Discovering Algebra An Investigative Approach | Discovering Geometry An Investigative Approach | Discovering Advanced Algebra An Investigative Approach

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INNOVATORS IN MATHEMATICS EDUCATION

Correlation of *Discovering Algebra, 2nd Edition, Discovering Geometry, 4th Edition,* and *Discovering Advanced Algebra, 2nd Edition* to Common Core State Standards (June 2010), Mathematics, High School

STANDARDS FOR MATHEMATICAL PRACTICE

Introduction	*Discovering Mathematics* General Comments
The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report Adding It Up: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).	***Discovering Mathematics* deeply addresses the Standards for Mathematical Practice.** The Standards for Mathematical Practice are addressed in both the daily Investigations and the Exercise Sets in every lesson of *Discovering Mathematics*. Because rich, non-routine problems are integral to the textbooks, students gain proficiency and comfort with analyzing problems, trying different solution methods, and evaluating their answers. A representative lesson is presented from each book to illustrate the correlation to each Standard for Mathematical Practice.

Standard	*Discovering Mathematics* Lessons
1. Make sense of problems and persevere in solving them. Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.	**Representative *Discovering Algebra* Lesson:** Lesson 9.1: Solving Quadratic Equations In Example A, the solution models the strategy of using a graph to understand the meaning of a problem. The Rocket Science Investigation scaffolds students in assigning variables (including units), understanding the meaning of different components of the projectile motion formula, and moving between the graphical and algebraic representations while making sense of the context. The purely symbolic solution to Example B is confirmed by a graph and a table, thus modeling the approach of checking a solution by using an alternative approach. On page 499, the textbook notes the importance of checking whether an answer to a real-world problem makes sense, and this point is supported by the emphasis on meaning in Exercises 5–7, 9, and 10. **Representative *Discovering Geometry* Lesson:** Lesson 1.9: A Picture is Worth a Thousand Words The problem solving skills of drawing diagrams and visualizing situations are developed throughout *Discovering Geometry*. Examples A, B, and C provided guided help in translating descriptions into diagrams and solving problems. In the Exercise Set, students make sense of a variety of problems, using diagrams to conceptualize a solution. Problems like Exercises 18–24 are posed throughout the textbook to help build students' visualization skills. **Representative *Discovering Advanced Algebra* Lesson:** Lesson 5.1: Exponential Functions In the Radioactive Decay Investigation, students collect data and then represent it using a table and graph. Students then model the data using a geometric sequence and evaluate and adjust their model. Students are able to draw connections between the recursive formula, the explicit formula, the graph, and the exponential model. In the exercises, students model real-world situations (Exercises 5 and 6), identify important features of functions, and make and verify predictions about graphs (Exercises 7, 8, 9, 10), and extend their knowledge of transformations to make a generalization about exponential functions (Exercise 11). In the Project The Cost of Living, students gather data, model it, and evaluate the accuracy of their model.

2. Reason abstractly and quantitatively. Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.	**Representative *Discovering Algebra* Lesson:** Lesson 5.3: Solving Systems of Equations Using Substitution Example A models the movement between verbal and symbolic expressions. First the scenario is modeled with an equation, and each element is explained in terms of the context. The algebraic substitution is also explained in terms of the problem context, and after showing the algebraic reasoning abstractly, the final solution is explained contextually. In the All Tied Up Investigation and Example B, students similarly move between the situation and the abstract algebraic representation, always attending to the units and meaning of the solution in summary. Exercise 10 illustrates students making sense of a system of equations given a context, solving the system, and then making sense of the fact that there is no solution to the system within the context. **Representative *Discovering Geometry* Lesson:** Lesson 9.3: Two Special Right Triangles Students move between finding relationships among the side lengths of isosceles right triangles and 30˚-60˚-90˚ triangles, and working abstractly with square roots. The use of isometric dot paper models helps students understand that side lengths can have irrational values, and provides a model for equivalent square root expressions. Students demonstrate their understanding by creating their own models (Exercises 12–13), writing an algebraic proof (Exercise 16), and constructing segments lengths that are square root multiples of a given length (Exercise 20). **Representative *Discovering Advanced Algebra* Lesson:** Lesson 5.4: Applications of Exponential and Power Equations Students move from problem situations to algebraic representations, and then interpret their results in terms of the original context. Students solve problems involving interest accrual, the motion of a pendulum, the intensity of light, a simulation of exponential decay, and the orbit of the moons of Saturn. In each exercise, students use the context to write an algebraic expression, and then use symbolic manipulation to answer questions. Students also make sense of the relationships through tables and graphs.

3. Construct viable arguments and critique the reasoning of others. Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.	**Representative *Discovering Algebra* Lessons:** Lesson 1.5: Exploring A Conjecture In this Activity Day, students deepen their understanding of ways to represent and analyze data by posing and then thoroughly testing a conjecture. This lesson occurs early in the textbook to promote careful and thorough reasoning about data, and to help students understand how data can be presented in misleading ways. **Representative *Discovering Geometry* Lessons:** Lesson 4.7: Flowchart Thinking In this lesson, students learn the flowchart format for proofs. The flowchart format visually illustrates the logical progression of a proof. In addition to completing flowchart proofs, students also verbalize the conclusion of a proof (Exercise 5), give a proof of a construction (Exercise 6), analyze a flawed argument (Exercise 7), and apply their reasoning to find a flaw in a diagram (Exercise 9). **Representative *Discovering Advanced Algebra* Lessons:** Lesson 3.6: Linear Systems Students use linear systems to model situations and to make mathematical arguments to support a decision or a prediction. For example, Example A uses equations, graphs, and tables to analyze which phone plan is best for different types of callers. The Investigation Population Trends has students choose several methods for estimating the year in which two populations were the same. In the Reason and Apply Exercises, students are repeatedly asked to justify their reasoning and explain their answers (Exercise 7c, 8b, 8d, and 9c).

4. Model with mathematics. Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.	**Representative *Discovering Algebra* Lesson:** Lesson 4.5: Writing Point-Slope Equations to Fit Data In the Life Expectancy Investigation, students model life expectancy data using a line of fit, make predictions, and then compare their results with those of other students. They gain insight into the limitations and variations of models for real-world data. In Exercises 4–8, students gain more experience modeling linear data. In each exercise, they use their model to make predictions and they assess the accuracy of their model. **Representative *Discovering Geometry* Lesson:** Lesson 10.2: Volume of Prisms and Cylinders Students develop volume formulas through models and generalizing their reasoning. They apply the formulas to find the volumes of pure geometric shapes, and also to estimate the weight of a Great Pyramid (Exercise 16), the volume of oil spilled during the Gulf War (Exercise 17), and the volume of the AIDS Memorial Quilt (Exercise 18). Additionally, students can apply their modeling skills to building the pieces of a Soma Cube in the project on page 537. **Representative *Discovering Advanced Algebra* Lesson:** Lesson 5.8: Applications of Logarithms This lesson focuses on using logarithms to model and solve real-world problems. Example B shows in detail the algebraic process of curve straightening, but maintains a focus on the goal of modeling the original data. In the Cooling Investigation, students are given general guidance, but must track their own progress and make sense of their equations within the context of an object cooling over time. Students explore and model a variety of contexts in the exercises, including the relationship between storage temperature and freshness of milk, the sales of a video game over time, the loudness of spoken words given the distance, and the visibility from a plane based on height. The Income by Gender Project provides an opportunity for students to gather data, identify a relationship to model, and analyze and assess their model.

5. Use appropriate tools strategically. Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.	**Representative *Discovering Algebra* Lesson:** Lesson 3.5: Linear Equations and Rate of Change Students use technology to generate recursive routines, graph data, and graph equations. Students are encouraged to use a variety of techniques to model and analyze data, including calculator lists and tables, tracing calculator graphs, and graphing and solving equations by hand. In Step 7of the Wind Chill Investigation, students make connections between the multiple representations, and in Step 8 they interpret the rate of change in context. In Exercises 1–4, students move among tables, equations, verbal descriptions, and calculator technology as they explore linear equations and rate of change. **Representative *Discovering Geometry* Lesson:** Lesson 5.3: Kite and Trapezoid Properties An optional Dynamic Geometry Exploration (available to students at www.keymath.com) is pictured for exploring the properties of the sides of a kite. Students then use patty paper to construct and explore other kite properties. In Investigation 2, students use a straightedge, protractor, and compass to explore the properties of trapezoids. They justify their conclusions with a proof. In Exercises 14–16, they choose the construction tools to use for each construction. **Representative *Discovering Advanced Algebra* Lessons:** Students make use of a variety of tools to explore mathematical concepts throughout the textbook. For example, they frequently use graphing calculators to graph data, observe patterns in tables, and graph functions. They also can use motion sensors to gather data (page 199), Dynamic Algebra Explorations to explore functions (page 209), dynamic geometry software to explore transformations (page 220), and dynamic data software to simulate an experiment (page 569).

6. Attend to precision. Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.	**Representative *Discovering Algebra* Lesson:** Lesson 1.7: Estimating In the Guesstimating Investigation, students review and practice the mathematical skills of plotting data, labeling and scaling axes, and graphing the line $y = x$ in the context of estimating measures and gauging their accuracy. The context promotes attention to the details of the graph, as a point above the line has a different meaning than a point below the line. In the exercises, students revisit the estimation context and also interpret points graphed around the line $y = x$ in new contexts. This allows them to assess a claim about SAT scores in Exercise 9. **Representative *Discovering Geometry* Lessons:** Lesson 1.3: What's a Widget Students learn the importance of giving precise definitions by exploring counterexamples. They write definitions and test definitions written by other students. Exercises 15–24 challenge students to rigorously test statements before confirming them as true. **Representative *Discovering Advanced Algebra* Lessons:** Chapter 2 Exploration: Precision, Accuracy, and Significant Figures In this early Exploration, students learn about the difference between precision and accuracy, and use significant figures as a way to express precision. Throughout the textbook, students must be attentive to units of measure, make and test predictions, and clearly communicate their reasoning.

7. Look for and make use of structure. Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.	**Representative *Discovering Algebra* Lesson:** Lesson 6.3: Multiplication and Exponents Students observe patterns and generalize their observations in the Investigation Moving Ahead. Because students have been working with exponents in the context of exponential equations, they can easily extend their knowledge of exponent properties to more complex equations, as in Exercise 12. **Representative *Discovering Geometry* Lesson:** Lesson 6.4: Proving Circle Conjectures Students synthesize properties of segments and angles in circles as they work on challenging proofs that involve breaking problems into parts, adding auxiliary lines, and working through algebraic arguments. They learn that reasoning strategies can help them break a complex problem into manageable parts. **Representative *Discovering Advanced Algebra* Lesson:** Lesson 8.5: The General Quadratic Students synthesize their knowledge of conic sections by attending to the structure of the general quadratic. Students recognize the role of each coefficient in a quadratic equation and become proficient at converting between general and standard form.

8. Look for and express regularity in repeated reasoning. Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.	**Representative *Discovering Algebra* Lesson:** Lesson 6.2: Exponential Equations In the Growth of the Koch Curve Investigation, students use the Koch curve as the basis for exploring exponential growth. Repeated multiplication is used to model the growth in the length of the fractal, and students generalize their results to write an exponential equation that models the growth. Real-world contexts promote sense-making and attention to the reasonableness of results, as demonstrated in Exercises 9, 12, and 13. **Representative *Discovering Geometry* Lesson:** Lesson 2.3: Mathematical Modeling Students observe patterns and model them with both geometric diagrams and algebraic expressions. In the Party Handshakes Investigation, students are encouraged to make connections, as the handshake problem is related to triangular and rectangular numbers. **Representative *Discovering Advanced Algebra* Lesson:** Lesson 9.2: Infinite Geometric Series In the Infinite Geometric Series Formula Investigation, students move from a concrete example to the derivation of the formula for the sum of a convergent infinite geometric series. Students make use of this type of reasoning and prepare for the derivation of the partial sums formula in the Exercises.

Number and Quantity

Standard	*Discovering Mathematics* Lessons
The Real Number System	
Extend the properties of exponents to rational exponents.	
1. Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5.*	***Discovering Algebra* Lesson:** Lesson 11.5: Operations with Roots (Exercise 16) ***Discovering Advanced Algebra* Lessons:** Lesson 5.2: Properties of Exponents and Power Functions Lesson 5.3: Rational Exponents and Roots

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2. Rewrite expressions involving radicals and rational exponents using the properties of exponents.	***Discovering Algebra*** **Lessons:** Lesson 6.3: Multiplication and Exponents Lesson 6.5: Looking Back with Exponents Lesson 6.6: Zero and Negative Exponents ***Discovering Advanced Algebra*** **Lessons:** Lesson 5.2: Properties of Exponents and Power Functions Lesson 5.3: Rational Exponents and Roots
Use properties of rational and irrational numbers.	
3. Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.	***Discovering Algebra*** **Lessons:** Lesson 9.1: Solving Quadratic Equations Lesson 11.5: Operations with Roots (*Teacher's Edition*) ***Discovering Advanced Algebra*** **Lessons:** (Partial) Lesson 7.5: Complex Numbers Lesson 7.7: Higher Degree Polynomials (Exercise 15)
Quantities*	
Reason quantitatively and use units to solve problems.	
1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.	***Discovering Algebra*** **Lessons:** Lesson 2.3: Proportions and Measurement Systems Lesson 2.4: Direct Variation Lesson 4.1: A Formula for Slope ***Discovering Advanced Algebra*** **Lesson:** Lesson 0.3: Organizing Information
2. Define appropriate quantities for the purpose of descriptive modeling.	***Discovering Algebra*** **Lessons:** Lesson 0.5: Out of Chaos Lesson 2.1: Proportions ***Discovering Advanced Algebra*** **Lessons:** Lesson 3.1: Linear Equations and Arithmetic Sequences Lesson 3.2: Revisiting Slope Lesson 3.3: Fitting a Line to Data Lesson 5.1: Exponential Functions
3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.	***Discovering Advanced Algebra*** **Lesson:** Chapter 2 Exploration: Precision, Accuracy, and Significant Figures

NUMBER AND QUANTITY

The Complex Number System

Perform arithmetic operations with complex numbers.

1. Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.	***Discovering Advanced Algebra*** **Lesson:** Lesson 7.5: Complex Numbers
2. Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.	***Discovering Algebra*** **Lesson:** Chapter 9 Review: Take Another Look 1 ***Discovering Advanced Algebra*** **Lesson:** Lesson 7.5: Complex Numbers
3. (+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.	***Discovering Advanced Algebra*** **Lesson:** Lesson 7.5: Complex Numbers The term *modulus* is not used, but students are introduced to *magnitude* in the Project The Mandelbrot Set.

Represent complex numbers and their operations on the complex plane.

4. (+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.	***Discovering Advanced Algebra*** **Lesson:** Lesson 7.5: Complex Numbers *Discovering Advanced Algebra Assessment Resources*, Chapter 7 Constructive Assessment Options Problem 7 This standard is covered completely in *Precalculus with Trigonometry: Concepts and Applications*, by Paul A. Foerster.
5. (+) Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. *For example, $(1 - \sqrt{3}i)^3 = 8$ because $(1 - \sqrt{3}i)$ has modulus 2 and argument 120°.*	***Discovering Advanced Algebra*** **Lessons:** Extension, *Teacher's Edition,* page 412 (Partial) Chapter 7 Review: Take Another Look 4 and 5
6. (+) Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.	Students learn about the complex plane in *Discovering Advanced Algebra* Lesson 7.5, and explore several properties of graphing on the complex plane in Take Another Look Exercises 4 and 5 on page 440. However, they do not explicitly study the properties noted in this standard.

Use complex numbers in polynomial identities and equations.

7. Solve quadratic equations with real coefficients that have complex solutions.	***Discovering Advanced Algebra*** **Lesson:** Lesson 7.5: Complex Numbers

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8. (+) Extend polynomial identities to the complex numbers. *For example, rewrite* $x^2 + 4$ *as* $(x + 2i)(x – 2i)$.	***Discovering Advanced Algebra*** **Lesson:** Lesson 7.5: Complex Numbers (Exercises 6, 11, and 12)
9. (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.	***Discovering Advanced Algebra*** **Lesson:** (Partial) Lesson 7.7: Higher-Degree Polynomials The corollaries of the Fundamental Theorem of Algebra are covered informally in *Discovering Advanced Algebra*. This standard is covered completely in *Precalculus with Trigonometry: Concepts and Applications*, by Paul A. Foerster.
Vector and Matrix Quantities	
Represent and model with vector quantities.	
1. (+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., $\boldsymbol{v}$, $\lvert\boldsymbol{v}\rvert$, $\lVert\boldsymbol{v}\rVert$, $\boldsymbol{v}$).	***Discovering Geometry*** **Lesson:** Lesson 5.5: Properties of Parallelograms ***Discovering Advanced Algebra*** **Lesson:** Lesson 12.5: Introduction to Vectors
2. (+) Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.	***Discovering Geometry*** **Lessons:** Lesson 7.2: Properties of Isometries Lesson 12.5: Problem Solving with Trigonometry ***Discovering Advanced Algebra*** **Lesson:** Lesson 12.5: Introduction to Vectors
3. (+) Solve problems involving velocity and other quantities that can be represented by vectors.	***Discovering Geometry*** **Lesson:** Lesson 12.5: Problem Solving with Trigonometry ***Discovering Advanced Algebra*** **Lesson:** Lesson 12.5: Introduction to Vectors
Perform operations on vectors.	
4. (+) Add and subtract vectors.	
4a. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.	***Discovering Geometry*** **Lessons:** Lesson 5.5: Properties of Parallelograms Lesson 12.5: Problem Solving with Trigonometry ***Discovering Advanced Algebra*** **Lesson:** Lesson 12.5: Introduction to Vectors

NUMBER AND QUANTITY	
4b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.	***Discovering Geometry*** **Lesson:** Lesson 12.5: Problem Solving with Trigonometry ***Discovering Advanced Algebra*** **Lesson:** Lesson 12.5: Introduction to Vectors
4c. Understand vector subtraction $\boldsymbol{v} - \boldsymbol{w}$ as $\boldsymbol{v} + (-\boldsymbol{w})$, where $-\boldsymbol{w}$ is the additive inverse of $\boldsymbol{w}$, with the same magnitude as $\boldsymbol{w}$ and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.	***Discovering Advanced Algebra*** **Lesson:** Lesson 12.5: Introduction to Vectors
5. (+) Multiply a vector by a scalar.	
5a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(vx, vy) = (cvx, cvy)$.	***Discovering Advanced Algebra*** **Lesson:** Lesson 12.5: Introduction to Vectors
5b. Compute the magnitude of a scalar multiple $c\boldsymbol{v}$ using $\|\|c\boldsymbol{v}\|\| = \|c\|\boldsymbol{v}$. Compute the direction of $c\boldsymbol{v}$ knowing that when $\|c\|\boldsymbol{v} \neq 0$, the direction of $c\boldsymbol{v}$ is either along $\boldsymbol{v}$ (for $c > 0$) or against $\boldsymbol{v}$ (for $c < 0$).	***Discovering Advanced Algebra*** **Lesson:** (Partial) Lesson 12.5: Introduction to Vectors This standard is covered completely in *Precalculus with Trigonometry: Concepts and Applications,* by Paul A. Foerster
Perform operations on matrices and use matrices in applications.	
6. (+) Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.	***Discovering Algebra*** **Lesson:** Lesson 1.8: Using Matrices to Organize and Combine Data ***Discovering Advanced Algebra*** **Lessons:** Lesson 6.1: Matrix Representations Lesson 6.2: Matrix Operations
7. (+) Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.	***Discovering Algebra*** **Lesson:** Lesson 1.8: Using Matrices to Organize and Combine Data ***Discovering Advanced Algebra*** **Lesson:** Lesson 6.2: Matrix Operations

NUMBER AND QUANTITY	
8. (+) Add, subtract, and multiply matrices of appropriate dimensions.	***Discovering Algebra*** **Lesson:** Lesson 1.8: Using Matrices to Organize and Combine Data Lesson 8.7: Transformations with Matrices ***Discovering Geometry*** **Lessons:** Lesson 7.3: Compositions of Transformations (Exercise 18) Lesson 7.7: Tessellations That Use Rotation (Exercise 14) Chapter 7 Review (Take Another Look Exercises 4 and 5) ***Discovering Advanced Algebra*** **Lessons:** Lesson 6.2: Matrix Operations Lesson 6.3: Solving Systems with Inverse Matrices
9. (+) Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.	***Discovering Advanced Algebra*** **Lesson:** Lesson 6.2: Matrix Operations
10. (+) Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.	***Discovering Advanced Algebra*** **Lessons:** Lesson 6.3: Solving Systems with Inverse Matrices Chapter 6 Review: Take Another Look 1
11. (+) Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.	***Discovering Advanced Algebra*** **Lessons:** (The term *vector* is used only in the *Teacher's Edition*, page 320, but these concepts are covered.) Lesson 6.2: Matrix Operations Chapter 6 Review: Take Another Look 2
12. (+) Work with 2 × 2 matrices as a transformations of the plane, and interpret the absolute value of the determinant in terms of area.	***Discovering Algebra*** **Lesson:** Lesson 8.7: Transformations with Matrices ***Discovering Geometry*** **Lesson:** Chapter 7 Review: Take Another Look 4 and 5 ***Discovering Advanced Algebra*** **Lesson:** Lesson 6.2: Matrix Operations Chapter 6 Review: Take Another Look 1 The determinant is introduced in *Discovering Advanced Algebra*, and covered thoroughly in *Precalculus with Trigonometry: Concepts and Applications,* by Paul A. Foerster.

ALGEBRA	
Standard	***Discovering Mathematics* Lessons**
Seeing Structure in Expressions	
Interpret the structure of expressions.	
1. Interpret expressions that represent a quantity in terms of its context.★	
1a. Interpret parts of an expression, such as terms, factors, and coefficients.	***Discovering Algebra* Lessons:** This standard is addressed throughout the book. Examples include: Lesson 2.4: Direct Variation Lesson 3.2: Linear Plots Lesson 6.1: Recursive Routines Lesson 9.2: Finding the Roots and the Vertex ***Discovering Geometry* Lesson:** Using Your Algebra Skills 12: Transforming Functions ***Discovering Advanced Algebra* Lessons:** This standard is addressed throughout the book. Examples include: Lesson 3.2: Revisiting Slope Lesson 3.3: Fitting a Line to Data Lesson 5.1: Exponential Functions Lesson 5.4: Applications of Exponential and Power Functions Lesson 5.8: Applications of Logarithms Lesson 6.5: Systems of Inequalities Lesson 6.6: Linear Programming Lesson 7.2: Equivalent Quadratic Expressions Lesson 7.6: Factoring Polynomials Lesson 8.6: Introduction to Rational Functions

ALGEBRA	
1b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret* $P(1+r)^n$ *as the product of P and a factor not depending on P.*	***Discovering Algebra*** **Lessons:** Lesson 4.3: Point-Slope Form of a Linear Equation Lesson 6.2: Exponential Equations Lesson 6.6: Zero and Negative Exponents Lesson 8.2: Translating Graphs Lesson 8.3: Reflecting Points and Graphs Lesson 8.4: Stretching and Shrinking Graphs ***Discovering Geometry*** **Lesson:** Using Your Algebra Skills 12: Transforming Functions ***Discovering Advanced Algebra*** **Lessons:** Lesson 4.7: Transformations and the Circle Family Lesson 5.8: Applications of Logarithms Lesson 7.7: Higher-Degree Polynomials Lesson 8.6: Introduction to Rational Functions
2. Use the structure of an expression to identify ways to rewrite it. *For example, see* $x^4 - y^4$ *as* $(x^2)^2 - (y^2)^2$*, thus recognizing it as a difference of squares that can be factored as* $(x^2 - y^2)(x^2 + y^2)$.	***Discovering Algebra*** **Lessons:** Lesson 9.2: Finding the Roots and the Vertex Lesson 9.3: From Vertex to General Form Lesson 9.4: Factored Form Lesson 9.7: The Quadratic Formula ***Discovering Advanced Algebra*** **Lessons:** Lesson 5.7: Properties of Logarithms Lesson 7.2: Equivalent Quadratic Forms Lesson 7.8: More About Finding Solutions Lesson 13.6: Fundamental Trigonometric Identities
Write expressions in equivalent forms to solve problems.	
3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.	
3a. Factor a quadratic expression to reveal the zeros of the function it defines.	***Discovering Algebra*** **Lesson:** Lesson 9.2: Finding the Roots and the Vertex ***Discovering Advanced Algebra*** **Lessons:** Lesson 7.2: Equivalent Quadratic Forms Lesson 7.5: Complex Numbers Lesson 7.6: Factoring Polynomials

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3b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.	***Discovering Algebra*** **Lesson:** Lesson 9.6: Completing the Square ***Discovering Advanced Algebra*** **Lessons:** Lesson 7.3: Completing the Square Lesson 7.5: Complex Numbers
3c. Use the properties of exponents to transform expressions for exponential functions. *For example the expression* 1.15^t *can be rewritten as* $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ *to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.*	***Discovering Algebra*** **Lessons:** Lesson 6.3: Multiplication and Exponents Lesson 6.5: Looking Back with Exponents Lesson 6.8: Decreasing Exponential Models and Half-Life ***Discovering Advanced Algebra*** **Lessons:** Lesson 5.3: Rational Exponents and Roots Lesson 5.4: Applications of Exponential and Power Functions Chapter 5 Exploration: The Number *e* Lesson 5.8: Applications of Logarithms
4. Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *For example, calculate mortgage payments.*	***Discovering Advanced Algebra*** **Lessons:** Lesson 9.2: Infinite Geometric Series Lesson 9.3: Partial Sums of Geometric Series Chapter 9 Review: Take Another Look 3
Arithmetic with Polynomials and Rational Expressions	
Perform arithmetic operations on polynomials.	
1. Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.	***Discovering Algebra*** **Lesson:** Lesson 8.6: Introduction to Rational Functions ***Discovering Advanced Algebra*** **Lesson:** Chapter 7 Refreshing Your Skills: Polynomial Expressions
Understand the relationship between zeros and factors of polynomials.	
2. Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.	***Discovering Advanced Algebra*** **Lesson:** Lesson 7.8: More About Finding Solutions

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3. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.	***Discovering Algebra*** **Lesson:** Lesson 9.8: Cubic Functions ***Discovering Advanced Algebra*** **Lessons:** Lesson 7.6: Factoring Polynomials Lesson 7.7: Higher-Degree Polynomials
Use polynomial identities to solve problems.	
4. Prove polynomial identities and use them to describe numerical relationships. *For example, the polynomial identity* $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ *can be used to generate Pythagorean triples.*	***Discovering Advanced Algebra*** **Lessons:** Lesson 7.3: Completing the Square Lesson 7.8: More About Finding Solutions (Exercise 12)
5. (+) Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.	***Discovering Advanced Algebra*** **Lesson:** Lesson 10.7: The Binomial Theorem and Pascal's Triangle
Rewrite rational expressions.	
6. Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.	***Discovering Advanced Algebra*** **Lesson:** Lesson 8.6: Introduction to Rational Functions Lesson 8.7: Graphs of Rational Functions
7. (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.	***Discovering Advanced Algebra*** **Lesson:** Lesson 8.6: Introduction to Rational Functions

ALGEBRA

Creating Equations*

Create equations that describe numbers or relationships.

1. Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*	***Discovering Algebra*** **Lessons:** This standard is addressed throughout the book. Examples include: Lesson 2.1: Proportions Lesson 2.2: Capture-Recapture Lesson 2.3: Proportions and Measurement Systems Lesson 2.4: Direct Variation Lesson 2.8: Undoing Operations Lesson 5.5: Inequalities in One Variable Lesson 9.1: Solving Quadratic Equations ***Discovering Geometry*** **Lessons:** Using Your Algebra Skills 5: Writing Equations ***Discovering Advanced Algebra*** **Lessons:** Lesson 3.1: Linear Equations and Arithmetic Sequences Lesson 3.2: Revisiting Slope Lesson 5.1: Exponential Functions Lesson 5.6: Logarithmic Functions Lesson 7.2: Equivalent Quadratic Forms Lesson 8.6: Introduction to Rational Functions

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2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.	***Discovering Algebra*** **Lessons:** This standard is addressed throughout the book. Examples include: Lesson 2.4: Direct Variation Lesson 3.2: Linear Plots Lesson 4.1: A Formula for Slope Lesson 6.1: Recursive Routines Lesson 6.2: Exponential Equations Lesson 9.2: Finding the Roots and the Vertex ***Discovering Geometry*** **Lessons:** Using Your Algebra Skills 6: Solving Systems of Linear Equations Using Your Algebra Skills 7: Finding Points of Concurrency ***Discovering Advanced Algebra*** **Lessons:** This standard is addressed throughout the book. Examples include Lesson 3.1: Linear Equations and Arithmetic Sequences Lesson 3.3: Fitting a Line to Data Lesson 5.1: Exponential Functions Lesson 5.6: Logarithmic Functions Lesson 7.2: Equivalent Quadratic Forms Lesson 8.6: Graphs of Rational Functions Lesson 13.5: Modeling with Trigonometric Equations
3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*	***Discovering Algebra*** **Lesson:** Chapter 5: Systems of Equations and Inequalities ***Discovering Geometry*** **Lessons:** Using Your Algebra Skills 6: Solving Systems of Linear Equations Using Your Algebra Skills 7: Finding Points of Concurrency ***Discovering Advanced Algebra*** **Lessons:** Lesson 3.6: Linear Systems Lesson 3.7: Substitution and Elimination Lesson 6.3: Solving Systems with Inverse Matrices Lesson 6.4: Row Reduction Method Lesson 6.5: Systems of Inequalities Lesson 6.6: Linear Programming

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4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law V = IR to highlight resistance R.*	***Discovering Algebra*** **Lessons:** Lesson 2.8: Undoing Operations Lesson 3.6: Solving Equations Using the Balancing Method (Exercise 11) ***Discovering Geometry*** **Lesson:** Using Your Algebra Skills 10: Solving for Any Variable ***Discovering Advanced Algebra*** **Lessons:** This standard is addressed throughout the book. Examples include: Lesson 3.7: Substitution and Elimination (Exercise 10) Lesson 4.5: Reflections and the Square Root Family (Exercise 13) Lesson 5.3: Rational Exponents and Roots (Exercise 13) Lesson 5.5: Building Inverses of Functions (Exercise 11)
Reasoning with Equations and Inequalities	
Understand solving equations as a process of reasoning and explain the reasoning.	
1. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.	***Discovering Algebra*** **Lessons:** This standard is addressed throughout the book. Examples include: Lesson 4.4: Equivalent Algebraic Expressions Lesson 9.1: Solving Quadratic Equations ***Discovering Geometry*** **Lessons:** Using Your Algebra Skills 4: Solving Equations Using Your Algebra Skills 6: Solving Systems of Linear Equations Using Your Algebra Skills 7: Finding Points of Concurrency ***Discovering Advanced Algebra*** **Lessons:** Chapter 3 Refreshing Your Skills: Linear Relationships Chapter 4 Refreshing Your Skills: Solving Equations Lesson 5.2: Properties of Exponents and Power Functions Lesson 5.3: Rational Exponents and Roots Lesson 5.6: Logarithmic Functions Lesson 5.7: Properties of Logarithms Chapter 6 Refreshing Your Skills: Properties of Real Numbers Lesson 7.3: Completing the Square Lesson 7.4: The Quadratic Formula Lesson 7.5: Complex Numbers

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2. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.	***Discovering Algebra*** **Lesson:** Lesson 8.6: Introduction to Rational Functions ***Discovering Advanced Algebra*** **Lessons:** Chapter 4 Refreshing Your Skills: Solving Equations Lesson 4.5: Reflections and the Square Root Family Lesson 8.6: Introduction to Rational Functions Lesson 8.7: Graphs of Rational Functions
Solve equations and inequalities in one variable.	
3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.	***Discovering Algebra*** **Lessons:** This standard is addressed throughout the book. Examples include: Lesson 2.8: Undoing Operations Lesson 3.6: Solving Equations Using the Balancing Method Lesson 4.2: Writing a Linear Equation to Fit Data Lesson 4.3: Point-Slope Form of a Linear Equation Lesson 4.4: Equivalent Algebraic Expressions Lesson 5.5: Inequalities in One Variable ***Discovering Geometry*** **Lessons:** Using Your Algebra Skills 4: Solving Equations Using Your Algebra Skills 10: Solving for Any Variable ***Discovering Advanced Algebra*** **Lessons:** Lesson 0.2: Symbolic Representation Chapter 3 Refreshing Your Skills: Linear Relationships Lesson 3.7: Substitution and Elimination Chapter 6 Refreshing Your Skills: Properties of Real Numbers
4. Solve quadratic equations in one variable.	
4a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.	***Discovering Algebra*** **Lessons:** Lesson 9.6: Completing the Square Lesson 9.7: The Quadratic Formula ***Discovering Advanced Algebra*** **Lessons:** Lesson 7.3: Completing the Square Lesson 7.4: The Quadratic Formula

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4b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.	***Discovering Algebra*** **Lessons:** Lesson 9.1: Solving Quadratic Equations Lesson 9.2: Finding the Roots and the Vertex Lesson 9.3: From Vertex to General Form Lesson 9.4: Factored Form Lesson 9.5: Activity Day: Projectile Motion Lesson 9.6: Completing the Square ***Discovering Geometry*** **Lesson:** (Partial) Using Your Algebra Skills 8: Products, Factors, and Quadratic Equations ***Discovering Advanced Algebra*** **Lessons:** Chapter 4 Refreshing Your Skills: Solving Equations Lesson 4.4: Translations and the Quadratic Family Lesson 7.3: Completing the Square Lesson 7.4: The Quadratic Formula Lesson 7.5: Complex Numbers
Solve systems of equations.	
5. Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.	***Discovering Algebra*** **Lesson:** Lesson 5.3: Solving Systems of Equations Using Elimination ***Discovering Geometry*** **Lesson:** Using Your Algebra Skills 6: Solving Systems of Linear Equations ***Discovering Advanced Algebra*** **Lesson:** Lesson 3.7: Substitution and Elimination

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6. Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.	***Discovering Algebra*** **Lessons:** Lesson 5.1: Solving Systems of Equations Lesson 5.2: Solving Systems of Equations Using Substitution Lesson 5.3: Solving Systems of Equations Using Elimination Lesson 5.4: Solving Systems of Equations Using Matrices ***Discovering Geometry*** **Lessons:** Using Your Algebra Skills 6: Solving Systems of Linear Equations Using Your Algebra Skills 7: Finding Points of Concurrency ***Discovering Advanced Algebra*** **Lessons:** Lesson 3.6: Linear Systems Lesson 3.7: Substitution and Elimination Lesson 6.3: Solving Systems with Inverse Matrices Lesson 6.4: Row Reduction Method
7. Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$.	***Discovering Algebra*** **Lessons:** Lesson 9.7: The Quadratic Formula (Exercise 11) Lesson 8.7: Graphs of Rational Functions (Exercise 15) ***Discovering Advanced Algebra*** **Lesson:** Lesson 6.5: Systems of Inequalities
8. (+) Represent a system of linear equations as a single matrix equation in a vector variable.	***Discovering Algebra*** **Lesson:** Lesson 5.4: Solving Systems of Equations Using Matrices ***Discovering Advanced Algebra*** **Lessons:** Lesson 6.3: Solving Systems with Inverse Matrices Lesson 6.4: Row Reduction Method
9. (+) Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater).	***Discovering Advanced Algebra*** **Lesson:** Lesson 6.3: Solving Systems with Inverse Matrices

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Represent and solve equations and inequalities graphically.

10. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).	***Discovering Algebra*** **Lessons:** Lesson 1.7: Estimating Lesson 2.4: Direct Variation Lesson 3.1: Recursive Sequences ***Discovering Geometry*** **Lesson:** Using Your Algebra Skills 12: Transforming Functions ***Discovering Advanced Algebra*** **Lesson:** Lesson 4.2: Function Notation
11. Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.★	***Discovering Algebra*** **Lessons:** Lesson 5.1: Solving Systems of Equations Lesson 6.2: Exponential Equations Lesson 9.1: Solving Quadratic Equations Lesson 9.2: Finding the Roots and the Vertex Lesson 9.4: Factored Form Lesson 9.5: Activity Day: Projectile Motion ***Discovering Advanced Algebra*** **Lessons:** Lesson 3.6: Linear Systems Lesson 4.6: Dilations and the Absolute-Value Family (Exercise 5) Lesson 4.7: Transformations and the Circle Family (Exercise 9) Lesson 5.6: Logarithmic Functions Lesson 8.5: The General Quadratic
12. Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.	***Discovering Algebra*** **Lessons:** Lesson 5.6: Graphing Inequalities in Two Variables Lesson 5.7: Systems of Inequalities ***Discovering Advanced Algebra*** **Lessons:** Lesson 6.5: Systems of Inequalities Lesson 6.6: Linear Programming

FUNCTIONS

Standard	*Discovering Mathematics* Lessons
Interpreting Functions	
Understand the concept of a function and use function notation.	
1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.	***Discovering Algebra* Lessons:** Lesson 7.1: Secret Codes Lesson 7.2: Functions and Graphs Lesson 7.4: Function Notation Lesson 9.1: Solving Quadratic Equations ***Discovering Advanced Algebra* Lessons:** Lesson 3.2: Revisiting Slope Lesson 4.2: Function Notation
2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.	***Discovering Algebra* Lessons:** Lesson 7.4: Function Notation (Exercise 7) Lesson 7.5: Defining the Absolute-Value Function (Exercise 11) Lesson 8.1: Translating Points (Exercise 11) Lesson 9.1: Solving Quadratic Equations Lesson 9.6: Completing the Square ***Discovering Geometry* Lessons:** Lesson 2.2: Finding the *n*th Term Using Your Algebra Skills 12: Transforming Functions ***Discovering Advanced Algebra* Lessons:** This standard is addressed throughout the book. Examples include: Lesson 4.2: Function Notation Lesson 4.5: Reflections and the Square Root Family Lesson 4.8: Compositions of Functions Lesson 5.5: Building Inverses of Functions

FUNCTIONS	
3. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1) for n ≥ 1.*	***Discovering Algebra* Lesson:** Lesson 3.1: Recursive Sequences ***Discovering Geometry* Lesson:** Lesson 2.2: Finding the *n*th Term ***Discovering Advanced Algebra* Lessons:** Lesson 1.1: Recursively Defined Sequences Lesson 1.2: Modeling Growth and Decay Lesson 3.1: Linear Equations and Arithmetic Sequences Lesson 5.1: Exponential Functions
Interpret functions that arise in applications in terms of the context.	
4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*★	***Discovering Algebra* Lessons:** Lesson 7.3: Graphs of Real-World Situations ***Discovering Advanced Algebra* Lessons:** Lesson 4.1: Interpreting Graphs Lesson 7.2: Equivalent Quadratic Forms Lesson 7.3: Completing the Square Lesson 7.7: Higher-Degree Polynomials Lesson 8.7: Graphs of Rational Functions Lesson 13.3: Graphing Trigonometric Functions Lesson 13.5: Modeling with Trigonometric Functions
5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*★	***Discovering Algebra* Lessons:** Lesson 1.6: Two-Variable Data Lesson 6.1: Recursive Routines Lesson 6.7: Fitting Exponential Models to Data Lesson 6.8: Activity Day: Decreasing Exponential Models and Half-Life Lesson 7.3: Graphs of Real-World Situations ***Discovering Advanced Algebra* Lessons:** Lesson 3.2: Revisiting Slope Lesson 4.8: Compositions of Functions Lesson 13.1: Defining Circular Functions

FUNCTIONS	
6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★	***Discovering Algebra*** **Lessons:** Lesson 4.1: A Formula for Slope Lesson 4.2: Writing a Linear Equations to Fit Data Lesson 4.3: Point-Slope Form of a Linear Equation Chapter 4 Reviews: Take Another Look ***Discovering Advanced Algebra*** **Lessons:** Lesson 3.1: Linear Equations and Arithmetic Sequences Lesson 3.2: Revisiting Slope
Analyze functions using different representations.	
7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★	
7a. Graph linear and quadratic functions and show intercepts, maxima, and minima.	***Discovering Algebra*** **Lessons:** Lesson 3.4: Linear Equations and Intercept Form Lesson 3.5: Linear Equations and Rate of Change Lesson 4.1: A Formula for Slope Lesson 4.2: Writing a Linear Equations to Fit Data Lesson 4.3: Point-Slope Form of a Linear Equation Lesson 9.2: Finding the Roots and the Vertex Lesson 9.6: Completing the Square ***Discovering Advanced Algebra*** **Lessons:** Lesson 3.3: Fitting a Line to Data Lesson 4.3: Lines in Motion Lesson 4.4: Translations and the Quadratic Family Lesson 7.2: Equivalent Quadratic Forms Lesson 7.3: Completing the Square Lesson 7.4: The Quadratic Formula
7b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.	***Discovering Algebra*** **Lessons:** Lesson 8.2: Translating Graphs Lesson 8.3: Reflecting Graphs ***Discovering Advanced Algebra*** **Lessons:** Lesson 3.3: Fitting a Line to Data Lesson 3.6: Linear Systems (Exercise 9) Chapter 4 Project: Step Functions Lesson 4.5: Reflections and the Square Root Family Lesson 4.6: Dilations and the Absolute-Value Family Lesson 5.3: Rational Exponents and Roots (Exercise 6)

FUNCTIONS	
7c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.	***Discovering Algebra*** **Lesson:** Lesson 9.8: Cubic Functions ***Discovering Advanced Algebra*** **Lessons:** Lesson 7.6: Factoring Polynomials Lesson 7.7: Higher-Degree Polynomials
7d. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.	***Discovering Algebra*** **Lesson:** Lesson 8.6: Introduction to Rational Functions ***Discovering Advanced Algebra*** **Lessons:** Lesson 8.6: Introduction to Rational Functions Lesson 8.7: Graphs of Rational Functions
7e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.	***Discovering Algebra*** **Lessons:** Lesson 6.2: Exponential Equations Lesson 6.7: Fitting Exponential Models to Data ***Discovering Advanced Algebra*** **Lessons:** Lesson 5.1: Exponential Functions Lesson 5.4: Applications of Exponential and Power Equations Lesson 5.6: Logarithmic Functions Lesson 13.3: Graphing Trigonometric Functions Lesson 13.5: Modeling with Trigonometric Functions
8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.	
8a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.	***Discovering Algebra*** **Lessons:** Lesson 9.1: Solving Quadratic Equations Lesson 9.2: Finding the Roots and the Vertex Lesson 9.3: From Vertex to General Form Lesson 9.4: Factored Form Lesson 9.6: Completing the Square ***Discovering Advanced Algebra*** **Lessons:** Lesson 7.2: Equivalent Quadratic Forms Lesson 7.3: Completing the Square

FUNCTIONS

8b. Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as* $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, *and classify them as representing exponential growth or decay.*	***Discovering Algebra*** **Lessons:** Lesson 6.3: Multiplication and Exponents Lesson 6.5: Looking Back with Exponents Lesson 6.6: Zero and Negative Exponents Lesson 6.7: Fitting Exponential Models to Data ***Discovering Advanced Algebra*** **Lessons:** Lesson 5.1: Exponential Functions Lesson 5.4: Applications of Exponential and Power Equations
9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*	***Discovering Algebra*** **Lessons:** This standard is addressed throughout the book. Examples include: Lesson 3.2: Linear Plots Lesson 6.1: Recursive Routines Lesson 9.1: Solving Quadratic Equations ***Discovering Advanced Algebra*** **Lessons:** This standard is addressed throughout the book. Examples include: Lesson 3.1: Linear Equations and Arithmetic Sequences Lesson 4.4: Translations and the Quadratic Family Lesson 5.1: Exponential Functions Lesson 7.2: Equivalent Quadratic Forms
Building Functions	
Build a function that models a relationship between two quantities.	
1. Write a function that describes a relationship between two quantities.★	
1a. Determine an explicit expression, a recursive process, or steps for calculation from a context.	***Discovering Algebra*** **Lessons:** This standard is addressed throughout the book. Examples include: Chapter 3: Linear Equations Chapter 6: Exponents and Exponential Models ***Discovering Geometry*** **Lesson:** Lesson 2.2: Finding the *n*th Term ***Discovering Advanced Algebra*** **Lessons:** This standard is addressed throughout the book. Examples include: Lesson 3.1: Linear Equations and Arithmetic Sequences Chapter 5: Exponential, Power, and Logarithmic Functions

FUNCTIONS	
1b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*	***Discovering Advanced Algebra*** **Lessons:** Lesson 5.4: Applications of Exponential and Power Equations Lesson 13.7: Combining Trigonometric Functions
1c. (+) Compose functions. *For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.*	***Discovering Advanced Algebra*** **Lesson:** Lesson 4.8: Compositions of Functions
2. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.⋆	***Discovering Algebra*** **Lessons:** Lesson 3.4: Linear Equations and the Intercept Form Lesson 3.5: Linear Equations and Rate of Change Lesson 6.2: Exponential Equations ***Discovering Advanced Algebra*** **Lessons:** Lesson 1.1: Recursively Defined Sequences Lesson 1.2: Modeling Growth and Decay Lesson 3.1: Linear Equations and Arithmetic Sequences Lesson 5.1: Exponential Functions
Build new functions from existing functions.	
3. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.	***Discovering Algebra*** **Lessons:** Lesson 8.2: Translating Graphs Lesson 8.3: Reflecting Points and Graphs Lesson 8.4: Stretching and Shrinking Graphs ***Discovering Geometry*** **Lesson:** Using Your Algebra Skills 12: Transforming Functions ***Discovering Advanced Algebra*** **Lessons:** Lesson 4.4: Translations and the Quadratic Family Lesson 4.5: Reflections and the Square Root Family Lesson 4.6: Dilations and the Absolute-Value Family Lesson 4.7: Transformations and the Circular Family Lesson 5.1: Exponential Functions Lesson 7.6: Factoring Polynomials Lesson 8.7: Graphs of Rational Functions Lesson 13.3: Graphing Trigonometric Functions

FUNCTIONS	
4. Find inverse functions.	
4a. Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse. *For example, $f(x) = 2x^3$ for $x > 0$ or $f(x) = (x+1)/(x-1)$ for $x \neq 1$.*	***Discovering Algebra* Lesson:** Chapter 7 Review: Take Another Look 1 ***Discovering Advanced Algebra* Lessons:** Lesson 5.5: Building Inverses of Functions Lesson 7.4: The Quadratic Formula (Exercise 14) Lesson 7.6: Factoring Polynomials (Exercise 14) Lesson 7.8: More About Finding Solutions (Exercise 14)
4b. (+) Verify by composition that one function is the inverse of another.	***Discovering Advanced Algebra* Lesson:** Lesson 5.5: Building Inverses of Functions
4c. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse.	***Discovering Advanced Algebra* Lesson:** Lesson 5.5: Building Inverses of Functions
4d. (+) Produce an invertible function from a non-invertible function by restricting the domain.	***Discovering Advanced Algebra* Lesson:** Lesson 13.4: Inverses of Trigonometric Functions
5. (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.	***Discovering Advanced Algebra* Lessons:** Lesson 5.6: Logarithmic Functions Lesson 5.8: Applications of Logarithms
Linear and Exponential Models	
Construct and compare linear and exponential models and solve problems.	
1. Distinguish between situations that can be modeled with linear functions and with exponential functions.	
1a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.	***Discovering Algebra* Lesson:** Lesson 6.1: Recursive Routines ***Discovering Advanced Algebra* Lessons:** Lesson 3.1: Linear Equations and Arithmetic Sequences Lesson 3.2: Revisiting Slope Lesson 5.1: Exponential Functions Students address this standard formally in *Precalculus with Trigonometry: Concepts and Applications,* by Paul A. Foerster.

FUNCTIONS	
1b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.	***Discovering Algebra*** **Lessons:** Lesson 3.4: Linear Equations and the Intercept Form Lesson 3.5: Linear Equations and Rate of Change Lesson 4.1: A Formula for Slope Lesson 4.2: Writing a Linear Equation to Fit Data ***Discovering Advanced Algebra*** **Lessons:** Lesson 3.2: Revisiting Slope Lesson 3.3: Time-Distance Relationships Lesson 3.4: Linear Equations and the Intercept Form
1c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.	***Discovering Algebra*** **Lessons:** Lesson 6.2: Exponential Equations Lesson 6.5: Looking Back with Exponents Lesson 6.6: Zero and Negative Exponents Lesson 6.7: Fitting Exponential Models to Data ***Discovering Advanced Algebra*** **Lessons:** Lesson 5.1: Exponential Functions Lesson 5.3: Rational Exponents and Roots Lesson 5.4: Applications of Exponential and Power Equations Lesson 5.8: Applications of Logarithms
2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).	***Discovering Algebra*** **Lessons:** Lesson 3.1: Recursive Sequences Lesson 3.3: Time-Distance Relationships Lesson 3.4: Linear Equations and the Intercept Form Lesson 3.5: Linear Equations and Rate of Change Lesson 4.2: Writing a Linear Equation to Fit Data Lesson 6.2: Exponential Equations Lesson 6.3: Multiplication and Exponents Lesson 6.7: Fitting Exponential Models to Data ***Discovering Geometry*** **Lesson:** Using Your Algebra Skills 5: Writing Linear Equations ***Discovering Advanced Algebra*** **Lessons:** Chapter 1: Sequences Lesson 3.1: Linear Equations and Arithmetic Sequences Lesson 3.2: Revisiting Slope Lesson 5.1: Exponential Functions Lesson 5.4: Applications of Exponential and Power Functions

FUNCTIONS	
3. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.	***Discovering Algebra*** **Lesson:** (Partial, exponential growth compared to linear only) Lesson 6.1: Recursive Routines ***Discovering Advanced Algebra*** **Lesson:** (Partial, exponential growth compared to linear only) Lesson 1.4: Graphing Sequences
4. For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.	***Discovering Advanced Algebra*** **Lesson:** Lesson 5.8: Applications of Logarithms
Interpret expressions for functions in terms of the situation they model.	
5. Interpret the parameters in a linear or exponential function in terms of a context.	***Discovering Algebra*** **Lessons:** Lesson 4.1: A Formula for Slope Lesson 4.2: Writing a Linear Equation to Fit Data Lesson 4.3: Point-Slope Form of a Linear Equation Lesson 4.4: Equivalent Algebraic Equations (Exercises 12, 13, and 14) Lesson 4.5: Writing Point-Slope Equations to Fit Data Lesson 4.7: Applications of Modeling Lesson 6.1: Recursive Routines Lesson 6.2: Exponential Equations Lesson 6.7: Fitting Exponential Models to Data ***Discovering Advanced Algebra*** **Lessons:** Lesson 3.1: Linear Equations and Arithmetic Sequences Lesson 3.2: Revisiting Slope Lesson 3.3: Fitting a Line to Data Lesson 5.1: Exponential Functions Lesson 5.4: Applications of Exponential and Power Equations
Trigonometric Functions	
Extend the domain of trigonometric functions using the unit circle.	
1. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.	***Discovering Advanced Algebra*** **Lesson:** Lesson 13.2: Radian Measure
2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.	***Discovering Advanced Algebra*** **Lessons:** Lesson 13.1: Defining the Circular Functions Lesson 13.2: Radian Measure Chapter 13 Exploration: Circular Functions

FUNCTIONS

3. (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for x, $\pi + x$, and $2\pi - x$ in terms of their values for x, where x is any real number.	***Discovering Advanced Algebra* Lesson:** Lesson 13.2: Radian Measure Lesson 13.3: Graphing Trigonometric Functions
4. (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.	***Discovering Advanced Algebra* Lessons:** Lesson 13.1: Defining the Circular Functions Chapter 13 Exploration: Circular Functions Lesson 13.6: Fundamental Trigonometric Identities
Model periodic phenomena with trigonometric functions.	
5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.★	***Discovering Advanced Algebra* Lesson:** Lesson 13.5: Modeling with Trigonometric Functions
6. (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.	***Discovering Advanced Algebra* Lesson:** Lesson 13.4: Inverses of Trigonometric Functions
7. (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.★	***Discovering Advanced Algebra* Lesson:** Lesson 13.5: Modeling with Trigonometric Functions
Prove and apply trigonometric identities.	
8. Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to calculate trigonometric ratios.	***Discovering Advanced Algebra* Lesson:** Lesson 13.6: Fundamental Trigonometric Identities
9. (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.	***Discovering Advanced Algebra* Lessons:** Lesson 13.6: Fundamental Trigonometric Identities Lesson 13.7: Combining Trigonometric Functions

MODELING

Modeling Standards

Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (★).

GEOMETRY	
Standard	***Discovering Mathematics* Lessons**
Congruence	
Experiment with transformations in the plane	
1. Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.	***Discovering Geometry* Lessons:** Lesson 1.1: Building Blocks of Geometry Lesson 1.3: What's a Widget Lesson 1.7: Circles
2. Model transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus stretch in a specific direction).	***Discovering Algebra* Lessons:** Lesson 8.1: Translating Points Lesson 8.2: Translating Graphs Lesson 8.3: Flipping Graphs Lesson 8.4: Stretching and Shrinking Graphs Lesson 8.7: Transformations with Matrices ***Discovering Geometry* Lessons:** Lesson 1.7: Circles (Exercises 19–21) Lesson 4.2: Properties of Isosceles Triangles (Exercises 24, 25) Lesson 7.1: Transformations and Symmetry Lesson 7.2: Properties of Isometries Lesson 7.3: Compositions of Transformations Lesson 11.1: Similar Polygons Lesson 11.2: Similar Triangles (Exercises 19, 20) Chapter 11 Exploration: Constructing a Dilation Design ***Discovering Advanced Algebra* Lessons:** Chapter 4 Exploration: Rotation as a Composition of Transformations
3. Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.	***Discovering Geometry* Lessons:** Symmetry is first introduced in Lesson 0.1, and then reviewed periodically as part of developing students' visualization skills before deeper coverage in Lesson 7.1. Lesson 0.1: Geometry in Nature and in Art Lesson 1.6: Special Quadrilaterals (Exercise 16) Lesson 2.1: Inductive Reasoning (Exercise 42) Lesson 2.5: Angle Relationships (Exercise 14) Lesson 3.1: Duplicating Segments and Angles (Exercise 14) Lesson 7.1: Transformation and Symmetry

GEOMETRY	
4. Develop definitions of rotations, reflections and translations in terms of angles, circles, perpendicular lines, parallel lines and line segments.	***Discovering Geometry*** **Lessons:** Lesson 7.1: Transformations and Symmetry Lesson 7.2: Properties of Isometries Lesson 7.3: Compositions of Transformations ***Discovering Advanced Algebra*** **Lesson:** Chapter 4 Exploration: Rotation as a Composition of Transformations
5. Given a specified rotation, reflection or translation and a geometric figure, construct the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Construct a sequence of transformations that will carry a given figure onto another.	***Discovering Algebra*** **Lessons:** Lesson 8.1: Translating Points Lesson 8.3: Flipping Graphs Lesson 8.4: Stretching and Shrinking Graphs Lesson 8.7: Transformations with Matrices ***Discovering Geometry*** **Lessons:** Lesson 7.1: Transformations and Symmetry Lesson 7.2: Properties of Isometries Lesson 7.3: Compositions of Transformations Lesson 11.1: Similar Polygons Lesson 11.2: Similar Triangles (Exercises 19, 20) Chapter 11 Exploration: Constructing a Dilation Design ***Discovering Advanced Algebra*** **Lesson:** Chapter 4 Exploration: Rotation as a Composition of Transformations
Understand congruence in terms of rigid motions	
6. Use geometric descriptions of rigid motions to transform figures and to predict the effect of a rigid motion on a figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.	***Discovering Geometry*** **Lessons:** Patty paper and compass-and-straightedge constructions prepare students for the formal introduction to rigid motions in Lesson 7.1. For example, see: Lesson 2.6: Special Angles on Parallel Lines Lesson 3.1: Duplicating Segments and Angles Lesson 3.6: Construction Problems Lesson 7.1: Transformations and Symmetry
7. Explain using rigid motions the meaning of congruence for triangles as the equality of all corresponding pairs of sides and all corresponding pairs of angles.	***Discovering Geometry*** **Lesson:** Lesson 3.6: Construction Problems

GEOMETRY	
8. Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence.	***Discovering Geometry* Lessons:** Lesson 3.6: Construction Problems Lesson 4.4: Are There Congruence Shortcuts? Lesson 4.5: Are There Other Congruence Shortcuts?
Prove geometric theorems	
9. Prove theorems about lines and angles. *Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.*	***Discovering Geometry* Lessons:** Lesson 2.5: Angle Relationships Lesson 2.6: Special Angles on Parallel Lines Lesson 3.2: Constructing Perpendicular Bisectors Lesson 13.2: Planning a Geometry Proof
10. Prove theorems about triangles. *Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.*	***Discovering Geometry* Lessons:** Lesson 4.1: Triangle Sum Conjecture Lesson 4.2: Properties of Isosceles Triangles Lesson 4.3: Triangle Inequalities Lesson 5.4: Properties of Midsegments Lesson 13.3: Triangle Proofs
11. Prove theorems about parallelograms. *Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other and conversely, rectangle are parallelograms with congruent diagonals.*	***Discovering Geometry* Lessons:** Lesson 5.5: Properties of Parallelograms Lesson 5.6: Properties of Special Parallelograms Lesson 5.7: Proving Quadrilateral Properties Lesson 13.4: Quadrilateral Proofs Using Your Algebra Skills 13: Coordinate Proof
Make geometric constructions	
12. Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc). *Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.*	***Discovering Geometry* Lessons:** Throughout *Discovering Geometry* students construct with compass and straightedge and patty paper folding. Dynamic geometry constructions are incorporated both as Explorations in the student book and as demonstrations and replacement lessons in the ancillary *Discovering Geometry with The Geometer's Sketchpad*. See, for example: Lesson 3.1: Duplicating Segments and Angles Lesson 3.2: Constructing Perpendicular Bisectors Lesson 3.3: Constructing Perpendiculars to a Line Lesson 3.4: Constructing Angle Bisectors Lesson 3.6: Construction Problems Chapter 6 Exploration: Intersecting Lines Through a Circle Chapter 13 Exploration: Proof as Challenge and Discovery

GEOMETRY	
13. Construct an equilateral triangle, a square and a regular hexagon inscribed in a circle.	***Discovering Geometry*** **Lessons:** Lesson 0.3: Circle Designs Lesson 1.7: Circles (Exercise 17) Lesson 3.1: Duplicating Segments and Angles (Exercises 8 and 9) Lesson 3.3: Constructing Perpendiculars to a Line (Exercise 10) Lesson 3.5: Constructing Parallel Lines (Exercise 3)
Similarity, Right Triangles, and Trigonometry	
Understand similarity in terms of similarity transformations	
1. Verify experimentally the properties of dilations:	
1a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.	***Discovering Geometry*** **Lesson:** (Partial) Lesson 11.1: Similar Polygons ***Discovering Advanced Algebra*** **Lesson:** Chapter 11 Exploration: Seeing the Sum of a Series
1b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.	***Discovering Geometry*** **Lessons:** Lesson 11.1: Similar Polygons Lesson 11.2: Similar Triangles (Exercises 19, 20) Chapter 11 Exploration: Constructing a Dilation Design ***Discovering Advanced Algebra*** **Lesson:** Chapter 11 Exploration: Seeing the Sum of a Series
2. Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all pairs of angles and the proportionality of all pairs of sides.	***Discovering Geometry*** **Lessons:** Lesson 11.1: Similar Polygons Lesson 11.2: Similar Triangles
3. Use the properties of similarity transformations to establish the AA criterion for similarity of triangles.	***Discovering Geometry*** **Lesson:** Lesson 11.2: Similar Triangles
Prove theorems involving similarity	
4. Prove theorems about triangles using similarity transformations. *Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean theorem proved using triangle similarity.*	***Discovering Geometry*** **Lessons:** Lesson 11.3: Indirect Measurement with Similar Triangles Lesson 11.4: Corresponding Parts of Similar Triangles Lesson 11.7: Proportional Segments Between Parallel Lines Lesson 13.7: Similarity Proofs

GEOMETRY	
5. Use triangle congruence and similarity criteria to solve problems and to prove relationships in geometric figures.	***Discovering Geometry*** **Lessons:** Lesson 4.4: Are There Congruence Shortcuts? Lesson 4.5: Are There Other Congruence Shortcuts? Lesson 4.6: Corresponding Parts of Congruent Triangles Lesson 4.7: Flowchart Thinking Lesson 4.8: Proving Special Triangle Conjectures Chapter 11: Similarity Lesson 13.3: Triangle Proofs Lesson 13.7: Similarity Proofs
Define trigonometric ratios and solve problems involving right triangles	
6. Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.	***Discovering Algebra*** **Lesson:** Lesson 11.7: Similar Triangles and Trigonometric Functions ***Discovering Geometry*** **Lesson:** Lesson 12.1: Trigonometric Ratios ***Discovering Advanced Algebra*** **Lessons:** Refreshing Your Skills for Chapter 12: Special Right Triangles Lesson 12.1: Right Triangle Trigonometry
7. Explain and use the relationship between the sine and cosine of complementary angles.	***Discovering Geometry*** **Lesson:** Lesson 12.1: Trigonometric Ratios ***Discovering Advanced Algebra*** **Lesson:** Lesson 12.1: Right Triangle Trigonometry
8. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.	***Discovering Algebra*** **Lessons:** Lesson 11.7: Similar Triangles and Trigonometric Functions Lesson 11.8: Trigonometry ***Discovering Geometry*** **Lessons:** Lesson 12.1: Trigonometric Ratios Lesson 12.2: Problem Solving with Right Triangles ***Discovering Advanced Algebra*** **Lessons:** Lesson 12.1: Right Triangle Trigonometry Lesson 12.2: The Law of Sines Lesson 12.3: The Law of Cosines

GEOMETRY

(+) Apply trigonometry to general triangles	
9. Derive the formula $A = \frac{1}{2} ab \sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.	***Discovering Geometry*** **Lesson:** Lesson 12.3: The Law of Sines ***Discovering Advanced Algebra*** **Lesson:** Chapter 12 Improving Your Geometry Skills, page 697
10. Prove the Laws of Sines and Cosines and use them to solve problems.	***Discovering Geometry*** **Lessons:** Lesson 12.3: The Law of Sines Lesson 12.4: The Law of Cosines Chapter 12 Review: Take Another Look 5 ***Discovering Advanced Algebra*** **Lessons:** Lesson 12.2: The Law of Sines Lesson 12.3: The Law of Cosines
11. Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).	***Discovering Geometry*** **Lessons:** Lesson 12.3: The Law of Sines Lesson 12.4: The Law of Cosines Lesson 12.5: Problem Solving with Trigonometry ***Discovering Advanced Algebra*** **Lessons:** Lesson 12.2: The Law of Sines Lesson 12.3: The Law of Cosines
Circles	
Understand and apply theorems about circles	
1. Prove that all circles are similar.	***Discovering Geometry*** **Lesson:** *Teacher's Edition*, p. 609
2. Identify and describe relationships among inscribed angles, radii, and chords. *Include the relationship between central, inscribed and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.*	***Discovering Geometry*** **Lessons:** Lesson 6.1: Tangent Properties Lesson 6.2: Chord Properties Lesson 6.3: Arcs and Angles
3. Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.	***Discovering Geometry*** **Lessons:** Lesson 3.7: Constructing Points of Concurrency Lesson 6.3: Arcs and Angles Lesson 13.6: Circle Proofs

GEOMETRY	
4. (+) Construct a tangent line from a point outside a given circle to the circle.	***Discovering Geometry*** **Lesson:** Lesson 6.1: Tangent Properties ***Discovering Advanced Algebra*** **Lesson:** Lesson 8.2: Circles and Ellipses
Find arc lengths and areas of sectors of circles	
5. Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.	***Discovering Geometry*** **Lessons:** Lesson 6.7: Arc Length Lesson 8.5: Areas of Circles Lesson 8.6: Any Way You Slice It ***Discovering Advanced Algebra*** **Lesson:** Lesson 13.2: Radian Measure
Expressing Geometric Properties with Equations	
Translate between the geometric description and the equation for a conic section	
1. Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.	***Discovering Geometry*** **Lesson:** Lesson 9.5: Distance in Coordinate Geometry ***Discovering Advanced Algebra*** **Lessons:** Lesson 4.7: Transformations and the Circle Family Lesson 8.2: Circles and Ellipses Lesson 8.5: The General Quadratic
2. Derive the equation of a parabola given a focus and directrix.	***Discovering Advanced Algebra*** **Lesson:** Lesson 8.3: Parabolas
3. (+) Derive the equations of ellipses and hyperbolas given two foci for the ellipse, and two directrices of a hyperbola.	***Discovering Advanced Algebra*** **Lessons:** Lesson 8.2: Circles and Ellipses Lesson 8.4: Hyperbolas
Use coordinates to prove simple geometric theorems algebraically	
4. Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0, 2).*	***Discovering Geometry*** **Lesson:** Using Your Algebra Skills 13: Coordinate Proof

GEOMETRY	
5. Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).	***Discovering Algebra*** **Lesson:** Lesson 11.1: Parallel and Perpendicular ***Discovering Geometry*** **Lessons:** Using Your Algebra Skills 3: Slopes of Parallel and Perpendicular Lines Using Your Algebra Skills 13: Coordinate Proof ***Discovering Advanced Algebra*** **Lesson:** Lesson 3.2: Revisiting Slope
6. Find the point on a directed line segment between two given points that divide the segment in a given ratio.	***Discovering Geometry*** **Lessons:** Lesson 11.6: Proportions with Volume (Exercise 20) Lesson 11.7: Proportional Segments Between Parallel Lines
7. Use coordinates to compute perimeters of polygons and areas for triangles and rectangles, e.g. using the distance formula.★	***Discovering Algebra*** **Lesson:** Lesson 11.6: A Distance Formula ***Discovering Geometry*** **Lesson:** Lesson 9.5: Distance in Coordinate Geometry
Geometric Measurement and Dimension	
Explain volume formulas and use them to solve problems	
1. Give an informal argument for the formulas for the volume of a cylinder, pyramid, and cone. *Use dissection arguments, Cavalieri's principle, and informal limit arguments.*	***Discovering Geometry*** **Lessons:** Lesson 10.2: Volume of Prisms and Cylinders Lesson 10.3: Volume of Pyramids and Cones
2. (+) Given an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.	Cavalieri's principle is discussed in the *Discovering Geometry Teacher's Edition* (pages 532, 539, and 559). ***Discovering Geometry*** **Lessons:** Lesson 10.2: Volume of Prisms and Cylinders Lesson 10.3: Volume of Pyramids and Cones Lesson 10.6: Volume of a Sphere
3. Use volume formulas for cylinders, pyramids, cones and spheres to solve problems.★	***Discovering Geometry*** **Lessons:** Lesson 10.2: Volume of Prisms and Cylinders Lesson 10.3: Volume of Pyramids and Cones Lesson 10.4: Volume Problems Lesson 10.5: Displacement and Density Lesson 10.6: Volume of a Sphere

GEOMETRY	
Visualize relationships between two-dimensional and three-dimensional objects	
4. Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.	***Discovering Geometry* Lessons:** Visualization skills are emphasized throughout the book. Examples include: Lesson 1.8: Space Geometry Lesson 1.9: A Picture is Worth a Thousand Words Chapter 1 Review Lesson 2.1: Inductive Reasoning (Exercises 23, 24) Lesson 10.2: Volume of Prisms and Cylinders (Exercises 22, 23)
Modeling with Geometry	
Apply geometric concepts in modeling situations	
1. Use geometric shapes, their measures and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).★	***Discovering Geometry* Lessons:** Lesson 10.1: The Geometry of Solids Lesson 10.2: Volume of Prisms and Cylinders Lesson 10.3: Volume of Pyramids and Cones Lesson 10.4: Volume Problems Lesson 10.5: Displacement and Density Lesson 10.6: Volume of a Sphere Lesson 10.7: Surface Area of a Sphere
2. Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).★	***Discovering Geometry* Lesson:** Lesson 10.5: Displacement and Density
3. Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy constraints or minimize cost; working with typographic grid systems based on ratios).★	***Discovering Geometry* Lessons:** Chapter 5 Project: Building an Arch, page 280 Chapter 5 Project: Japanese Puzzle Quilts, page 303 Chapter 6 Project: Racetrack Geometry, page 354 Chapter 7 Project: Kaleidoscopes, page 402 Chapter 11 Project: Making a Mural, page 588 Chapter 12 Project: Light for All Seasons, page 651

STATISTICS AND PROBABILITY

Standard	*Discovering Mathematics* Lessons
Interpreting Categorical and Quantitative Data	
Summarize, represent, and interpret data on a single count or measurement variable	
1. Represent data with plots on the real number line (dot plots, histograms, and box plots).	***Discovering Algebra* Lessons:** Lesson 1.1: Bar Graphs and Dot Plots Lesson 1.2: Summarizing Data with Measures of Center Lesson 1.3: Five-Number Summaries and Box Plots Lesson 1.4: Histograms and Stem-and-Leaf Plots ***Discovering Advanced Algebra* Lessons:** Lesson 2.1: Box Plots Lesson 2.2: Measures of Spread Lesson 2.3: Histograms and Percentile Ranks
2. Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.	***Discovering Algebra* Lessons:** Lesson 1.2: Summarizing Data with Measures of Center Lesson 1.3: Five-Number Summaries and Box Plots Lesson 1.4: Histograms and Stem-and-Leaf Plots Lesson 1.5 Activity Day: Exploring a Conjecture Lesson 7.5: Defining the Absolute-Value Function Chapter 7 Review: Take Another Look 2 ***Discovering Advanced Algebra* Lessons:** Lesson 2.1: Box Plots Lesson 2.2: Measures of Spread Lesson 2.3: Histograms and Percentile Ranks
3. Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).	***Discovering Advanced Algebra* Lessons:** Lesson 2.1: Box Plots Lesson 2.2: Measures of Spread Lesson 2.3: Histograms and Percentile Ranks
4. Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets and tables to estimate areas under the normal curve.	***Discovering Advanced Algebra* Lessons:** Lesson 11.2: Probability Distributions Lesson 11.3: Normal Distributions Lesson 11.4: z-Values and Confidence Intervals

STATISTICS AND PROBABILITY

Summarize, represent, and interpret data on two categorical and quantitative variables

5. Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal and conditional relative frequencies). Recognize possible associations and trends in the data.	***Discovering Advanced Algebra*** **Lessons:** Lesson 2.3: Histograms and Percentile Ranks Lesson 11.2: Probability Distributions Lesson 11.5: Bivariate Data and Correlation
6. Represent data on two quantitative variables on a scatter plot and describe how the variables are related.	***Discovering Algebra*** **Lessons:** This standard is addressed throughout the book. Examples include: Lesson 1.6: Two-Variable Data Lesson 3.5: Linear Equations and Rate of Change Lesson 4.2: Writing a Linear Equation to Fit Data Lesson 4.6: More on Modeling Lesson 4.7: Applications of Modeling Lesson 4.8: Data Collection and Modeling Lesson 6.1: Recursive Routines Lesson 6.7: Fitting Exponential Models to Data ***Discovering Advanced Algebra*** **Lessons:** Lesson 3.3: Fitting a Line to Data Lesson 3.4: The Median-Median Line Lesson 5.1: Exponential Functions Lesson 5.8: Applications of Logarithms Lesson 11.5: Bivariate Data and Correlation

STATISTICS AND PROBABILITY	
6a. Use a model function fitted to the data to solve problems in the context of the data. *Use given model functions or choose a function suggested by the context. Emphasize linear and exponential models.*	***Discovering Algebra*** **Lessons:** This standard is addressed throughout the book. Examples include: Lesson 3.3: Time-Distance Relationships Lesson 3.4: Linear Equations and the Intercept Form Lesson 3.5: Linear Equations and Rate of Change Lesson 4.2: Writing a Linear Equation to Fit Data Lesson 4.6: More on Modeling Lesson 4.7: Applications of Modeling Lesson 6.7: Fitting Exponential Models to Data ***Discovering Advanced Algebra*** **Lessons:** This standard is addressed throughout the book. Examples include: Lesson 3.3: Fitting a Line to Data Lesson 3.4: The Median-Median Line Lesson 3.5: Prediction and Accuracy Lesson 5.1: Exponential Functions Lesson 5.8: Applications of Logarithms
6b. Informally assess the fit of a model function by plotting and analyzing residuals.	***Discovering Advanced Algebra*** **Lessons:** Lesson 3.5: Prediction and Accuracy Chapter 3 Exploration: Residual Plots and Least Squares
6c. Fit a linear function for scatter plots that suggest a linear association.	***Discovering Algebra*** **Lessons:** Lesson 1.6: Two-Variable Data Lesson 1.7: Estimating Lesson 2.4: Direct Variation Chapter 4: Fitting a Line to Data ***Discovering Geometry*** **Lessons:** (Partial) Chapter 2 Project: Best-Fit Lines, page 107 ***Discovering Advanced Algebra*** **Lessons:** Lesson 3.3: Fitting a Line to Data Lesson 3.4: The Median-Median Line Lesson 3.5: Prediction and Accuracy Chapter 3 Exploration: Residual Plots and Least Squares

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Interpret linear models	
7. Interpret the slope (rate of change) and the intercept (constant term) of a linear fit in the context of the data.	***Discovering Algebra*** **Lessons:** Lesson 3.3: Time-Distance Relationships Lesson 3.4: Linear Equations and the Intercept Form Lesson 3.5: Linear Equations and Rate of Change Chapter 4: Fitting a Line to Data ***Discovering Advanced Algebra*** **Lessons:** Lesson 3.3: Fitting a Line to Data Lesson 3.4: The Median-Median Line Lesson 3.5: Prediction and Accuracy
8. Compute (using technology) and interpret the correlation coefficient of a linear fit.	***Discovering Advanced Algebra*** **Lesson:** Lesson 11.5: Bivariate Data and Correlation
9. Distinguish between correlation and causation.	***Discovering Advanced Algebra*** **Lessons:** Lesson 11.1: Experimental Design Lesson 11.5: Bivariate Data and Correlation
Making Inferences and Justifying Conclusions	
Understand and evaluate random processes underlying statistical experiments	
1. Understand that statistics is a process for making inferences about population parameters based on a random sample from that population.	***Discovering Advanced Algebra*** **Lessons:** Lesson 11.1: Experimental Design Lesson 11.2: Probability Distributions
2. Decide if a specified model is consistent with results from a given data-generating process, e.g. using simulation. *For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?*	***Discovering Algebra*** **Lessons:** Lesson 10.2: Probability Outcomes and Trials Lesson 10.3: Random Outcomes ***Discovering Advanced Algebra*** **Lessons:** Lesson 10.1: Randomness and Probability Chapter 10 Exploration: The Law of Large Numbers
Make inferences and justify conclusions from sample surveys, experiments and observational studies	
3. Recognize the purposes of and differences among sample surveys, experiments and observational studies; explain how randomization relates to each.	***Discovering Advanced Algebra*** **Lesson:** Lesson 11.1: Experimental Design

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4. Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.	***Discovering Advanced Algebra* Lesson:** Lesson 11.4: *z*-Values and Confidence Intervals
5. Use data from a randomized experiment to compare two treatments; justify significant differences between parameters through the use of simulation models for random assignment.	Students explore results of simulation models for random assignment in *Discovering Advanced Algebra* Chapter 11: Application of Statistics. However, students do not explicitly address this standard.
6. Evaluate reports based on data.	***Discovering Advanced Algebra* Lessons:** Chapter 11 Project: Simpson's Paradox Chapter 11 Project: Correlation vs. Causation Lesson 11.6: The Least Squares Line
Conditional Probability and the Rules of Probability	
Understand independence and conditional probability and use them to interpret data	
1. Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").	***Discovering Algebra* Lesson:** Lesson 10.5: Multiple-Stage Experiments ***Discovering Advanced Algebra* Lesson:** Lesson 10.3: Mutually Exclusive Events and Venn Diagrams
2. Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.	***Discovering Algebra* Lesson:** Lesson 10.5: Multiple-Stage Experiments ***Discovering Advanced Algebra* Lesson:** Lesson 10.2: Counting Outcomes and Tree Diagrams
3. Understand the conditional probability of A given B as P(A and B)/P(B), and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.	***Discovering Algebra* Lesson:** Lesson 10.5: Multiple-Stage Experiments ***Discovering Advanced Algebra* Lessons:** Lesson 10.2: Counting Outcomes and Tree Diagrams Lesson 10.3: Mutually Exclusive Events and Venn Diagrams

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4. Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science and English. Estimate the probability that a randomly selected student from your class will favor science given that the student is a boy. Do the same for other subjects and compare the results.*	***Discovering Advanced Algebra* Lesson:** Lesson 10.2: Counting Outcomes and Tree Diagrams (Exercise 16)
5. Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of being unemployed if you are female with the chance of being female if you are unemployed.*	***Discovering Algebra* Lesson:** Lesson 10.5: Multiple-Stage Experiments ***Discovering Advanced Algebra* Lessons:** Lesson 10.2: Counting Outcomes and Tree Diagrams Lesson 10.3: Mutually Exclusive Events and Venn Diagrams
Use the rules of probability to compute probabilities of compound events in a uniform probability model	
6. Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A and interpret the answer in terms of the model.	***Discovering Algebra* Lesson:** Lesson 10.5: Multiple-Stage Experiments ***Discovering Advanced Algebra* Lessons:** Lesson 10.2: Counting Outcomes and Tree Diagrams Lesson 10.3: Mutually Exclusive Events and Venn Diagrams
7. Apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B), and interpret the answer in terms of the model.	***Discovering Advanced Algebra* Lesson:** Lesson 10.3: Mutually Exclusive Events and Venn Diagrams
8. (+) Apply the general Multiplication Rule in a uniform probability model, P(A and B) = P(A)P(B|A) = P(B)P(A|B), and interpret the answer in terms of the model.	***Discovering Advanced Algebra* Lessons:** Lesson 10.2: Counting Outcomes and Tree Diagrams Lesson 10.3: Mutually Exclusive Events and Venn Diagrams
9. (+) Use permutations and combinations to compute probabilities of compound events and solve problems.	***Discovering Algebra* Lesson:** Lesson 10.4: Counting Techniques ***Discovering Advanced Algebra* Lessons:** Lesson 10.5: Permutations and Probability Lesson 10.6: Combinations and Probability

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Using Probability to Make Decisions

Calculate expected values and use them to solve problems

1. Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.	***Discovering Advanced Algebra*** **Lesson:** Lesson 11.2: Probability Distributions
2. Calculate the expected value of a random variable; interpret it as the mean of the probability distribution.	***Discovering Advanced Algebra*** **Lesson:** Lesson 11.2: Probability Distributions
3. Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. *For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.*	***Discovering Advanced Algebra*** **Lessons:** Lesson 10.4: Random Variables and Expected Value Lesson 11.2: Probability Distributions
4. Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. *For example, find a current data distribution on the number of TV sets per household in the United States and calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly selected households?*	***Discovering Advanced Algebra*** **Lesson:** Lesson 11.2: Probability Distributions

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Use probability to evaluate outcomes of decisions

5. Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values.

5a. Find the expected payoff for a game of chance. *For example, find the expected winnings from a state lottery ticket or a game at a fast-food restaurant.*	***Discovering Algebra*** **Lesson:** Lesson 10.6: Expected Value ***Discovering Advanced Algebra*** **Lessons:** Chapter 10 Exploration: The Law of Large Numbers Lesson 10.4: Random Variables and Expected Value Lesson 10.6: Combinations and Probability
5b. Evaluate and compare strategies on the basis of expected values. *For example, compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident.*	***Discovering Advanced Algebra*** **Lesson:** Chapter 10 Exploration: The Law of Large Numbers Lesson 10.4: Random Variables and Expected Value
6. Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).	***Discovering Advanced Algebra*** **Lessons:** Lesson 10.1: Randomness and Probability Lesson 10.4: Random Variables and Expected Value Lesson 10.6: Combinations and Probability
7. Analyze decisions and strategies using probability concepts (e.g. product testing, medical testing, pulling a hockey goalie at the end of a game).	***Discovering Advanced Algebra*** **Lessons:** Lesson 10.4: Random Variables and Expected Value Lesson 10.7: The Binomial Theorem and Pascal's Triangle Lesson 11.1: Experimental Design Lesson 11.3: Normal Distributions Lesson 11.4: z-Values and Confidence Intervals Chapter 11 Exploration: Prediction Intervals