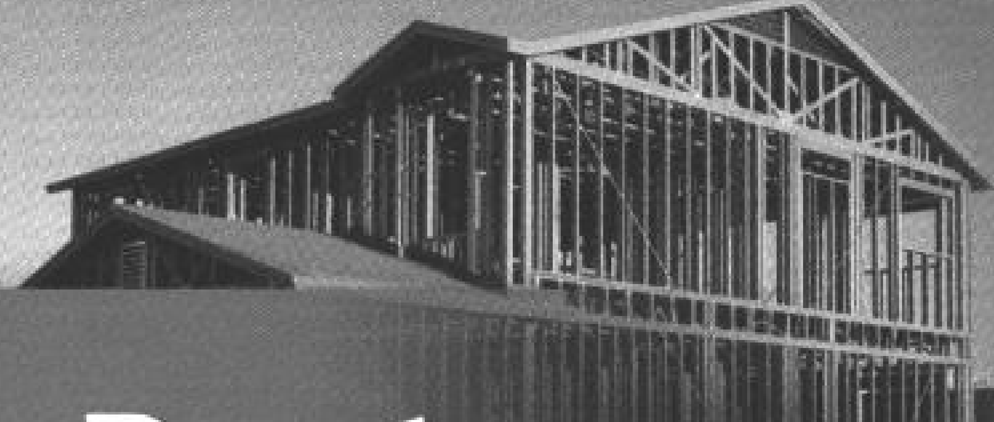
UNIT 8

Square Roots and Pythagoras



Some of the greatest builders are also great mathematicians. Geometry is their specialty. Look at the architecture on these pages.
What aspects of geometry do you see?

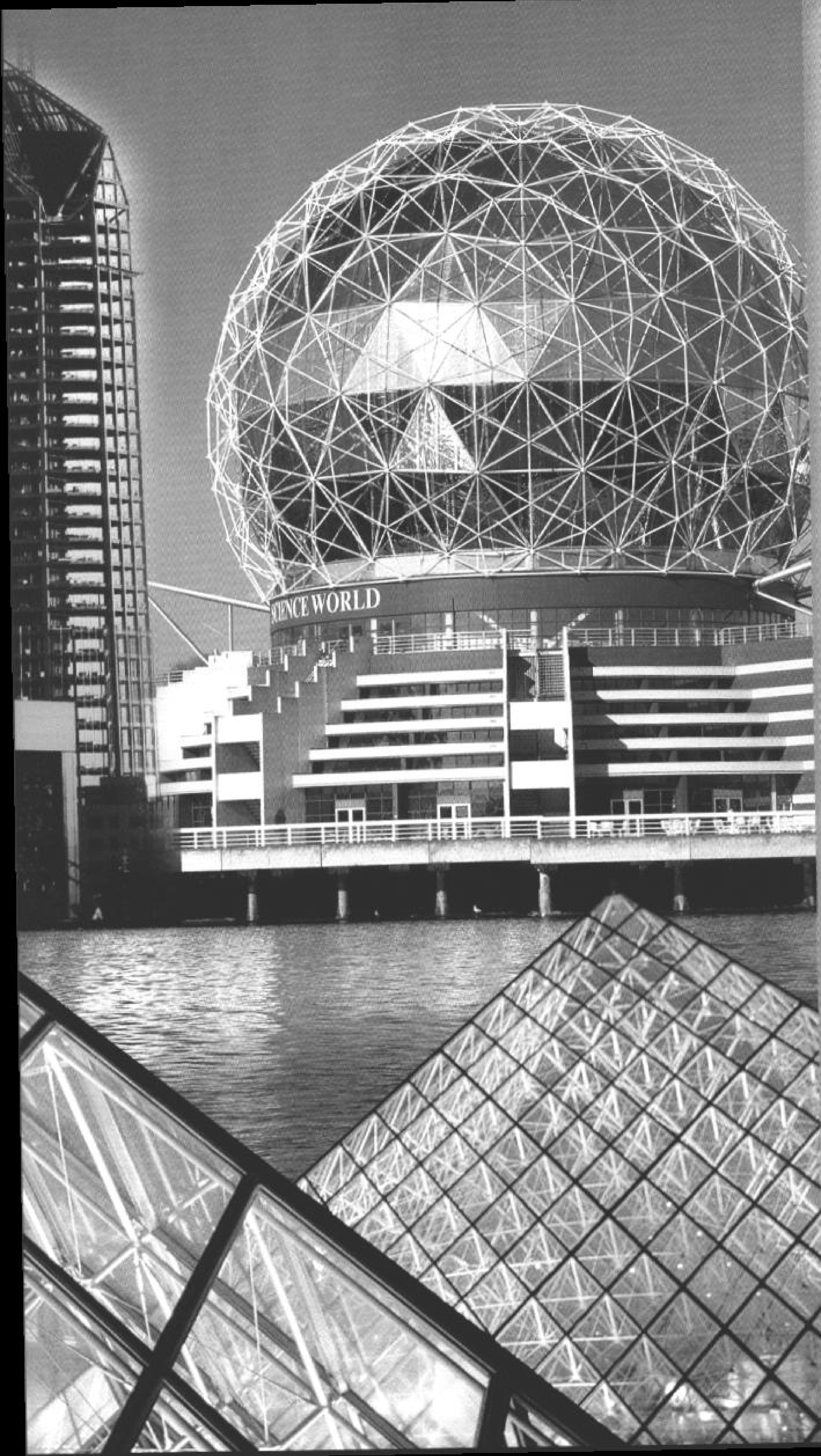
In this unit, you will develop strategies to measure distances that cannot be described exactly, using whole numbers, fractions, or decimals.

What You'll Learn

- Relate the area of a square to the length of its side.
- Understand that the square root of a non-perfect square is approximate.
- Estimate and calculate the square root of a whole number.
- Draw a circle, given its area.
- Investigate and apply the Pythagorean Theorem.

Why It's Important

The Pythagorean Theorem enables us to measure distances that would be impossible to measure using only a ruler. It enables a construction worker to make a square corner without using a protractor.



Key Words

- square number
- perfect square
- square root
- irrational number
- leg
- hypotenuse
- Pythagorean Theorem
- Pythagorean triple

UNIT

8

Skills You'll Need

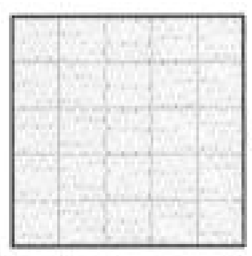
Areas of a Square and a Triangle

Area is the amount of surface a figure covers. It is measured in square units.

Example 1

Find the area of each figure.

a)



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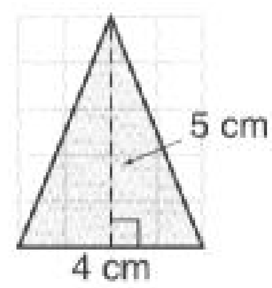


Solution

a) The figure is a square.
The area of a square is: $A = s^2$
Substitute: $s = 5$

$A = 5^2$
$= 5 \times 5$
$= 25$

The area is 25 cm^2.

b) The figure is a triangle.
The area of a triangle is: $A = \frac{1}{2}bh$
Substitute: $b = 4$ and $h = 5$

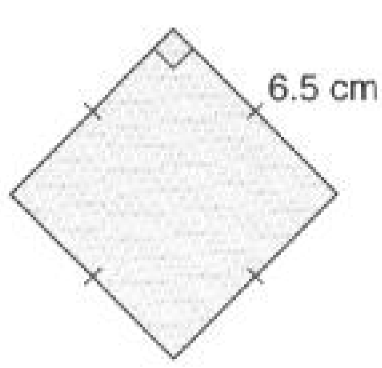
$A = \frac{1}{2}(4 \times 5)$
$= \frac{1}{2}(20)$
$= 10$

The area is 10 cm^2.

✓ Check

1. Find the area of each figure.

a)



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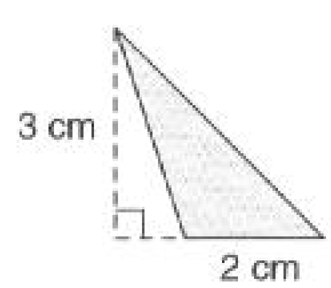

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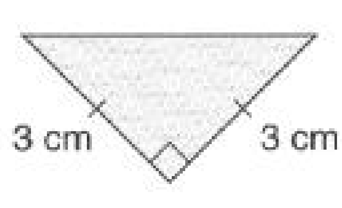

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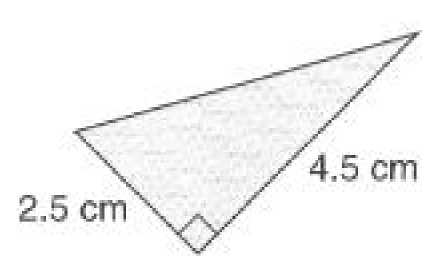


Square Numbers

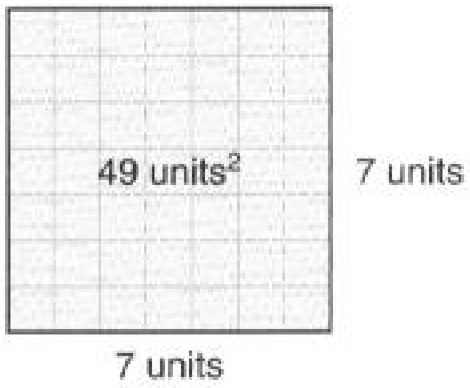
When we multiply a number by itself, we square the number.
We can use exponents to write a **square number.**
4^2 means: $4 \times 4 = 16$
We say, "Four squared is sixteen."
16 is a square number, or a **perfect square.**
One way to model a square number is to draw a square whose area is equal to the square number.

Example 2

Show that 49 is a square number. Use symbols, words, and a diagram.

Solution

With symbols: $49 = 7 \times 7 = 7^2$
With words: "Seven squared is forty-nine."



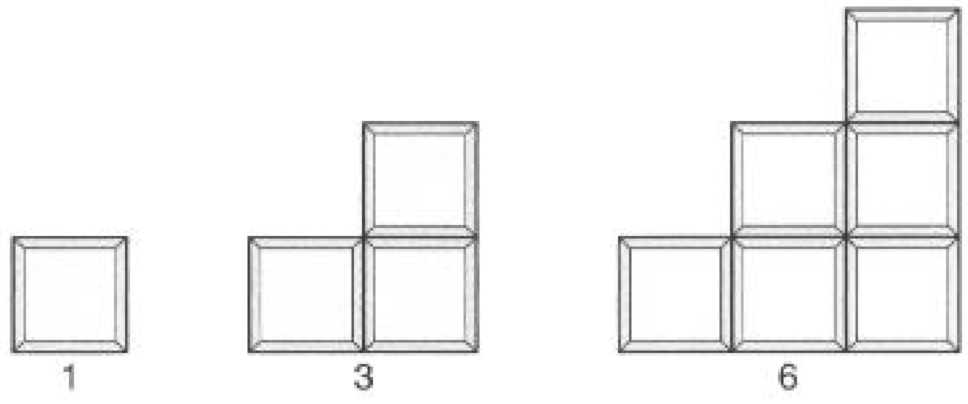
✓ Check

2. Show that 36 is a square number. Use a diagram, symbols, and words.

3. Write each number in exponent form.
a) 25 **b)** 81 **c)** 64 **d)** 169

4. List the first 15 square numbers.

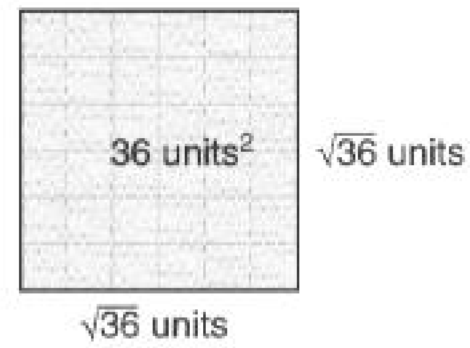
5. Here are the first 3 triangular numbers.



a) Write the next 3 triangular numbers.
b) Add consecutive triangular numbers. What do you notice? Explain.

Square Roots

Squaring and finding a **square root** are inverse operations.
For example, $7^2 = 49$ and $\sqrt{49} = 7$
We can model a square root with a diagram.
The *area* of a square shows the *square number*.
The *side length* of the square shows a *square root* of the square number.
We say, "A square root of 36 is 6."
We write: $\sqrt{36} = 6$



Example 3

Find a square root of 64.

Solution

Method 1

Think of a number that, when multiplied by itself, produces 64.

$8 \times 8 = 64$

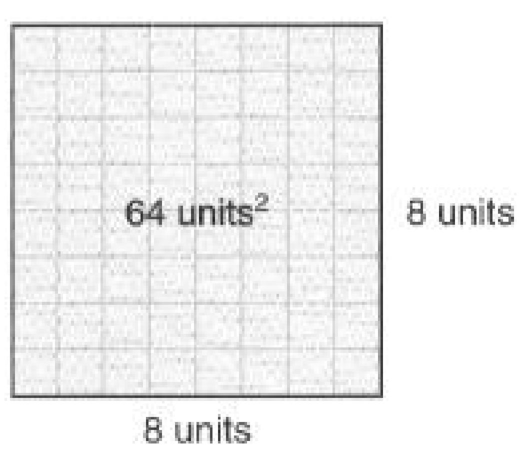
So, $\sqrt{64} = 8$

Method 2

Visualize a square with an area of 64 units².
Find its side length.

$64 = 8 \times 8$

So, $\sqrt{64} = 8$



✓ Check

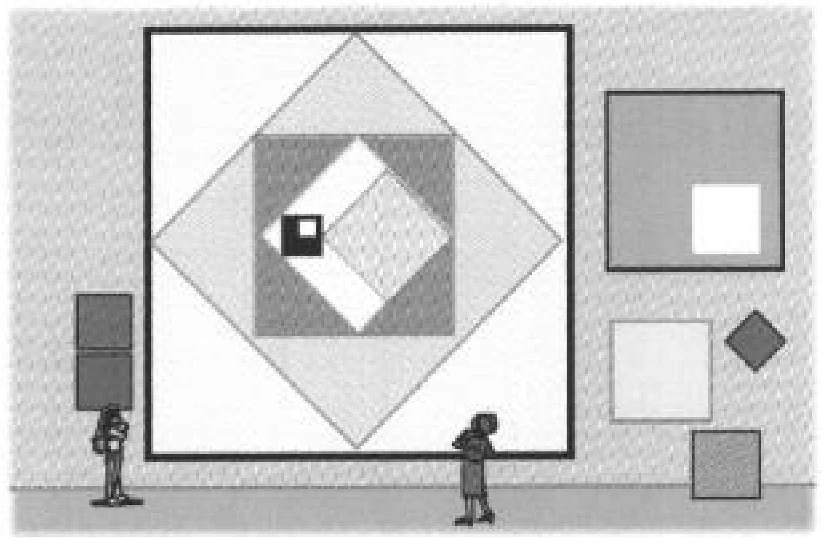
6. Find each square root.

a) $\sqrt{1}$ b) $\sqrt{25}$ c) $\sqrt{81}$ d) $\sqrt{9}$
e) $\sqrt{16}$ f) $\sqrt{100}$ g) $\sqrt{121}$ h) $\sqrt{225}$

8.1 Constructing and Measuring Squares

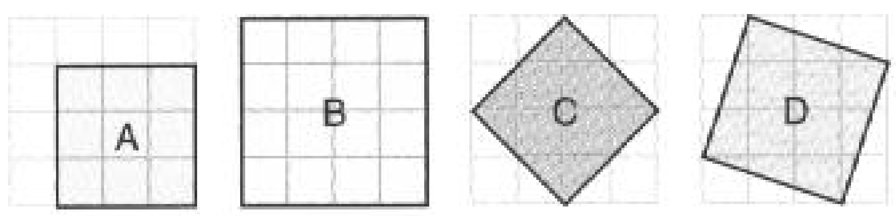
Focus Use the area of a square to find the length of a line segment.

All squares are *similar*.
They come in many sizes, but always have the same shape.
How many ways can you describe a square?



Explore

Work with a partner.
You will need 1-cm grid paper.
Copy the squares below.
Without using a ruler, find the area and side length of each square.



What other squares can you draw on a 4 by 4 grid?
Find the area and side length of each square.
Write all your measurements in a table.

Reflect & Share

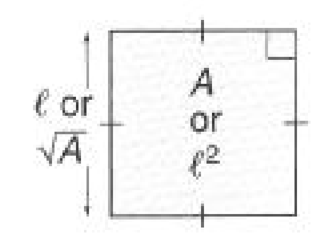
How many squares did you draw?
Describe any patterns in your measurements.
How did you find the area and side length of each square?
How did you write the side lengths of squares C and D?

Connect

We can use the properties of a square to find its area or side length.

Area of a square = length × length
= $(\text{length})^2$

When the side length is l, the area is l^2.
When the area is A, the side length is $\sqrt{A}$.



Example 1

A square has side length 10 cm.
What is the area of the square?

Solution

$$\text{Area} = (\text{length})^2 \text{ or } A = l^2$$
$$A = 10^2$$
$$= 100$$

The area is 100 cm^2.

Example 2

A square has area 81 cm^2.
What is the side length of the square?

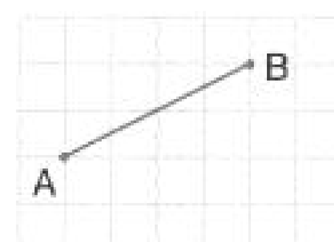
Solution

$$\text{Length} = \sqrt{\text{Area}} \text{ or } l = \sqrt{A}$$
$$l = \sqrt{81}$$
$$= 9$$

The side length is 9 cm.

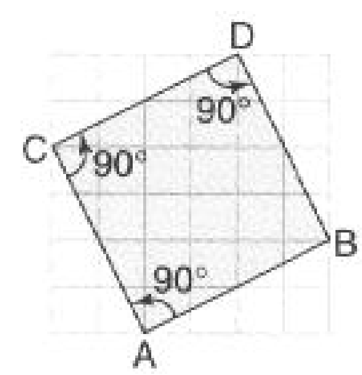
We can calculate the length of any segment on a grid by thinking of it as the side length of a square.

To find the length of the line segment AB:



Construct a square on the segment.
Find the area of the square.
Then, the length of the segment is the square root of the area.

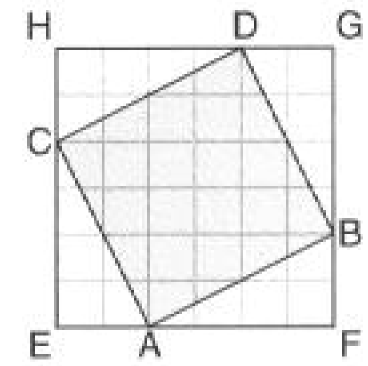
To construct a square on segment AB:
Rotate segment AB 90° counterclockwise about A, to get segment AC.
Rotate segment AC 90° counterclockwise about C, to get segment CD.
Rotate segment CD 90° counterclockwise about D, to get segment DB.



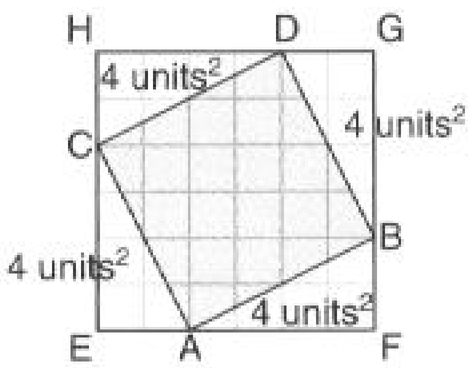
Here are two methods to find the area of the square.

Method 1

Draw an enclosing square and subtract areas.
Draw square EFGH along grid lines so each vertex of ABDC lies on one side of the enclosing square.



The area of EFGH $= 6^2$ units^2
$= 36$ units^2

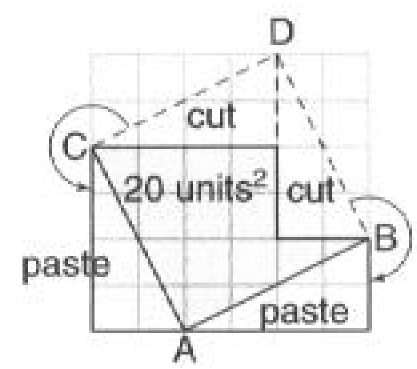


The triangles formed by the enclosing square are congruent.
Each triangle has area: $\frac{1}{2}(4)(2)$ units^2 $= 4$ units^2
So, the 4 triangles have area 4×4 units^2 $= 16$ units^2

The area of ABDC = Area of EFGH − Area of triangles
$= 36$ units^2 $- 16$ units^2
$= 20$ units^2
So, the side length of ABDC is: AB $= \sqrt{20}$ units

Method 2

Cut the square into smaller figures, then rearrange.
Cut and move two triangles to form a figure with side lengths along grid lines.
Count squares to find the area.



The area of the new figure is 20 units^2.
So, the area of square ABDC = 20 units^2
And, the side length of the square, AB $= \sqrt{20}$ units

Since 20 is not a square number, we cannot write $\sqrt{20}$ as a whole number.
Later in this unit, you will learn how to find an approximate value for $\sqrt{20}$ as a decimal.

Practice

1. Simplify.

a) 3^2	**b)** $\sqrt{1}$	**c)** 4^2	**d)** $\sqrt{64}$
e) 7^2	**f)** $\sqrt{144}$	**g)** 10^2	**h)** $\sqrt{169}$
i) 6^2	**j)** $\sqrt{121}$	**k)** 12^2	**l)** $\sqrt{625}$

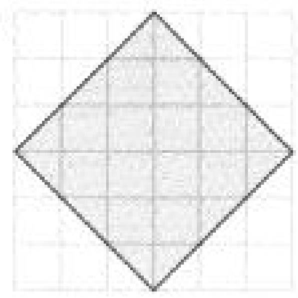
Number Strategies

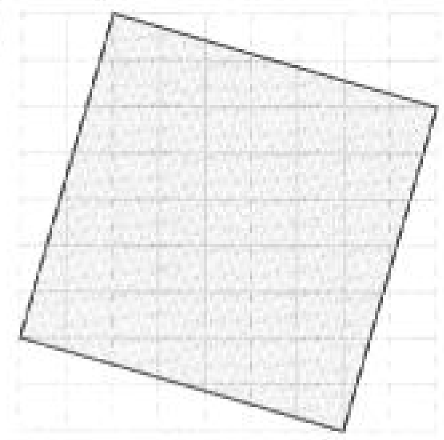
The surface area of a cube is 96 cm².

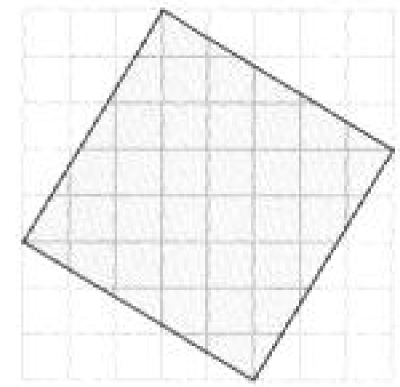
What is the area of one face of the cube?

What is the length of one edge of the cube?

2. Copy each square on grid paper. Find its area.
   Then write the side length of the square.

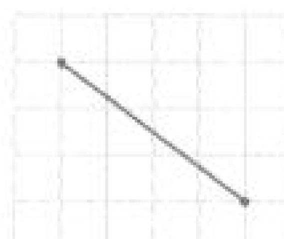
a) 

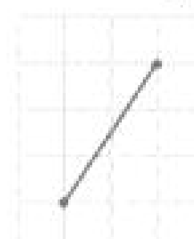
b) 

c) 

3. The area A of a square is given. Find its side length.
   Which side lengths are whole numbers?
   a) $A = 36\text{ cm}^2$ b) $A = 49\text{ m}^2$ c) $A = 95\text{ cm}^2$ d) $A = 108\text{ m}^2$

4. Copy each segment on grid paper.
   Draw a square on each segment.
   Find the area of the square and the length of the segment.

a) 

b) 

c) 

d) 

5. The Great Pyramid at Giza is the largest pyramid in the world.
   The area of its square base is about 52 441 m².
   What is the length of each side of the base?

6. **Assessment Focus**
   On square dot paper, draw a square with an area of 2 units².
   Write to explain how you know the square does have this area.

Take It Further

7. Suppose you know the length of the diagonal of a square.
   How can you find the side length of the square? Explain.

Reflect

How are square roots related to exponents?
How is the area of a square related to its side length?
How can we use this relationship to find the length of a line segment? Include an example in your explanation.

8.2 Estimating Square Roots

Focus Develop strategies for estimating a square root.

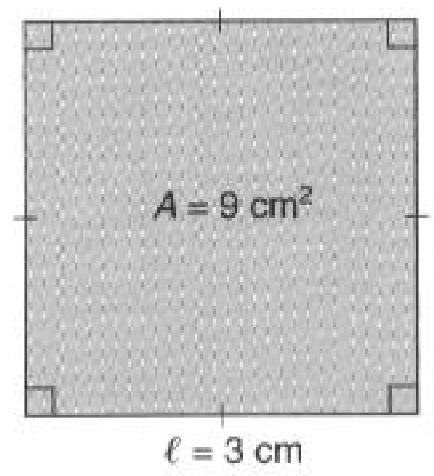
You know that the square root of a given number is a number which, when multiplied by itself, results in the given number;

for example, $\sqrt{121} = \sqrt{11 \times 11}$

$= 11$

You also know that the square root of a number is the side length of a square with area that is equal to the number.

For example, $\sqrt{9} = 3$



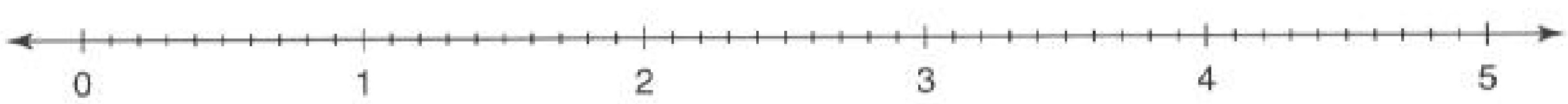
Explore

Work with a partner.

Use a copy of the number line below.

Place each square root on the number line to show its approximate value as a decimal: $\sqrt{2}$, $\sqrt{5}$, $\sqrt{9}$, $\sqrt{18}$, $\sqrt{24}$

Use grid paper if it helps.



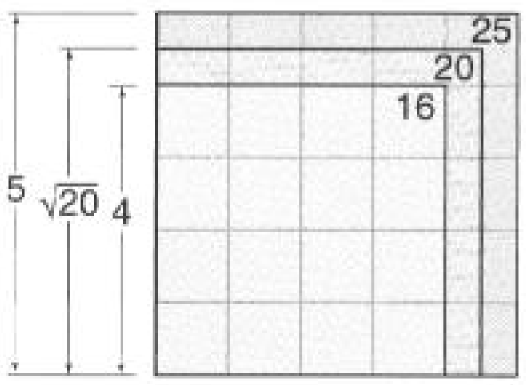
Reflect & Share

Compare your answers with those of another pair of classmates.

What strategies did you use to estimate the square roots?

How could you use a calculator to check your square roots?

Connect



Here is one way to estimate the value of $\sqrt{20}$:
Find the square number closest to 20, but greater than 20.
The number is 25.
On grid paper, draw a square with area 25.
Its side length is: $\sqrt{25} = 5$
Find the square number closest to 20, but less than 20.
The number is 16.
Draw a square with area 16.
Its side length is: $\sqrt{16} = 4$
Draw the squares so they overlap.

A square with area 20 lies between these two squares.
Its side length is $\sqrt{20}$.
20 is between 16 and 25, but closer to 16.
$\sqrt{20}$ is between $\sqrt{16}$ and $\sqrt{25}$, but closer to $\sqrt{16}$.
So, $\sqrt{20}$ is between 4 and 5, but closer to 4.
An estimate of $\sqrt{20}$ is 4.4.

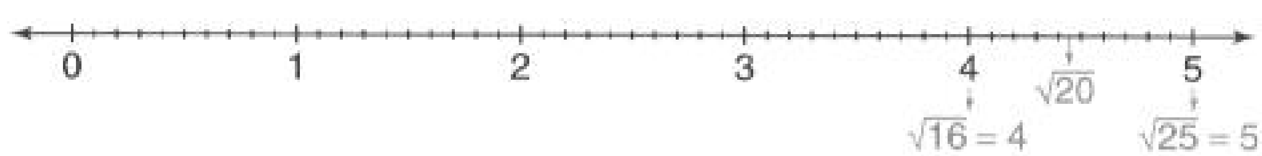
The *Example* illustrates another method to estimate $\sqrt{20}$.

Example

Use a number line and a calculator to estimate $\sqrt{20}$.

Solution

Think of the perfect squares closest to $\sqrt{20}$.

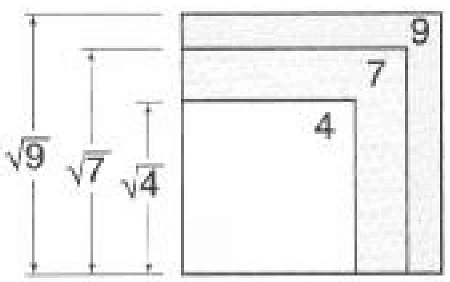


$\sqrt{20}$ is between 4 and 5, but closer to 4.
With a calculator, use guess and check to refine the estimate.

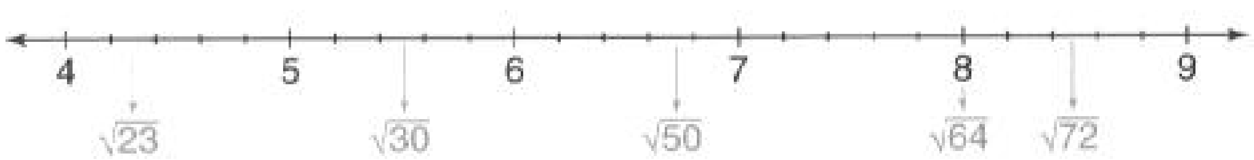
Try 4.4: $4.4 \times 4.4 = 19.36$ (too small)
Try 4.5: $4.5 \times 4.5 = 20.25$ (too large)
Try 4.45: $4.45 \times 4.45 = 19.8025$ (too small)
Try 4.46: $4.46 \times 4.46 = 19.8916$ (too small)
Try 4.47: $4.47 \times 4.47 = 19.9809$ (very close)
A close estimate of $\sqrt{20}$ is 4.47.

Practice

1. Copy this diagram on grid paper. Then estimate the value of $\sqrt{7}$.



2. Use the number line below.
   a) Which placements are good estimates of the square roots? Explain your reasoning.
   b) Use the number line to estimate the value of each square root that is incorrectly placed.



Mental Math

Estimate.
What strategies did you use?

- $\frac{3}{4}$ of 70
- $\frac{2}{3}$ of 55
- $\frac{5}{8}$ of 100
- $\frac{1}{5}$ of 299

3. Which two consecutive whole numbers is each square root between? How do you know?
   a) $\sqrt{5}$ b) $\sqrt{11}$ c) $\sqrt{57}$ d) $\sqrt{38}$ e) $\sqrt{171}$

4. Write five square roots whose values are between 9 and 10. Explain your strategy.

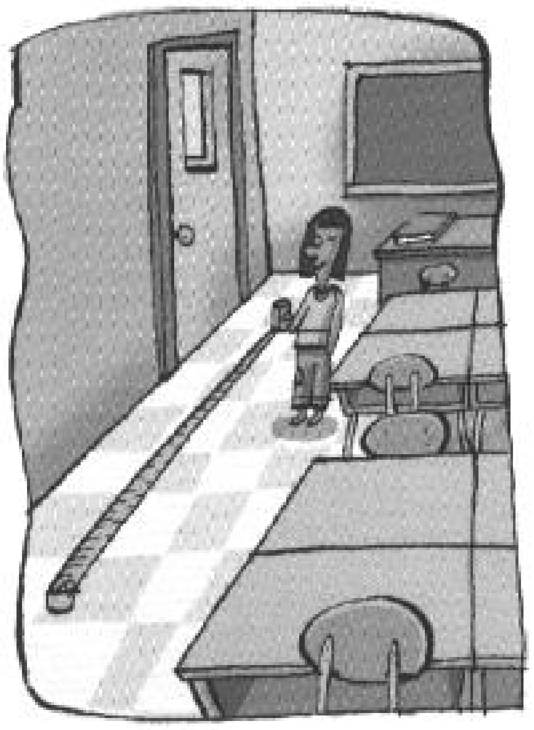
5. Is each statement true or false? Explain.
   a) $\sqrt{17}$ is between 16 and 18.
   b) $\sqrt{5} + \sqrt{5}$ is greater than $\sqrt{10}$.
   c) $\sqrt{131}$ is between 11 and 12.

6. Use guess and check to estimate the value of each square root. Record each trial.
   a) $\sqrt{23}$ b) $\sqrt{13}$ c) $\sqrt{78}$ d) $\sqrt{135}$ e) $\sqrt{62}$

To round a length in centimetres to the nearest millimetre, round to the nearest tenth.

7. Find the approximate side length of the square with each area. Give your answer to the nearest millimetre.
   a) 92 cm^2 b) 430 m^2 c) 150 cm^2 d) 29 m^2

8. A square garden has an area of 138 m^2.
   a) What are the approximate dimensions of the garden?
   b) About how much fencing would be needed to go around the garden?

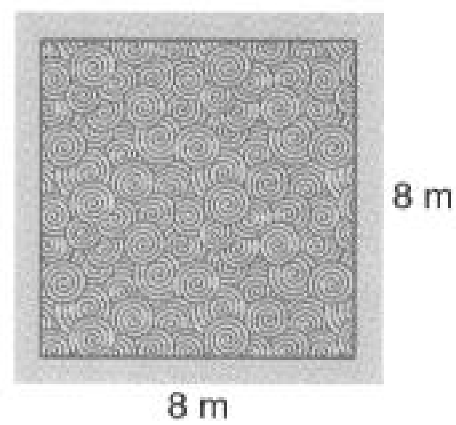


9. **Assessment Focus** A student uses a 1-m square canvas for her painting. After framing, she wants her artwork to have an area twice the area of the canvas.
What are the dimensions of the square frame?
Show your work.

10. Most classrooms are rectangles.
Measure the dimensions of your classroom.
Calculate its area.
What if your classroom was a square.
What would its dimensions be?

Take It Further

11. A square carpet covers 75% of the area of a floor.
The floor is 8 m by 8 m.



a) What are the dimensions of the carpet?
b) What area of the floor is not covered by carpet?

12. Is the product of two perfect squares sometimes a perfect square? Always a perfect square?
Investigate to find out. Write about your findings.

A palindrome is a number that reads the same forward and backward.

13. a) Find the square root of each palindrome.
i) $\sqrt{121}$
ii) $\sqrt{12\,321}$
iii) $\sqrt{1\,234\,321}$
iv) $\sqrt{123\,454\,321}$
b) Continue the pattern.
Write the next 4 palindromes and their square roots.

Reflect

How can you find the perimeter of a square if you know its area?
What is your favourite method for estimating a square root of a number that is not a perfect square? Explain your choice.

Fitting In

Game

HOW TO PLAY THE GAME:

Your teacher will give you 3 sheets of game cards. Cut out the 54 cards.

1. Place the 1, 5, and 9 cards on the table.
Spread them out so there is room for several cards between them.
Shuffle the remaining cards.
Give each player six cards.

2. All cards laid on the table must be arranged from least to greatest.
Take turns to place a card so it touches another card on the table.
- It can be placed to the right of the card if its value is greater.
- It can be placed to the left of the card if its value is less.
- It can be placed on top of the card if its value is equal.
- However, it cannot be placed between two cards that are already touching.

YOU WILL NEED

1 set of FITTING IN game cards; scissors

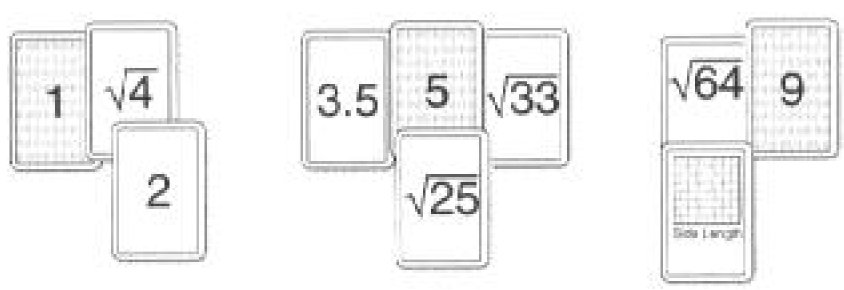
NUMBER OF PLAYERS

2 to 4

GOAL OF THE GAME

To get the lowest score

In this example, the $\sqrt{16}$ card cannot be placed because the 3.5 and the 5 cards are touching.
The player cannot play that card in this round.



What other games could you play with these cards? Try out your ideas.

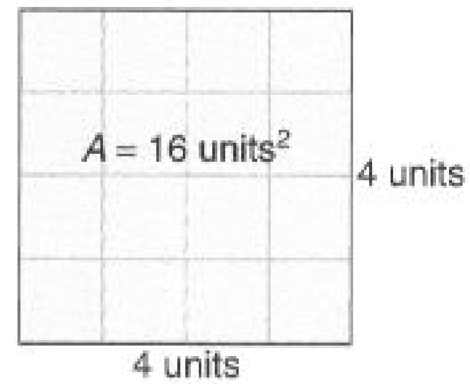
3. Place as many of your cards as you can. When no player can place any more cards, the round is over.
Your score is the number of cards left in your hand.
At the end of five rounds, the player with the lowest score wins.

Investigating Square Roots with a Calculator

Technology

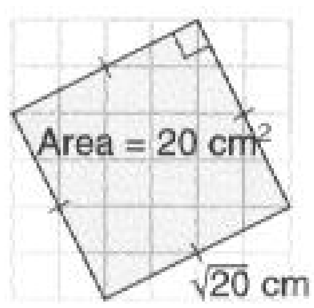
Focus Use a calculator to investigate square roots.

➢ We can use a calculator to calculate a square root.
To find a square root of 16:
On a calculator,
press: [√] 16 [)] [ENTER =] to display 4
A square root of 16 is 4.



Check by multiplying.
Press: 4 [×] 4 [ENTER =] to display 16

If you use a different calculator,
what keystrokes do you use to find square roots?



➢ Many square roots are not whole numbers.
To find a square root of 20:
On a calculator,
press: [√] 20 [)] [ENTER =] to display 4.472135955
A square root of 20 is approximately 4.5.

➢ We investigate what happens when we check our answer.
Compare using a scientific calculator
with a 4-function calculator.
On a 4-function calculator, press: 20 [√] [×] 20 [√] [=]
What do you see in the display?
On a scientific calculator,
press: [√] 20 [)] [×] [√] 20 [)] [ENTER =]
What do you see in the display?
Which display is accurate? How do you know?

➢ Check what happens when you enter
4.472135955 × 4.472135955 into both calculators.
What if you multiplied using pencil and paper.
Would you expect a whole number or a decimal? Explain.

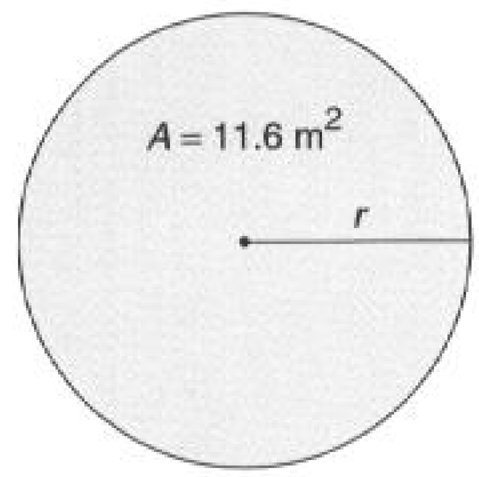
Recall from *Unit 6* that π is another irrational number.

➢ $\sqrt{20}$ cannot be described exactly by a decimal.
The decimal for $\sqrt{20}$ never repeats and never terminates.
A number like $\sqrt{20}$ is called an **irrational number**.

When we know the area of a circle, we can use square roots to calculate its radius.

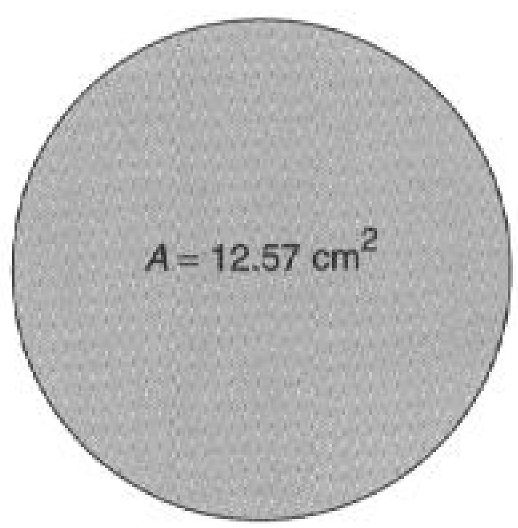
The area of a circle, A, is about $3r^2$.
So, r^2 is about $\frac{1}{3}$ of A, or $\frac{A}{3}$.
Similarly, the area $A = \pi r^2$; so $r^2 = \frac{A}{\pi}$
To find r, we take the square root.
So, $r = \sqrt{\frac{A}{\pi}}$
We can use this formula to calculate the radius of a circle when we know its area.

A circular rug has area 11.6 m^2.
To calculate the radius of the rug, use: $r = \sqrt{\frac{A}{\pi}}$
Substitute: $A = 11.6$
$r = \sqrt{\frac{11.6}{\pi}}$
Use a calculator.
Key in: [√] 11.6 [÷] [π] [)] [ENTER =]
to display 1.92156048
$r \doteq 1.92$
The radius of the rug is about 1.92 m, to the nearest centimetre.



✓ Check

1. Calculate the radius and diameter of each circle.
Give the answers to 1 decimal place.

a)


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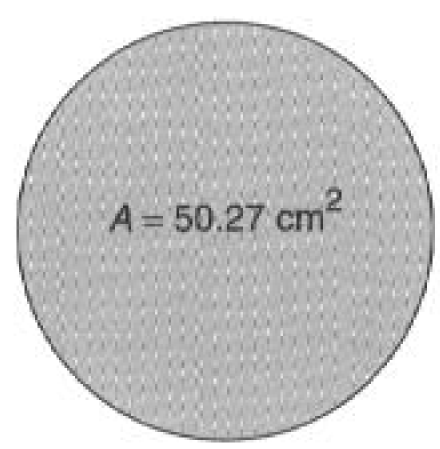

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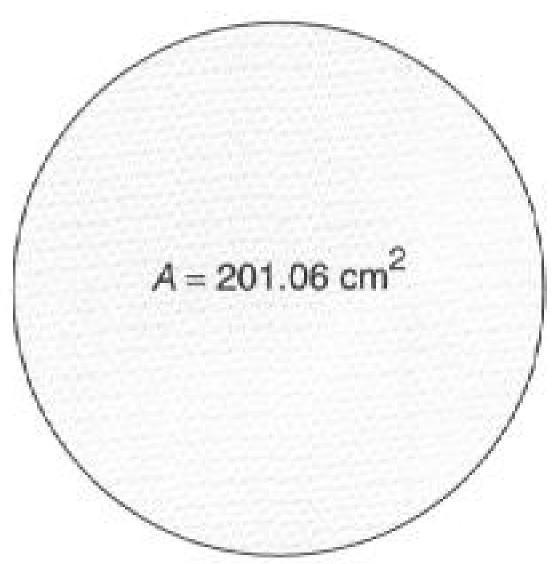

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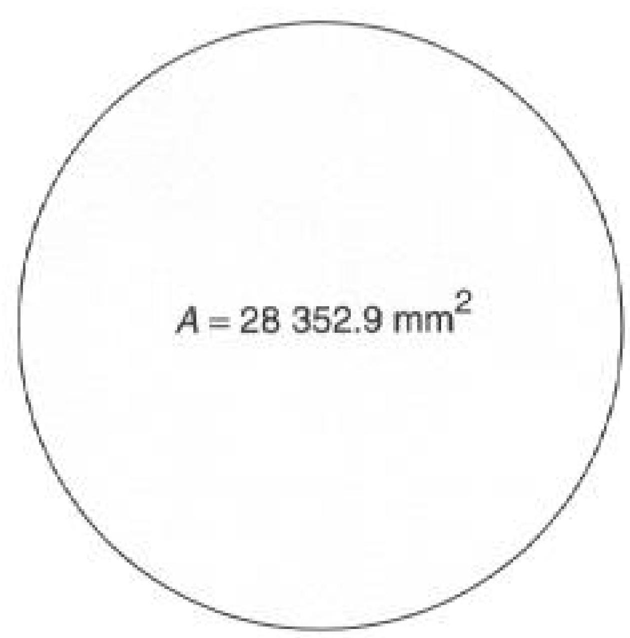


2. Draw each circle in question 1.

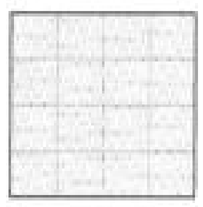
Mid-Unit Review

LESSON

8.1 **1.** Copy each square onto 1-cm grid paper.

i) Find the area of each square.
ii) Write the side length of each square as a square root.
iii) Which areas can be written using exponents? Explain.

a)



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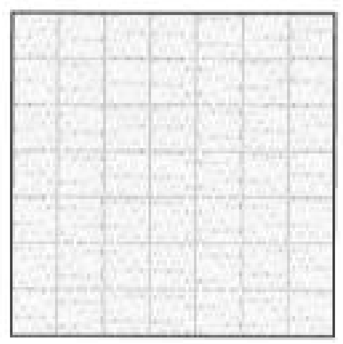

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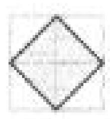
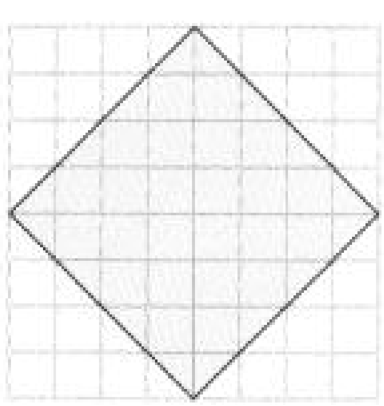


2. a) The area of a square is 24 cm^2. What is its side length?
b) The side length of a square is 9 cm. What is its area?
c) Explain the relationship between square roots and square numbers.
Use diagrams, symbols, and words.

8.1
8.2 **3.** Copy this square onto 1-cm grid paper.



a) What is the area of the square?
b) Write the side length of the square as a square root.
c) Estimate the side length to the nearest millimetre.

8.2 **4.** Between which two consecutive whole numbers does each square root lie? How do you know?

a) $\sqrt{3}$
b) $\sqrt{65}$
c) $\sqrt{57}$
d) $\sqrt{30}$

5. What is a square root of 100?
Use this fact to predict the square root of each number.
Use a calculator to check.

a) 900
b) 2500
c) 400
d) 8100
e) 10 000
f) 1 000 000

6. a) Draw a circle with area 113 cm^2.
b) Does the circle in part a have an area of exactly 113 cm^2?
How do you know?

7. The opening of the fresh air intake pipe for a furnace is circular.
Its area is 550 cm^2.
What are the radius and diameter of the pipe? Give the answers to the nearest millimetre.

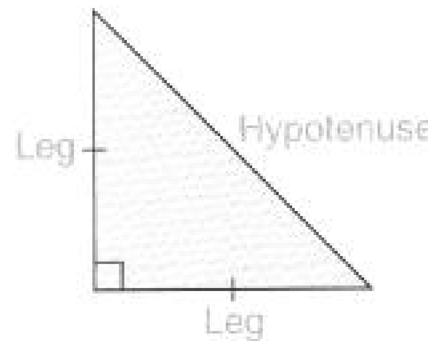
8. The top of a circular concrete footing has an area of 4050 cm^2.
What is the radius of the circle?
Give the answer to the nearest millimetre.

8.3 The Pythagorean Relationship

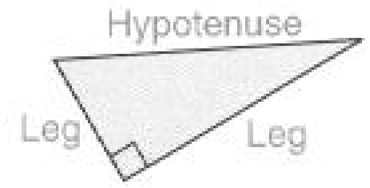
Focus Discover a relationship among the side lengths of a right triangle.

In *Lesson 8.1*, you learned how to use the properties of a square to find the length of a line segment.

We will now use the properties of a right triangle to find the length of a line segment. A right triangle has two **legs** that form the right angle. The third side of the right triangle is called the **hypotenuse**.



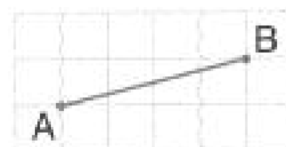
Isosceles right triangle



Scalene right triangle

Explore

Work on your own.
You will need grid paper.



➢ Copy segment AB.
Find the length of the segment by drawing a square on it.

➢ Copy segment AB again.
Draw a right triangle that has segment AB as its hypotenuse.
Draw a square on each side.
Find the area and side length of each square.

Use the corner of a sheet of paper or a protractor to check that the angles in the square are right angles.

➢ Draw 3 different right triangles, with a square on each side.
Find the area and side length of each square.
Record your results in a table.

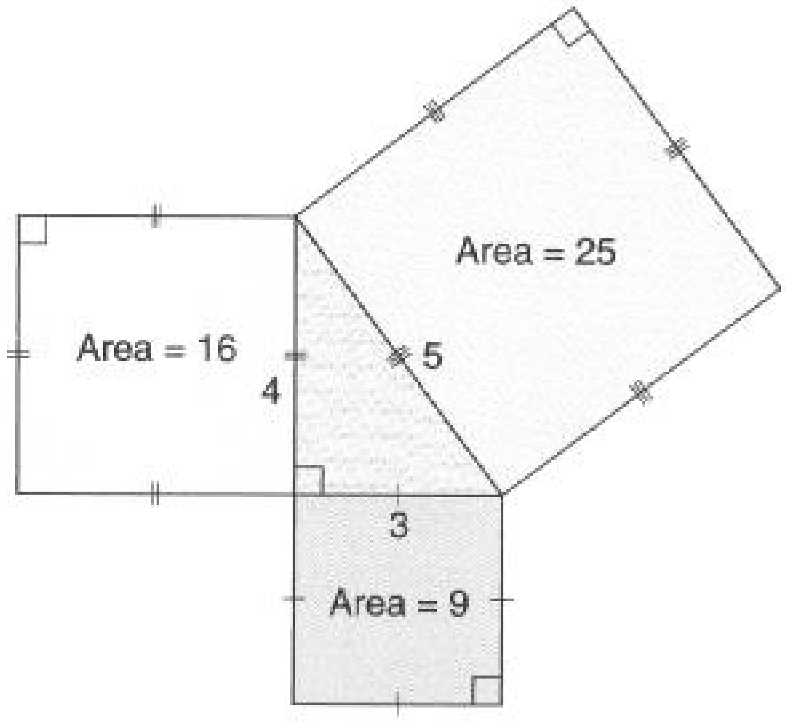
	Area of Square on Leg 1	Length of Leg 1	Area of Square on Leg 2	Length of Leg 2	Area of Square on Hypotenuse	Length of Hypotenuse
Triangle 1						
Triangle 2						
Triangle 3						
Triangle 4						

Reflect & Share

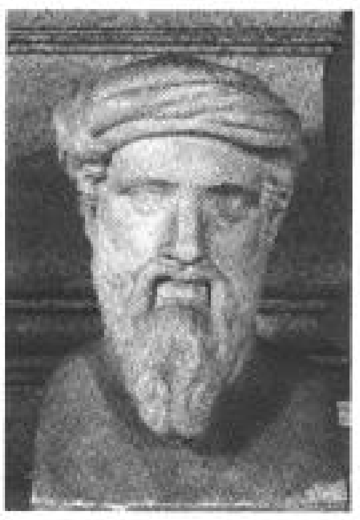
Compare your results with those of another classmate.
What relationship do you see among the areas of the squares on the sides of a right triangle? How could this relationship help you find the length of a side of a right triangle?

Connect

Here is a right triangle, with a square drawn on each side.



Later in this unit, we will use *The Geometer's Sketchpad* to verify that this relationship is true for all right triangles.



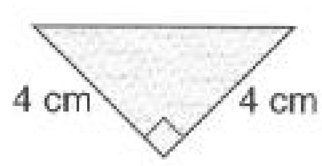
The area of the square on the hypotenuse is 25.
The areas of the squares on the legs are 9 and 16.
Notice that: $25 = 9 + 16$
This relationship is true for all right triangles:
The area of the square on the hypotenuse is equal to the sum of the areas of the squares on the legs.
This relationship is called the **Pythagorean Theorem**.
This theorem is named for the Greek mathematician, Pythagoras, who first wrote about it.

We can use this relationship to find the length of any side of a right triangle, when we know the lengths of the other two sides.

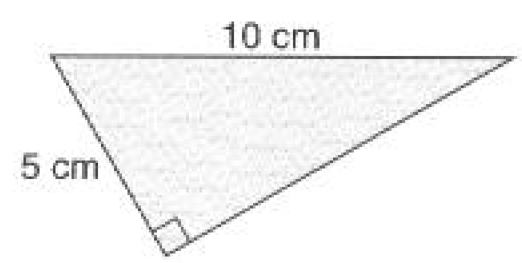
Example

Find the length of the unmarked side in each right triangle.
Give the lengths to the nearest millimetre.

a)

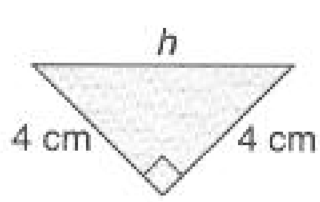


b)

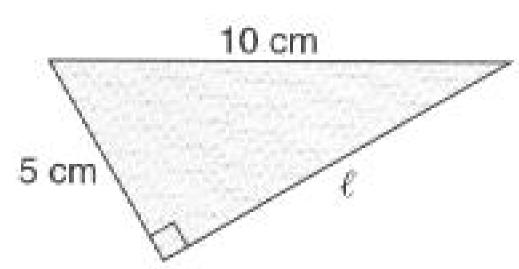


Solution

a) The unmarked side is the hypotenuse. Label it h.



b) The unmarked side is a leg. Label it l.



The area of the square on the hypotenuse is h^2.
The area of the squares on the legs are 4^2 and 4^2.
So, $h^2 = 4^2 + 4^2$
Use the order of operations.
Square, then add.

$$h^2 = 16 + 16$$
$$h^2 = 32$$

The area of the square on the hypotenuse is 32.
So, the side length of the square is: $h = \sqrt{32}$
Use a calculator.
$h \doteq 5.6569$
So, the hypotenuse is approximately 5.7 cm.

The area of the square on the hypotenuse is 10^2.
The areas of the squares on the legs are l^2 and 5^2.
So, $10^2 = l^2 + 5^2$
Square each number.
$100 = l^2 + 25$
To solve this equation, subtract 25 from each side.

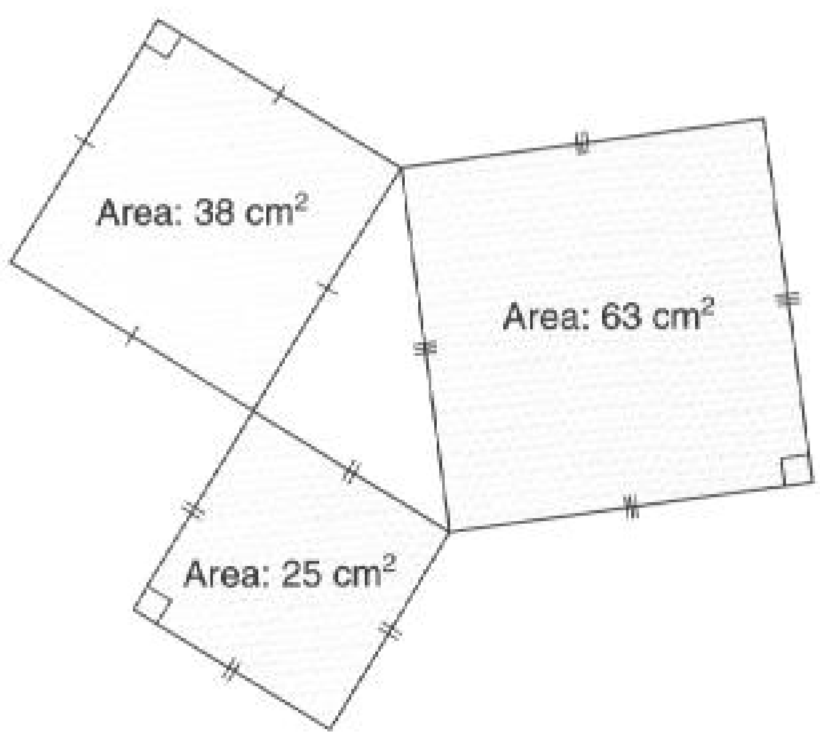
$$100 - 25 = l^2 + 25 - 25$$
$$75 = l^2$$

The area of the square on the leg is 75.
So, the side length of the square is: $l = \sqrt{75}$
$l \doteq 8.66025$
So, the leg is approximately 8.7 cm.

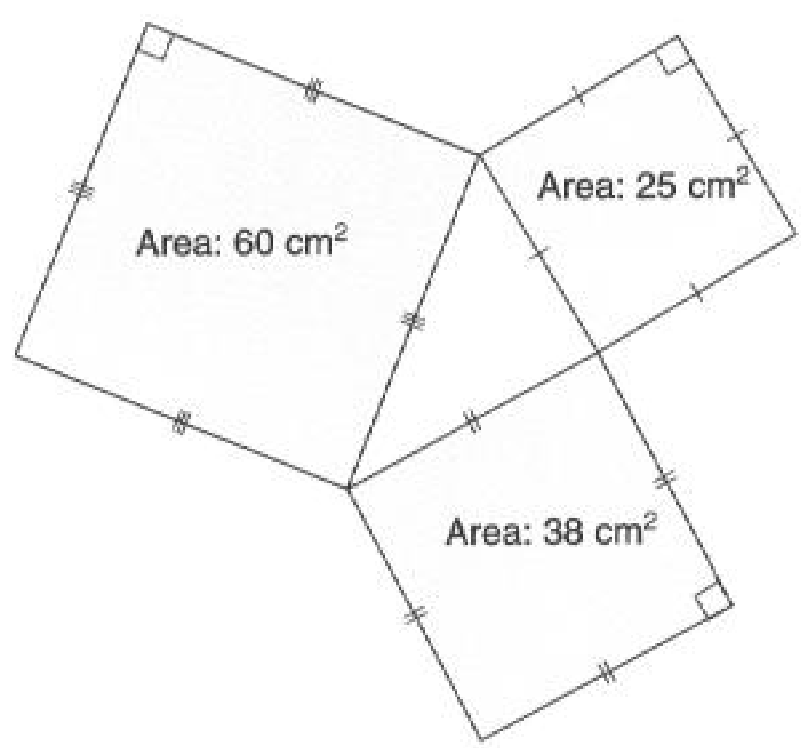
Practice

1. The area of the square on each side of a triangle is given.
Is the triangle a right triangle? How do you know?

a)



b)



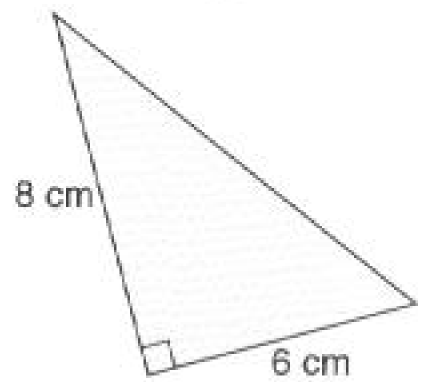
Number Strategies

Write the first 6 multiples of each number:
4, 9, 11, 12

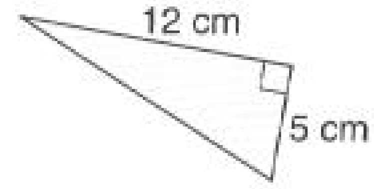
Find the lowest common multiple of these numbers.

2. Find the length of the hypotenuse in each right triangle.

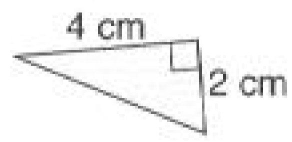
a)



b)



c)



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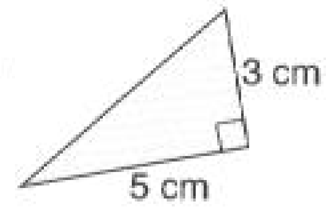
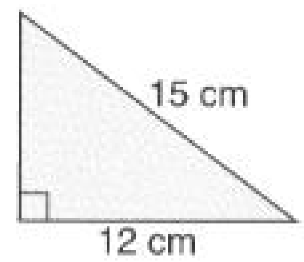


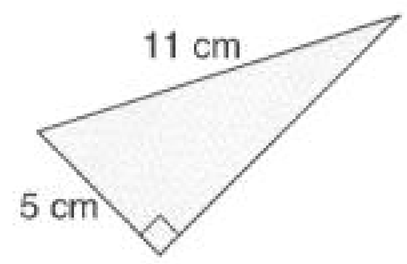
3. Find the length of the unmarked leg in each right triangle.

a)

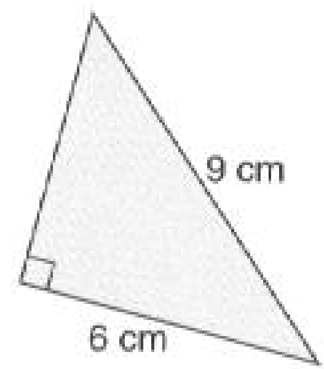


b)

c)

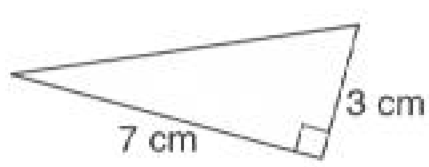


d)



4. Find the length of the unmarked side in each right triangle.

a)



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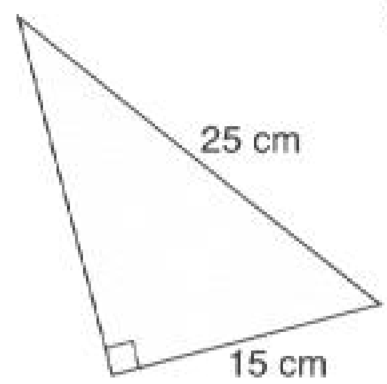
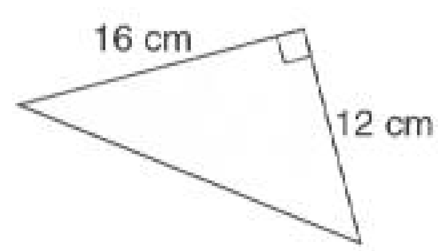


c)



Math Link

History

"Numbers Rule the Universe!" That was the belief held by a group of mathematicians called the Brotherhood of Pythagoreans. Their power and influence became so strong that fearful politicians forced them to disband. Nevertheless, they continued to meet in secret and teach Pythagoras' ideas.

The 3 whole-number side lengths of a right triangle are called a *Pythagorean triple*.

5. Look at the answers to questions 1 to 4, and at the triangles in *Connect*. Identify the triangles that have all 3 side lengths that are whole numbers.

a) List the lengths of the legs and the hypotenuse for each of these triangles. Try to arrange the measures to show patterns.

b) What patterns do you see? Explain the patterns.

c) Extend the patterns. Explain your strategy.

6. Mei Lin uses a ruler and compass to construct a triangle with side lengths 3 cm, 5 cm, and 7 cm. Before Mei Lin constructs the triangle, how can she tell if the triangle will be a right triangle? Explain.

7. Assessment Focus

The hypotenuse of a right triangle is $\sqrt{18}$ units.
What are the lengths of the legs of the triangle?
How many different answers can you find?
Sketch a triangle for each answer. Explain your strategies.

8. On grid paper, draw a line segment with each length.
Explain how you did it.

a) $\sqrt{5}$ b) $\sqrt{10}$ c) $\sqrt{13}$ d) $\sqrt{17}$

Take It Further

9. Use grid paper.
Draw a right triangle with a hypotenuse with each length.

a) $\sqrt{20}$ units b) $\sqrt{89}$ units c) $\sqrt{52}$ units

10. a) Sketch a right triangle with side lengths: 3 cm, 4 cm, 5 cm

b) Imagine that each side is a diameter of a semicircle. Sketch a semicircle on each side.

c) Calculate the area of each semicircle you drew. What do you notice? Explain.

Reflect

When you know the side lengths of a triangle, how can you tell if it is a right triangle? Use examples in your explanation.

Using *The Geometer's Sketchpad* to Verify the Pythagorean Theorem

Technology

Focus Use a computer to investigate the Pythagorean relationship.

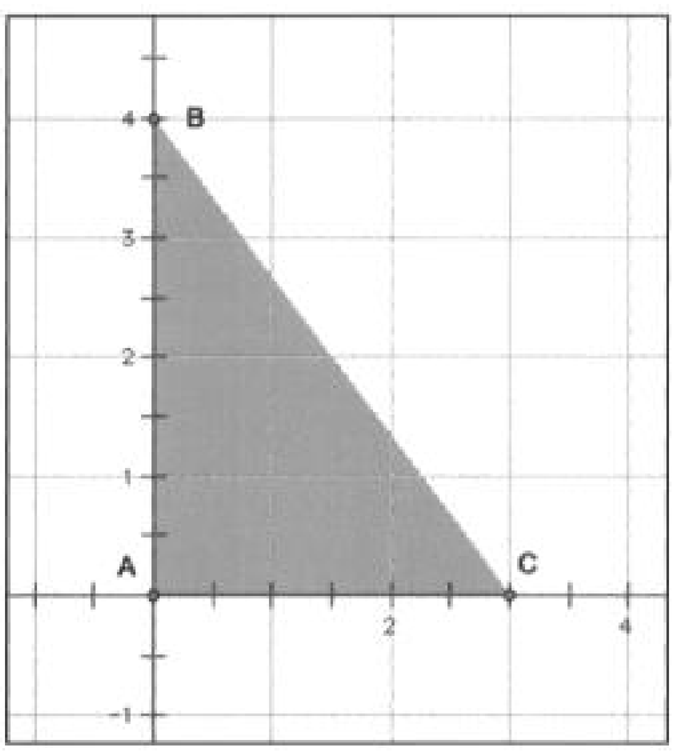
1. Open *The Geometer's Sketchpad*.
2. From the **Graph** menu, select **Show Grid**.
3. From the **Graph** menu, select **Snap Points**.

To construct right △ABC:

4. From the **Toolbox**, choose .
   Construct points at (0, 0), (0, 4), and (3, 0).

   From the **Toolbox**, choose **A**.
   Click the 3 points in the order listed above to label them A, B, and C.

5. From the **Toolbox**, choose .
   Click the 3 points to highlight them.
   From the **Construct** menu, choose **Segments**.
   You now have a right triangle.
   Click and drag any vertex, and it remains a right triangle.



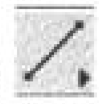
6. Click points A, B, and C.
   From the **Construct** menu, choose **Triangle Interior**.
   From the **Display** menu, choose **Color**.
   From the pull-down menu, pick a colour.
   Click the triangle to deselect it.

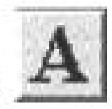
To construct a square on each side of △ABC:

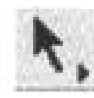
7. From the **Toolbox**, choose .
   Double-click point B.
   The flash shows this is now a centre of rotation.
   Click point B, point C, and segment BC.
   From the **Transform** menu, choose **Rotate**.
   Enter 90 degrees. Click **Rotate**.

The minus sign in front of the angle measure means a clockwise rotation.

8. Click anywhere on the screen to deselect the rotated segment.
   Make sure points B and C, and segment BC, are still selected.
   Double-click point C to mark a new centre of rotation.
   From the **Transform** menu, choose **Rotate**.
   Enter –90 degrees. Click **Rotate**.

9. From the **Toolbox**, choose  .
Join the points at the ends of the rotated segments to form a square on side BC.

10. From the **Toolbox**, choose  .
Double-click one unlabelled vertex of the square. Type D. Click [OK].
Double-click the other unlabelled vertex. Type E. Click [OK].

11. From the **Toolbox**, choose  .
Click the vertices of the square. If other points or segments are highlighted, deselect them by clicking them.
From the **Construct** menu, choose **Quadrilateral Interior.**
From the **Display** menu, choose **Color.**
From the pull-down menu, pick a colour.

12. From the **Measure** menu, choose **Area.**
The area of square BDEC appears.

13. Repeat *Steps 7* to *12* to construct and measure a square on each of the other two sides of the triangle.
Decide the direction of rotation for each line segment; it could be 90 degrees or –90 degrees.
Label the vertices F, G, and H, I.

14. Drag a vertex of the triangle and observe what happens to the area measurements.
What relationship is shown?

To use *The Geometer's Sketchpad* calculator:

15. From the **Measure** menu, choose **Calculate.**
Click the area equation for the smallest square.
Click [+].
Click the area equation for the next smallest square.
Click [OK].

16. Drag a vertex of the triangle. How do the measurements change?
How does *The Geometer's Sketchpad* verify the Pythagorean Theorem?

Use *The Geometer's Sketchpad* to investigate "what if" questions.

17. What if the triangle was not a right triangle.
Is the relationship still true?

Communicating Solutions

The draft solution to a problem is often messy. To communicate a solution clearly, the draft solution must be tidy.
Communicating means talking, writing, drawing, or modelling to describe, explain, and justify your ideas to others. The draft solution is revised and edited for clear communication. The final solution presents only the steps needed to arrive at the answer. The steps are listed in the correct order.

Communicate Solutions – Represent, justify, and prove to others

Revise and edit draft solutions:

1. Show all the math information you *used*; that is, words, numbers, drawings, tables, graphs, and/or models.
2. Remove any information that you did not use.
3. Arrange the steps in a logical order.
4. Create a concluding statement.
5. Check the criteria to assess your communication.

Here are some criteria for good communication of math solutions:

- The solution is complete. All steps are shown. Other students can follow the steps and come to the same conclusion.
- The steps are in a logical order.
- The calculations are accurate.
- The spelling and grammar are correct.
- The math conventions are correct; for example, units, position of equal sign, labels and scales on graphs/diagrams, symbols, brackets, and so on.
- Where appropriate, more than one possible solution is described.
- The strategies are reasonable and are explained.
- Where appropriate, words, numbers, drawings, tables, graphs, and/or models are used to support the solution.
- The concluding statement fits the context and clearly answers the problem.

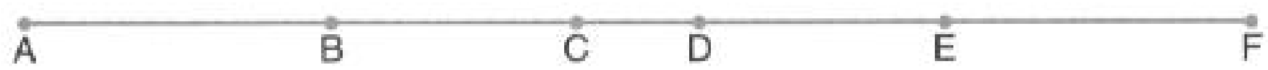
✓ Check

Solve these problems. Share your work with a classmate for feedback and suggestions. Use the criteria as a guide.

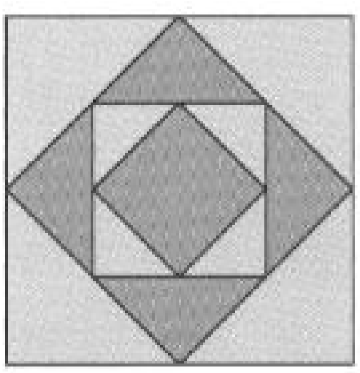
1. Find this sum:
23 + 25 + 27 + 23 + 25 + 27 + 23 +
25 + 27 + 23 + 25 + 27 + 23 + 25 + 27
Explain three different ways to solve this problem.

2. **a)** Six people met at a party.
All of them exchanged handshakes.
How many handshakes were there?

b) How many different line segments can be named using the labelled points as end points? List them.



c) How are these problems similar?



3. The side length of the largest square is 20 cm.

a) What is the area of each purple section?

b) What is the area of each orange section?

Explain how you got your answers.

4. There are 400 students at a school.
Is the following statement true?
There will always be at least two students in the school whose birthdays fall on the same day of the year.
Explain.

5. Camden has a custard recipe that needs 6 eggs, 1 cup of sugar, 750 mL of milk, and 5 mL of vanilla. He has 4 eggs.
He adjusts the recipe to use the 4 eggs.
How much of each other ingredient will he need?

6. Lo Choi wants to buy a dozen doughnuts. She has a coupon.
This week, the doughnuts are on sale for $3.99 a dozen.
If Lo Choi uses the coupon, each doughnut is $0.35.
Should Lo Choi use the coupon? Explain.

7. How many times in a 12-h period does the sum of the digits on a digital clock equal 6?

8.4 Applying the Pythagorean Theorem

Focus Solve problems using the Pythagorean Theorem.

Explore

Work with a partner.
Solve this problem:
A doorway is 2.0 m high and 1.0 m wide.
A square piece of plywood has side length 2.2 m.
Can the plywood fit through the door?
How do you know?
Show your work.

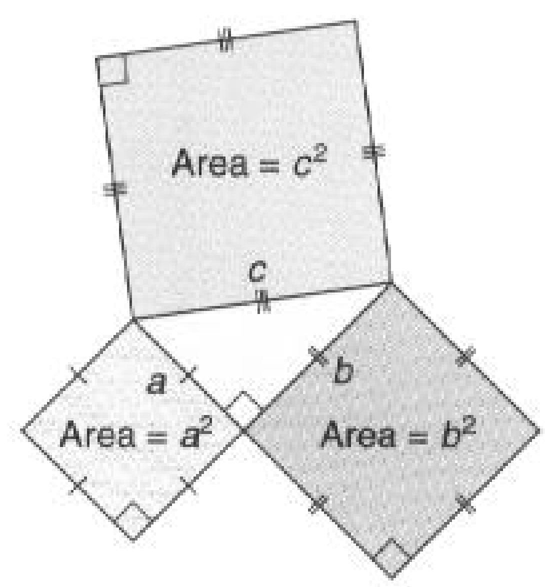
Reflect & Share

Compare your solution with that of another pair of classmates.
If the solutions are different, find out which is correct.
What strategies did you use to solve the problem?

Connect

Since the Pythagorean Theorem is true for all right triangles, we can write an algebraic equation to describe it.

In the triangle at the right, the hypotenuse has length c, and the legs have lengths a and b.



The area of the square on the hypotenuse is $c \times c$, or c^2.

The areas of the squares on the legs are $a \times a$ and $b \times b$, or a^2 and b^2.

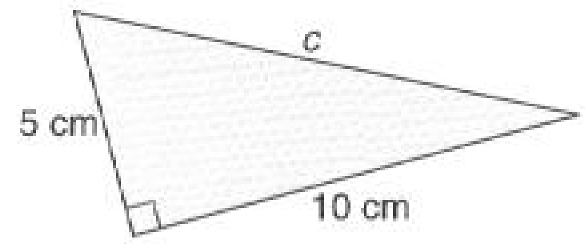
So, we can say: $c^2 = a^2 + b^2$

When we use this equation, remember that the lengths of the legs are represented by a and b, and the length of the hypotenuse by c.

Example 1

Find the length of each side labelled with a variable.
Give the lengths to the nearest millimetre.

a)



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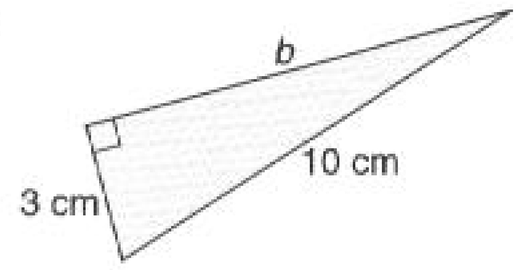


Solution

Use the Pythagorean Theorem: $c^2 = a^2 + b^2$

a) Substitute: $a = 5$ and $b = 10$

$c^2 = 5^2 + 10^2$

Square, then add.

$c^2 = 25 + 100$

$c^2 = 125$

The area of the square with side length c is 125.

So, $c = \sqrt{125}$

$c \doteq 11.180\,34$

c is approximately 11.2 cm.

b) Substitute: $a = 3$ and $c = 10$

$10^2 = 3^2 + b^2$

Square, then add.

$100 = 9 + b^2$

Subtract 9 from each side to isolate b^2.

$100 - 9 = 9 + b^2 - 9$

$91 = b^2$

The area of the square with side length b is 91.

So, $b = \sqrt{91}$

$b \doteq 9.539\,39$

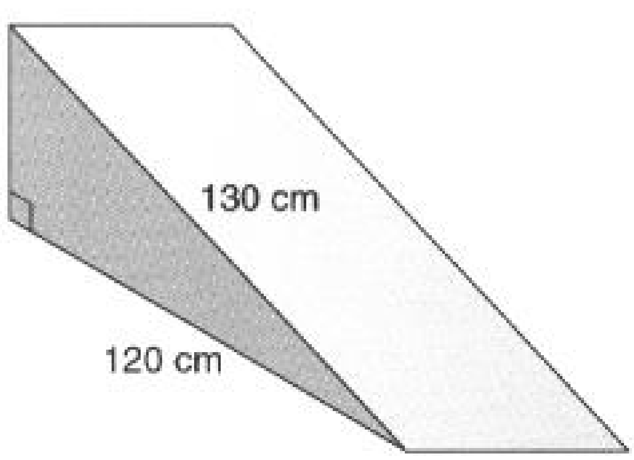
b is approximately 9.5 cm.

Use a calculator to calculate each square root.

We can use the Pythagorean Theorem to solve problems that involve right triangles.

Example 2

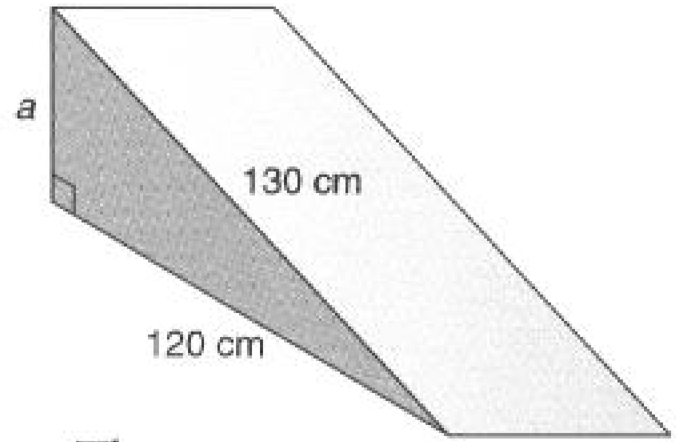
A ramp has horizontal length 120 cm and sloping length 130 cm.
How high is the ramp?



Solution

The height of the ramp is vertical, so the front face of the ramp is a right triangle.
The hypotenuse is 130 cm.

One leg is 120 cm.
The other leg is the height. Label it a.



Use the Pythagorean Theorem.
$c^2 = a^2 + b^2$
Substitute: $c = 130$ and $b = 120$
$130^2 = a^2 + 120^2$ Use a calculator.
$16\,900 = a^2 + 14\,400$
Subtract 14 400 from each side to isolate a^2.
$16\,900 - 14\,400 = a^2 + 14\,400 - 14\,400$
$2500 = a^2$
The area of the square with side length a is 2500.
$a = \sqrt{2500}$
$= 50$
The ramp is 50 cm high.

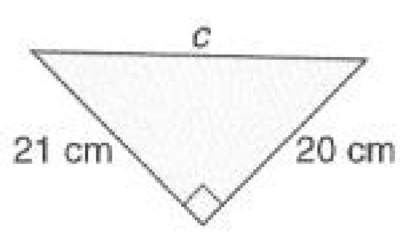
Practice

Calculator Skills

Suppose your calculator does not have a $\boxed{\sqrt{\ }}$ key. How can you find $\sqrt{1089}$?

1. Find the length of each hypotenuse labelled with a variable.

a)



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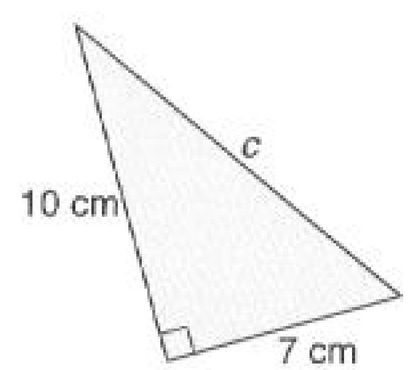

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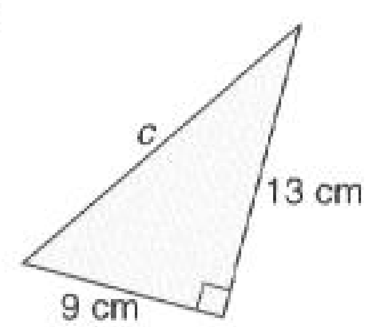
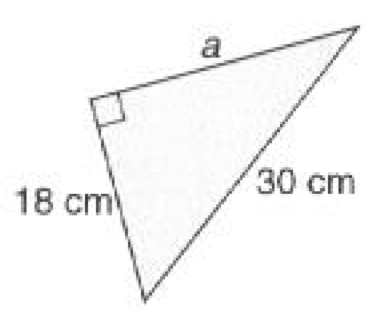


2. Find the length of each leg labelled with a variable.

a)



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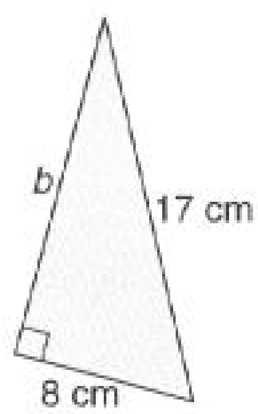

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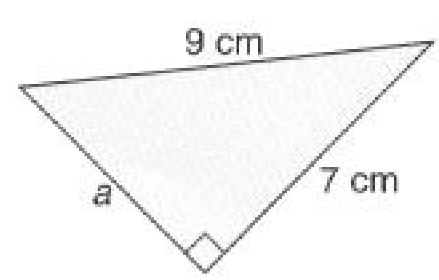
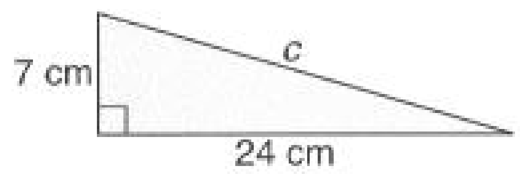
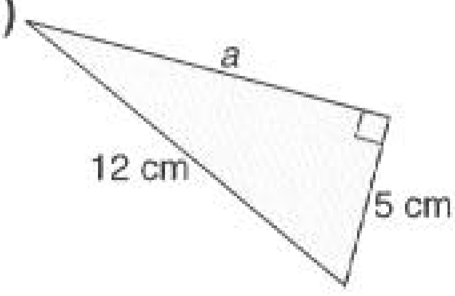
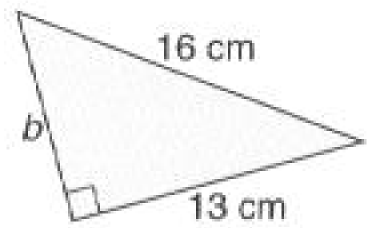
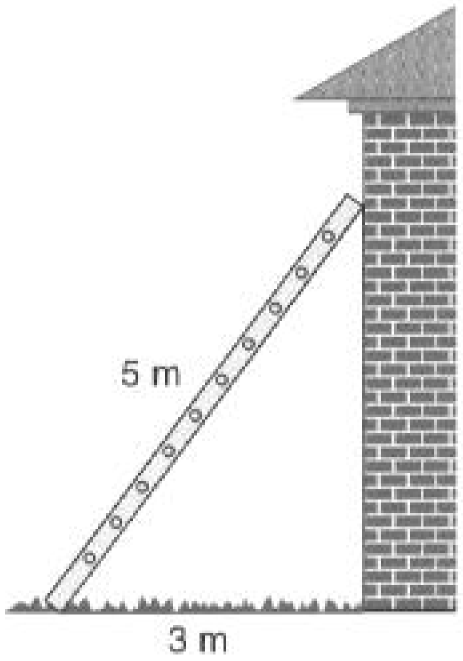


3. Find the length of each side labelled with a variable.

a) 

b) 

c) 



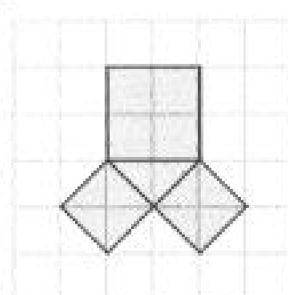
4. A 5-m ladder is leaning against a house.
It is 3 m from the base of the wall.
How high does the ladder reach?

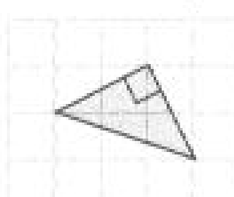
5. Brandon constructed a right triangle with sides 10 cm and 24 cm.

a) How long is the third side?

b) Why are there two answers to part a?

6. Copy each diagram on grid paper.
Explain how each diagram illustrates the Pythagorean Theorem.

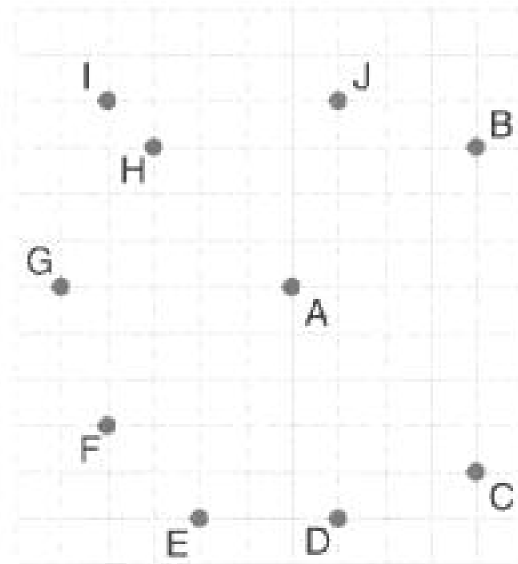
a) 

b) 

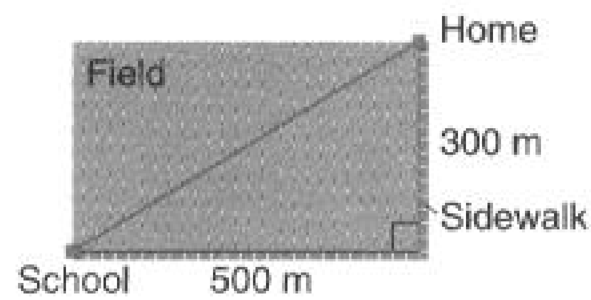
7. Alyssa has made a picture frame.
The frame is 60 cm long and 25 cm wide.
To check that the frame has square corners,
Alyssa measures a diagonal.
How long should the diagonal be?
Sketch a diagram to illustrate your answer.

8. The size of a TV set is described by the length of a diagonal of the screen.
One TV is labelled as size 70 cm.
The screen is 40 cm high.
What is the width of the screen?
Draw a diagram to illustrate your answer.

9. Assessment Focus
Look at the grid.
Without measuring, find another point that is the same distance from A as B is.
Explain your strategy.
Show your work.

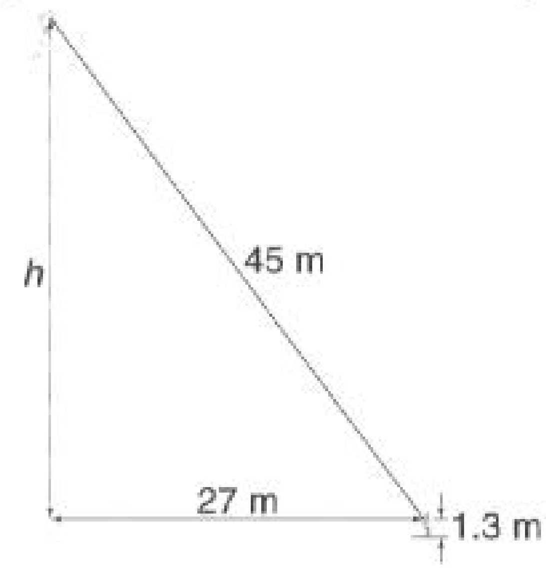


10. Joanna usually uses the sidewalk when she walks home from school. Today she is late, and so cuts through the field. How much shorter is Joanna's shortcut?

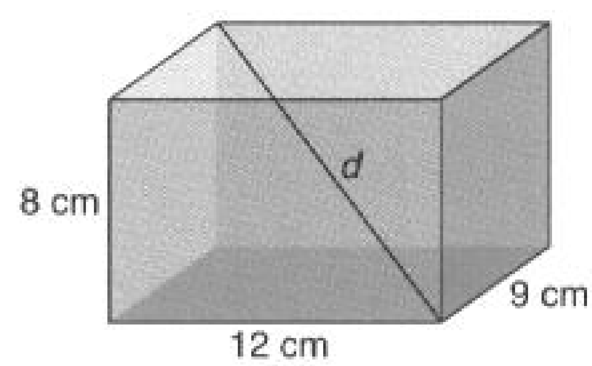


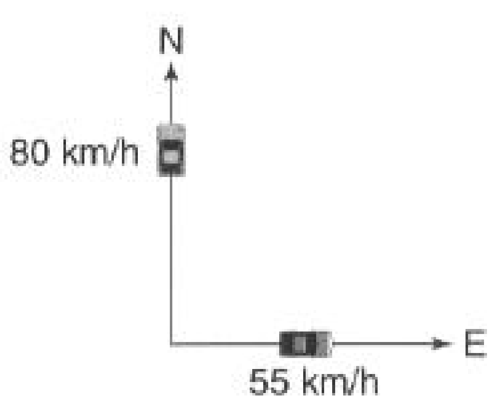
Take It Further

11. How high is the kite above the ground?



12. What is the length of the diagonal in this rectangular prism?





13. Two cars meet at an intersection.
One travels north at an average speed of 80 km/h.
The other travels east at an average speed of 55 km/h.
How far apart are the cars after 3 h?

Reflect

When can you use the Pythagorean Theorem to solve a problem?
Use examples in your explanation.

8.5 Special Triangles

Focus Apply the Pythagorean Theorem to isosceles and equilateral triangles.

Explore

Work with a partner.
An isosceles triangle has two equal sides.
Use this information to find the area of an isosceles triangle with side lengths 6 cm, 5 cm, and 5 cm.

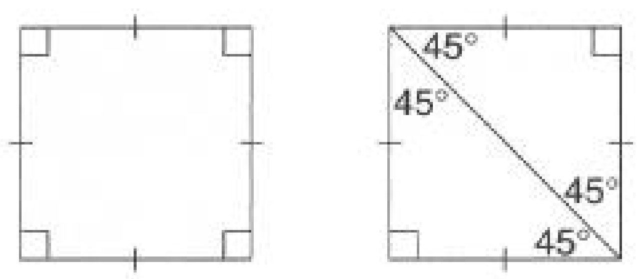
Reflect & Share

Share your results with another pair of classmates.
Compare strategies.
How could you use the Pythagorean Theorem to help you find the area of the triangle?

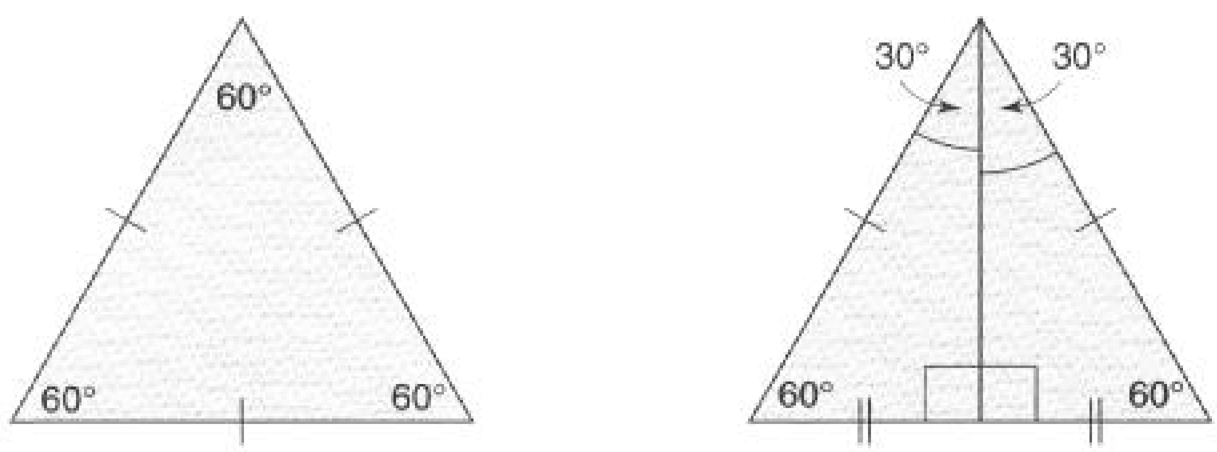
Connect

To apply the Pythagorean Theorem to new situations, we look for right triangles within other figures.

- A square has four equal sides and four 90° angles.
  A diagonal creates two congruent isosceles right triangles.
  Any isosceles right triangle has two equal sides and angles of 45°, 45°, and 90°.



- An equilateral triangle has three equal sides and three 60° angles.
  A line of symmetry creates two congruent right triangles.
  Each congruent right triangle has angles 30°, 60°, and 90°.



We can use the area of an equilateral triangle to find the surface area and volume of a hexagonal prism when the base is a regular hexagon.

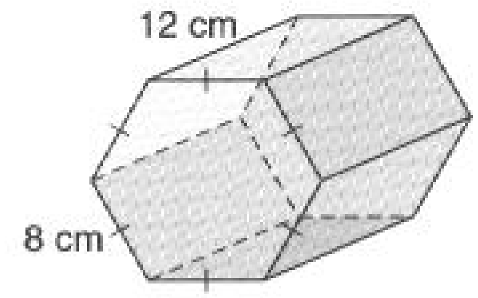
Example

The base of a prism is a regular hexagon with side length 8 cm. The length of the prism is 12 cm.

a) Find the area of the hexagonal base.

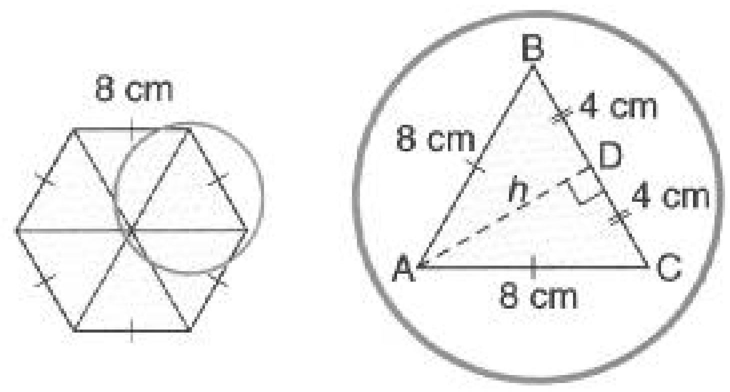
b) Find the volume of the prism.

c) Find the surface area of the prism.



Solution

a) The diagonals through the centre of a regular hexagon divide it into 6 congruent equilateral triangles.
One of these triangles is $\triangle ABC$.
Draw the perpendicular from A to BC at D.
AD bisects BC, so: BD = DC = 4 cm
Label h, the height of $\triangle ABC$.



Use the Pythagorean Theorem in $\triangle ABD$.

$c^2 = a^2 + b^2$

Substitute: $c = 8$, $a = 4$, $b = h$

$$8^2 = 4^2 + h^2$$
$$64 = 16 + h^2$$
$$64 - 16 = 16 + h^2 - 16$$
$$48 = h^2$$

So, $h = \sqrt{48}$

The height of $\triangle ABD$ is $\sqrt{48}$ cm.

The base of the triangle is 8 cm.

So, the area of $\triangle ABC = \frac{1}{2} \times 8 \times \sqrt{48}$

There are 6 congruent triangles.

And, the area of the hexagon $= 6 \times \frac{1}{2} \times 8 \times \sqrt{48}$

$= 24 \times \sqrt{48}$ Use a calculator.

$\doteq 166.28$

The area of the hexagonal base is approximately 166 cm^2.

b) The volume of the prism is: $V =$ base area $\times$ length
Use the exact value of the base area: $24 \times \sqrt{48}$
The length of the prism is 12 cm.

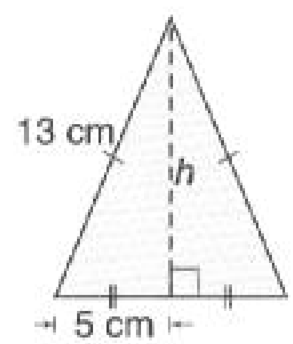
So, $V = 24 \times \sqrt{48} \times 12$ Use a calculator.
$\doteq 1995.32$
The volume of the prism is approximately 1995 cm^3.

c) The surface area A of the prism is the sum of the areas of the 6 rectangular faces and the two bases.
The rectangular faces are congruent.
So, $A = 6 \times (8 \times 12) + 2 \times (24 \times \sqrt{48})$
$\doteq 576 + 332.55$
$= 908.55$
The surface area of the prism is approximately 909 cm^2.

Practice

1. Find each length indicated.
Sketch and label the triangle first.

a)



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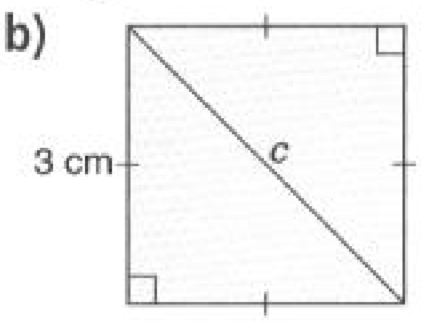

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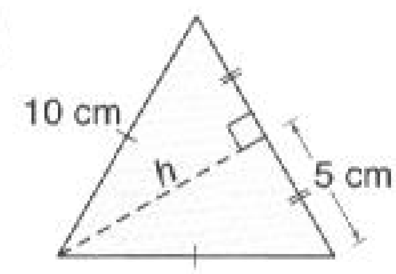
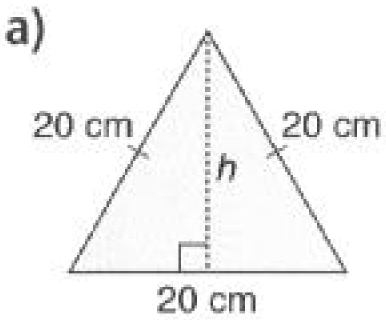


2. Find each length indicated.
Sketch and label the triangle first.

a)



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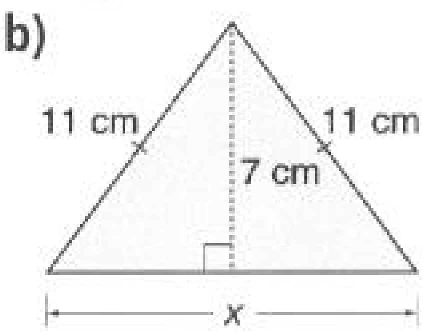

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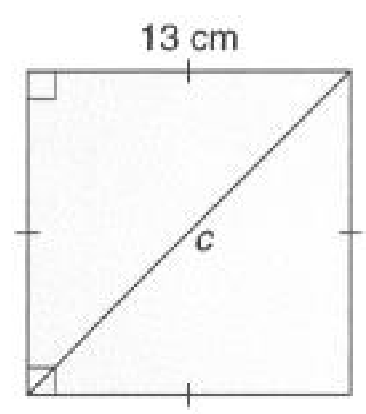
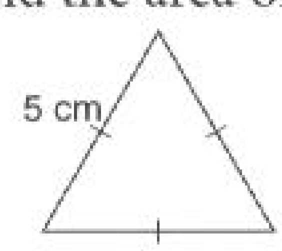
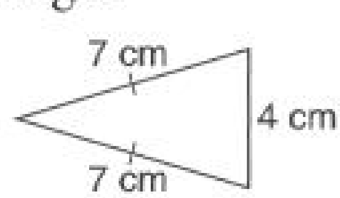


3. Find the area of each triangle.

a)



b)



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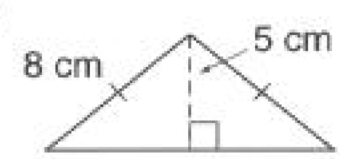
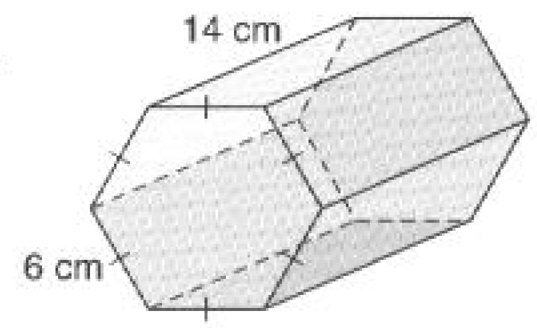
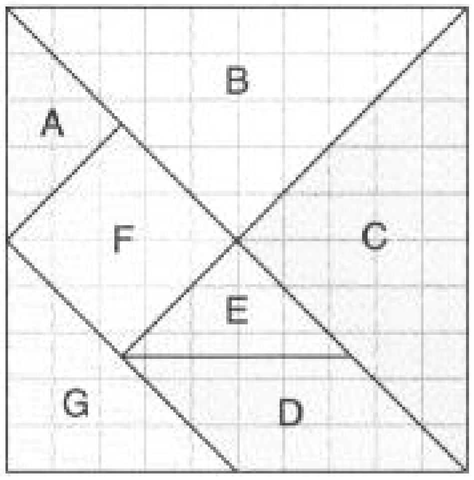


4. A prism has a base that is a regular hexagon with side length 6 cm. The prism is 14 cm long.
   a) Find the area of the base of the prism.
   b) Find the volume of the prism.
   c) Find the surface area of the prism.



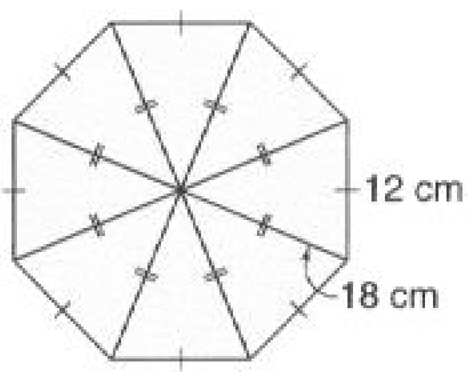
5. **Assessment Focus**
   Here is a tangram.
   Its side length is 10 cm.
   a) What is the area of Figure F?
      How long is each side of the square?
   b) What is the perimeter of Figure B?
   c) What is the perimeter of Figure D?
   d) How can you use Figure D to find the perimeter of Figure E?
   Show your work.



Number Strategies

A rectangular pool has length 12 m and width 7 m. A circular pool has the same area as the rectangular pool. What is the circumference of the circular pool?

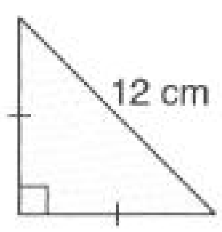
6. Here is one base of an octagonal prism. The prism is 30 cm long.



   a) Find the volume of the prism.
   b) Find the surface area of the prism.

Take It Further

7. Find the area and perimeter of this right isosceles triangle.



Reflect

How can the Pythagorean Theorem be used in isosceles and equilateral triangles?
Include examples in your explanation.

Unit Review

Review any lesson with etext online tutorial

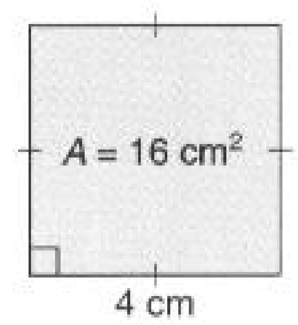
What Do I Need to Know?

✓ Side Length and Area of a Square

The side length of a square is equal to the square root of its area.

Length $= \sqrt{\text{Area}}$

Area $= (\text{Length})^2$

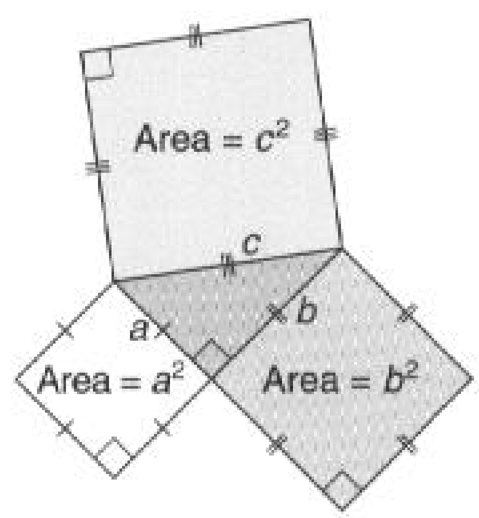


✓ The Pythagorean Theorem

In a right triangle, the area of the square on the hypotenuse is equal to the sum of the areas of the squares on the two legs.

$c^2 = a^2 + b^2$

Use the Pythagorean Theorem to find the length of a side in a right triangle, when two other sides are known.



What Should I Be Able to Do?

For extra practice, go to page 495.

LESSON

8.1, 8.2

1. Estimate each square root to the nearest whole number.

a) $\sqrt{6}$ b) $\sqrt{11}$
c) $\sqrt{26}$ d) $\sqrt{35}$
e) $\sqrt{66}$ f) $\sqrt{86}$

2. Estimate each square root to 1 decimal place.
Show your work.

a) $\sqrt{55}$ b) $\sqrt{75}$
c) $\sqrt{95}$ d) $\sqrt{105}$
e) $\sqrt{46}$ f) $\sqrt{114}$

3. Use a calculator to write each square root to 1 decimal place.

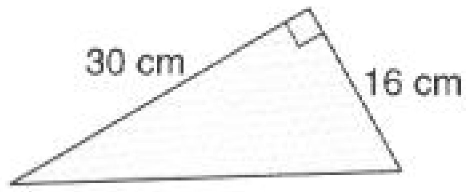
a) $\sqrt{46}$ b) $\sqrt{84}$
c) $\sqrt{120}$ d) $\sqrt{1200}$

4. A square blanket has an area of 16 900 cm^2. How long is each side of the blanket?

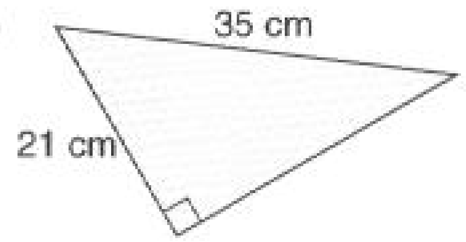
LESSON

8.3 **5.** Find the length of the unmarked side in each right triangle.

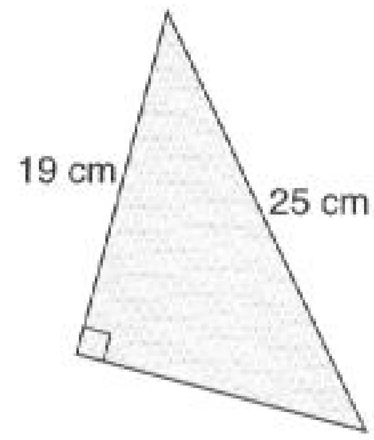
a)



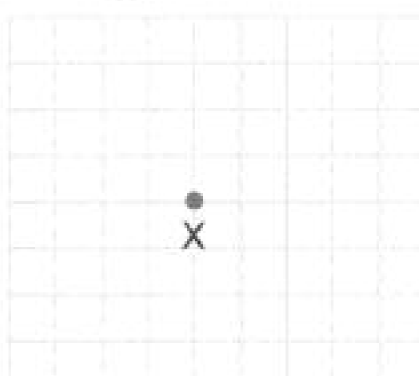
b)



c)



6. There is buried treasure at one of the points of intersection of the grid lines shown below.
Copy the grid.

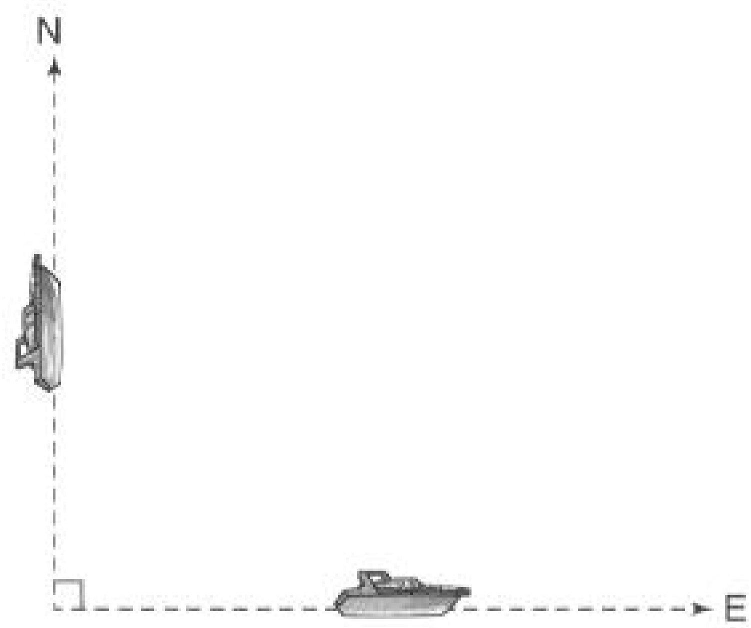


The treasure is $\sqrt{13}$ units from the point marked X.

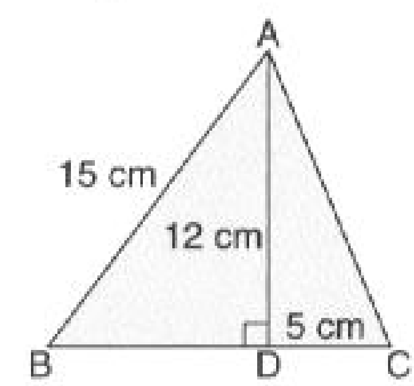
a) Where might the treasure be? Explain how you located it.

b) Could there be more than one position? Explain.

8.4 **7.** A boat travels due east at an average speed of 10 km/h.
At the same time, another boat travels due north at an average speed of 12 km/h.
After 2 h, how far apart are the boats? Explain your thinking.

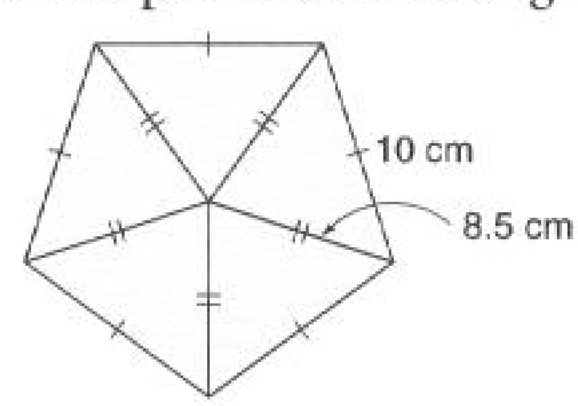


8.5 **8.** Find the perimeter of $\triangle ABC$.



9. Find the area of an equilateral triangle with side length 15 cm.

10. Here is one base of a pentagonal prism. It comprises five isosceles triangles, with the measures given. The prism is 7 cm long.



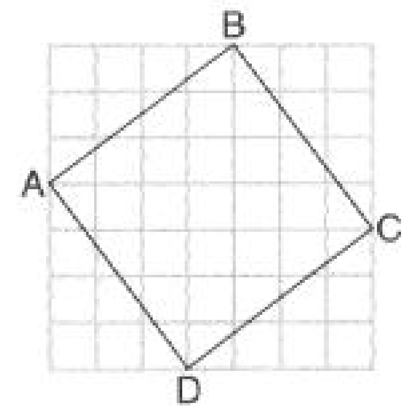
a) Sketch the prism.

b) Find the surface area of the prism.

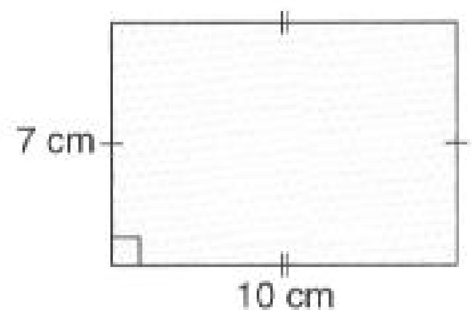
c) Find the volume of the prism.

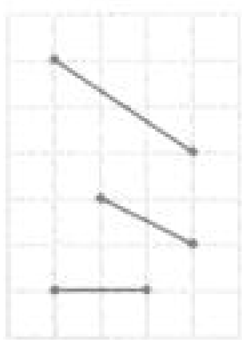
Practice Test

1. a) What is the area of square ABCD?
   b) What is the length of line segment AB? Explain your reasoning.



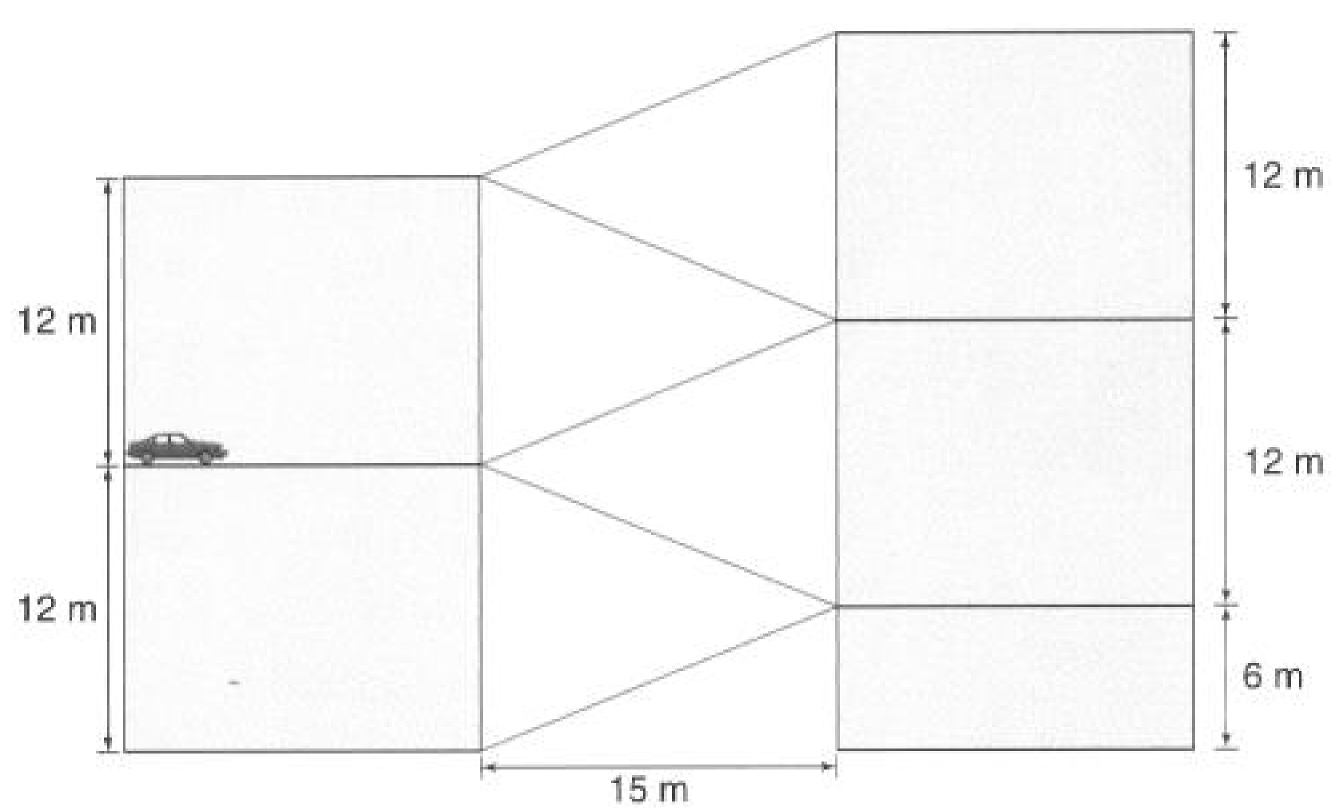
2. Find the side length of a square that has the same area as this rectangle.





3. Draw these 3 line segments on 1-cm grid paper.
   a) Find the length of each line segment to the nearest millimetre.
   b) Could these segments be arranged to form a triangle?
   If your answer is no, explain why not.
   If your answer is yes, could they form a right triangle? Explain.

4. A parking garage has ramps from one level to the next.
   a) How long is each ramp?
   b) What is the total length of the ramps?



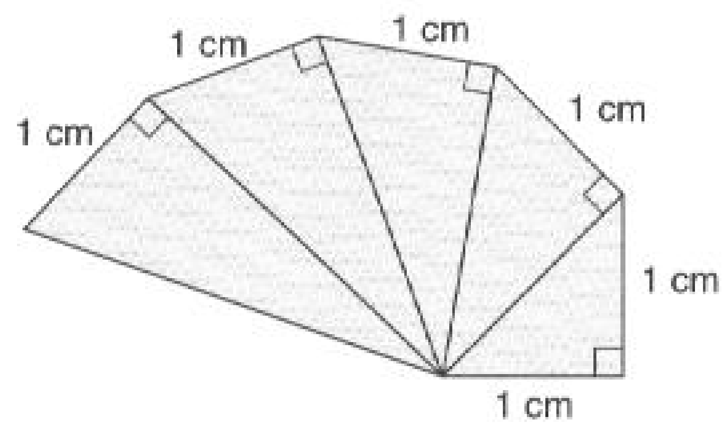
Unit Problem Pythagoras through the Ages

Throughout the ancient world, mathematicians were fascinated by right triangles. You will explore some of their discoveries.

Ancient Greece, 400 B.C.E.

Theodorus was born about 100 years after Pythagoras.
Theodorus used right triangles to create a spiral.
Today it is known as the Wheel of Theodorus.

➢ Follow these steps to draw the Wheel of Theodorus. You will need a ruler and protractor. Your teacher will give you a copy of a 10-cm ruler.

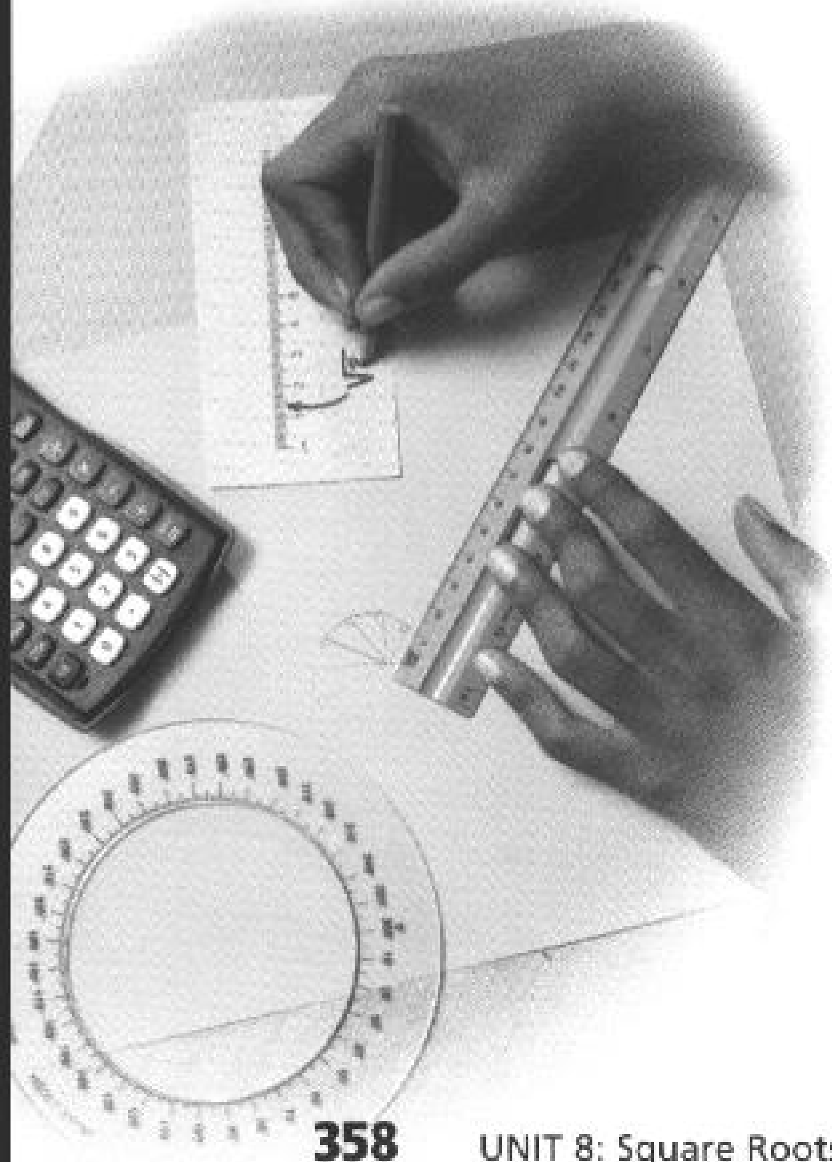


Step 1 Draw a right triangle with legs 1 cm.

Step 2 The hypotenuse of this triangle is one leg of the next triangle.
Draw the other leg of the next triangle 1 cm long.
Draw the hypotenuse.

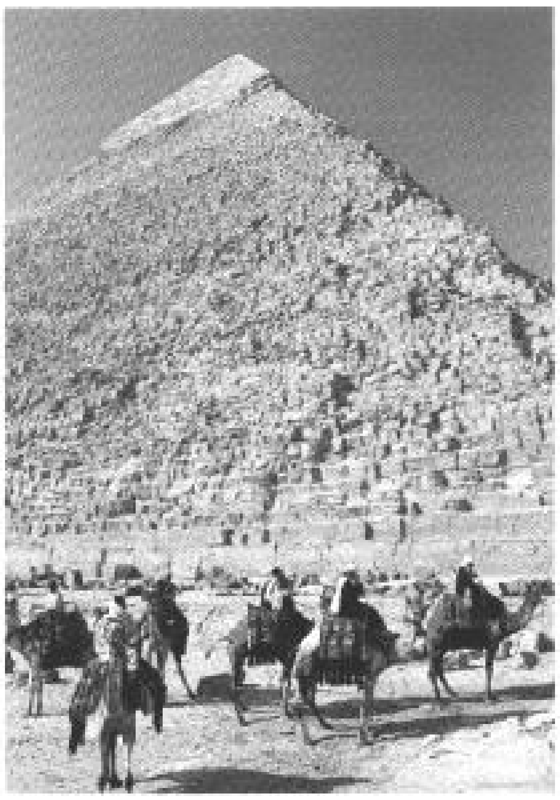
Step 3 Repeat *Step 2* until you have at least ten triangles.

➢ Use the Pythagorean Theorem to find the length of each hypotenuse.
Label each hypotenuse with its length as a square root.
What patterns do you see?



➢ Use a ruler to measure the length of each hypotenuse to the nearest millimetre.
Use the copy of the 10-cm ruler. Mark the point on the ruler that represents the value of each square root.
Compare the two ways to measure the hypotenuse.
What do you notice?

➢ Without using a calculator or extending the Wheel of Theodorus, estimate $\sqrt{24}$ as a decimal.
Label $\sqrt{24}$ cm on your ruler.
Explain your reasoning and any patterns you see.



Ancient Egypt, 2000 B.C.E.

In ancient Egypt, the Nile River overflowed every year and destroyed property boundaries.
Because the land plots were rectangular,
the Egyptians needed a way to mark a right angle.

The Egyptians tied 12 evenly spaced knots along a piece of rope and made a triangle from it.
Explain how you think the Egyptians used the knotted rope to mark a right angle.

Check List

Your work should show:

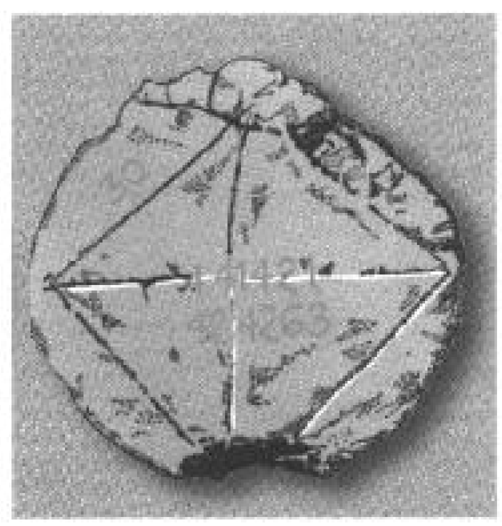
- ✓ All constructions and diagrams correctly labelled
- ✓ Detailed and accurate calculations
- ✓ Clear descriptions of the patterns observed
- ✓ Reasonable explanations of your thinking and your conclusions about the Pythagorean Theorem

Ancient Babylon, 1700 B.C.E.

Archaeologists have discovered evidence that the ancient Babylonians knew about the Pythagorean Theorem over 1000 years before Pythagoras!

The archaeologists found this tablet.

When the tablet is translated, it looks like this.



What do you think the diagram on the tablet means?
Explain your reasoning.

Reflect on the Unit

What is the Pythagorean Theorem? How is it used?
Include examples in your explanation.

Units 1–8 Cumulative Review

UNIT

1 **1.** Evaluate.

a) $3.8 + 5.7 \div 1.9$

b) $2.4^2 - (4.2 - 3.7)^2$

c) $(1.5 + 4.2) + 2.8 \times 7.2$

d) $1.5 + (4.2 + 2.8) \times 7.2$

2 **2.** Which is the better buy?

a) 8 cheese slices for \$1.49 or 24 cheese slices for \$3.29

b) 1.89 L of cranberry-raspberry drink for \$3.27 or 3.78 L for \$5.98

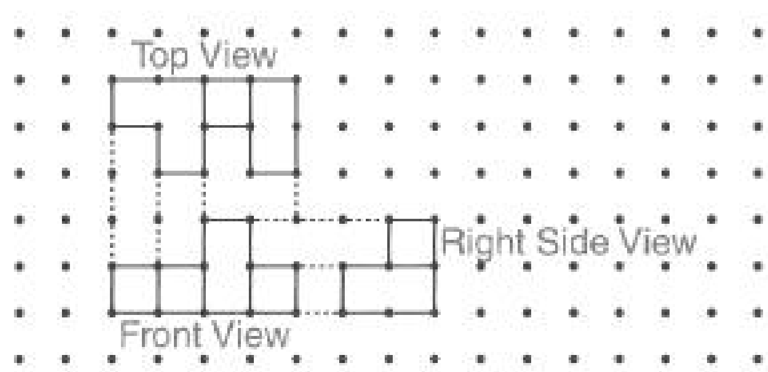
c) 100 g of iced tea mix for \$0.29 or 500 g for \$1.69

d) 1 can of chicken soup for \$0.57 or a 12-pack for \$5.99

3 **3.** Use linking cubes.

a) Build the object for the set of views below.

b) Sketch the object on isometric dot paper.



4 **4.** Write as many division questions as you can that have each fraction below as their quotient.

a) $\frac{1}{2}$ **b)** $\frac{2}{3}$ **c)** $\frac{4}{5}$ **d)** $\frac{5}{6}$

Write about the strategies you used to find the division questions.

5 **5.** In each case, identify the collected data as from a sample or a census. Justify your answer.

a) To find the mean number of chocolate chips in a cookie, every 100th cookie was tested.

b) To find what type of movie the family wanted to rent, all 6 family members were asked.

c) To find the number of Ontario families that enjoy skiing, one in 10 households was surveyed by phone.

d) To find out if the city should increase funding for snow removal, a questionnaire was enclosed in every tax bill.

6. Adam recorded the heights of a bean plant and sunflower plant.

Time (days)	Height (cm)	
	Bean	Sunflower
0	2	6
3	6	8
6	9	11
9	13	14
12	20	17
15	28	21
18	35	26
21	41	33
24	47	38
27	56	42
30	63	48
33	68	53

a) Display the data using the most suitable method. Justify your choice.

b) Describe any trends in the graph.

c) Predict the height of each plant after 39 days. Explain the method you used to make your prediction.

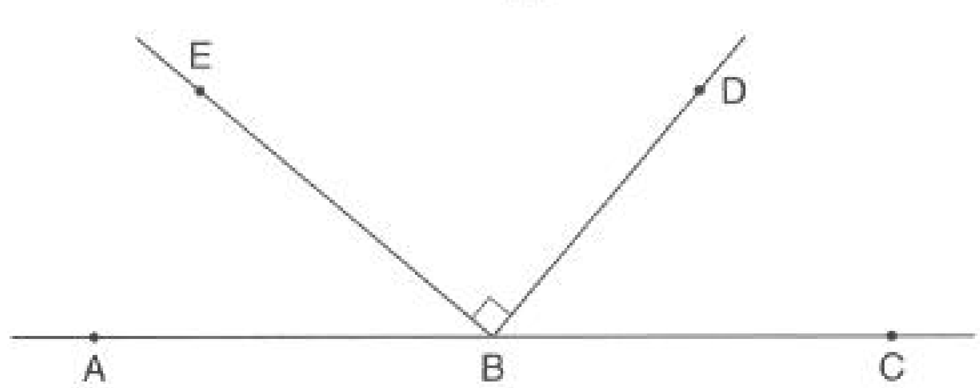
6 **7.** Draw a circle. Label its centre C. Choose two points G and H on the circle that are not the endpoints of a diameter. Join CG, GH, and CH. What type of triangle is $\triangle CGH$? How do you know?

8. Which has the greater area: a circle with circumference 1 m or a circle with radius 30 cm? Justify your answer.

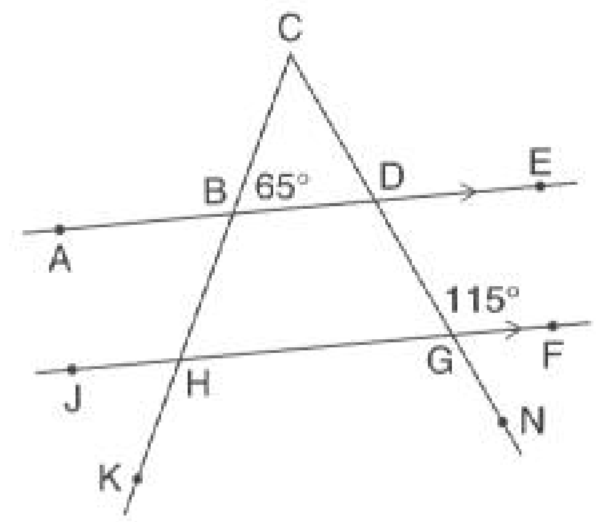
7 **9.** A cardboard tube is used to send a poster by mail. The tube is 0.8 m long with diameter 7 cm. The ends of the tube are closed with tape. What is the area of cardboard in the tube?

10. a) Name the complement of $\angle ABE$.

b) Name the supplement of $\angle ABE$.



11. Look at this diagram.



a) Name two pairs of:

i) interior angles

ii) alternate angles

iii) corresponding angles

b) Find the measures of $\angle CHG$, $\angle JHK$, $\angle CGH$, and $\angle FGN$.

c) Find the measure of $\angle BCD$. What kind of triangle is $\triangle BCD$?

8 **12.** Estimate each square root to 1 decimal place. Show your work. Then check with a calculator.

a) $\sqrt{52}$ b) $\sqrt{63}$

c) $\sqrt{90}$ d) $\sqrt{76}$

13. The area of the square on each side of a triangle is given. Is the triangle a right triangle? How do you know?

a) 16 cm^2, 8 cm^2, 30 cm^2

b) 16 cm^2, 8 cm^2, 24 cm^2

14. The dimensions of a rectangle are 3 cm by 4 cm. What is the length of a diagonal? Explain your reasoning.