Standard Form v. Vertex Form

The **Standard Form** of a quadratic equation is: $y = ax^2 + bx + c$.

The **Vertex Form** of a quadratic equation is $y = a(x-h)^2 + k$ where (h, k) represents the vertex of an equation and a is the same a value used in the Standard Form equation.

Converting from Standard Form to Vertex Form: Determine the vertex (h, k) of your original Standard Form equation and substitute the a, h, and k into the Vertex Form of the equation.

You can find the vertex of an equation by finding the axis of symmetry $x = \frac{-b}{2a}$ and substituting this x value into the original equation to find the y coordinate of the vertex.

Example: Convert $y = 2x^2 - 4x + 5$ to Vertex Form.

1) **Find the vertex** $x = \frac{-b}{2a} = \frac{-(-4)}{2(2)} = \frac{4}{4} = 1$

$y = 2(1)^2 - 4(1) + 5 = 2 - 4 + 5 = 3$

Vertex: (1, 3)

2) **Substitute** a, h, and k **into** $y = a(x-h)^2 + k$

$$\boxed{y = \mathbf{2}(x - \mathbf{1})^2 + \mathbf{3}}$$

Converting from Vertex Form to Standard Form: Use the FOIL Method to find the product of the squared polynomial. Simplify using order of operations and arrange in descending order of power.

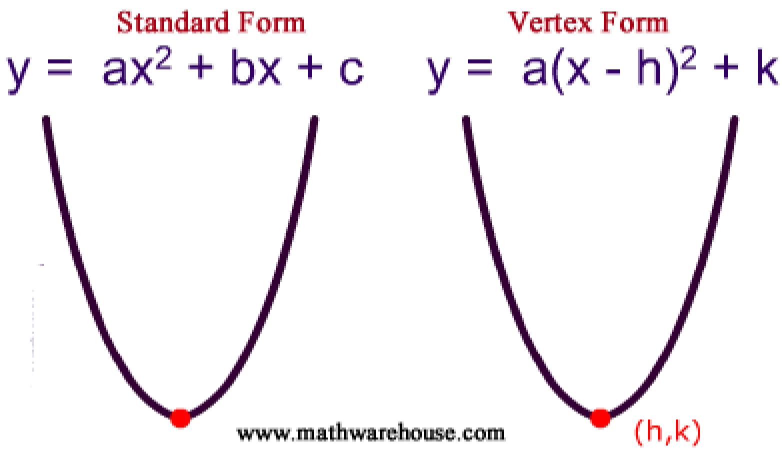
Example: Convert $y = 2(x-1)^2 + 3$ to Standard Form.

1) **FOIL Method** $y = 2(x-1)^2 + 3$
2) **Simplify using order of operations**

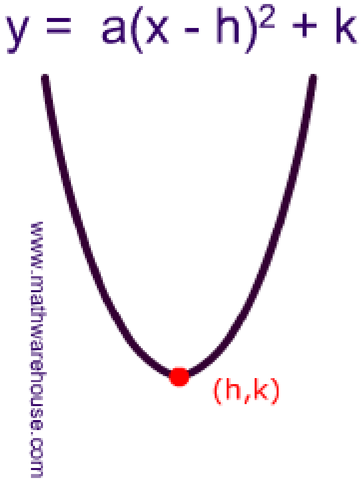
$y = 2(x^2 - 2x + 1) + 3$

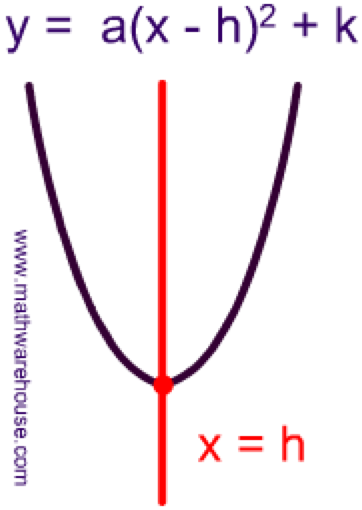
$y = 2x^2 - 4x + 2 + 3$

$$\boxed{y = 2x^2 - 4x + 5}$$



The vertex form of a quadratic function is given by
$f(x) = a(x - h)^2 + k$, where (h, k) is the vertex of the parabola.





Definition:

The vertex form of a quadratic function
$f(x) = a(x - h)^2 + k$,
where (h, k) is the vertex of the parabola.

Chapter 9 Supplement: Vertex Form - Translations

Guided Practice:

Convert the following Standard Form equations into Vertex Form.

1. $y = x^2 - 4x + 6$

2. $y = 4x^2 + 8x - 5$

***3.** $y = 2x^2 - 2x + 7$

4. $y = -x^2 + 6x + 2$

5. $y = -2x^2 + 4x - 5$

6. $y = 5x^2 + 10x - 12$

7. $y = 2x^2 + 8x - 7$

8. $y = x^2 - 8x + 15$

***9.** $y = 0.5x^2 + 2x + 7$

Guided Practice:

Convert the following Vertex Form equations into Standard Form.

1. $y = 3(x - 4)^2 + 5$

***2.** $y = -(x + 5)^2 - 3$

3. $y = (x - 2)^2 - 7$

***4.** $y = 0.5(x + 6)^2 - 11$

5. $y = -2(x + 1)^2 + 2$

6. $y = (x - 4)^2 - 20$

Chapter 9 Supplement: Vertex Form - Translations

Independent Practice:

Convert the following Standard Form equations into Vertex Form.

1. $y = 3x^2 - 6x + 5$

2. $y = x^2 + 10x - 8$

*__3.__ $y = -x^2 + 2x + 6$

4. $y = x^2 + 2x + 11$

5. $y = -2x^2 + 8x + 1$

6. $y = 3x^2 + 12x - 13$

7. $y = -x^2 - 6x + 1$

8. $y = -5x^2 - 10x + 12$

*__9.__ $y = 0.5x^2 - 4x + 2$

Independent Practice:

Convert the following Vertex Form equations into Standard Form.

1. $y = -3(x-1)^2 + 6$

*__2.__ $y = (x-2)^2 + 3$

3. $y = 2(x+7)^2 - 12$

***4.** $y = 0.5(x+8)^2 - 1$

5. $y = -(x+4)^2 + 7$

6. $y = (x-9)^2 + 10$

9.3 *Graphing Quadratic Functions ~ Tech Lab*

Let us look at the graph of $y = x^2$

We can analyze the "**parent function**" for special points and behavior.

$$y = x^2$$

Domain:

Range:

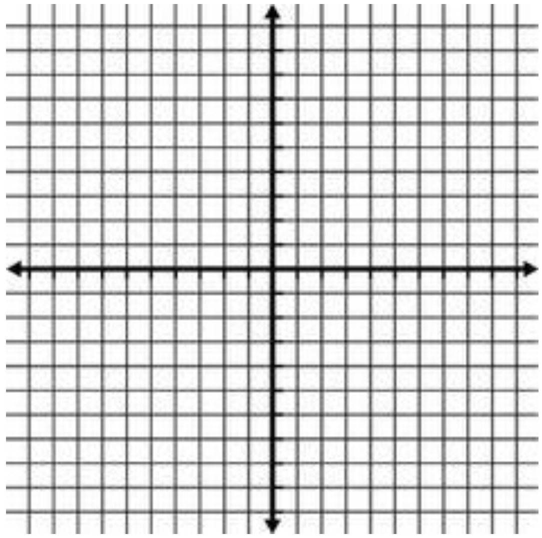
Y-Intercept:

Vertex:

X-Intercepts (Zeros/Roots/Solutions):

Increasing/Decreasing:

x	y



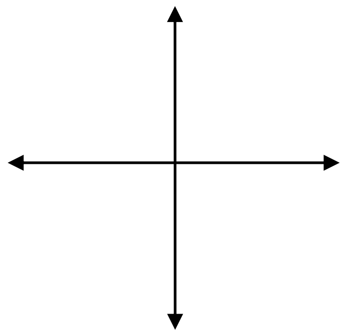
In these notes, we will learn a new technique for graphing a function- shifting it up, down, left, or right. So we can eventually graph any function knowing given parent shape.

Exploration of Transformations - Vertical Shifts

1) Graph on your calculator in Y_1.

    a. Sketch a graph of the function $y = x^2$

    b. What is the vertex of the graph? _________

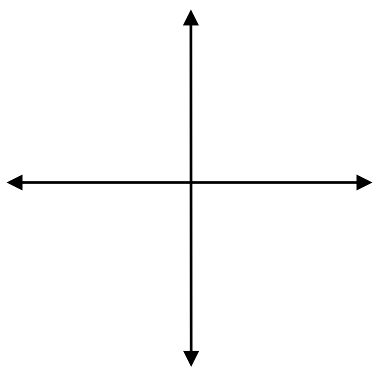


2) Graph $y = x^2 + 2$ on your calculator in Y_2.

    a. Sketch a graph of the function.

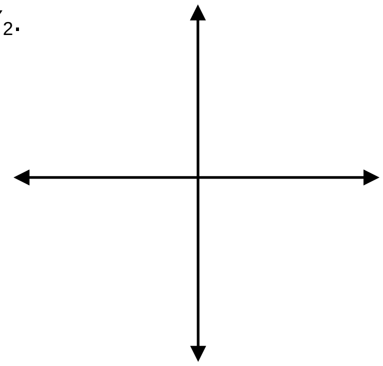
    b. What is the vertex of the graph? _________

    c. How has the graph moved? (up or down) _________



Chapter 9 Supplement: Vertex Form - Translations

3) Clear your previous Y_2 and graph $y = x^2 - 5$ on your calculator in Y_2.



   a. Sketch a graph of the function and the function.

   b. What is the vertex of the graph? _________

   c. How has the graph moved? (up or down) _________

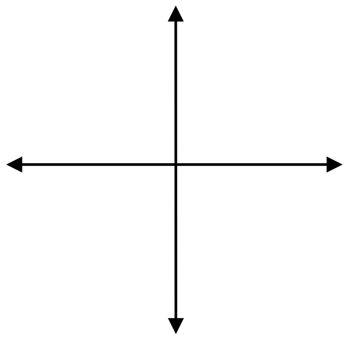
Rule:

Given that $y = a(x-h)^2 + k$ is the symbolic form of a quadratic function, how does changing value of ***k*** change the graph of the function?

If **k is positive**, what direction will the function move? If **k is negative**, what direction will it move?

Exploration of Transformations – Horizontal Shifts

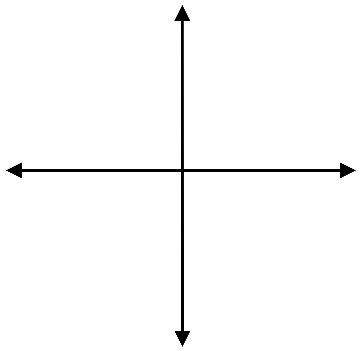
1) Graph $y = x^2$ on your calculator in Y_1.



   a. Sketch a graph of the function.

   b. What is the vertex of the graph? _________

2) Graph $y = (x-2)^2$ on your calculator in Y_2.



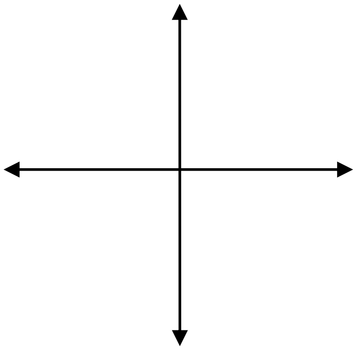
   a. Sketch a graph of the function and the function.

   b. What is the vertex of the graph? _________

   c. How has the graph moved? (left or right) _________

Chapter 9 Supplement: Vertex Form - Translations

3) Graph $y = (x+5)^2$ on your calculator in Y_2.



a. Sketch a graph of the function.

b. What is the vertex of the graph? ________

c. How has the graph moved? (left or right) ________

Rule:

Given that $y = a(x-h)^2 + k$ is the symbolic form of a quadratic function, how does changing value of ***h*** change the graph of the function?

When we have $(x-h)$ what direction does the graph move? _______________

When we have $(x+h)$ what direction does the graph move? _______________

Exploration of Transformations – Vertical Stretch or Shrink / Narrower or Wider Graphs

1) Graph $y = x^2$ on your calculator in Y_1.

a. What direction does the graph open? ________

b. What is the vertex of the graph? ________

c. Fill in the table to the right. These coordinates are the basic ordered pairs of the absolute value function.

x	***y***
-2	
-1	
0	
1	
2	

2) Graph $y = 2x^2$ on your calculator in Y_2.

a. What direction does the graph open? ________

b. What is the vertex of the graph? ________

c. Fill in the table to the right. How do these y-coordinates compare with the y-coordinates in question #1?
Is the graph wider or narrower?

x	***y***
-2	
-1	
0	
1	
2	

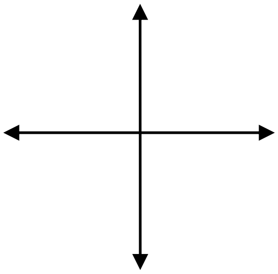
Chapter 9 Supplement: Vertex Form - Translations

3) Graph $y = \frac{1}{2}x^2$ on your calculator in Y_2.

x	*y*
-2	
-1	
0	
1	
2	

a. What direction does the graph open? _________
b. What is the vertex of the graph? _________
c. Fill in the table to the right. How do these y-coordinates compare with the y-coordinates in question #1?
   Is the graph wider or narrower?

4) Graph $y = -2x^2$ on your calculator in Y_2.



a. Sketch the graph of the function and the function in #1.

b. How has the graph of the function changed?

Rule:

Given that $y = a(x-h)^2 + k$ is the symbolic form of the vertex function, how does changing value of ***a*** change the graph of the function?

If ***a*** is positive, the graph ________________

If ***a*** is negative, the graph ________________

If $0 < a < 1$, the graph is _____________

If $a > 1$, the graph is _______________

Chapter 9 Supplement: Vertex Form - Translations

2.5a Exploration of ALL Transformations (Homework)

1) Graph $y = x^2$ on your calculator in Y_1.

2) Graph $y = (x-3)^2 - 4$ on your calculator in Y_2.

   a. What direction does the graph open? ___________

   b. How does the graph move? (left/right, up/down) _________________

   c. Does the graph become narrower or wider? _____________

   d. What is the vertex of the graph? ____________

3) Graph $y = -\frac{1}{2}(x-2)^2 - 3$ on your calculator in Y_2.

   a. What direction does the graph open? ___________

   b. How does the graph move? (left/right, up/down) _________________

   c. Does the graph become narrower or wider? _____________

   d. What is the vertex of the graph? ____________

4) Graph $y = -2(x+5)^2 + 8$ your calculator in Y_2.

   a. What direction does the graph open? ___________

   b. How does the graph move? (left/right, up/down) _________________

   c. Does the graph become narrower or wider? _____________

   d. What is the vertex of the graph? ____________

5) Graph $y = (x+6)^2 - 4$ on your calculator in Y_2.

   a. What direction does the graph open? ___________

   b. How does the graph move? (left/right, up/down) _________________

   c. Does the graph become narrower or wider? _____________

   d. What is the vertex of the graph? ____________

Chapter 9 Supplement: Vertex Form - Translations

Review:

Given the vertex function is $y = a(x-h)^2 + k$ …

6) If a>0, does the graph open up or down? _______

7) If a<0, does the graph open up or down? _______

8) If $a > 1$, does the graph become narrower or wider? _______

9) If 0<a<1, does the graph become narrower or wider? _______

10) How does changing value of **k** change the graph of the function? ____________________

11) How does changing value of ***h*** change the graph of the function? ____________________

Translations:

A **translation** is a change in the position of a figure either up, down, left, right, or diagonal. Adding or subtracting constants in the equations of functions translates the graphs of the functions.

When written in vertex form: (h, k) is the vertex of the parabola, and $x = h$ is the axis of symmetry.

The graph of $\boldsymbol{g}(\boldsymbol{x}) = \boldsymbol{x}^2 + k$ translates the graph of $\boldsymbol{f}(\boldsymbol{x}) = x^2$ vertically.

If $k > 0$, the graph of $\boldsymbol{f}(\boldsymbol{x}) = \boldsymbol{x}^2$ is translated k units up.

If $k < 0$, the graph of $\boldsymbol{f}(\boldsymbol{x}) = \boldsymbol{x}^2$ is translated $|k|$ units down.

The graph of $\boldsymbol{g}(\boldsymbol{x}) = (\boldsymbol{x} - h)^2$ is the graph of $\boldsymbol{f}(\boldsymbol{x}) = \boldsymbol{x}^2$ translated horizontally.

If $h > 0$, the graph of $\boldsymbol{f}(\boldsymbol{x}) = \boldsymbol{x}^2$ is translated h units to the right.

If $h < 0$, the graph of $\boldsymbol{f}(\boldsymbol{x}) = \boldsymbol{x}^2$ is translated $|h|$ units to the left.

Notice that the h value is subtracted in this form, and that the k value is added.

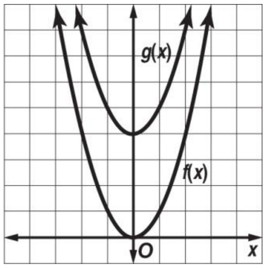
If the equation is $y = 2(x - 1)^2 + 5$, the value of h is 1, and k is 5.

If the equation is $y = 3(x + 4)^2 - 6$, the value of h is -4, and k is -6.

Examples:

Describe how the graph of each function is related to the graph of $f(x) = x^2$.

a. $g(x) = x^2 + 4$

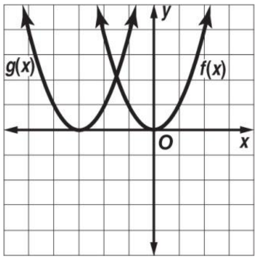


The value of k is 4, and 4 > 0. Therefore,

the graph of $g(x) = x^2 + 4$ is a translation of the

graph of $f(x) = \boldsymbol{x}^2$ up 4 units

b. $g(x) = (x + 3)^2$



The value of h is –3, and –3 < 0. Thus, the

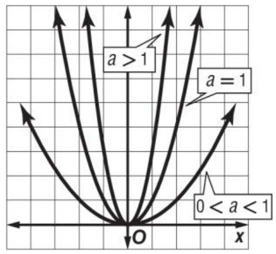
graph of $g(x) = (x + 3)^2$ is a translation of the

graph of $f(x) = \boldsymbol{x}^2$ to the left 3 units.

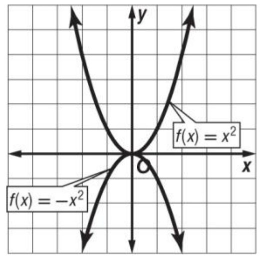
Dilations and Reflections:

A **dilation** is a transformation that makes the graph narrower or wider than the parent graph. A **reflection** flips a figure over the *x*- or *y*-axis.

The graph of $\boldsymbol{f(x) = ax^2}$ stretches or compresses the graph of $\boldsymbol{f(x) = x^2}$.
If $|a| > 1$, the graph of $\boldsymbol{f(x) = x^2}$ is stretched vertically.
If $0 < |a| < 1$, the graph of $\boldsymbol{f(x) = x^2}$ is compressed vertically.



The graph of the function $\boldsymbol{-f(x)}$ flips the graph of $\boldsymbol{f(x) = x^2}$ across the *x*-axis.
The graph of the function $\boldsymbol{f(-x)}$ flips the graph of $\boldsymbol{f(x) = x^2}$ across the *y*-axis.

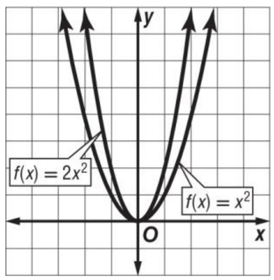


Example: Describe how the graph of each function is related to the graph of $f(x) = x^2$.

a. $g(x) = 2x^2$

The function can be written as $f(x) = ax^2$ where $a = 2$.

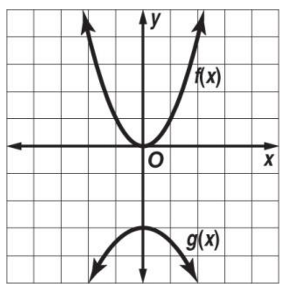
Because $|a| > 1$, the graph of $y = 2x^2$ is the graph of $y = x^2$ that is stretched vertically.



b. $g(x) = -\frac{1}{2}x^2 - 3$

The negative sign causes a reflection across the *x*-axis.

Then a dilation occurs in which $a = \frac{1}{2}$ and a translation in which $k = -3$. So the graph of $g(x) = -\frac{1}{2}x^2 - 3$ is reflected across the *x*-axis, dilated wider than the graph of $f(x) = x^2$, and translated down 3 units.



Chapter 9 Supplement: Vertex Form - Translations

Guided Practice:

Describe how the graph of each function is related to the graph of $f(x) = x^2$. Also draw a sketch to illustrate the translation.

1. $g(x) = x^2 + 1$

2. $g(x) = (x - 6)^2$

3. $g(x) = (x + 1)^2$

4. $g(x) = 20 + x^2$

5. $g(x) = (-2 + x)^2$

6. $g(x) = -\frac{1}{2} + x^2$

7. $g(x) = x^2 + \frac{8}{9}$

8. $g(x) = x^2 - 0.3$

9. $g(x) = (x + 4)^2$

Chapter 9 Supplement: Vertex Form - Translations

Independent Practice:

Describe how the graph of each function is related to the graph of *f*(*x*) = x^2.

1. $g(x) = -5x^2$

2. $g(x) = -(x+1)^2$

3. $g(x) = -\frac{1}{4}x^2 - 1$

4. $g(x) = (x+10)^2$

****5.** $g(x) = -\frac{2}{5} + x^2$

***6.** $g(x) = 9 - x^2$

7. $g(x) = 2x^2 + 2$

8. $g(x) = -\frac{3}{4}x^2 - \frac{1}{2}$

9. $g(x) = -3(x-4)^2$

10. $g(x) = x^2 + 2$

11. $g(x) = (x-1)^2$

12. $g(x) = x^2 - 8$

13. $g(x) = 7x^2$

14. $g(x) = \frac{1}{5}x^2$

15. $g(x) = -6x^2$

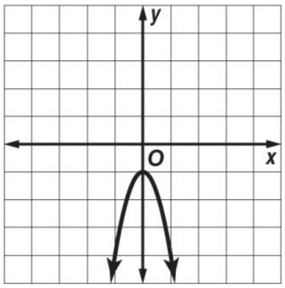
16. $g(x) = -x^2 + 3$

17. $g(x) = 5 - \frac{1}{5}x^2$

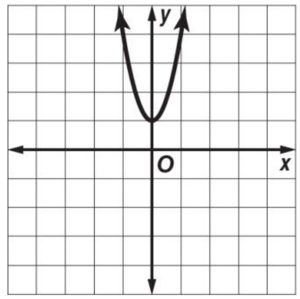
18. $g(x) = 4(x-1)^2$

Match each equation to its graph.

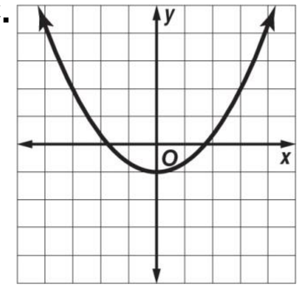
A.



B.



C.



20. $y = -3x^2 - 1$

21. $y = \frac{1}{3}x^2 - 1$

22. $y = 3x^2 + 1$

Match each equation to its graph.

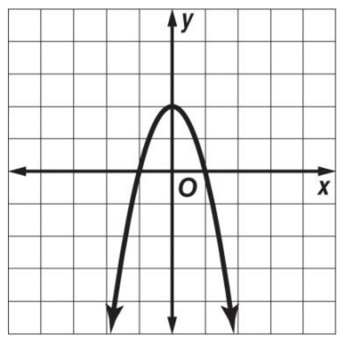
23. $y = 2x^2 - 2$

24. $y = \frac{1}{2}x^2 - 2$

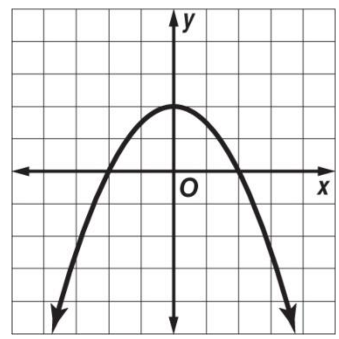
25. $y = -\frac{1}{2}x^2 + 2$

26. $y = -2x^2 + 2$

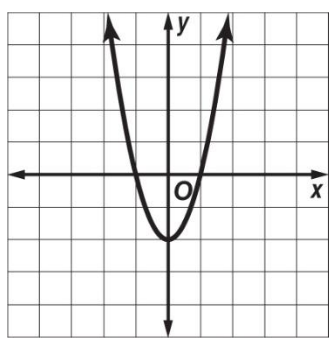
A.



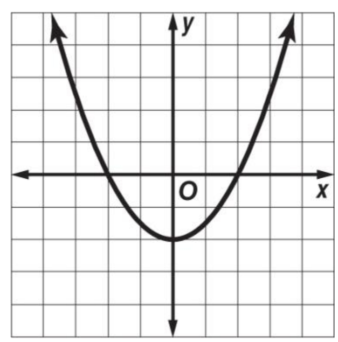
C.



B.



D.



List the functions in order from the most vertically stretched to the least vertically stretched graph.

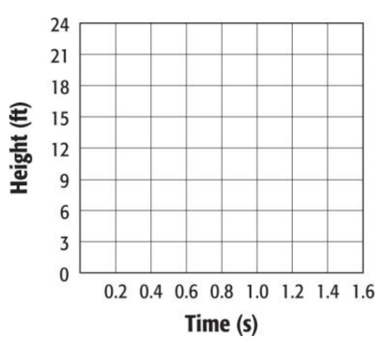
27. $f(x) = 3x^2$, $g(x) = \frac{1}{2}x^2$, $h(x) = -2x^2$

28. $f(x) = \frac{1}{2}x^2$, $g(x) = -\frac{1}{6}x^2$, $h(x) = 4x^2$

9-3 Word Problem Practice ~ Transformations of Quadratic Functions

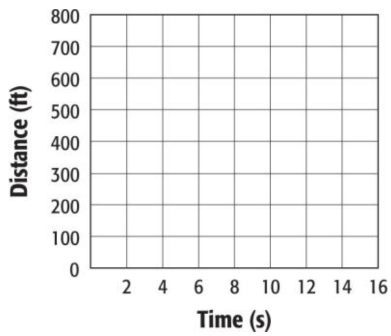
29. SPRINGS The potential energy stored in a spring is given by $U_s = \frac{1}{2}kx^2$ where *k* is a constant known as the spring constant, and *x* is the distance the spring is stretched or compressed from its initial position. How is the graph of the function for a spring where *k* = 2 newtons/meter related to the graph of the function for a spring where *k* = 10 newtons/meter?

30. PHYSICS A ball is dropped from a height of 20 feet. The function $h = -16t^2 + 20$ models the height of the ball in feet after *t* seconds. Graph the function and compare this graph to the graph of its parent function.



31. ACCELERATION The distance *d* in feet a car accelerating at 6 ft/s^2 travels after *t* seconds is modeled by the function $d = 3t^2$. Suppose that at the same time the first car begins accelerating, a second car begins accelerating at 4 ft/s^2 exactly 100 feet down the road from the first car. The distance traveled by second car is modeled by the function $d = 2t^2 + 100$.

a. Graph and label each function on the same coordinate plane.



b. Explain how each graph is related to the graph of $d = t^2$.

c. After how many seconds will the first car pass the second car?

32. PARACHUTING Two parachutists jump at the same time from two different planes as part of an aerial show. The height h_1 of the first parachutist in feet after *t* seconds is modeled by the function $h_1 = -16t^2 + 5000$. The height h_2 of the second parachutist in feet after *t* seconds is modeled by the function $h_2 = -16t^2 + 4000$.

a. What is the parent function of the two functions given?

b. Describe the transformations needed to obtain the graph of h_1 from the parent function.

c. Which parachutist will reach the ground first?