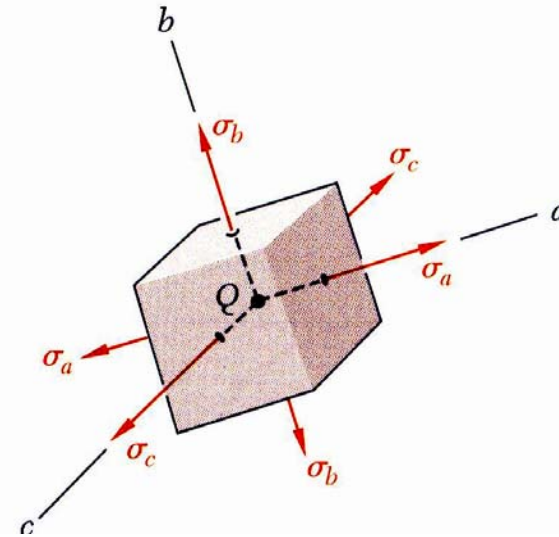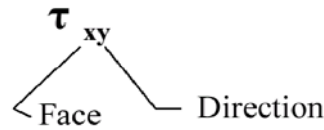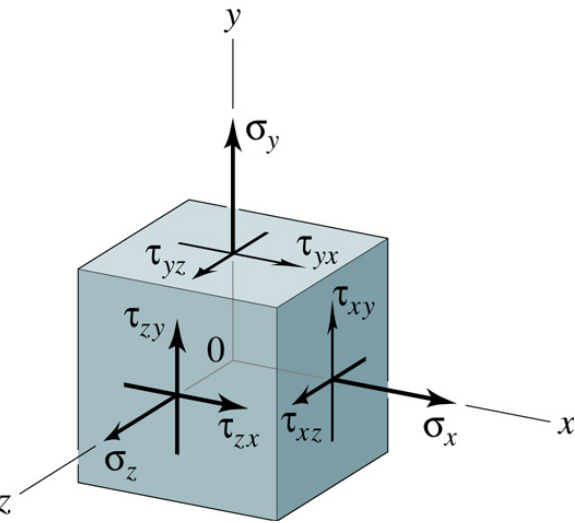
From equilibrium principles:

$$\tau_{xy} = \tau_{yx} \quad, \tau_{xz} = \tau_{zx} \quad, \tau_{zy} = \tau_{yz}$$

Normal Stresses $\quad \sigma_x \quad \sigma_y \quad \sigma_z$

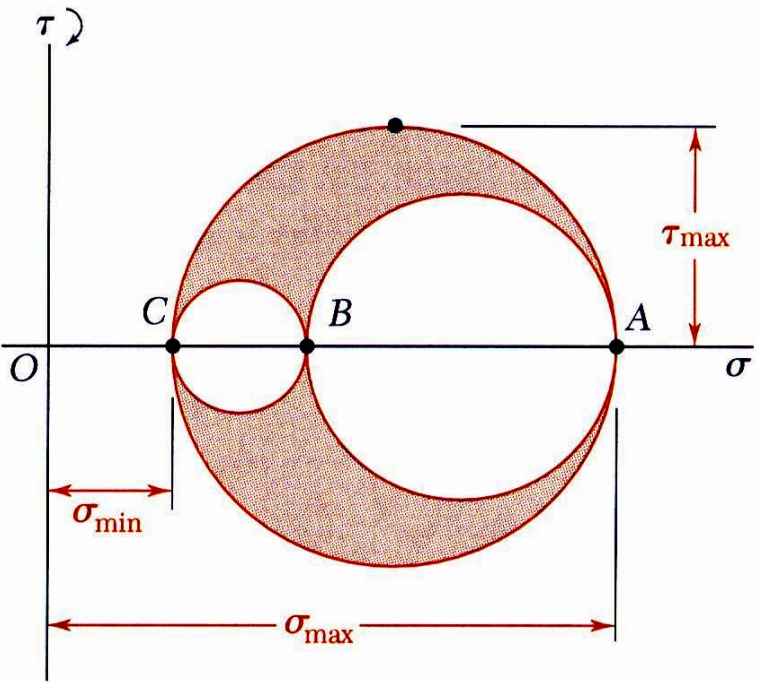**Shear Stresses** $\quad \tau_{xy} \quad \tau_{yz} \quad \tau_{xz}$

These are the principal axes and principal planes and the normal stresses are the principal stresses.

$$\boldsymbol{\sigma} = \sigma_{ij} = \begin{bmatrix} \sigma_x & \tau_{yx} & \tau_{zx} \\ \tau_{xy} & \sigma_y & \tau_{zy} \\ \tau_{xz} & \tau_{yz} & \sigma_z \end{bmatrix}$$

Eigen values $\longrightarrow$

$$\boldsymbol{\sigma}' = \sigma'_{ij} = \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{bmatrix}$$
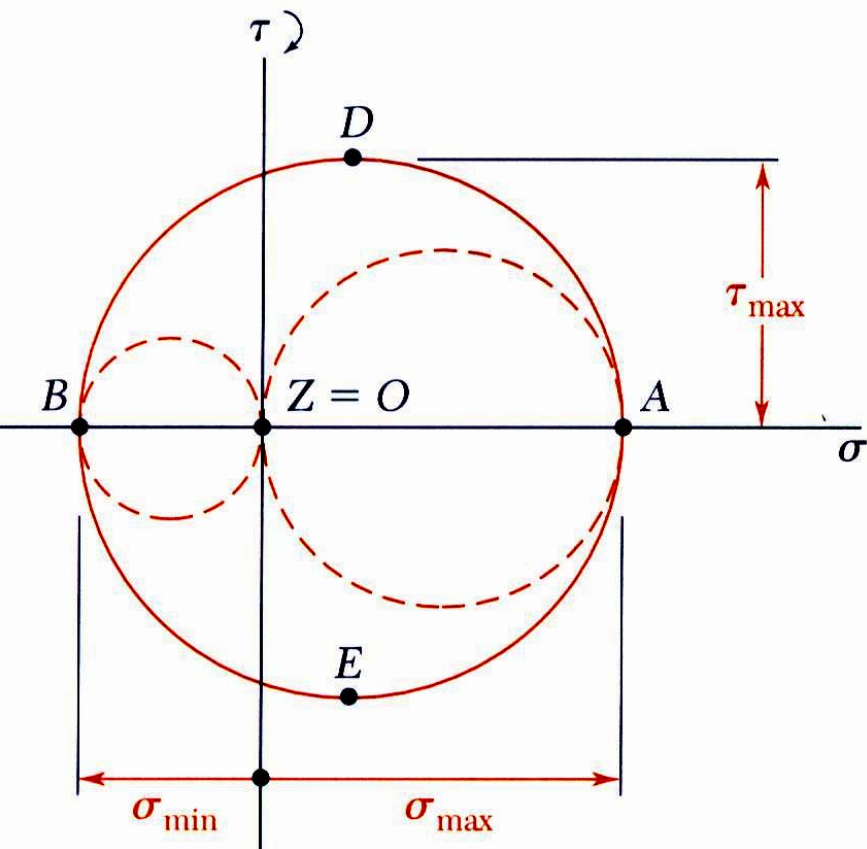

The three circles represent the normal and shearing stresses for rotation around each principal axis.

Points A, B, and C represent the principal stresses on the principal planes (shearing stress is zero)

Radius of the largest circle yields the maximum shearing stress.

$$\tau_{max} = \frac{1}{2} \left| \sigma_{max} - \sigma_{min} \right|$$

In the case of plane stress, the axis perpendicular to the plane of stress is a principal axis (shearing stress equal zero).
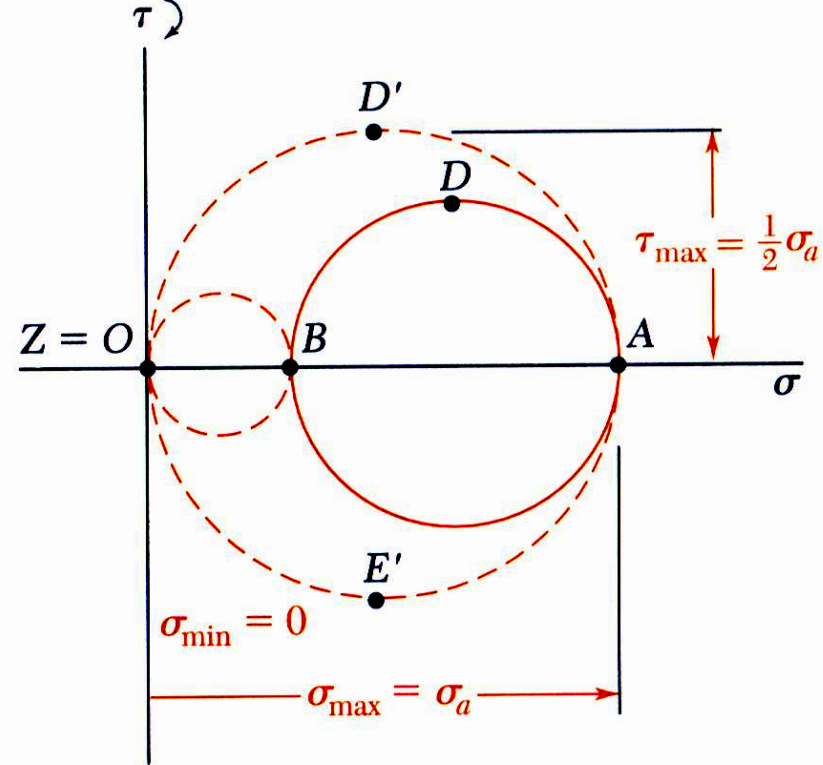
If the points *A* and *B* (representing the principal planes) are on opposite sides of the origin, then

the corresponding principal stresses are the maximum and minimum normal stresses for the element

the maximum shearing stress for the element is equal to the maximum "in-plane" shearing stress

planes of maximum shearing stress are at 45° to the principal planes.
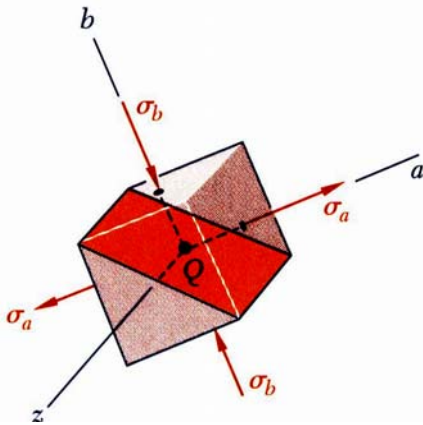
If A and B are on the same side of the origin (i.e., have the same sign), then

the circle defining σ_{max}, σ_{min}, and τ_{max} for the element is not the circle corresponding to transformations within the plane of stress

maximum shearing stress for the element is equal to half of the maximum stress

planes of maximum shearing stress are at 45 degrees to the plane of stress
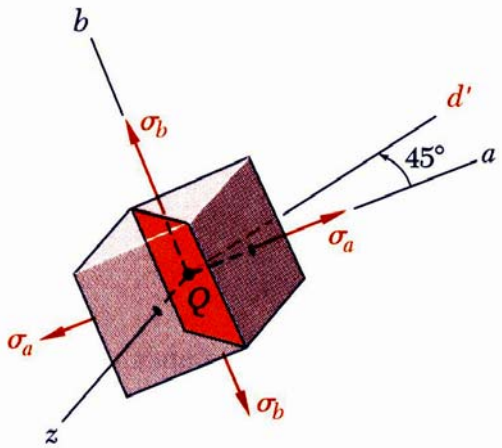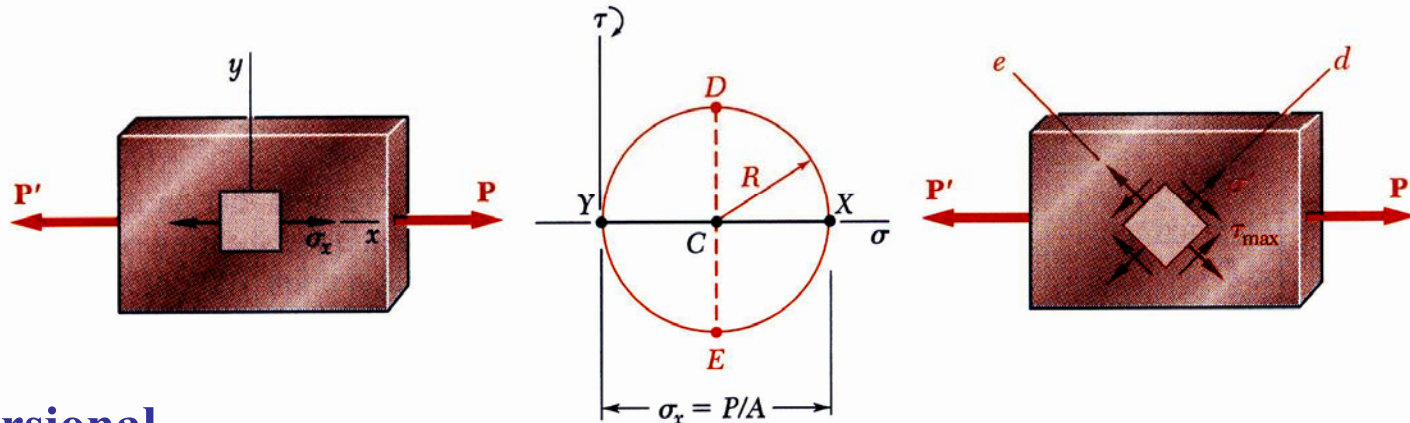
Failure Theories

Why do mechanical components fail? Mechanical components fail because the applied stresses exceeds the material's strength (Too simple).

What kind of stresses cause failure? Under any load combination, there is always a combination of normal and shearing stresses in the material.

Mohr's circle for centric axial loading:

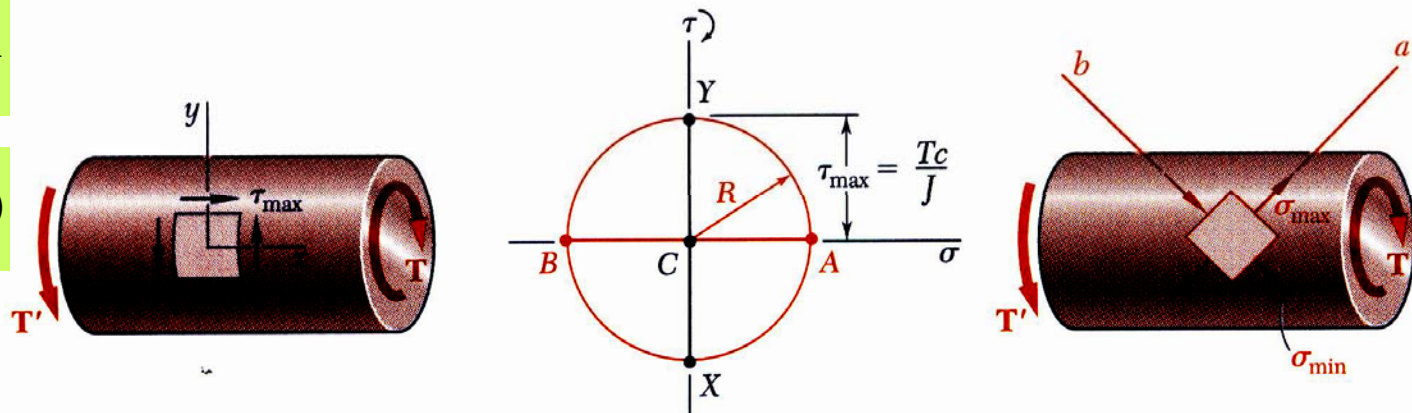$$\sigma_x = \frac{P}{A}, \quad \sigma_y = \tau_{xy} = 0$$

$$\sigma_x = \sigma_y = \tau_{xy} = \frac{P}{2A}$$

Mohr's circle for torsional loading:

$$\sigma_x = \sigma_y = 0 \quad \tau_{xy} = \frac{Tc}{J}$$

$$\sigma_x = \sigma_y = \frac{Tc}{J} \quad \tau_{xy} = 0$$

What is the definition of Failure?
Obviously fracture but in some components yielding can also be considered as failure, if yielding distorts the material in such a way that it no longer functions properly

Which stress causes the material to fail?
Usually *ductile materials* are limited by their *shear strengths*. While *brittle materials* (ductility < 5%) are limited by their *tensile strengths*.

Stress at which point?
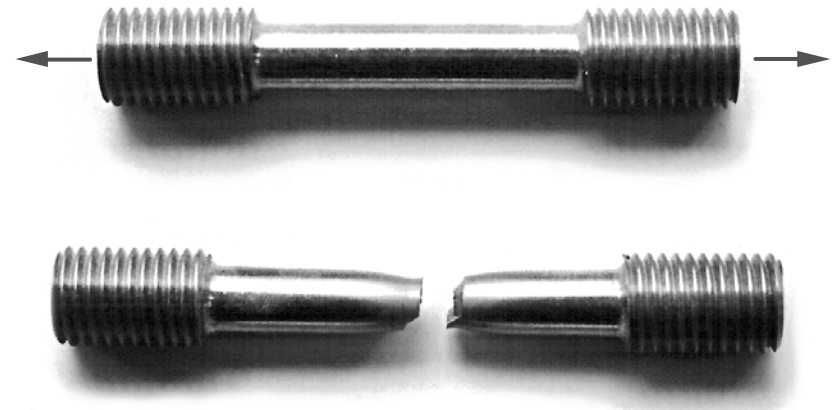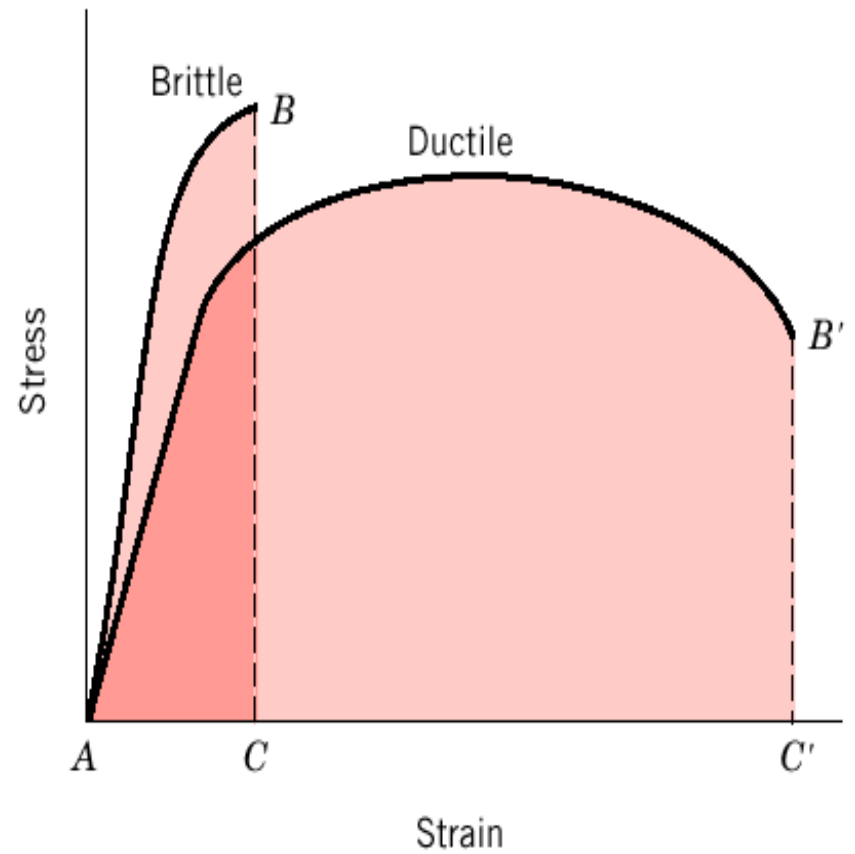
Maximum Shear Stress Theory (MSST)

Failure occurs when the maximum shear stress in the part (subjected to plane stress) exceeds the shear stress in a tensile test specimen (of the same material) at yield.

Maximum shear stress criteria: The structural component is safe as long as the maximum shear stress is less than the maximum shear stress in a tensile test specimen at yield, i.e.,
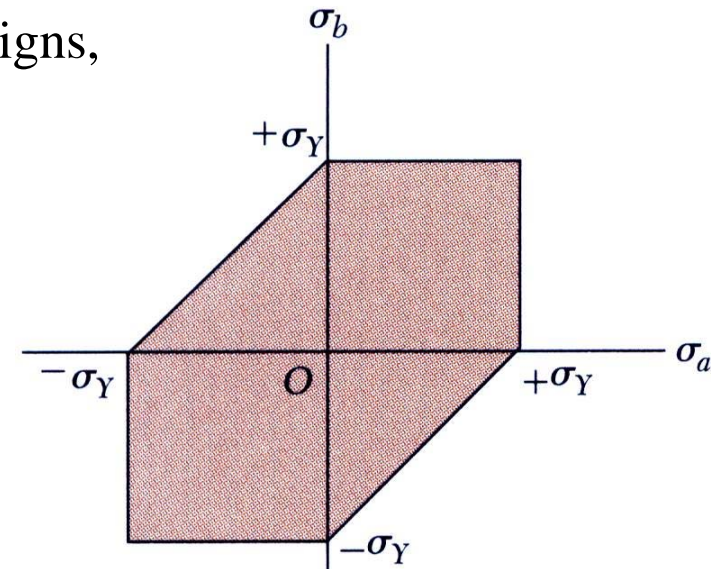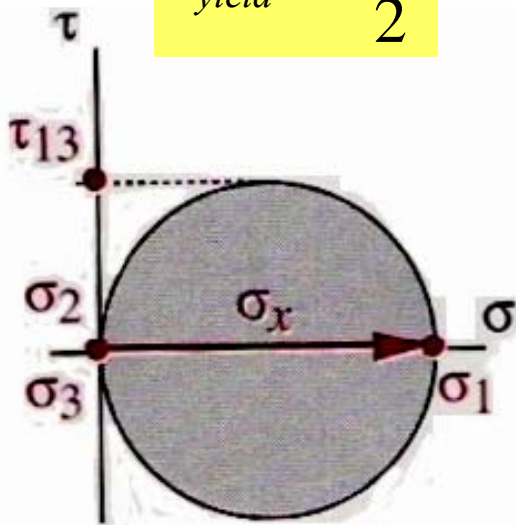
$$\tau_{yield} = \frac{S_y}{2}$$

$$\tau_{max} < \tau_y = \frac{\sigma_y}{2}$$

For σ_a and σ_b with the same sign,

$$\tau_{max} = \frac{|\sigma_a|}{2} \text{ or } \frac{|\sigma_b|}{2} < \frac{\sigma_y}{2}$$

For σ_a and σ_b with opposite signs,

$$\tau_{max} = \frac{|\sigma_a - \sigma_b|}{2} < \frac{\sigma_Y}{2}$$

The maximum absolute shear stress is always the radius of the largest of the Mohr's circle.

Distortion Energy Theory (DET)

Based on the consideration of angular distortion of stressed elements. The theory states that failure occurs when the distortion strain energy in the material exceeds the distortion strain energy in a tensile test specimen (of the same material) at yield.

Maximum distortion energy criteria: Structural component is safe as long as the distortion energy per unit volume is less than that occurring in a tensile test specimen at yield.

Von Mises effective stress : Defined as the uniaxial tensile stress that creates the same distortion energy as any actual combination of applied stresses.

$$u_d < u_Y$$

$$\frac{1}{6G}\left(\sigma_a^2 - \sigma_a\sigma_b + \sigma_b^2\right) < \frac{1}{6G}\left(\sigma_Y^2 - \sigma_Y \times 0 + 0^2\right)$$
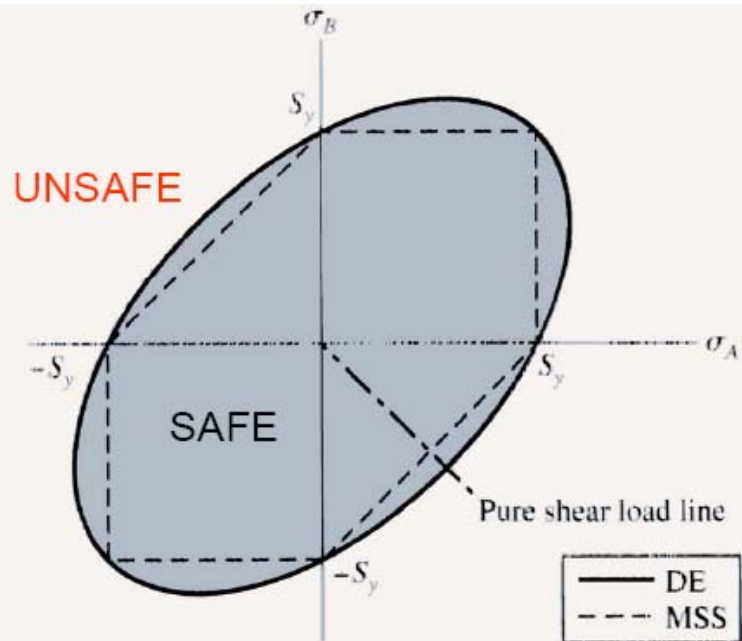
$$\sigma_a^2 - \sigma_a\sigma_b + \sigma_b^2 < \sigma_Y^2$$

$$\sigma_{VM} = \sqrt{\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - \sigma_1\sigma_2 - \sigma_2\sigma_3 - \sigma_3\sigma_1}$$

$$\sigma_{VM} = \sqrt{\sigma_1^2 + \sigma_3^2 - \sigma_3\sigma_1}$$

2D

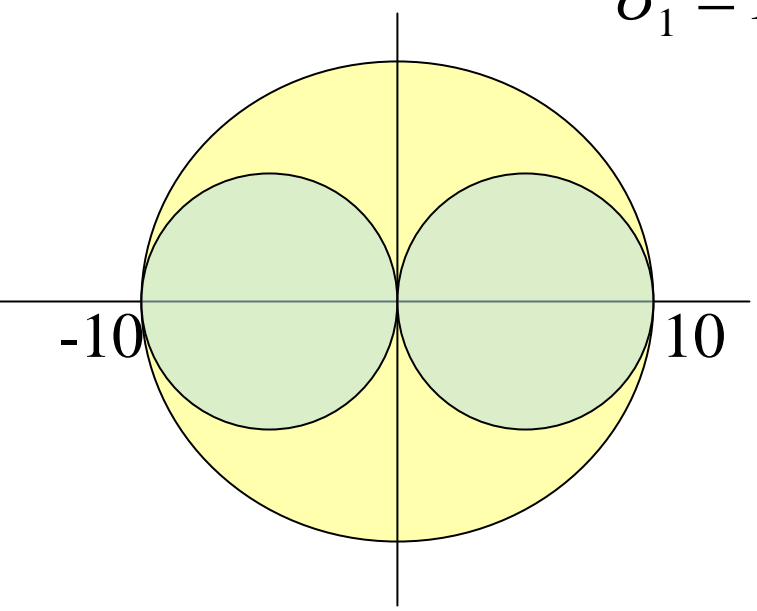$$\sigma_{VM} = \sqrt{\sigma_x^2 + \sigma_y^2 - \sigma_x\sigma_y + 3\tau_{xy}^2}$$

UNSAFE

SAFE

Pure shear load line

—— DE
--- MSS

Given the material S_Y, σ_x, σ_v and τ_{xy} find the safety factors for all the applicable criteria.

a. Pure aluminum $S_Y = 30MPa$ $\sigma_x = 10MPa$ $\sigma_y = -10MPa$ $\tau_{xy} = 0MPa$

$$\sigma_1 = 10MPa \qquad \sigma_3 = -10MPa \qquad \tau_{Max} = 10MPa$$

Ductile

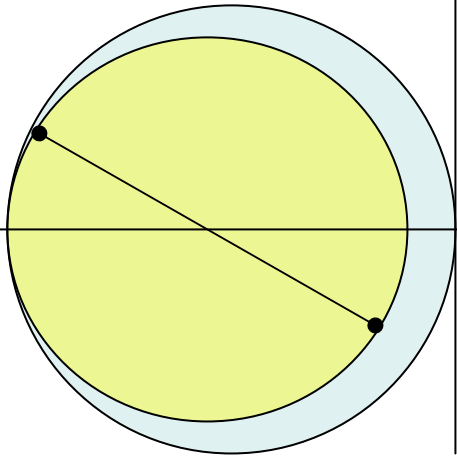Use either the Maximum Shear Stress Theory (MSST) or the Distortion Energy Theory (DET)

MSST Theory

$$n = \frac{S_y}{\sigma_1 - \sigma_3} = \frac{30}{10 - (-10)} = \frac{30}{20} = 1.5$$

DET Theory

$$\sigma_{VM} = \sqrt{\sigma_x^2 + \sigma_y^2 - \sigma_x\sigma_y + 3\tau_{xy}^2} = \sqrt{300} = 17.32MPa$$

$$n = \frac{S_y}{\sigma_{VM}} = \frac{30MPa}{17.32MPa} = 1.73$$

b. 0.2%C Carbon Steel $\quad S_Y = 65 Ksi \quad \sigma_x = -5 Ksi \quad \sigma_y = -35 Ksi \quad \tau_{xy} = 10 Ksi$

In the plane XY the principal stresses are -1.973Ksi and -38.03Ksi with a maximum shear stress in the XY plane of 18.03Ksi

In any orientation

$$\sigma_1 = 0 Ksi \quad \sigma_2 = -1.973 Ksi \quad \sigma_3 = -38.03 Ksi$$

$$\tau_{Max} = 19.01 Ksi$$

Ductile

MSST Theory $\quad n = \dfrac{S_y}{\sigma_1 - \sigma_3} = \dfrac{65}{0 - (-38.03)} = 1.71$

DET Theory $\quad \sigma_{VM} = \sqrt{\sigma_1^2 + \sigma_3^2 - \sigma_1\sigma_3} = 38.03 Ksi$

$$n = \dfrac{S_y}{\sigma_{VM}} = \dfrac{65 Ksi}{38.03 MPa} = 1.71$$