St. Augustine Prep

Summer Packet

Geometry

The intention of this packet is to review your Algebra I skills needed for Geometry. You may print out the packet and write your answers on the worksheets. If you are unable to print, please write on loose leaf paper. **Please write neatly!** We will go over the packet the first week of school and you will be tested on the review material.

Make your first homework and test grade the best it can be!

Section 1- Order of Operations- PEMDAS

PEMDAS is a standard used for combining Real numbers. It is the order in which you combine numbers.

What does **PEMDAS** stand for?
P-parenthesis
E-exponents
(MD)- Multiplication and Division
(AS)- Addition and Subtraction

You will always do parenthesis first, then exponents, but for multiplication and division, it is done left to right, as well as, addition and subtraction.

Examples:

1. $100 \div 2 \cdot 3 \div (8+2)$
$100 \div 2 \cdot 3 \div (10)$
$50 \cdot 3 \div (10)$
$150 \div 10$
15

2. $6 \div 2 \cdot 3 - 3(3+2) - (-25+5)$
$6 \div 2 \cdot 3 - 3(5) - (-20)$
$3 \cdot 3 - 3(5) - (-20)$
$9 - 15 + 20$
$-6 + 20$
14

Need Additional Help?

http://www.khanacademy.org,

Your Turn- These must be completed on another sheet of paper, making sure you write the problem as well.

1. $50 \div 2 + 5 \cdot 10 \div 2 + 7$
2. $-3^2 + (12 \div 3)^2 \cdot 2 - 5$
3. $\left(\frac{18 \div 6 + 12}{3}\right)^2$
4. $-\frac{10}{5} \div \left(\frac{9}{4}\right) \div 2$

Section 2- Combining Like Terms

To combine like terms, you must first make sure the expression contains no parenthesis. Get rid the parenthesis by using the **distributive property**. Then, you combine the same "family" of terms by adding or subtracting the **coefficients**.

Distributive property- $a(b+c)=ab+ac$

A **Coefficient** is a real number that is multiplied to a variable. If there is no number visible with a variable, then the coefficient is assumed to be a 1.

A "**family**" must have the same variable(s) and the same exponent(s). Look below to see examples of families and examples that aren't families.

Families	*Not Families*
$3x, x, -5x$	$3x, -x^2$
$3xy, 5xy, -7xy$	$3x, 3y$

Examples of Combining Like Terms:

1. $6x-5x+3x-x$
   $(6-5+3-1)x$
   $3x$

2. $-2x+3x^2+9x-6x^2$
   $(-2+9)x+(3-6)x^2$
   $7x+(-3)x^2$
   $7x-3x^2$

3. $4(2x-5)-2(x^2-3)+5x(x-1)$
   $8x-20-2x^2+6+5x^2-5x$
   $(-2+5)x^2+(8-5)x+(-20+6)$
   $3x^2+3x+(-14)$
   $3x^2+3x-14$

Need Additional Help?

http://www.khanacademy.org/math/arithmetic/number-properties/v/distributive-property-example-1

Your Turn- These must be completed on another sheet of paper, making sure you write the problem as well.

1. $10x-6y+3x-7x+9y$
2. $2(2x+3)-4(5x-6)$
3. $6x(2x-3y)+5y(2x+y)$

Section 3- Solving Equations

To solve a linear equation, use the properties of equality and properties of real numbers to find the value of the variable that satisfies the equation. In the case of **literal equations** (equations that have more than one variable), you solve for the variable being asked, and your answer will be another expression.

Remember to always use the **Golden Rule of Equality**- what you do to one side of the equation, must be done to the other side.

You can ALWAYS check your answer for equations. Substitute the answer value back into the original equation, and you get a TRUE statement, then the answer is correct.

Examples:

1.
$$5x-3=2$$
$$5x-3+3=2+3$$
$$5x=5$$
$$x=1$$

2.
$$1-2(x+1)=x+6$$
$$1-2x-2=x+6$$
$$-1-2x=x+6$$
$$1-1-2x=x+6-1$$
$$-2x=x+5$$
$$-2x-x=-x+x+5$$
$$-3x=5$$
$$x=-\frac{5}{3}$$

3. Solve for l
$$P=2(l+w)$$
$$P=2l+2w$$
$$P-2w=2l+2w-2w$$
$$P-2w=2l$$
$$\frac{P-2w}{2}=l$$

4.
$$\frac{1}{3}(x-4)+6=\frac{2}{3}(2x-1)+2$$
$$3\left(\frac{1}{3}(x-4)+6=\frac{2}{3}(2x-1)+2\right)$$
$$1(x-4)+18=2(2x-1)+6$$
$$x-4+18=4x-2+6$$
$$x+14=4x+4$$
$$x+14-4=4x+4-4$$
$$x+10=4x$$
$$-x+x+10=4x-x$$
$$10=3x$$
$$\frac{10}{3}=x$$

Need Additional Help?

http://www.khanacademy.org/math/algebra/solving-linear-equations/v/solving-one-step-equations-2

http://www.khanacademy.org/math/algebra/solving-linear-equations/v/multi-step-equations-2

http://www.khanacademy.org/math/algebra/solving-linear-equations/v/solving-equations-with-the-distributive-property

http://www.khanacademy.org/math/algebra/solving-linear-equations/v/solving-for-a-variable

You Turn:

1. $7x - 3 = 19 + 5x$

2. $12\left(2x - \left(5 + 6\right)\right) = 3x + \left(6 \cdot 7\right)$

3. $\frac{1}{2}\left(2x - 6\right) + 5 = \frac{2}{3}x + 9 + \frac{7}{3}x$

4. $2\left(x + 1\right) + 3\left(2x + 1\right) = -7$

5. $Solve\ for\ h:$ $S = 2\pi r^2 + 2\pi rh$

6. $Solve\ for\ l:$ $S = B + \frac{1}{2}l$

7. $\frac{2x - 1}{4} = \frac{x - 7}{3}$

Section 4- System of Equations

Normally, there are many ordered pairs that satisfy a given equation. For example, (3,4), (4,5), (5,6), and infinitely many other pairs all satisfy the equation $y = x + 1$. In solving a system of two linear equations, however, you need to find ordered pairs that satisfy BOTH equations at once. Ordinarily, there is just one such ordered pairs; it is the point where the graphs of the two lines intersect.

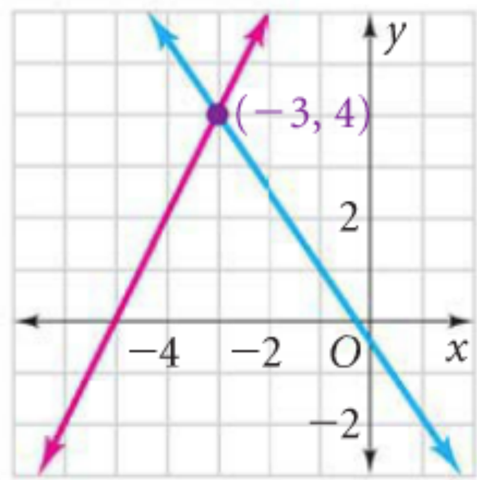
Two methods used for solving system of equations (other than graphing_ are the substitution method and the linear combination method. Each are demonstrated below.

EXAMPLE

Algebra Solve the system.

$$2x - y = -10$$
$$-3x - 2y = 1$$

Solve one of the equations for a variable. Looking at the two equations, it seems easiest to solve the first equation for y.



$2x - y = -10$

$-y = -2x - 10$ **Subtract 2*x* from each side.**

$y = 2x + 10$ **Multiply each side by −1.**

Now substitute $2x + 10$ for y in the other equation.

$-3x - 2y = 1$ **Write the other equation.**

$-3x - 2(2x + 10) = 1$ **Substitute (2*x* + 10) for *y*.**

$-3x - 4x - 20 = 1$ **Use the Distributive Property.**

$-7x = 21$ **Simplify and add 20 to each side.**

$x = -3$ **Divide each side by −7.**

So $x = -3$. To find y, substitute -3 for x in either equation.

$2x - y = -10$ **Write one of the equations.**

$2(-3) - y = -10$ **Substitute −3 for x.**

$-6 - y = -10$ **Simplify.**

$-y = -4$ **Add 6 to each side.**

$y = 4$ **Multiply each side by −1.**

So the solution is $x = -3$ and $y = 4$, or $(-3, 4)$. If you graph $2x - y = -10$ and $-3x - 2y = 1$, you'll find that the lines intersect at $(-3, 4)$.

EXAMPLE

Solve by elimination. $4x + 2y = 14$
$7x - 3y = -8$

Step 1 Eliminate one variable.

Start with the given system.		To prepare for eliminating *y*, multiply one equation by 3 and the other equation by 2.		Add the equations to eliminate *y*.
$4x + 2y = 14$	$\rightarrow$	$3(4x + 2y = 14)$	$\rightarrow$	$12x + 6y = 42$
$7x - 3y = -8$	$\rightarrow$	$2(7x - 3y = -8)$	$\rightarrow$	$14x - 6y = -16$
				$26x + 0 = 26$

Step 2 Solve for x.

$$26x = 26$$
$$x = 1$$

Step 3 Solve for the eliminated variable y using either of the original equations.

$4x + 2y = 14$ **Use the first equation.**

$4(1) + 2y = 14$ **Substitute 1 for *x*.**

$2y = 10$

$y = 5$

The solution is $(1, 5)$.

Need Additional Help?

http://www.khanacademy.org/math/algebra/systems-of-eq-and-ineq/v/solving-systems-by-substitution-2

http://www.khanacademy.org/math/algebra/ck12-algebra-1/v/solving-systems-of-equations-by-multiplication

Your Turn:

Solve each system by substitution:

1. $5x - y = 4$
   $x + 5y = -4$

2. $x + y = 4$
   $y = 7x + 4$

3. $x - y = 7$
   $3x + 2y = 6$

Solve each system by elimination:

4. $4x - y = 105$
   $x + 7y = -10$

5. $6x + y = 13$
   $-x + y = -8$

6. $4x - 9y = 61$
   $10x + 3y = 25$

Section 5- Factoring Quadratics

Factoring a quadratic expression (an expression that has the highest exponent of 2) simply means rewriting the expression with prime expressions.

Examples:

1. Factor $x^2 + 7x + 12$.

Find the factors of 12. Identify the pair that has a sum of 7.

Factors of 12	Sum of Factors
1 and 12	13
2 and 6	8
3 and 4	7 ✓

$x^2 + 7x + 12 = (x + 3)(x + 4)$

Check $x^2 + 7x + 12 \stackrel{?}{=} (x + 3)(x + 4)$

$= x^2 + 4x + 3x + 12$

$= x^2 + 7x + 12$ ✓

2. Factor $d^2 - 17d + 42$.

Since the middle term is negative, find the negative factors of 42 until you find the pair that has a sum of -17.

Factors of 42	Sum of Factors
-1 and -42	-43
-2 and -21	-23
-3 and -14	-17 ✓

$d^2 - 17d + 42 = (d - 3)(d - 14)$

3.

a. Factor $m^2 + 6m - 27$.

Identify the pair of factors of -27 that has a sum of 6.

Factors of −27	Sum of Factors
1 and −27	−26
27 and −1	26
3 and −9	−6
9 and −3	6 ✓

$m^2 + 6m - 27 = (m - 3)(m + 9)$

4.

b. Factor $p^2 - 3p - 18$.

Identify the pair of factors of -18 that has a sum of -3.

Factors of −18	Sum of Factors
1 and −18	−17
18 and −1	17
−6 and 3	−3 ✓

$p^2 - 3p - 18 = (p + 3)(p - 6)$

5.

Factor $6n^2 + 23n + 7$.

	$6n^2$	+	$23n$	+	7	
	F		O I		L	
	$1 \cdot 6$		$1 \cdot 7 + 1 \cdot 6 = 13$		$1 \cdot 7$	
factors of a			$1 \cdot 1 + 7 \cdot 6 = 43$		$7 \cdot 1$	factors of c
	$2 \cdot 3$		$2 \cdot 7 + 1 \cdot 3 = 17$		$1 \cdot 7$	
			$2 \cdot 1 + 7 \cdot 3 = 23$ ✓		$7 \cdot 1$	

$$6n^2 + 23n + 7 = (2n + 7)(3n + 1)$$

6.

Factor $7x^2 - 26x - 8$.

$7x^2$	$-26x$	-8
$1 \cdot 7$	$(1)(-8) + (1)(7) = -1$	$(1)(-8)$
	$(1)(1) + (-8)(7) = -55$	$(-8)(1)$
	$(1)(-4) + (2)(7) = 10$	$(2)(-4)$
	$(1)(2) + (-4)(7) = -26$ ✓	$(-4)(2)$

$$7x^2 - 26x - 8 = (1x + -4)(7x + 2)$$

7.

Factor $20x^2 + 80x + 35$ completely.

$20x^2 + 80x + 35 = 5(4x^2 + 16x + 7)$ **Factor out the GCF.**

Factor $4x^2 + 16x + 7$.

$4x^2$	$16x$	7
$1 \cdot 4$	$1 \cdot 7 + 1 \cdot 4 = 11$	$1 \cdot 7$
	$1 \cdot 1 + 7 \cdot 4 = 29$	$7 \cdot 1$
$2 \cdot 2$	$2 \cdot 7 + 1 \cdot 2 = 16$ ✓	$1 \cdot 7$

$4x^2 + 16x + 7 = (2x + 1)(2x + 7)$

$20x^2 + 80x + 35 = 5(2x + 1)(2x + 7)$ **Include the GCF in your final answer.**

Need Additional Help?

http://www.khanacademy.org/math/algebra/quadtratics/v/factoring-quadratic-expressions

http://www.khanacademy.org/math/algebra/polynomials/v/factoring-trinomials-with-a-leading-1-coefficient

http://www.khanacademy.org/math/algebra/polynomials/v/factoring-trinomials-with-a-non-1-leading-coefficient-by-grouping

Your Turn:

Factor each of the following trinomials. Make sure you factor any GCF out first.

1. $x^2 + 7x + 10$
2. $x^2 + 21x + 20$
3. $x^2 - 10x + 25$
4. $x^2 - 15x + 36$
5. $x^2 + 8x - 20$
6. $x^2 - 3x + 40$
7. $x^2 - x - 56$
8. $2x^2 + 5x + 2$
9. $6x^2 - 23x + 7$
10. $5x^2 - 14x - 13$
11. $2x^2 + x - 3$
12. $20x^2 - 31x - 9$
13. $2x^2 - 12x + 10$
14. $4x^2 + 14x + 6$
15. $18x^2 - 12x - 6$

Section 6: Radical expressions

Radical expressions play a huge part in Geometry. Whether is it finding the distance between two point, the length of the side of a polygon, determining Pythagorean triples, or performing many different solving situations, you will need to know how to simplify, add, subtract, and rationalize denominators.

Simplifying radicals: A simplified radical expression is said to be in simplest form when all three of the following statements are true:

1. The expression under the radical sign (the radicand) contains no perfect square factors (other than 1).
2. The radicand doesn't contain a fraction.
3. The denominator does not contain a radical expression.

Examples: Simplify the following radical expressions

1. $\sqrt{192}$

   $\sqrt{64}\sqrt{3}$

   $8\sqrt{3}$

2. $\sqrt{45a^3}$

   $\sqrt{9a^2}\sqrt{5a}$

   $3\text{a}\sqrt{5a}$

3. $\sqrt{x^3y^4z^9}$

   $\sqrt{x^2y^4z^8}\sqrt{xz}$

   $xy^2z^4\sqrt{xz}$

Adding and Subtracting Radical Expressions: Remember that you can only add or subtract radicals if the radicands are the same. If the radicands are the not the same, you might be able to simplify one or all to make them the same. Once the radicands are the same, you simply add the coefficients, and leave the like radicand.

Examples:

4. $\sqrt{5} + 3\sqrt{5}$

   $4\sqrt{5}$

5. $6\sqrt{10} - 11\sqrt{10}$

   $-5\sqrt{10}$

6. $\sqrt{18} - \sqrt{48} + \sqrt{32} + \sqrt{147}$

   $\sqrt{9}\sqrt{2} - \sqrt{16}\sqrt{3} + \sqrt{16}\sqrt{2} + \sqrt{49}\sqrt{3}$

   $3\sqrt{2} - 4\sqrt{3} + 4\sqrt{2} + 7\sqrt{3}$

   $7\sqrt{2} + 3\sqrt{3}$

Multiplying and Dividing Radicals- you do not have to have like radicands to multiply or divide, but you may not leave a radical in the denominator of a fraction. You will have to rationalize to rid the radical. **Rationalizing** means to multiply both the numerator and denominator by the radical in the denominator.

Examples:

7. $2\sqrt{3}\cdot 5\sqrt{7}$

$$10\sqrt{21}$$

8. $\sqrt{\frac{108}{6}}$

$$\sqrt{18}$$

$$\sqrt{9}\sqrt{2}$$

$$3\sqrt{2}$$

9. $\sqrt{\frac{92}{3}} = \frac{\sqrt{92}}{\sqrt{3}}$

$$\frac{\sqrt{4}\sqrt{23}}{\sqrt{3}}$$

$$\frac{2\sqrt{23}}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}$$

$$\frac{2\sqrt{69}}{3}$$

Need Additional Help?

http://www.khanacademy.org/math/algebra/exponents-radicals/v/simplifying-radicals

http://www.khanacademy.org/math/algebra/exponents-radicals/v/more-simplifying-radical-expressions

http://www.khanacademy.org/math/algebra/exponents-radicals/v/how-to-rationalize-a-denominator

Your Turn: Simplify each of the following.

1. $\sqrt{200}$
2. $\sqrt{98}$
3. $\sqrt{28n^3}$
4. $\sqrt{22} \cdot \sqrt{11}$
5. $2\sqrt{18} \cdot 7\sqrt{6}$
6. $\sqrt{7x} \cdot \sqrt{21x^3}$
7. $\sqrt{\dfrac{21}{49}}$
8. $\dfrac{\sqrt{30a^5}}{\sqrt{40a}}$
9. $\dfrac{9}{\sqrt{8}}$
10. $2\sqrt{12} - 7\sqrt{3}$
11. $3\sqrt{7} - \sqrt{28}$
12. $\sqrt{2}\left(\sqrt{10} - \sqrt{3}\right)$