Spin Density Wave Glasses

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Thanks: Leon Balents, T. Giamarchi, D. Huse

Happy Birthday, Steve!

An old and important question

Fate of incommensurate density wave order in the presence of impurities?

This talk: focus mainly on Spin Density Waves (SDW).

Weak non-magnetic disorder → ``Spin Density Wave Glass”

Many interesting properties distinct from conventional spin glass.

Related questions: impurity effects on ``pair density wave” superconductors (Berg, Fradkin, Kivelson, 08)

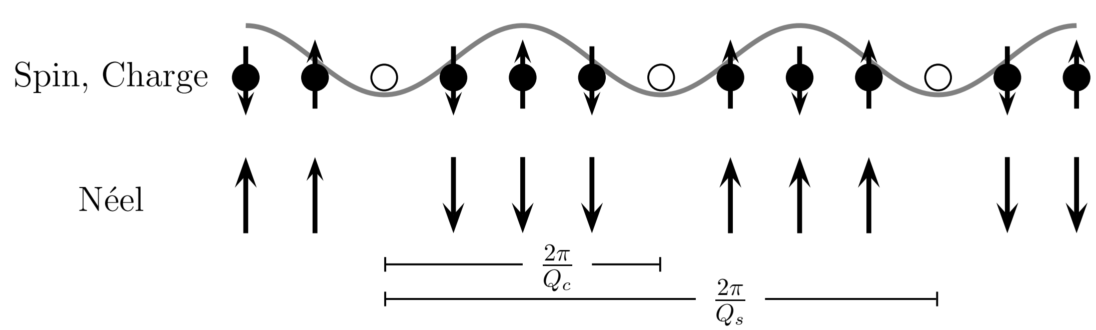
Spin Density Wave (and Pair Density Wave) order:
Order parameters, topological defects.

Some simplifying assumptions

For concreteness consider

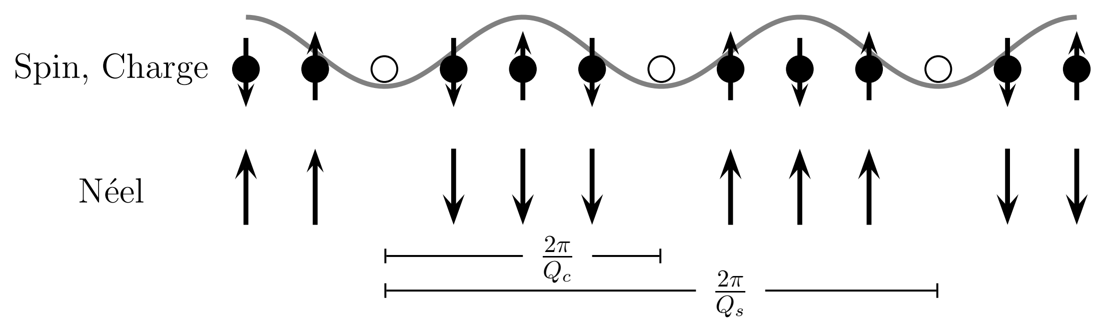
1. Uniaxial SDW (not spiral)
2. Unidirectional in real space with direction chosen by symmetry of underlying crystal.
3. Incommensurate order

``Spin stripes''



``Anti-phase'' stripes, common in La-based cuprates.

Stripes and charge order



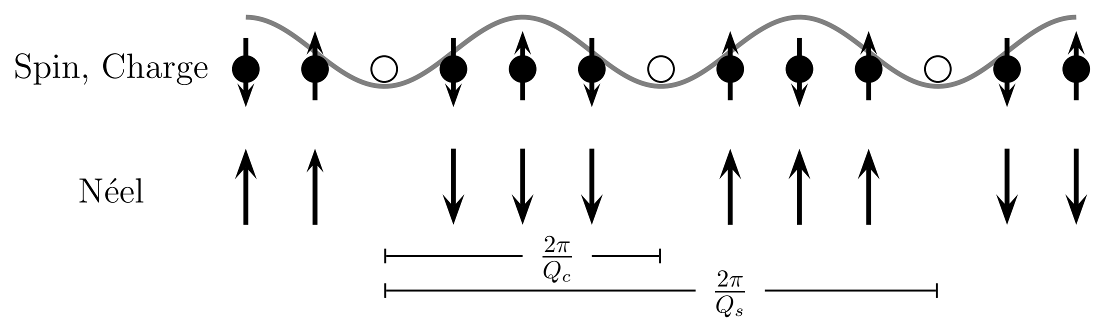
``Anti-phase'' stripes, common in La-based cuprates.

Typically spin stripe implies charge stripe (Landau argument) but charge stripe does not imply spin order.

Term in free energy $\propto \vec{S}^2 \rho$

=> spin order at wave vector $\mathbf{Q}$ accompanied by charge order at wave vector $2\mathbf{Q}$.

SDW order parameter

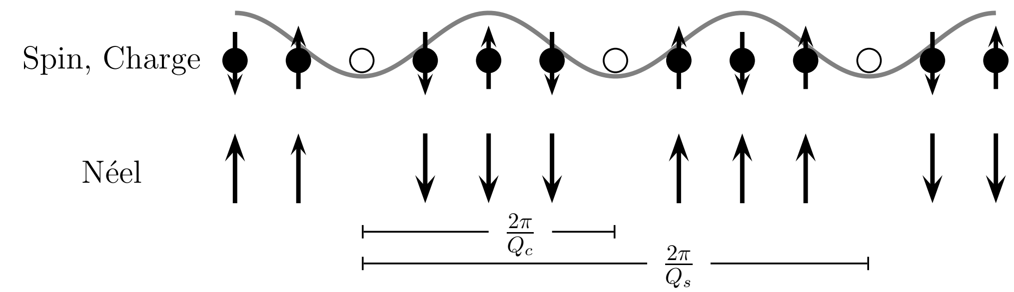


$$\vec{S}_r \sim e^{i\mathbf{Q}.\mathbf{r}} e^{i\theta_s} \vec{N} + c.c$$

SDW order parameter $= e^{i\theta_s}\vec{N}$.
θ_s: stripe displacement
$\vec{N}$: local Neel vector.
CDW order parameter $= e^{2i\theta_s}$.

A convenient formal description

SDW order parameter $= e^{i\theta_s}\vec{N}$.
CDW order parameter $= e^{2i\theta_s}$.



$$\vec{S}_r \sim e^{i\mathbf{Q}.\mathbf{r}} e^{i\theta_s}\vec{N} + c.c$$

Both order parameters are composites of fields $b = e^{i\theta_s}, \vec{N}$.

SDW order parameter $\vec{S}_{\mathbf{Q}} = b\vec{N}$.

CDW order parameter $\rho_{\mathbf{2Q}} \sim b^2$.

Formulation in terms of $b, \vec{N}$ has Z_2 "gauge" redundancy $(b, \vec{N} \to -b, -\vec{N})$.

Zaanen, Nussinov, 2000
Sachdev,.....2001

No fractionalization, etc in this talk. `Slave' formulation useful nevertheless.

Closely related order: Pair Density Wave (PDW) superconductor

Cooper pairing at non-zero momentum (eg, FFLO).

$$\Delta(x) \sim \Delta_0 \cos(\mathbf{Q} \cdot \mathbf{x})$$

Many names:
Larkin-Ovchinnikov SC, PDW SC,
Striped SC, Amperean paired SC

Proposed for LBCO (Berg, Kivelson, ...)

Very similar to SDW with XY spin anisotropy.

However: different action of time reversal.

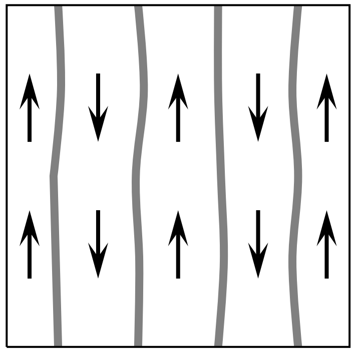
SDW breaks time reversal; PDW preserves it.

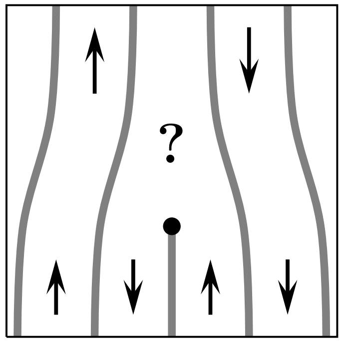
(Formal: Spin U(1) commutes with T-reversal while charge U(1) does not.)

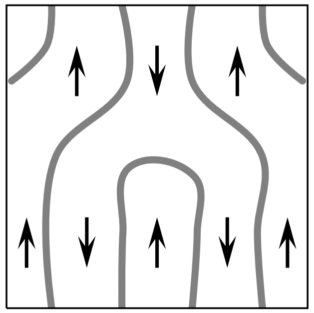
Topological defects in SDW phases

Important class of defects: dislocations in CDW order.

Point defect in 2d, line defect in 3d.







Spin order is frustrated at a strength-1 dislocation.

Spin order not frustrated at a double dislocation.

Strength-1 dislocation necessarily accompanied by 'disclination' where $\vec{N}$ twists by π.

Alternate: Single dislocation bound to Z_2 gauge vortex in $b, \vec{N}$ description.

Fate of incommensurate SDW to non-magnetic impurities?

Impurity: linear coupling to CDW order parameter

$$\int d^d\mathbf{x} v(\mathbf{x}) \rho_{\mathbf{2Q}}(\mathbf{x}) + c.c$$

$$v(\mathbf{x}) \text{ random.}$$

But no linear coupling to SDW order parameter

Larkin/Imry-Ma (1970s): destroy long range CDW order at long scales for d < 4.

Elastic energy lost by following random potential overwhelmed by potential energy gain.

SDW => CDW;

Therefore no CDW => no SDW long range order.

SDW glasses in $d = 3$

Pinned Charge Density Waves: Elastic/Bragg glass

Giamarchi, Le Doussal, 94
Gingras, Huse, 95
D. S. Fisher, 97

Weak disorder in d = 3: CDW enters an `elastic glass' phase where long dislocation loops do not occur.

Describe by random field XY model without vortices.

Many approximate treatments (eg, `Functional' RG near d = 4):

Power law order for CDW order parameter:

$$\overline{\rho^*_{\mathbf{2Q}}(\mathbf{x})\rho_{\mathbf{2Q}}(\mathbf{x}')} \sim \frac{1}{|\mathbf{x}-\mathbf{x}'|^{\eta_c}}$$

Delta function Bragg peaks replaced by power law peaks.

Estimate: $\eta_c \approx 1.1$ (extrapolate from leading order Functional RG

Fate of spin order?

Mross, TS, 14

No long range SDW order.

But no long dislocations => no frozen Z_2 gauge vortices (i.e no π-disclinations of N-vector).

$\vec{N}$ has true long range order.
=> SDW order has power law Bragg peaks.

$$\begin{aligned} \overline{\vec{S}^*_{\mathbf{Q}}(\mathbf{x}) \cdot \vec{S}_{\mathbf{Q}}(\mathbf{x}')} &= \overline{e^{i\theta_s(\mathbf{x})} e^{-i\theta_s(\mathbf{x}')} \vec{N}(\mathbf{x}) \cdot \vec{N}(\mathbf{x}')} \\ &\sim \overline{e^{i\theta_s(\mathbf{x})} e^{-i\theta_s(\mathbf{x}')}} \\ &\sim \frac{1}{|\mathbf{x} - \mathbf{x}'|^{\eta_S}} \end{aligned}$$

Estimate: $\eta_S \approx 0.27$ (leading order Functional RG)

(In general θ_s not Gaussian => $\eta_c \neq 4\eta_s$). See, eg, Federenko, LeDoussal, Wiese, 2014

Long range spin nematic order

Mross,TS, 14

Meaning of ordering of $\vec{N}$?

$\vec{N}$ not gauge invariant and hence not observable.
$\vec{N}$ order => Long range spin quadrupole order (= spin nematic).

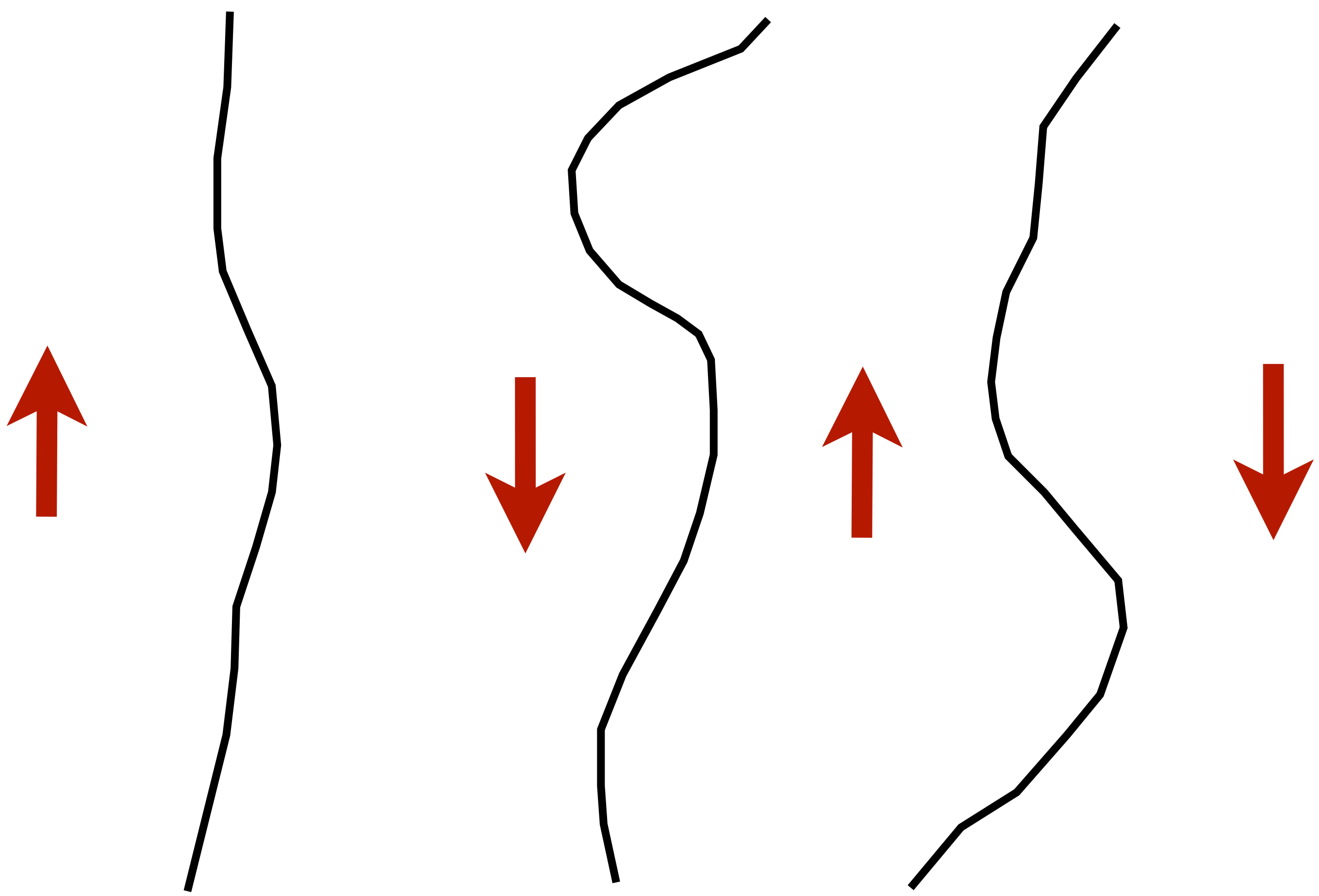
Order parameter

$$Q_{\alpha\beta} = N_\alpha N_\beta - \delta_{\alpha\beta}\frac{\vec{N}^2}{3}$$

Spontaneous spin anisotropy without long range SDW order.

Similar: thermal/quantum melting of spin stripes → spin nematic (Zaanen 2000).

Physical picture



Spins are frozen but SDW phase is randomly disordered.

Spins retain common axis along which they point up or down (spin nematic LRO).

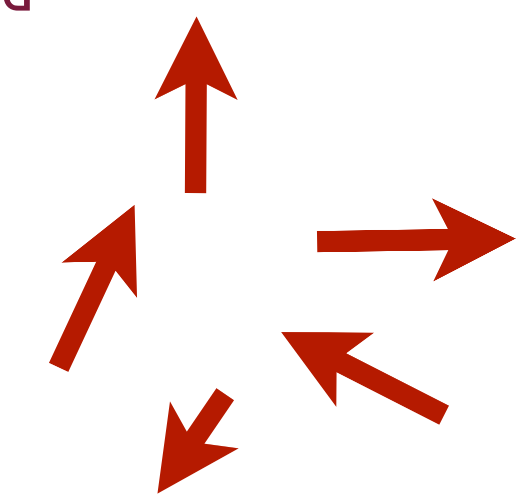
Uniaxial spin glass

Spin freezing => Edwards-Anderson spin glass order parameter.

$$lt_{t\to\infty}\overline{\vec{S}(\mathbf{x},t)\cdot\vec{S}(\mathbf{x},0)} = q_{EA} \neq 0.$$

Uniaxial spin glass in a Heisenberg system with easy axis determined spontaneously.

Clearly distinct from the `conventional' Heisenberg spin glass.



Heisenberg spin glass

Contrast with the `standard' Heisenberg spin glass

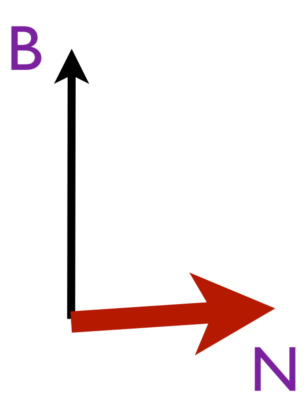
Spin Density Wave Glass

1. Power law correlations of SDW order coexisting with spin glass order parameter.

2. Spontaneous spin anisotropy, propagating gapless nematic director waves (Goldstone modes of spin nematic order).

3. Different magnetic field behavior.
Director orients perpendicular to field.

Weak intrinsic spin anisotropy can pin director.



Example: B along an easy axis
$B \ll B_p$: glassy as stripes reorganize to accommodate magnetization
$B \gg B_p$: glass effects reduced in magnetization as spins reorient.
B_p: field required to 'depin' director.

Comments

1. SDW glass presumably relevant to all linear polarized incommensurate SDW materials in 3d, for instance in $Cr_{1-x}V_x$.

2. SDW systems - opportunity to study Bragg glass physics in experiments?

3. Even some classic spin glasses (eg CuMn) show strong short range SDW order. SDW glass physics relevant starting point?

SDW and PDW glasses in $d = 2$

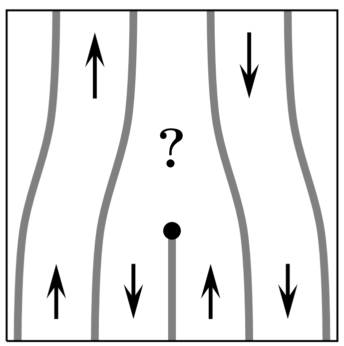
Pinned SDW (or PDW) in 2d

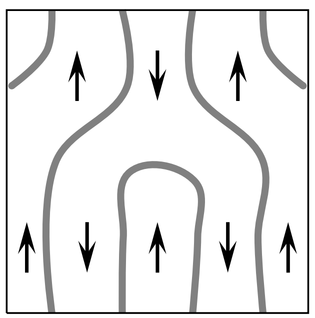
Weak disorder:

Larkin-Imry-Ma: Accompanying CDW disordered beyond finite length scale ξ_L

Fate of dislocations?

Single and double dislocations possibly different.





Competing effects:
Single dislocation: Elastic energy of both CDW distortion and spin disclination versus optimal potential energy gain

Doubled dislocation: No spin disclination => weaker elastic energy cost for dislocation

Doubled dislocations

Same analysis as for ordinary pinned CDW

Zeng, Leath, D. Fisher, 1999
Le Doussal, Giamarchi, 2000

Length scales $L \gg \xi_L$:
Average elastic energy cost $\overline{E_{elast}} \sim K \ln(L)$.
Optimal potential energy gain for isolated dislocations $E_V \sim (\ln(L))^{\frac{3}{2}}$
Always favor isolated strength-2 dislocations at long scales.

=> Destroy elastic glass phase in 2d always.

Exponential decay of CDW and hence SDW correlations at long scales.

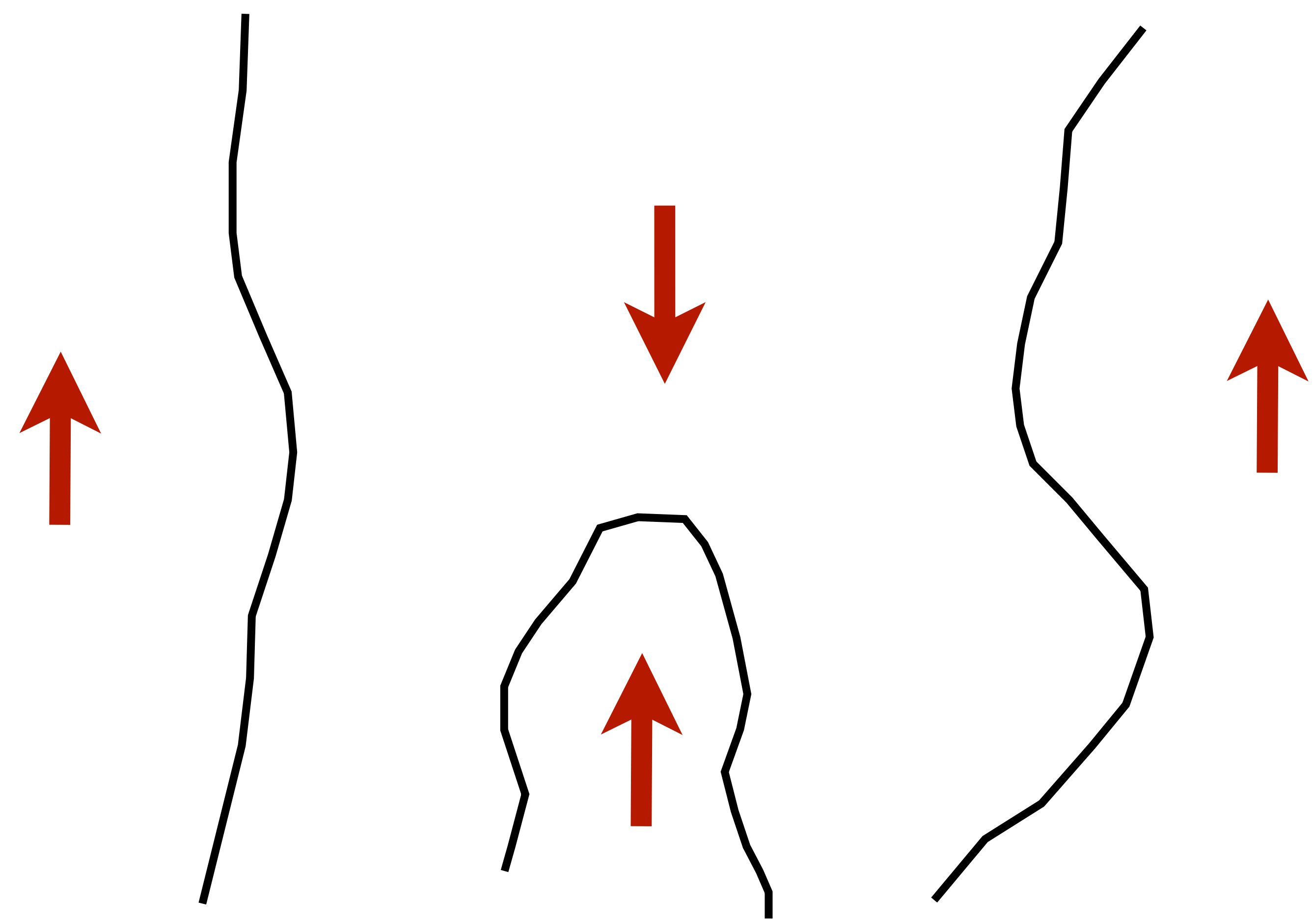
Weak disorder: Long separation between Larkin length and dislocation scale (Giamarchi, Le Doussal, 1994, 2000)

More subtle: fate of single dislocations?

Are single dislocations ***necessarily*** generated at long scales for weak disorder?

Due to their higher elastic cost might expect these are less likely than double dislocations.

Single dislocations absent => long range spin nematic order survives even in 2d in the SDW glass.



Meaning in Pair Density Wave context

Berg, Fradkin, Kivelson 2009

Spin Density Wave	Pair Density Wave
Spin Nematic	Charge-4 superconductivity
Strength-1 CDW dislocation bound to π-disclination of spin	Strength-1 CDW dislocation bound to **hc/4e SC vortex**

Does weakly disordered 2d PDW necessarily generate strength-1 dislocations
=> ***random hc/4e SC vortices of either sign?***

For PDW this breaks T-reversal symmetry locally.

(Detect through local vortex probes in LBCO?)

A conjecture

Mross, TS

Weak non-magnetic disorder: Strength-1 dislocations are not generated at long scales.

=>

1. Spin nematic LRO persists in 2d in SDW glass

2. Weakly disordered PDW preserves time reversal symmetry (no random half-vortices of SC order).

Hints-I
Qualitative argument

Suppress single dislocations by hand in the SDW glass, and examine stability of resulting state to introducing them.

Long range spin nematic order: elastic energy $\approx K_s \ln(L)$.

Random energy gain in fully disordered CDW state presumably just a constant

=> Single dislocations not favored

SDW glass coexisting with spin nematic LRO stable.

Hints-II
Numerics

Mross, TS, 14

Simple model of 2 coupled XY orders
(one for CDW and the other for SDW).

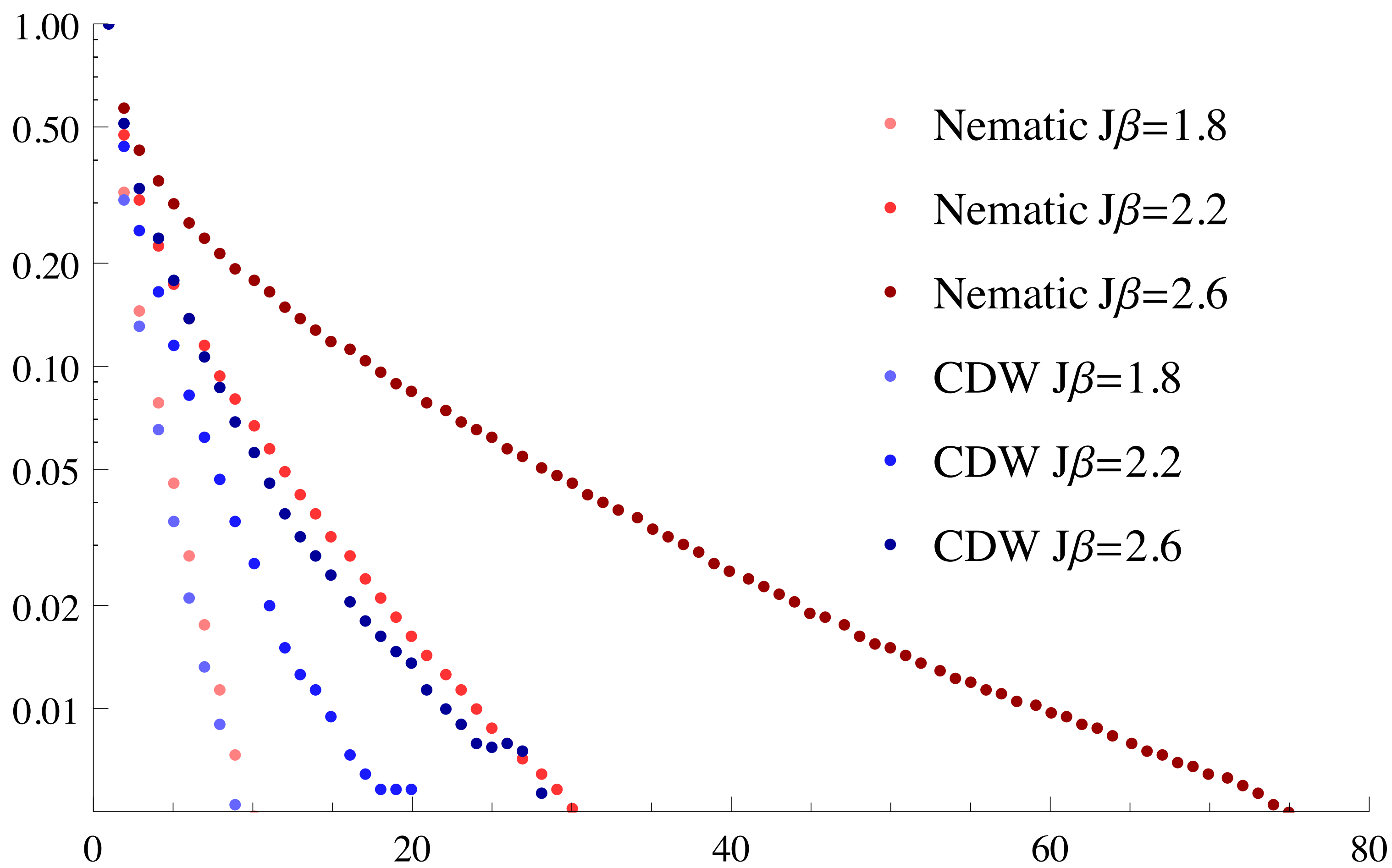
$$H = -J \sum_{<ij>} \cos(\nabla\theta_N)\cos(\nabla\theta_s) - h\sum_i \cos(2\theta_{si} - \alpha_i)$$

CDW order parameter $= e^{2i\theta_s}$
(XY) SDW order parameter $= e^{i(\theta_N + \theta_s)}$
$\alpha_i \in [0, 2\pi]$, random.

Study through Monte Carlo simulations on 100 x100 lattice at not too low temperature.

Persistence of nematic correlations.

Mross, TS, 14



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To be settled.....

Summary

Weakly disordered uniaxial incommensurate SDW → `Spin Density Wave Glass' without long range SDW order.

In 3d (and may be also 2d) spin nematic order survives: uniaxial spin glass in a Heisenberg spin system.

In 3d, but not 2d, SDW order has power law correlations (SDW Bragg glass)

Interesting experimental opportunity to probe Bragg glass physics?

Similar phenomena in PDWs.