

STATISTICAL SIZE EFFECT IN QUASI-BRITTLE STRUCTURES: I. IS WEIBULL THEORY APPLICABLE?

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ABSTRACT: The classical applications of Weibull statistical theory of size effect in quasi-brittle structures such as reinforced concrete structures, rock masses, ice plates, or tough ceramic parts are being reexamined in light of recent results. After a brief review of the statistical weakest-link model, distinctions between structures that fail by initiation of macroscopic crack growth (metal structures) and structures that exhibit large macroscopic crack growth prior to failure (quasi-brittle structures) are pointed out. It is shown that the classical Weibull-type approach ignores the stress redistributions and energy release due to stable large fracture growth prior to failure, which causes a strong deterministic size effect. Further, it is shown that, according to this classical theory, every structure is equivalent to a uniaxially loaded bar of variable cross section, which means that the mechanics of the failure process is ignored. Discrepancies with certain recent test data on the size effect are also pointed out. Modification of the Weibull approach that can eliminate these shortcomings is left for a subsequent paper.

INTRODUCTION

Quasi-brittle structures are those in which: (1) Failure is caused by fracture rather than plastic yield; and (2) the fracture front is surrounded by a large fracture-process zone in which progressive distributed cracking or other damage takes place. Brittle structures, which can be analyzed according to linear elastic fracture mechanics (LEFM), are the limiting special case of quasi-brittle structures for which the size of the fracture-process zone at failure becomes negligible compared to the structure size. Quasi-brittle structures do not follow LEFM, and nonlinear fracture mechanics is required. Both structure types are characterized by post-peak softening and absence of yield plateau on the load-deflection diagram. For quasi-brittle structures, the peak of the diagram is rounded, while for brittle structures there is an almost sudden change from rising slope to descending slope of the load-displacement diagram. Quasi-brittle failures are typical of reinforced concrete structures (here the established term is *brittle* failure, but *quasi-brittle* would be a better term since purely brittle failures are not seen in reinforced concrete structures). They are also typical of certain kinds of rock, ice, modern tough ceramics, various composites, etc.

Traditionally, the size effect in failure of concrete structures has been explained in Weibull's (1939, 1951) statistical theory, which extended to multidimensional solids the weakest-link model for a chain proposed by Peirce (1926) and used the extreme value statistics originated by Tippett (1925), Fréchet (1927), and others; see also Freudenthal (1968), Mihashi and Zaitsev (1981), Zech and Wittmann (1977), Carpinteri (1989), Mihashi (1983), Mihashi and Izumi (1977); for recent works and reviews, see Kittl

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and Diaz (1988, 1989, 1990). In Weibull's theory, the failure is determined by the minimum value of the strength of the material, and the statistical size effect is due to the fact that the larger the structure, the smaller is the strength value likely to be encountered in the structure. This explanation is certainly valid for one-dimensional structures such as a long chain or a long fiber, but extension to multidimensional structures depends on certain simplifying hypotheses that do not have to be satisfied for all types of structures. These hypotheses appear to be applicable to structures consisting of ceramics, glasses, and fatigue-embrittled metals. But they do not apply to quasi-brittle structures, such as concrete structures, because of their ability to develop large fractures in a stable manner prior to failure. The central idea in Weibull-type statistical analysis of failure and size effect is that the survival probability of the structure is the joint probability of survival of all its elementary parts. Implementation of this idea, however, is clear and simple only for a long fiber or a long chain. It is difficult for two-dimensional and three-dimensional structures.

This paper will examine the limitations of the classical Weibull theory of size effect from the viewpoint of quasi-brittle structures, such as reinforced concrete structures. A companion paper will present a new formulation which overcomes the main limitations. The basic idea of the present formulation has been briefly outlined in a previous conference paper (Bažant 1987).

REVIEW OF WEIBULL THEORY

We need to start by reviewing the principles of Weibull theory. First, we consider a one-dimensional structure consisting of many elements coupled in series, for example a chain [Fig. 1(a)]. All of the elements (links of the chain) have the same distribution of strength σ , characterized by the cumulative probability distribution $P_1(\sigma)$, which represents the probability of failure of one element, i.e., the probability that the strength in the element is less than the applied stress σ . The survival probability of one element is $1 - P_1$. If the whole chain should survive, all of its elements must survive. This means that the probability of survival of the chain is the joint probability of the survival of all of the elements. According to the joint-probability theorem, the survival probability of a chain of N elements is, $1 - P_f = (1 - P_1)(1 - P_1) \dots (1 - P_1)$, or

$$1 - P_f = (1 - P_1)^N \quad \dots \dots \dots (1)$$

where P_f = failure probability of the chain. Now

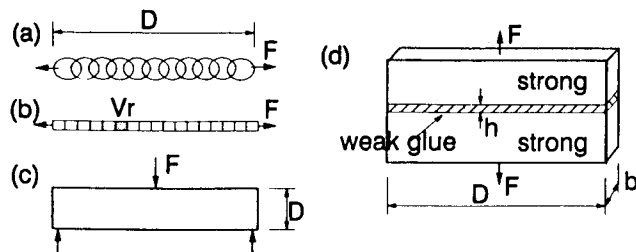


FIG. 1. Some Different Cases for Weibull Distribution

$$\ln(1 - P_f) = N \ln(1 - P_1) \quad \dots \dots \dots (2)$$

and since P_1 is very small in practical application, $\ln(1 - P_1) \approx -P_1$. Therefore

$$\ln(1 - P_f) = -NP_1 \quad \dots \dots \dots (3)$$

This may be extended to a continuous, homogeneously stressed solid [e.g., a long fiber, Fig. 1(b)] by setting $N = V/V_r$; thus $\ln(1 - P_f) = -(V/V_r)P_1$ or

$$P_f(\sigma) = 1 - \exp \left[-\frac{V}{V_r} P_1(\sigma) \right] \quad \dots \dots \dots (4)$$

Here V = volume of the body; and V_r = representative volume of the material. In the greatest generality, V_r represents the smallest volume for which the material can be treated as a continuum (and for which the concept of stress on the macroscopic scale makes sense). In this definition, V_r is defined as the smallest volume for which the main statistical characteristics of the microstructure do not change substantially if the volume is displaced within the material. The size of the representative volume is the characteristic length l of the material—a central concept in the nonlocal-continuum theory. The foregoing definition of V_r coincides with that used in nonlocal or statistical theories of microinhomogeneous materials. For practical purposes, though, V_r can be taken as any material volume for which the strength distribution has been experimentally determined.

To describe the statistical distribution of P_1 , Weibull (1939) [also Weibull (1951)] introduced the following empirical form (Fig. 2)

$$P_1(\sigma) = \left(\frac{\sigma - \sigma_u}{\sigma_0} \right)^m ; \text{ for } \sigma > \sigma_u$$

$$P_1(\sigma) = 0; \text{ for } \sigma \leq \sigma_u \quad \dots \dots \dots (5)$$

in which σ_0 and σ_u = empirical material parameters; m = shape parameter (Weibull modulus); σ_0 = scale parameter; and σ_u = datum parameter

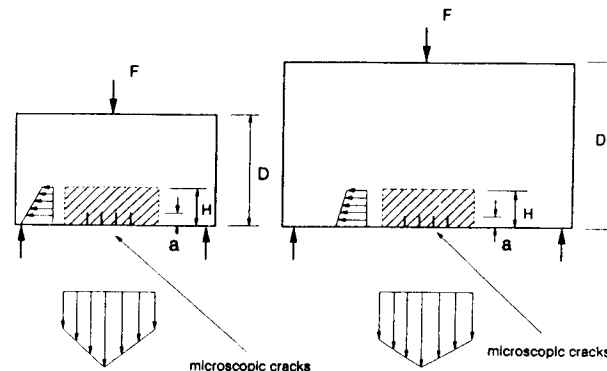


FIG. 2. Geometrically Similar Structures with Microscopic Cracks

(strength threshold). For calculations it is convenient to assume that $\sigma_u = 0$, and then the results of direct tensile tests of concrete indicate approximately $m = 12$ (Zech and Wittmann 1977). In reality, the threshold σ_u is of course nonzero, but it is hard to determine σ_u reliably due to scatter of test results. Unless the strength range of data is very broad, and random scatter is small, different σ_u -values (with different corresponding values of σ_u and m) allow almost equally good fits of the data on $P_1(\sigma)$.

To generalize (4) for a structure with nonuniform stress [(e.g., a beam, Fig. 1(c)), one may imagine the structure to consist of many parts of small volumes, $\Delta V_{(j)}$, each with uniform stress $\sigma_j (j = 1, \dots, \bar{n})$. The probability of survival of the structure is the joint probability of survival of all its parts, and so

$$1 - P_f = \exp\left\{-\frac{\Delta V_{(1)}}{V_r} P_1[\sigma_{(1)}]\right\} \exp\left\{-\frac{\Delta V_{(2)}}{V_r} P_1[\sigma_{(2)}]\right\} \dots \exp\left\{-\frac{\Delta V_{(\bar{n})}}{V_r} P_1[\sigma_{(\bar{n})}]\right\} = \exp\left\{-\sum_{j=1}^{\bar{n}} \frac{\Delta V_{(j)}}{V_r} P_1[\sigma_{(j)}]\right\} \dots \dots \dots (6)$$

Now, if the volume of each part tends to zero and the number of the parts tends to infinity, one obtains a structure with continuously variable stress $\sigma(\mathbf{x})$. Eq. (6) thus becomes

$$1 - P_f = \exp\left[-\int_V \left\langle \frac{\sigma(\mathbf{x}) - \sigma_u}{\sigma_0} \right\rangle^m \frac{dV(\mathbf{x})}{V_r}\right] \dots \dots \dots (7)$$

where $\langle \rangle$ denotes the positive part of the argument, i.e., $\langle x \rangle = x$ if $x > 0$, and $\langle x \rangle = 0$ if $x \leq 0$; and \mathbf{x} = the coordinate vectors of material points.

Eq. (7) needs to be further generalized to triaxial-stress states. For this, in principle, the triaxial failure surface in the stress space needs to be used. However, for the sake of simplicity, one may assume that the cracks may form only in the planes normal to the principal stresses $\sigma_i(\mathbf{x}) (i = 1, 2, 3)$ and that the formation of any crack depends only on the principal stresses normal to it, but not on the other principal stresses, as proposed by Freudenthal (1968). Considering the joint probability of survival of the material on all three principal stress planes, similarly to (6), one gets, for a structure with triaxial stresses, the following probability of survival (P_f = probability of failure)

$$1 - P_f = \exp\left[-\int_V \sum_{i=1}^n \left\langle \frac{\sigma_i(\mathbf{x}) - \sigma_u}{\sigma_0} \right\rangle^m \frac{dV(\mathbf{x})}{V_r}\right] \dots \dots \dots (8)$$

where n = number of dimensions (1, 2, or 3). In anisotropic materials, furthermore, subscript i needs to be also attached to m , σ_0 and σ_u .

It may be noted that more sophisticated multiaxial formulations have been proposed. For example, Petrovic (1987) considered that a crack can form on a plane of any orientation and the failure probability depends only on the normal stress on that plane. In that case the joint probability of survival for the planes of all orientations leads to an integral over all spatial directions at each point. However, this might be no more realistic than (8), since the fact that the cracking probability may depend also on the stresses that act on the planes of other orientations (as well as on the strain and the invariants of stress and strain) is still neglected.

STATISTICAL SIZE EFFECT DUE TO RANDOM STRENGTH

The size effect is defined by comparing geometrically similar structures of different characteristic dimensions (sizes) D . For this purpose, one introduces the nominal strength (nominal stress at maximum load),

$$\sigma_N = \frac{F}{bD} \dots \dots \dots (9)$$

where F = failure load (maximum load); and b = thickness of the structure, which may either be constant (two-dimensional similarity), or proportional to D (three-dimensional similarity). Now the basic property of all structures is that, according to elastic analysis with allowable stress limit as well as plastic analysis or any analysis based on some material failure criterion in terms stress or strain, σ_N is independent of structure D , i.e., there is no size effect. This is not true, however, when the material properties are random, as shown by Weibull. To demonstrate it, we may represent the stress distributions in the structure as

$$\sigma_i(\mathbf{x}) = \sigma_N S_i(\xi), \quad \xi = \frac{1}{D} \mathbf{x} \dots \dots \dots (10)$$

in which S_i = functions of the relative coordinate vectors ξ , calculated according to elasticity or plasticity or any other suitable theory. Substituting $dV = dx_1 \dots dx_n = D^n d\xi_1 \dots d\xi_n$, where x_1, \dots, x_n = Cartesian-coordinate components, and ξ_1, \dots, ξ_n = corresponding elastic-coordinate components of the vector $\xi = (\xi_1, \dots, \xi_n)$, we obtain

$$\ln(1 - P_f) = -\frac{D^n}{V_r} \int_{\Omega} \int_{i=1}^n \left\langle \frac{\sigma_N S_i(\xi) - \sigma_u}{\sigma_0} \right\rangle^m d\xi_1 \dots d\xi_n \dots \dots \dots (11)$$

in which Ω = the domain of the structure in relative coordinates ξ , which is the same for similar structures of all sizes.

The strength threshold is usually neglected in practical calculations, that is $\sigma_u = 0$, not only for the sake of simplicity but also because it is very difficult to identify the value of σ_u from test data reliably, as already pointed out. With this simplification, (11) may be rewritten as follows

$$\ln(1 - P_f) = -\frac{D^n}{V_r} \left(\frac{\sigma_N}{\sigma_0}\right)^m I_n, \quad \text{with}$$

$$I_n = \int_{\Omega} \int_{i=1}^n [S_i(\xi)]^m d\xi_1 \dots d\xi_n \dots \dots \dots (12)$$

Here I_n , σ_0 , and V_r = constants when the structure size D is varied. Thus, when the failure probability P_f is specified, the following dependence of the nominal strength of the structure on its size D or volume $V (V \propto D^n)$ results:

$$\sigma_N = k_0 D^{-n/m} \dots \dots \dots (13)$$

in which $k_0 = \sigma_0 [-V_r \ln(1 - P_f)/I_n]^{1/m}$ = constant. When one substitutes $P_f = 0.5$, one gets the size dependence of the median strength, which is known for concrete to be almost the same as the mean strength, $\bar{\sigma}_N$ [for the calculation of $\bar{\sigma}_N$, see the companion paper (Bažant and Xi 1991)].

From direct tensile tests, the typical value of m for concrete is about 12. Then, for two-dimensional similarity ($n = 2$)

$$\sigma_N \propto D^{-1/6} \dots \dots \dots (14)$$

and for three-dimensional similarity ($n = 3$)

$$\sigma_N \propto D^{-1/4} \dots \dots \dots (15)$$

Eq. (13) is also valid for one-dimensional similarity. This case is obtained, e.g., for the body in Fig. 1(d), which consists of two strong blocks joined by a thin layer of weak glue (of Weibull distribution with modulus $m = 12$). The blocks cannot fail and failure is assumed to occur in the glue layer as soon as one elementary volume of the glue fails. In that case $n = 1$ and (13) yields

$$\sigma_N \propto D^{-1/12} \dots \dots \dots (16)$$

provided that block thickness b and glue layer thickness h are not varied.

BASIC CRITICISMS OF CLASSICAL WEIBULL-TYPE APPROACH

Stress Redistribution

The key to the calculation of failure probability of the structure is the function $S_i(\xi)$, characterizing the stress at point $x = D\xi$. In this regard, one must distinguish two types of structures: (1) Those failing at the initiation of the macroscopic crack growth (i.e., the structure just before failure contains only microscopic cracks or other flaws, as is typical of many ceramics and fatigue-embrittled metal structures); and (2) those failing only after a large stable macroscopic crack growth (which is the case of reinforced concrete structures).

For the first type of structure, the key point is that function $S_i(\xi)$ just before failure is known, since microscopic flaws have negligible influence on the overall stress distribution within the structure. In such structures there exists a region of size H (Fig. 2) such that

$$H \ll D \text{ and } H \gg a \dots \dots \dots (17)$$

where $D =$ structure dimension; and $a =$ crack or flaw size. The condition $H \ll D$ means that the stress distribution within region H would be nearly uniform if the flaw did not exist. If the size of the flaw or initial crack is very small, $a \ll D$, it is a characteristic of the state of the material. It is related to the inhomogeneity size and is independent of the structure dimension D . Since $H \ll D$, the presence of the flaw of size a affects the stress distribution only locally, and the situation is nearly the same as that of a crack in an infinite space with a uniform-stress state at infinity equal to the stress in region H . Thus, the only effect of the flaw of size a is a reduction of the effective macroscopic strength of the material. This permits that the random variation of the sizes of the initial flaws can be related to the random variation of the material strength, as described by Weibull distribution.

For the second type of structure, for example, reinforced concrete structures, the behavior is completely different. Due to reinforcement as well as to the existence of strain softening in a large zone of microcracking and crack bridging near the front of a continuous fracture, reinforced concrete structures do not fail at crack initiation. Large cracks, typically cutting through 50% to 90% of the cross section, grow in a stable manner before

the maximum load is reached (see the typical macroscopic crack patterns in Fig. 3). The design codes, in fact, require the failure load to be significantly higher than the crack-initiation load (for bending, at least 1.25 times higher, according to ACI Standard 318, but in practice this ratio is usually much larger). Consequently, a reinforced concrete structure normally undergoes pronounced inelastic deformation with large macroscopic stable crack growth prior to reaching the failure load (maximum load). This inevitably engenders stress redistributions, such that the stress distribution $\sigma(x)$ at incipient failure is very different from the elastic stress distribution, which has commonly been assumed in the previous studies of the statistical size effect. The existence of macroscopic crack growth is also documented by the load-deflection diagram, in which the start of macroscopic crack growth is manifested by a significant reduction of slope, as seen in Fig. 4(a). This contrasts with metal or some kinds of rock structures, for which the load-deflection diagram typically looks as shown in Fig. 4(b).

To sum up, Weibull theory cannot be applied to the failure of reinforced concrete structures unless the effect of stable macroscopic crack growth on the stress distribution function $S_i(\xi)$ is taken into account (Bažant 1988).

Equivalence to Uniaxially Stressed Bar

Another limitation of the existing Weibull-type formulations is revealed by realizing that, if the stress-distribution function $S_i(\xi)$ is known a priori, every structure is equivalent to a uniaxially stressed bar of variable cross-sectional area, A . To demonstrate it, consider the bar in Fig. 5, where x is the longitudinal coordinate. For the real structure, one can calculate the

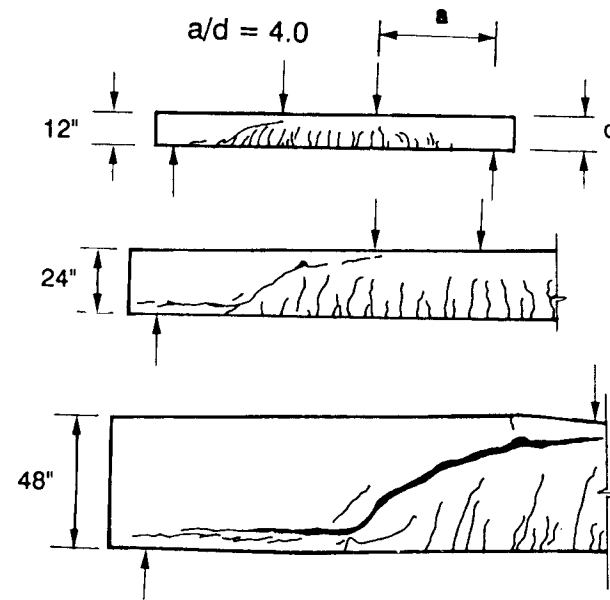


FIG. 3. Example of Geometrically Similar Reinforced Concrete Structures with Stable Macroscopic Fracture Growth before Failure [adapted from Kani (1967)]

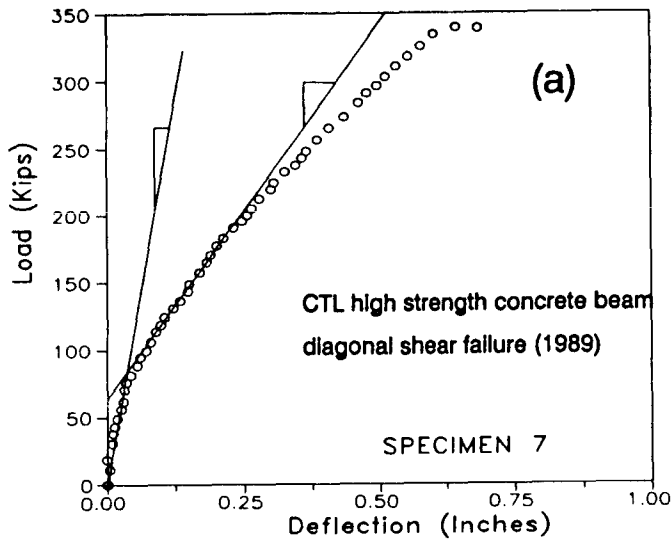


FIG. 4(a). Typical Load-Deflection Diagram of Reinforced Concrete Structures in which Slope Reduction Indicates Start of Actual Growth of Macroscopic Cracks

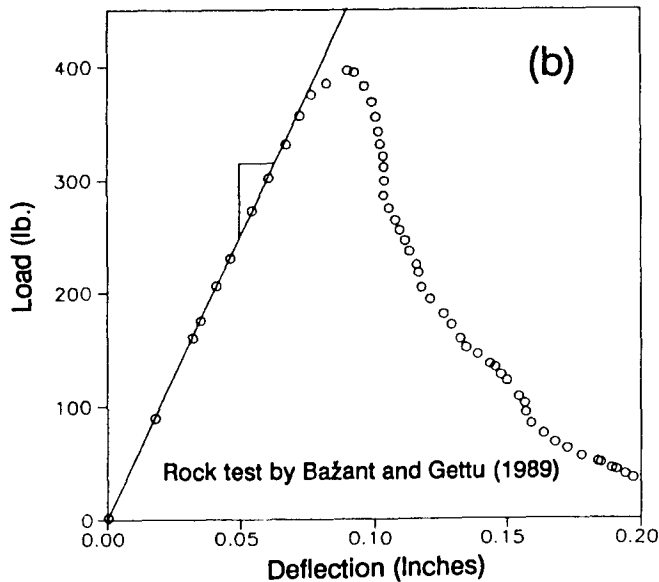


FIG. 4(b). Geometrically Similar Structures in which Fracture Length at Maximum Load is Small (Microscopic), and Typical Load-Deflection Diagram

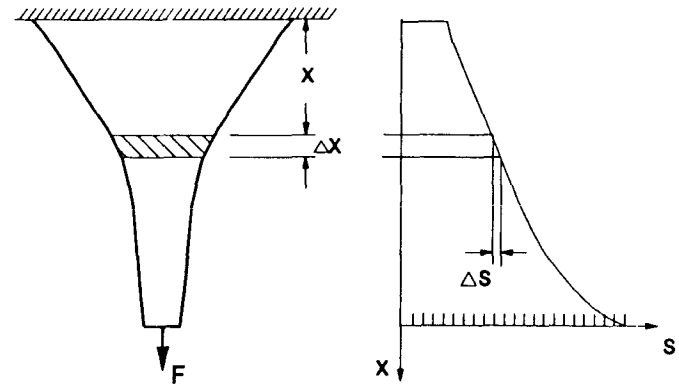


FIG. 5. Uniaxially Loaded Bar of Variable Cross Section

volume $V(S)$ of the part of the structure in which the equivalent uniaxial stress, $\bar{S}(\xi) = [\sum_{i=1}^n S_i^n(\xi)]^{1/n}$, exceeds the value S , representing the stress at some cross section of a uniaxial bar (Fig. 5)

$$V(S) = \int_V H[\sigma_N \bar{S}_i(\xi) - S] dV(\xi) \dots \dots \dots (18)$$

in which H = Heaviside step function. Now one may subdivide the stress range into small steps ΔS with S representing the value at the middle of each step. Then one can calculate function $V(S)$ from (18), and also get the derivative $V'(S) = \Delta V(S)/\Delta S$. Beginning with $S = 0$, one must satisfy for each step ΔS (with the value S at the middle of the step) the condition of equal volume in the real structure, i.e., $V'(S)\Delta S = A\Delta x = \sigma_N \Delta x/S$. The cross-sectional area of the bar corresponding to each stress value S is then calculated as $A = \sigma_N/S$ (in which σ_N is imagined as the load applied on the uniaxial bar, Fig. 5), and the length of the bar element having this cross-sectional area is

$$\Delta x = V'(S)\Delta S \frac{S}{\sigma_N} \dots \dots \dots (19)$$

Putting all these segments Δx with areas $A = \sigma_N/S$ together, one obtains the profile of the equivalent bar. So, we see that, indeed, for every multidimensional structure for which the stress-distribution function at incipient failure is known a priori, an equivalent uniaxial bar of variable cross section can be found (Bažant 1988). This bar, according to the classical application of Weibull theory, behaves in the same way as the actual structure.

Obviously, in this approach, all information about the mechanics of failure is lost, and the structural geometry becomes irrelevant. Of course this cannot be true. So the Weibull-type approach cannot be regarded as realistic, unless the stress-distribution function realistically describes the stress field at imminent failure.

Differences between Two- and Three-Dimensional Geometric Similarities

Another questionable aspect of the classical Weibull-type approach is the effect of the number of dimensions, n , implied by (13). Consider, for ex-

ample, that the beam dimensions are increased in the ratio D_2/D_1 according to either two-dimensional similarity, in which case, the beam thickness b is kept constant, or according to three-dimensional similarity, in which case, the beam thickness is also increased in proportion to D . According to (13), the nominal strength σ_N should change in the ratio $(D_2/D_1)^{-2/m}$ or $(D_2/D_1)^{-3/m}$, respectively. Now, although systematic data on the effect of thickness is unavailable, it appears from experience that there is no significant difference between these two cases.

No significant difference is manifested by comparing the slopes of the plots of $\log(\sigma_N)$ versus $\log(D)$ for tests with two-dimensional similarity and three-dimensional similarity. The effect of the number of dimensions, n , can be checked by using Bažant and Kazemi's (1989) tests of diagonal shear failure of concrete beam (reinforced by longitudinal bars with hooks at the ends), which were similar to two dimensions (same thickness), and the pull-out tests of bars by Bažant and Sener (1988) or the torsional shear fracture tests by Bažant et al. (1990), which were similar in three dimensions; see the data points in Fig. 6. Taking the diagonal shear test as reference, the fact that the slope of the mean trend of the data is approximately $-1/2$ implies that $m = 4$ (let us pretend we do not know any uniaxial test data that indicate a much larger m). Then, for three-dimensional similarity, the slope of the line should be $-n/m = -3/4$ (Bažant 1988). But the pull-out tests made with the same concrete indicate the slope to be also $-1/2$, which does clearly disagree with the classical Weibull-type analysis [but agrees with the modified statistical theory presented in the companion paper (Bažant and Xi 1991)].

The classical Weibull-type theories are further put in question when one tries to compare the results of tests on bars of different sizes failing in uniaxial tension with the diagonal shear tests. The former tests indicate that, approximately, $m = 12$ (Zech and Wittmann 1977). But if $m = 12$, then the slope of the line in Fig. 6 on the left would have to be $-1/6$ rather than $-1/2$. This is a serious discrepancy indeed [it is remedied in the companion paper (Bažant and Xi 1991)].

Energy Release Due to Large Stable Crack Growth

From the mechanics viewpoint, the basic problem with the classical Weibull-type approaches to reinforced concrete structures is that they generally ignore the effect of macroscopic fracture growth on the energy-release rate of the structure. Experiments confirm that in concrete structures, the fracture length at maximum load is usually proportional to the structure dimension, while the width of the fracture process zone, h , is almost the same for any size and is a material property, as shown by the example of rectangular panel in Fig. 7 (Bažant 1984). The fracture length a_0 before failure of this panel may be imagined to release the stress from the sparsely shaded triangular areas in the figure. When, during failure, the fracture extends by Δa , the stress is further relieved from the densely cross-hatched narrow strips, the area of which gets larger as the structure size gets larger. This means that the release of the stored energy of the structure into the fracture extension Δa , which comes from the strip, is larger for a larger structure if the nominal stress is the same. However, fracture extension requires roughly the same energy per unit length of extension, regardless of the structure size. Therefore, the nominal stress at failure must get smaller if the structure gets larger, so that the strain energy density in the densely cross-hatched strip would be smaller in a larger structure, thus making it possible to obtain the same energy release per unit length of the fracture.

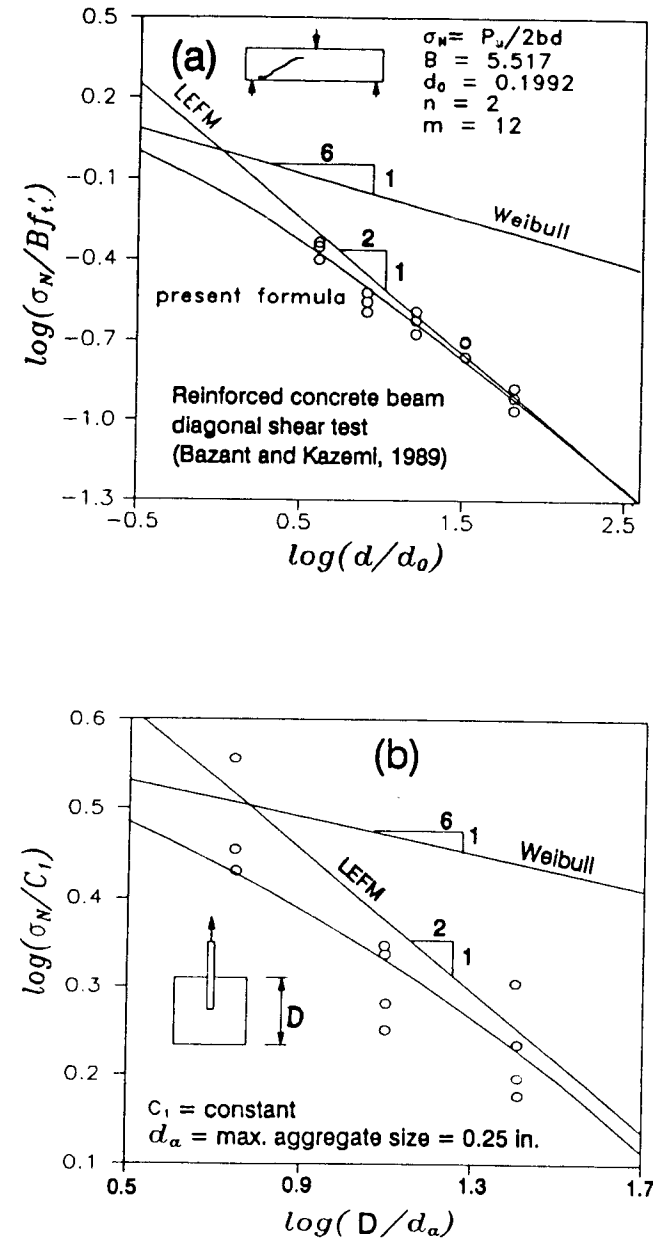


FIG. 6. Test Results of Bažant and Kazemi (1989) for Diagonal Shear Failure (a) and of Bažant and Sener (1988) for Pull-Out (b), and Comparisons with Size Effect Lines Obtained from Classical Weibull-Type Theory

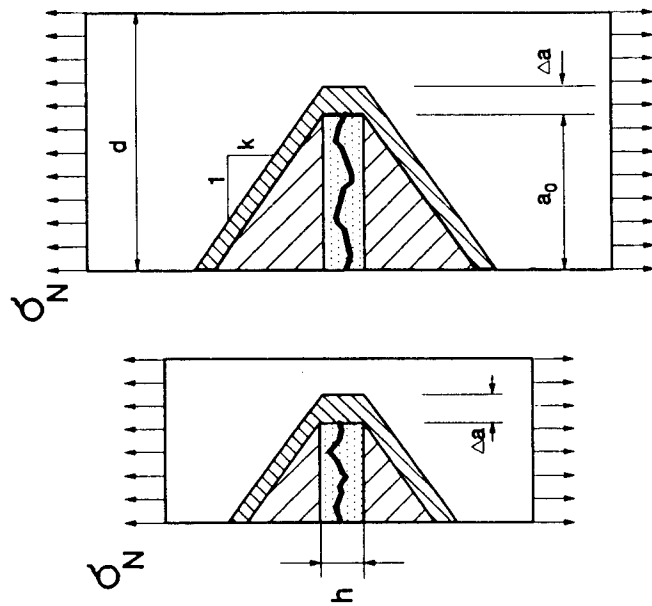


FIG. 7. Stress Relief Zone in Geometrically Similar Panels with Large Similar Fractures

The foregoing argument has been used to derive a deterministic size effect law that is different from that in (13) and agrees quite well with a broad range of test results (Bažant 1984; Bažant and Kazemi 1990). That law has also been supported by certain other, more general, arguments.

Spatial Correlation

Another questionable aspect of classical Weibull-type theories based on (2) is the neglect of spatial correlation. This might be justified for the links in a chain, but not for continuous bodies of concrete cast at one time. If the strength value realized in one small material element is on the low side of the average strength, the strength value realized in the adjacent material elements is more likely to be also on the low side than on the high side of average strength. The standard way to deal with spatial correlation would be to introduce a spatial autocorrelation function for material behavior, such as strength, but that approach would be rather complicated for the present purpose. There is, nevertheless, another simpler way to introduce spatial correlation—the nonlocal concept, advanced in the subsequent paper.

CONCLUSION

The classical applications of Weibull theory to failure of reinforced concrete structures suffer from several serious shortcomings.

1. The stress-distribution function used in the applications cannot be assumed as the elastic stress distribution, but must take into account the stress redistributions caused by large macroscopic stable crack growth prior to reaching maximum load. That growth causes a strong deterministic (or systematic) size effect, which prevails over the statistical size effect due to random strength.

2. According to classical applications of Weibull theory, every structure is equivalent to a uniaxial bar with a variable cross section, which means that all information on the failure mechanism and structure geometry is lost.

3. According to classical theories, the differences in the size effect between two-dimensional similarity and three-dimensional similarity are predicted to be too strong, contradicting experience.

4. Tests of geometrically similar concrete structures, e.g., diagonal shear tests, show a much stronger size than that predicted by classical Weibull-theory applications (provided that the Weibull modulus value is taken the same as that obtained from direct tension test).

5. The classical Weibull-type theories neglect spatial statistical correlation of random material properties.

Modifications of the Weibull theory required to eliminate the aforementioned shortcomings are relegated to the subsequent paper.

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