St. Augustine Prep

Algebra I

Summer Assignment

The following packet contains topics that you will be required to know in order to succeed in Algebra I. Please complete all problems. You may print out the packet and write your answers on the worksheets. If you are unable to print, please write on loose leaf paper. **Please write neatly!** The topics should have been discussed in your eighth grade math course. Your work will be collected the first day of class. We will go over the packet the first week of school and you will be tested on the review material.

Make your first homework and test grade the best it can be!

Section 1 – Order of Operations

The order of operations in mathematics is a set of rules to follow to determine which operation to do first when there are different operations within a single problem. The order to perform combined operations is called the **PEMDAS** rule. A common mnemonic for **PEMDAS** is **P**lease **E**xcuse **M**y **D**ear **A**unt **S**ally.

- Always work on the calculations within ***parenthesis*** first (if any)
- Next, calculate the ***exponents***
- Then, carry out ***multiplication*** or ***division***, working from ***left to right***
- Lastly, do ***addition*** and ***subtraction***, working from ***left to right***

Example 1

Evaluate $16 - 8 \div 2^2 + 14$.

$16 - 8 \div \mathbf{2^2} + 14 = 16 - 8 \div \mathbf{4} + 14$	Evaluate powers.
$= 16 - 2 + 14$	Divide 8 by 4.
$= 14 + 14$	Subtract 2 from 16.
$= 28$	Add 14 and 14.

Example 2

Evaluate each expression.

a. $4 \div 2 + 5(10 - 6)$

$4 \div 2 + 5(10 - 6) = 4 \div 2 + 5(4)$	Evaluate inside parentheses.
$= 2 + 5(4)$	Divide 4 by 2.
$= 2 + 20$	Multiply 5 by 4.
$= 22$	Add 2 to 20.

b. $6\left[32 - (2 + 3)^2\right]$

$6\left[32 - (2 + 3)^2\right] = 6\left[32 - (5)^2\right]$	Evaluate innermost expression first.
$= 6[32 - 25]$	Evaluate power.
$= 6[7]$	Subtract 25 from 32.
$= 42$	Multiply.

Section 1 – Exercises

Evaluate and simplify each expression.

1. 3^5	2. $10+8^3 \div 16$	3. $(12-6)\bullet 5^2$
4. $18 \div 9+2\bullet 6$	5. $3[10-(27 \div 9)]$	6. $4[(6^3-9) \div 23]$
7. $\dfrac{8+3^3}{12-7}$	8. $3[4-8+4^2(2+5)]$	9. $25+\left[(16-3\bullet 5)+\dfrac{12+3}{5}\right]$

Section 2 – Fractions

We use fractions or ratios every day. A fraction is part of an entire object. It consists of two numbers, a number on the top called a numerator and a number on the bottom called a denominator. To add or subtract a fraction, you must have a common denominator.

- Find the Least Common Denominator (LCD) of the fractions
- Rename the fractions to have the LCD
- Add (or subtract) the numerators
- Keep the LCD
- Simplify the fraction

Example 1

Find each sum or difference. Write in simplest form.

a. $\frac{1}{2}+\frac{2}{3}$

$\frac{1}{2}+\frac{2}{3}=\frac{3}{6}+\frac{4}{6}$ The LCD for 2 and 3 is 6. Rename $\frac{1}{2}$ as $\frac{3}{6}$ and $\frac{2}{3}$ as $\frac{4}{6}$.

$=\frac{3+4}{6}$ Add the numerators.

$=\frac{7}{6}$ or $1\frac{1}{6}$ Simplify.

b. $\frac{3}{8}-\frac{1}{3}$

$\frac{3}{8}-\frac{1}{3}=\frac{9}{24}-\frac{8}{24}$ The LCD for 8 and 3 is 24. Rename $\frac{3}{8}$ as $\frac{9}{24}$ and $\frac{1}{3}$ as $\frac{8}{24}$.

$=\frac{9-8}{24}$ Subtract the numerators.

$=\frac{1}{24}$ Simplify.

c. $\frac{2}{5}-\frac{3}{4}$

$\frac{2}{5}-\frac{3}{4}=\frac{8}{20}-\frac{15}{20}$ The LCD for 5 and 4 is 20. Rename $\frac{2}{5}$ as $\frac{8}{20}$ and $\frac{3}{4}$ as $\frac{15}{20}$.

$=\frac{8-15}{20}$ Subtract the numerators.

$=-\frac{7}{20}$ Simplify.

Section 2 – Exercises

Find each sum or difference. Write your answer in simplest form. (Leave as an improper fraction.)

10. $\frac{2}{3}+\frac{7}{8}$	11. $\frac{13}{20}-\frac{2}{5}$	12. $\frac{5}{6}-\frac{8}{9}$

To multiply fractions, multiply the numerators and multiply the denominators. If the numerators and denominators have common factors, you can simplify before you multiply by cross canceling.

Example 2

Find each product.

a. $\frac{2}{5} \cdot \frac{1}{3}$

$\frac{2}{5} \cdot \frac{1}{3} = \frac{2 \cdot 1}{5 \cdot 3}$ Multiply the numerators. Multiply the denominators.

$= \frac{2}{15}$ Simplify.

b. $\frac{3}{5} \cdot 1\frac{1}{2}$

$\frac{3}{5} \cdot 1\frac{1}{2} = \frac{3}{5} \cdot \frac{3}{2}$ Write $1\frac{1}{2}$ as an improper fraction.

$= \frac{3 \cdot 3}{5 \cdot 2}$ Multiply the numerators. Multiply the denominators.

$= \frac{9}{10}$ Simplify.

c. $\frac{1}{4} \cdot \frac{2}{9}$

$\frac{1}{4} \cdot \frac{2}{9} = \frac{1}{\cancel{4}_{2}} \cdot \frac{\overset{1}{\cancel{2}}}{9}$ Divide by the GCF, 2.

$= \frac{1 \cdot 1}{2 \cdot 9}$ or $\frac{1}{18}$ Multiply the numerators. Multiply the denominators and simplify.

Section 2 – Exercises

Find each product. Write your answer in simplest form. (Leave as an improper fraction.)

13. $\frac{3}{5} \bullet \frac{5}{6}$	14. $\frac{11}{3} \bullet \frac{9}{44}$	15. $3\frac{1}{2} \bullet 1\frac{1}{2}$
16. $-\frac{2}{7} \bullet 4\frac{2}{3}$	17. $-\frac{1}{3} \bullet -7\frac{1}{2}$	18. $\frac{1}{4} \bullet -3\frac{5}{6}$

To divide one fraction by another, you multiply the first fraction by the reciprocal of the second fraction.

Example 3

Find each quotient.

a. $\frac{1}{3} \div \frac{1}{2}$

$\frac{1}{3} \div \frac{1}{2} = \frac{1}{3} \cdot \frac{2}{1}$ Multiply $\frac{1}{3}$ by $\frac{2}{1}$, the reciprocal of $\frac{1}{2}$.

$= \frac{2}{3}$ Simplify.

b. $\frac{3}{8} \div \frac{2}{3}$

$\frac{3}{8} \div \frac{2}{3} = \frac{3}{8} \cdot \frac{3}{2}$ Multiply $\frac{3}{8}$ by $\frac{3}{2}$, the reciprocal of $\frac{2}{3}$.

$= \frac{9}{16}$ Simplify.

c. $\frac{3}{4} \div 2\frac{1}{2}$

$\frac{3}{4} \div 2\frac{1}{2} = \frac{3}{4} \div \frac{5}{2}$ Write $2\frac{1}{2}$ as an improper fraction

$= \frac{3}{4} \cdot \frac{2}{5}$ Multiply $\frac{3}{4}$ by $\frac{2}{5}$, the reciprocal of $2\frac{1}{2}$.

$= \frac{6}{20}$ or $\frac{3}{10}$ Simplify.

d. $-\frac{1}{5} \div \left(-\frac{3}{10}\right)$

$-\frac{1}{5} \div \left(-\frac{3}{10}\right) = -\frac{1}{5} \cdot \left(-\frac{10}{3}\right)$ Multiply $-\frac{1}{5}$ by $\frac{10}{3}$, the reciprocal of $-\frac{3}{10}$.

$= \frac{10}{15}$ or $\frac{2}{3}$ Same sign ⟶ positive quotient; simplify.

Section 2 – Exercises

Find each quotient. Write your answer in simplest form. (Leave as an improper fraction.)

19. $\frac{3}{25} \div \frac{2}{15}$	20. $2\frac{1}{4} \div \frac{1}{2}$	21. $-\frac{9}{10} \div 3$

Section 3 – Real Number Comparison

An inequality is a mathematical sentence that compares the value of two expressions using an inequality symbol.

Inequality Symbol	Pronounced	Example
$<$	Less than	$4 < 9$
$\leq$	Less than or equal to	$-3 \leq 2$
$>$	Greater than	$-4 > -7$
$\geq$	Greater than or equal to	$5 \geq 5$
$\neq$	Not equal to	$7 \neq 11$

Example 1

Which one is greater, $\frac{4}{9} or \frac{5}{12}$?

Rewrite each fraction using the LCD.

$\frac{4}{9} = \frac{16}{36}$ and $\frac{5}{12} = \frac{15}{36}$

$\frac{16}{36} > \frac{15}{36}$ So $\frac{4}{9} > \frac{5}{12}$

Section 3 – Exercises

Use <, =, or > to compare the numbers.

22. -12_____-15	23. 0.63_____0.6	24. 0.88_____$\frac{8}{9}$
25. $\frac{2}{3}$_____$\frac{1}{6}$	26. $\frac{3}{4}$_____$\frac{12}{16}$	27. $-2\frac{5}{8}$_____$-2\frac{1}{2}$

Section 4 – Variables & Verbal Expressions

Translating in mathematics usually involves changing a verbal phrase into a mathematical phrase. The following are common phrases used in mathematics.

Phrase	**Sign**
sum, increased, added to, more than, plus, totals, combined, perimeter	+
difference, minus, less than, used, remain, subtracted from, decreased by	–
product, of, times, area, doubles, multiplied by	•
quotient, division, average, half, divided by, per	÷
is, is the same as, equal, was, were, has, costs, becomes	=

Section 4 – Exercises

Write an algebraic expression for each phrase.

28. 7 increased by x	29. the difference of 8 and n
30. the product of 2 and t	31. 10 decreased by m
32. 32 divided by d	33. 12 less than p
34. the sum of 7 and h	35. 9 plus the quotient of y and 15

Section 5 – Evaluating Algebraic Expressions

A ***variable*** is a letter that represents an unspecified number. To evaluate an algebraic expression, replace the variables with their values. Then find the value of the numerical expression using the order of operations.

Example 1

Evaluate $3x^2 + (2y + z^3)$ if $x = 4$, $y = 5$, $z = 3$.

$3x^2 + (2y + z^3)$

$= 3(4)^2 + (2 \cdot 5 + 3^3)$	Replace x with 4, y with 5, and z with 3.
$= 3(4)^2 + (2 \cdot 5 + 27)$	Evaluate 3^3.
$= 3(4)^2 + (10 + 27)$	Multiply 2 by 5.
$= 3(4)^2 + (37)$	Add 10 to 27.
$= 3(16) + 37$	Evaluate 4^2.
$= 48 + 37$	Multiply 3 by 16.
$= 85$	Add 48 to 37.

Section 5 – Exercises

Evaluate each expression.

36. xy for $x = 3$, $y = 16$	37. $n + 2$ for $n = -7$
38. $10 - r + 5$ for $r = 23$	39. $t + u \div 6$ for $t = 12$, $u = 18$
40. $4p - 26$ for $p = 10$	41. $m^2 - 7$ for $m = 11$
42. $3ab - c$ for $a = -4$, $b = 2$, $c = 5$	43. $\frac{ab}{2} - 4c$ for $a = 6$, $b = 5$, $c = 3$

Section 6 – Solving One-Step Equations

In an equation, the variable represents the number that satisfies the equation. To solve an equation means to find the value of the variable that makes the equation true. You will only need to perform one step in order to solve a **one-step** equation.

The strategy for getting the variable by itself involves using opposite operations. The most important thing to remember in solving a linear equation is that whatever you do to one side of the equation, you MUST do to the other side.

Example 1

$-2 = k - 14$	Solve
$-2 + 14 = k - 14 + 14$	Since 14 is subtracted from k, you must add 14 to **each** side of the equation
$12 = k$ or $k = 12$	Answer

Example 2

$\frac{x}{-7} = 15$	Solve
$(-7)\frac{x}{-7} = 15(-7)$	Since x is divided by –7, you must multiply **both** sides by –7
$x = -105$	Answer

Example 3

$\frac{3}{4}u = -24$	Solve
$\left(\frac{4}{3}\right)\frac{3}{4}u = -24\left(\frac{4}{3}\right)$	Multiply both sides by the reciprocal of $\frac{3}{4}$ and cancel any common factors
$u = -32$	Answer

Section 6 – Exercises

Solve each equation.

44. $37 = x - 72$	45. $5p = 325$
46. $d + 1.5 = 3.7$	47. $102 + t = 36$
48. $\frac{2}{3}y = 8$	49. $\frac{h}{7} = -12$
50. $\frac{3}{5}g = -6$	51. $\frac{1}{4}m = \frac{5}{8}$

Section 7 – Measures of Central Tendency

In working with statistical data, it is often useful to determine a single quantity that best describes a set of data. The best quantity to choose is usually one of the most popular measures of central tendency: mean, median, mode, or range.

Mean The mean is the sum of the data items in a set divided by the number of data items in the set.

Median The median is the middle value in a set of data when the numbers are arranged in numerical order. If the set has an even number of data items, the median is the mean of the two middle data values.

Mode The mode is the data item that occurs most often in a set of data.

Range The range is the difference between the greatest and least values in a set of data.

Example 1

Set of data: 34, 46, 31, 40, 33, 40, 35

In order: 31, 33, 34, 35, 40, 40, 46

Mean	$\frac{(31+33+34+35+40+40+46)}{7}$	Answer: 37
Median	35 is the middle number when written in numerical order	Answer: 35
Mode	40 is the only number that occurs more than once	Answer: 40
Range	46 – 31	Answer: 15

Example 2

Set of data: 41, 28, 37, 56, 34, 61

In order: 28, 34, 37, 41, 56, 61

Mean	$\frac{(28+34+37+41+56+61)}{6}$	Answer: $42.8\overline{3}$
Median	$\frac{37+41}{2}$ (There are an even number of numbers in the data set)	Answer: 39
Mode	No number repeats more than once	Answer: None
Range	61 – 28	Answer: 33

Section 7 – Exercises

Find the mean, median, mode, and range of each set of data.

52. Daily sales from a store $834, $1099, $775, $900, $970 Mean = Median = Mode = Range =	53. Goals scored in a soccer game 3, 2, 0, 11, 7, 6, 4, 10 Mean = Median = Mode = Range =
54. Number of days above 50° in last 5 months 6, 8, 15, 22, 8 Mean = Median = Mode = Range =	55. Height of players on a basketball team (inches) 72, 74, 70, 77, 76, 72 Mean = Median = Mode = Range =

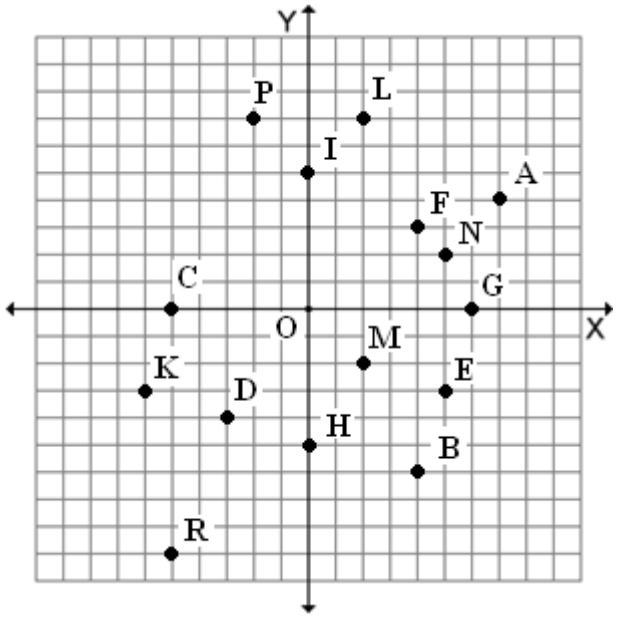
Section 8 – Plotting on the Coordinate Plane

You can graph points on a coordinate plane. Use an ordered pair (x, y) to record the coordinates. The first number in the pair is the *x-coordinate*. The second number is the *y-coordinate*. To graph a point, start at the origin (0, 0). Move **horizontally** according to the value of x. Then move **vertically** according to the value of y.

Section 8 – Exercises

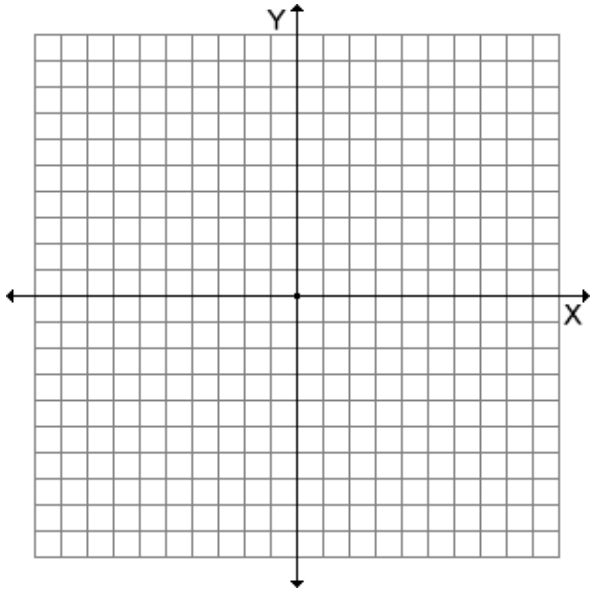
List the ordered pair for each letter, then identify the quadrant or axes the point lies in.

56. C	
57. A	
58. M	
59. P	
60. F	
61. I	
62. R	
63. E	



Plot & label the following ordered pairs.

64. F = (–8, 6)
65. R = (6, –1)
66. I = (–5, –7)
67. E = (4, 9)
68. N = (2, –3)
69. D = (–4, 0)
70. S = (0, 7)



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Review for Mastery

Variables and Expressions

To translate words into algebraic expressions, find words like these that tell you the operation.

$+$	$-$	$\cdot$	$\div$
add	subtract	multiply	divide
sum	difference	product	quotient
more	less	times	split
increased	decreased	per	ratio

Kenny owns *v* video games. Stan owns 7 more video games than Kenny. Write an expression for the number of video games Stan owns.

v represents the number of video games Kenny owns.

$v + 7$ *Think: The word "more" indicates addition.*

Order does not matter for addition. The expression $7 + v$ is also correct.

Jenny is 12 years younger than Candy. Write an expression for Jenny's age if Candy is *c* years old.

c represents Candy's age.

The word "younger" means "less," which indicates subtraction.

$c - 12$ *Think: Candy is older, so subtract 12 from her age.*

Order does matter for subtraction. The expression $12 - c$ is incorrect.

1. Jared can type 35 words per minute. Write an expression for the number of words he can type in *m* minutes. _____

2. Mr. O'Brien's commute to work is 0.5 hour less than Miss Santos's commute. Write an expression for the length of Mr. O'Brien's commute if Miss Santos's commute is *h* hours. _____

3. Mrs. Knighten bought a box of *c* cookies and split them evenly between the 25 students in her classroom. Write an expression for the number of cookies each student received. _____

4. Enrique collected 152 recyclable bottles, and Latasha collected *b* recyclable bottles. Write an expression for the number of bottles they collected altogether. _____

5. Tammy's current rent is *r* dollars. Next month it will be reduced by $50. Write an expression for next month's rent in dollars. _____

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Review for Mastery

Variables and Expressions *continued*

The value of $\square - 9$ depends on what number is placed in the box.

Evaluate $\square - 9$ when 20 is placed in the box.

$$\square - 9$$

$$\boxed{20} - 9$$

$$11$$

In algebra, variables are used instead of boxes.

Evaluate $x \div 7$ for $x = 28$.

$$x \div 7$$

$$28 \div 7$$

$$4$$

Sometimes, the expression has more than one variable.

Evaluate $x + y$ for $x = 6$ and $y = 2$.

$$x + y$$

$$6 + 2$$

$$8$$

Evaluate $5 + \square$ when each number is placed in the box.

6. 3 _____

7. 5 _____

8. 24 _____

Evaluate each expression for $x = 4$, $y = 6$, and $z = 3$.

9. $x + 15$ _____

10. $3y$ _____

11. $15 - z$ _____

Evaluate each expression for $x = 2$, $y = 18$, and $z = 9$.

12. $x \cdot z$ _____

13. $y - x$ _____

14. $y \div z$ _____

15. $\frac{y}{x}$ _____

16. xy _____

17. $z - x$ _____

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Review for Mastery

Solving Equations by Adding or Subtracting

Use counters to model solving equations.

Solve $x + 2 = 5$.

Using counters	Using numbers
$x +$ ○ ○ $=$ ○ ○ ○	$x + 2 = 5$
$x +$ ⊘ ⊘ $=$ ○ ○ ⊘	$x + 2 = 5$ $\underline{\quad -2 \quad -2}$
$x + 0 \quad =$ ○ ○	$x + 0 = 3$
$x =$ ○ ○	$x = 3$
Check: ○ ○ $+$ ○ ○ $=$ ○ ○ ○	***Check:*** $x + 2 = 5$ $3 + 2 \stackrel{?}{=} 5$ $5 \stackrel{?}{=} 5$ ✓

Solve the following by drawing counters. Check your answers.

1. $x + 1 = 5$

2. $7 = x + 2$

_____ _____

Solve each equation. Check your answers.

3. $x + 4 = 12$

4. $21 = x + 2$

5. $x + 3 = 8$

_____ _____ _____

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Review for Mastery

Solving Equations by Adding or Subtracting *continued*

Any addition equation can be solved by adding the opposite. If the equation involves subtraction, it helps to first rewrite the subtraction as addition.

Solve $x + 4 = 10$.

Find the opposite of this number.

$x + 4 = 10$ *The opposite of 4 is -4.*

$\underline{-4 \quad -4}$ *Add -4 to each side.*

$x = 6$

Check: $x + 4 = 10$

$6 + 4 \stackrel{?}{=} 10$

$10 \stackrel{?}{=} 10$ ✓

Solve $-5 = x - 8$.

Find the opposite of this number.

$-5 = x + -8$ *Rewrite subtraction as addition.*

The opposite of -8 is 8.

$\underline{+8 \quad +8}$ *Add 8 to each side.*

$3 = x$

Check: $-5 = x - 8$

$-5 \stackrel{?}{=} 3 - 8$

$-5 \stackrel{?}{=} -5$ ✓

Solve $x - (-6) = 2$.

Find the opposite of this number.

$x + 6 = 2$ *Rewrite subtraction as addition.*

The opposite of 6 is -6.

$\underline{-6 \quad -6}$ *Add -6 to each side.*

$x = -4$

Check: $x - (-6) = 2$

$4 - (-6) \stackrel{?}{=} 2$

$2 \stackrel{?}{=} 2$ ✓

Rewrite each equation with addition. Then state the number that should be added to each side.

6. $x - 7 = 12$ _____

7. $x - (-1) = -5$ _____

8. $-4 = x - 2$ _____

Solve each equation. Check your answers.

9. $x + 10 = -6$ _____

10. $-8 = x - 2$ _____

11. $x - (-5) = -2$ _____

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Review for Mastery

Solving Equations by Multiplying or Dividing

Solve equations involving multiplication and division by performing the inverse operation.

Solve $\frac{x}{5} = 4$. ***Check:*** $\frac{x}{5} = 4$

$\frac{x}{5} = 4$	*x is divided by 5.*	$\frac{20}{5} \stackrel{?}{=} 4$
$5 \cdot \frac{x}{5} = 4 \cdot 5$	*Multiply both sides by 5.*	$4 \stackrel{?}{=} 4$ ✓
$\frac{5x}{5} = 20$	*Simplify.*	
$x = 20$		

Solve $-3x = 27$. ***Check:*** $-3x = 27$

$-3x = 27$	*x is multiplied by −3.*	$-3(-9) \stackrel{?}{=} 27$
$\frac{-3x}{-3} = \frac{27}{-3}$	*Divide both sides by −3.*	$27 \stackrel{?}{=} 27$ ✓
$x = -9$	*Simplify.*	

Circle the correct word in each sentence. Then solve the equation.

1. $\frac{x}{-2} = 7$

   x is multiplied/divided by -2.

   To solve, multiply/divide both sides by -2.

   $x =$ _____

2. $5m = -40$

   m is multiplied/divided by 5.

   To solve, multiply/divide both sides by 5.

   $m =$ _____

Solve each equation. Check your answers.

3. $-2x = -20$

4. $\frac{w}{5} = -7$

5. $6z = -42$

_____ _____ _____

Name _____ Date _____ Class _____

Review for Mastery

Solving Equations by Multiplying or Dividing ***continued***

Equations with fractions can be solved by multiplying by the reciprocal.

Solve $\frac{2}{3}x = 8$. ***Check:*** $\frac{2}{3}x = 8$

$\frac{2}{3}x = 8$	*x is multiplied by* $\frac{2}{3}$.	$\frac{2}{3}(12) \stackrel{?}{=} 8$
$\frac{3}{2} \cdot \frac{2}{3}x = 8 \cdot \frac{3}{2}$	*Multiply both sides by* $\frac{3}{2}$.	$\frac{24}{3} \stackrel{?}{=} 8$
$\frac{6}{6}x = \frac{24}{2}$	*Simplify.*	$8 \stackrel{?}{=} 8$ ✓
$x = 12$		

Solve $-\frac{3}{4}x = \frac{2}{5}$. ***Check:*** $-\frac{3}{4}x = \frac{2}{5}$

$-\frac{3}{4}x = \frac{2}{5}$	*x is multiplied by* $-\frac{3}{4}$.	$-\frac{3}{4}\left(-\frac{8}{15}\right) \stackrel{?}{=} \frac{2}{5}$
$-\frac{4}{3} \cdot -\frac{3}{4}x = \frac{2}{5} \cdot -\frac{4}{3}$	*x is multiplied by* $-\frac{4}{3}$.	$\frac{24}{60} \stackrel{?}{=} \frac{2}{5}$
$\frac{12}{12}x = -\frac{8}{15}$	*Simplify.*	$\frac{2}{5} \stackrel{?}{=} \frac{2}{5}$ ✓
$x = -\frac{8}{15}$		

Find the reciprocal.

6. $\frac{2}{5}$ _____

7. $-\frac{5}{7}$ _____

8. 7 _____

Solve each equation. Check your answers.

9. $\frac{5}{6}x = 10$ _____

10. $6 = -\frac{3}{5}x$ _____

11. $\frac{2}{3}x = -\frac{3}{5}$ _____