

# Standards for Probability

## *General Outcome*

Develop critical thinking skills related to uncertainty.

### **General Notes:**

- The concept of sample space could be introduced at the start of this topic and form the basis for all probability work to follow. Sample spaces may exceed two events.
- Teachers may wish to present the specific outcomes in an order different from that listed in the program of studies.
- In probability, the word “or” is inclusive (i.e., it means “and/or”).
- Teachers should be aware that Venn diagrams and logical reasoning symbols may be used in probability.
- Teachers should be aware that specific outcomes 4, 5, and 6 do not require the simplification of factorial expressions (e.g., Simplify  $\frac{n!}{(n-2)!}$ ).

## *Specific Outcome 1*

Interpret and assess the validity of odds and probability statements. [C, CN, ME]

### **Notes:**

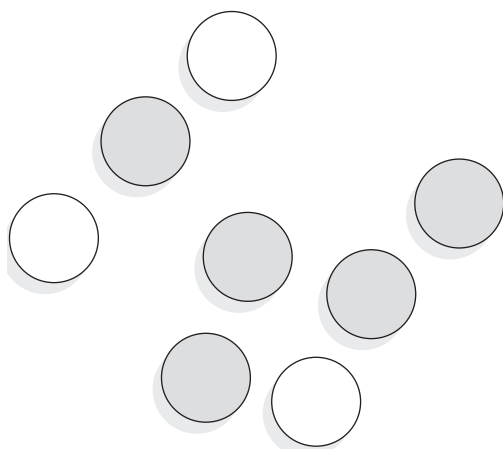
- Teachers may wish to use the analogy that odds can be considered “part-part” and probability can be considered “part-whole”.
- Students should be familiar with the terms “odds for”, “odds in favour of”, and “odds against”.
- Odds and probabilities can be represented in many ways, including but not limited to the following.

Odds in favour of A = # of outcomes for A : # of outcomes against A

Odds in favour of A =  $\frac{\text{\# of outcomes for A}}{\text{\# of outcomes against A}}$

$P(\text{event A}) = \frac{\text{\# of favourable outcomes}}{\text{total \# of outcomes}}$

- Concrete models like the one shown below can be used to model probability and odds.



5 grey parts  
4 white parts  
The whole consists of 9 parts.

- Teachers may wish to combine or embed this specific outcome with other specific outcomes in this topic.

*(Refer to examples 1, 2, 3, 4, and 5)*

### ***Specific Outcome 2***

Solve problems that involve the probability of mutually exclusive and non-mutually exclusive events. [CN, PS, R, V] [ICT: C6–2.3]

#### **Notes:**

- Students can use a variety of strategies to solve mutually exclusive and non-mutually exclusive problems. Some possible strategies are graphic organizers (e.g., tree diagrams, Venn diagrams, tables, organized lists) and probability formulas.
- Since complementary events are also mutually exclusive, this specific outcome does include the concept of complementary events.
- Problems for this outcome should be limited to a maximum of two events.

*(Refer to examples 6, 7, 8, 9, 10, and 11)*

### ***Specific Outcome 3***

Solve problems that involve the probability of two events. [CN, PS, R]

#### **Notes:**

- This outcome includes a study of independent events and dependent events.
- Students can use a variety of strategies to determine probabilities of independent events and dependent events. Some possible strategies are graphic organizers and probability formulas.
- Teachers may wish to address this outcome prior to addressing specific outcome 2.
- Teachers may wish to make connections between this outcome and specific outcomes 4, 5, and 6.

*(Refer to examples 12, 13, 14, 15, 24, and 28c)*

### ***Specific Outcome 4***

Solve problems that involve the fundamental counting principle. [PS, R, V] [ICT: C6–2.3]

#### **Notes:**

- Students can use a variety of organizational strategies (e.g., tree diagrams, charts, and other visual cues) to solve problems involving the fundamental counting principle.
- Factorial notation could be introduced in the development of this outcome.
- Teachers may wish to make connections between this outcome and specific outcome 3.
- Teachers should be aware that repetition of elements is not considered a restriction when distinguishing between Acceptable Standard and Standard of Excellence.

*(Refer to examples 16, 17, 18, 19, 22, 23, and 24)*

### ***Specific Outcome 5***

Solve problems that involve permutations. [ME, PS, R, T, V]

#### **Notes:**

- If not introduced previously, factorial notation could be discussed in this outcome in conjunction with the  ${}_nP_r$  formula. However, other strategies can also be used to solve problems involving permutations.
- When questions involving permutations with identical elements are solved, all of the elements within the context should be arranged.
- Simple 2-D pathways (e.g., no gaps in the grid, no overlapping regions) are applications of permutations with identical elements.
- Circular and ring permutations are beyond the scope of Mathematics 30–2.
- Teachers should consider probability problems that involve permutations. This is a connection to specific outcome 3.

*(Refer to examples 20, 21, 22, 23, 24, and 25)*

### ***Specific Outcome 6***

Solve problems that involve combinations. [ME, PS, R, T, V]

#### **Notes:**

- Students should be familiar with both notations for combinations as shown on the formula sheet.
- A problem that requires both permutations and combinations is beyond the scope of Mathematics 30–2.
- Teachers should consider probability problems that involve combinations. This is a connection to specific outcome 3.

*(Refer to examples 25, 26, 27, 28, 29, and 30)*

### Acceptable Standard

The student can

- determine the probability of an event
- determine odds in favour of or odds against an event
- determine the odds in favour of an event given the odds against the event and vice versa
- express odds in favour of or odds against an event as a probability
- distinguish between mutually exclusive events and non-mutually exclusive events
- determine  $P(A \cup B)$  for events that are mutually exclusive
- determine  $P(A)$  given  $P(A \cup B)$  and  $P(B)$  for mutually exclusive events
- interpret a model that represents any combination of mutually exclusive and non-mutually exclusive events
- determine the probability of the complement of an event, given the probability of the event
- distinguish between independent and dependent events
- determine  $P(A \cap B)$  for independent events
- determine  $P(A \cap B)$  for dependent events, given the order of the events
- apply the fundamental counting principle to problems with at most one restriction

### Standard of Excellence

The student can also

- provide an explanation for the validity of a probability statement
- provide an explanation for the validity of an odds statement
- express probability of an event as odds in favour of or odds against
- determine  $P(A \cup B)$  for events that are non-mutually exclusive
- determine  $P(A)$  given  $P(A \cup B)$ ,  $P(A \cap B)$ , and  $P(B)$  for non-mutually exclusive events
- represent events that are non-mutually exclusive using a graphic organizer
- determine  $P(A)$  when given  $P(A \cap B)$  and  $P(B)$  for independent events
- determine  $P(A \cap B)$  for dependent events when the order of the events is not given
- apply the fundamental counting principle to problems with more than one restriction
- apply the fundamental counting principle to problems that involve two or three cases (e.g., at least, at most, or)

- determine the number of permutations of  $n$  objects with some identical elements taken  $n$  at a time
- determine the number of permutations of  $n$  distinct objects taken  $r$  at a time
- determine the number of permutations of  $n$  objects taken  $r$  at a time with at most one restriction
- solve combination problems that involve a single case
- distinguish between problems that describe permutations and problems that describe combinations
- solve probability problems that describe permutations
- solve probability problems that describe single combinations in the numerator  
 (e.g.,  $\frac{{}_5C_3}{{}_{11}C_3}$ )
- participate in and contribute toward the problem-solving process for problems that require the analysis of probability studied in Mathematics 30–2

- solve permutation problems that involve two or three cases (e.g., at least, at most, or)
- determine the number of permutations of  $n$  objects taken  $r$  at a time with more than one restriction
- solve combination problems that involve two or three cases (e.g., at least, at most, or)
- solve probability problems that involve two or three combinations in the numerator  
 (e.g.,  $\frac{{}_5C_3 \cdot {}_4C_2}{{}_9C_5}$ )
- complete the solution to problems that require the analysis of probability studied in Mathematics 30–2

## Sample Questions

Students who achieve the *acceptable standard* should be able to answer all of the following questions, except for any part of a question labelled **SE**. Parts labelled **SE** are appropriate examples for students who achieve the *standard of excellence*.

**Note:** In the multiple-choice questions that follow, \* indicates the correct answer. Please be aware that the worked solutions shown are possible strategies; there may be other strategies that could be used.

1. The odds in favour of the Renegades winning the season final in a football league are listed as 10 : 7. The odds against the Renegades winning the season final are
  - A. 3 : 7
  - B. 3 : 10
  - \*C. 7 : 10
  - D. 10 : 3

### Numerical Response

2. Statistics show that 6 out of 25 car accidents are weather-related. The odds in favour of a car accident being weather-related can be expressed in the form  $a : b$ . The values of  $a$  and  $b$  are, respectively, \_\_\_\_\_ and \_\_\_\_\_.

**Solution:**

6 and 19

**Note:** This question could be adapted for use on a digital test. Please consult the site <https://questaplus.alberta.ca/> for more examples of this item type.

**SE**

3. A class of 35 students has 17 males. One student will be selected at random from the class. Jeanette suggested that the odds in favour of selecting a male student are 17 : 35. Is Jeannette correct? Justify your answer.

**Possible Solution:**

Jeanette is incorrect in stating that the odds are 17 : 35. Odds are measured as one “part” versus another “part.” In this situation, there are two parts: male and female. As such, the odds in favour of selecting a male should be stated as the ratio of the number of males to the number of females. The odds in favour of selecting a male student would therefore be 17 : 18.

Use the following information to answer question 4.

A television game show has listed the following odds in favour of winning for three of their games.

Game	Odds of Winning
Flip'em	1 : 3
Central Eye	2 : 5
Minefield	1 : 4

4. a. What is the probability of winning the Flip'em game?

**Solution:**

$$\frac{1}{4} \text{ or } 0.25$$

- b. Which of the three games is a contestant **most likely** to win? Justify your answer.

**Possible Solution:**

The probabilities of winning for each game are

Flip'em	0.25
Central Eye	0.29
Minefield	0.20

The game with the highest probability of winning is Central Eye; therefore, this is the game that a contestant is **most likely** to win.



Use the following information to answer question 5.

Across Alberta, there has been a great deal of debate about the cause and effect of climate change with respect to the environment, human activity, economics, and society. It is rarely refuted that climate patterns around the world appear to be changing. This change is a major concern for the insurance industry. It is important for insurance managers and researchers to anticipate the odds in favour of weather-related disasters, such as the 2013 wildfire near Nordegg. Such disasters often require enormous monetary payouts and must be considered in the cost of obtaining insurance.

Traditionally, insurance companies have used past history to determine the probability of weather-related disasters. However, with the observed changes to weather patterns, this is not necessarily the best method of managing the risk of large monetary payouts that could bankrupt the company.

One country dealing with the problems caused by a changing climate is the United Kingdom (UK). As an island nation, the UK must be aware of the effect of changing climates on oceans and other bodies of water. Historically, the odds in favour of a 100-year flood plain experiencing flooding in any given year are 1 : 99. When considering flood insurance on a 100-year flood plain, it has been estimated by scientific agencies that “... *the world with climate change has a 20 percent greater chance of experiencing those floods than the world without.*” (New York Times).

**SE**

5. Based on the information from the New York Times, Randall claimed that because of climate change, the new odds in favour of a flood occurring on a 100-year flood plain in the UK would be 21 : 99. Explain why you agree or disagree with Randall’s claim. Include an explanation of odds and probability in your response.

**Possible Solution:**

I disagree with Randall’s claim that the new odds in favour of a flood on a 100-year floodplain in the UK will be 21 : 99. The original odds were 1 : 99, which means that the probability of a flood occurring is  $\frac{1}{100}$  or 0.01. The quote suggests that there is a “20% greater chance” of a flood occurring. This can be interpreted in two different ways. The quote can mean that the new probability will be the old probability increased by 20%, to 0.21 (0.01 + 0.20). In this case, the odds in favour of the flood are 21 : 79. The quote could also mean that the new probability will be  $0.01 \cdot 1.20$  (a 20% increase) = 0.012, which means that the odds in favour of a flood are  $12 : 988 = 3 : 247$ . Neither of these odds statements match Randall’s claim; therefore, Randall’s claim is incorrect.

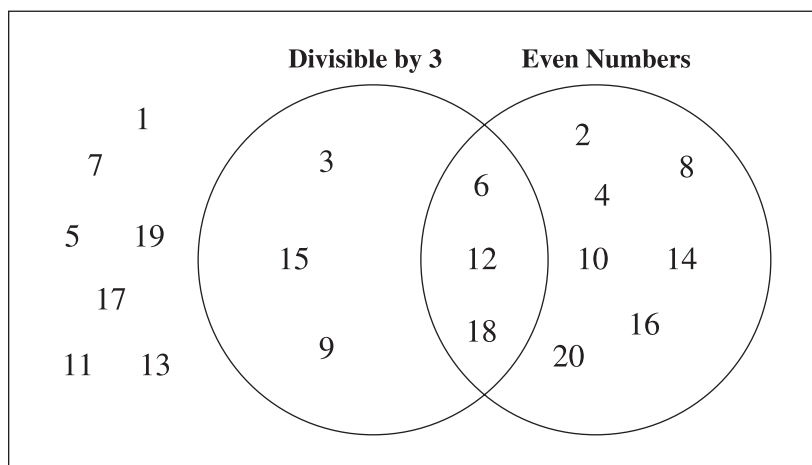
Use the following information to answer question 6.

For the set of whole numbers from 1 to 20 inclusive, Theresa knows that some numbers are divisible by 3 and some numbers are even. She is going to write each number on a different ball and place the balls in a box.

**SE**

6. If one ball is randomly selected from the box, what is the probability that the number written on it is divisible by 3 or is an even number?

**Possible Solution:**



The favourable outcomes are the outcomes within the circles, so the number of favourable outcomes is 13.  $P(\text{Divisible by 3 or Even Number}) = \frac{13}{20}$

**or**

$$\begin{aligned} &P(\text{Divisible by 3} \cup \text{Even number}) \\ &= P(\text{Divisible by 3}) + P(\text{Even Number}) - P(\text{Divisible by 3} \cap \text{Even Number}) \\ &= \frac{6}{20} + \frac{10}{20} - \frac{3}{20} \\ &= \frac{13}{20} \end{aligned}$$

7. A particular traffic light at the outskirts of a town is red for 30 s, green for 25 s, and yellow for 5 s in every minute. When a vehicle approaches the traffic light, the probability that the light will be red or yellow is

\*A.  $\frac{7}{12}$

B.  $\frac{1}{2}$

C.  $\frac{1}{12}$

D.  $\frac{1}{24}$

*Use the following information to answer numerical-response question 8.*

Malaga, Spain, lies in a region of Europe known as the Costa Del Sol (Coast of the Sun). The probability of sunshine on any given day in the region is approximately 0.89.

### **Numerical Response**

8. In a non-leap year of 365 days, the average number of days of the year that a tourist could expect to experience weather other than sunshine, to the nearest whole number, is \_\_\_\_\_ days.

**Possible Solution:**

$$1 - 0.89 = 0.11$$

$$0.11 \times 365 = 40.15$$

$$\approx 40 \text{ days}$$

Use the following information to answer numerical-response question 9.

Some possible events for rolling a regular six-sided die are listed below.

- 1 An even number
- 2 A number less than 3
- 3 A number that is a multiple of 3
- 4 A number that is greater than or equal to 2

### Numerical Response

9. From the list above, the two events that are mutually exclusive are numbered \_\_\_\_\_ and \_\_\_\_\_.

**Solution:**

23 or 32

Use the following information to answer question 10.

A recent survey determined that 85% of a population watches TV at least once a day, 35% of the population uses a computer at least once a day, and 25% of the population do both.

**SE**

10. a. What is the probability that a person chosen at random from the population watches TV at least once a day or uses a computer at least once a day?

**Possible Solution:**

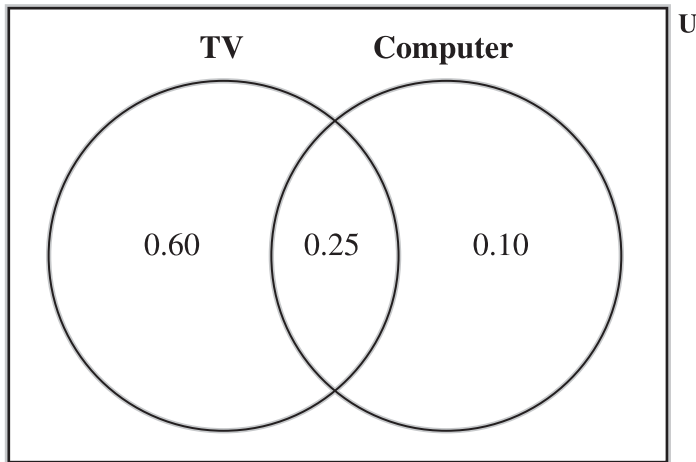
$$P(\text{TV}) = 0.85$$

$$P(\text{Computer}) = 0.35$$

$$P(\text{TV} \cap \text{Computer}) = 0.25$$

$$\begin{aligned} P(\text{TV} \cup \text{Computer}) &= P(\text{TV}) + P(\text{Computer}) - P(\text{TV} \cap \text{Computer}) \\ &= 0.85 + 0.35 - 0.25 \\ &= 0.95 \end{aligned}$$

or



$$P(\text{TV} \cup \text{Computer}) = 0.60 + 0.25 + 0.10 = 0.95$$

- b. Are the events of watching TV at least once a day and using the computer at least once a day mutually exclusive events? Justify your answer.

**Possible Solution:**

These events are not mutually exclusive because some of the survey participants do both activities.

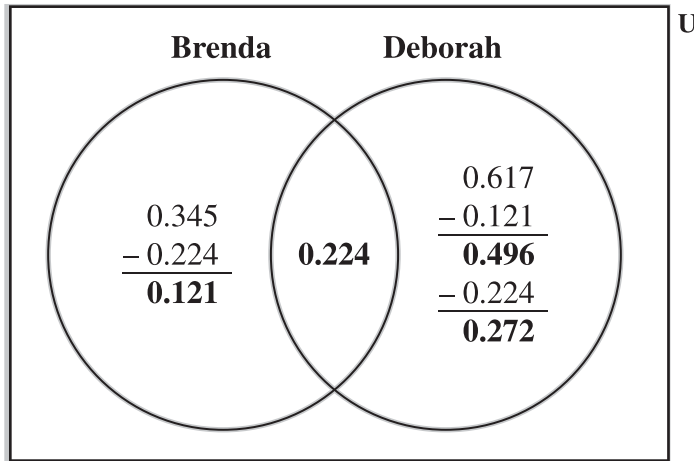
Use the following information to answer question 11.

The probability of Brenda getting a hit in a baseball game is 0.345. The probability of Brenda or Deborah getting a hit during the game is 0.617. The probability of both Brenda and Deborah getting hits during the game is 0.224.

**SE**

11. Determine the probability of Deborah getting a hit in the game.

**Possible Solution:**



$$\begin{aligned} P(D) &= 0.224 + 0.272 \\ &= 0.496 \end{aligned}$$

**or**

$$\begin{aligned} P(B \cup D) &= P(B) + P(D) - P(B \cap D) \\ 0.617 &= 0.345 + P(D) - 0.224 \\ 0.496 &= P(D) \end{aligned}$$

Use the following information to answer question 12.

Alan places five white marbles and five black marbles into a bag. He then performs the two experiments described below to select two marbles from the bag.

**First Experiment**

One marble is selected from the bag and replaced before a second marble is selected.

**Second Experiment**

One marble is selected from the bag and **not** replaced before a second marble is selected.

The following two events are the same for each experiment:

**Event X:** The first marble selected is black.

**Event Y:** The second marble selected is white.

12. In the first experiment, Event X and Event Y are   *i*  , and in the second experiment, Event X and Event Y are   *ii*  .

The statement above is completed by the information in row

Row	<i>i</i>	<i>ii</i>
A.	dependent	independent
B.	dependent	dependent
C.	independent	independent
*D.	independent	dependent

13. A box contains 6 blue balls and 4 red balls. Two balls are drawn from the box, one after the other, without replacement. The probability, to the nearest hundredth, that the first ball drawn is blue and the second ball drawn is red is \_\_\_\_\_.

**Possible Solution:**

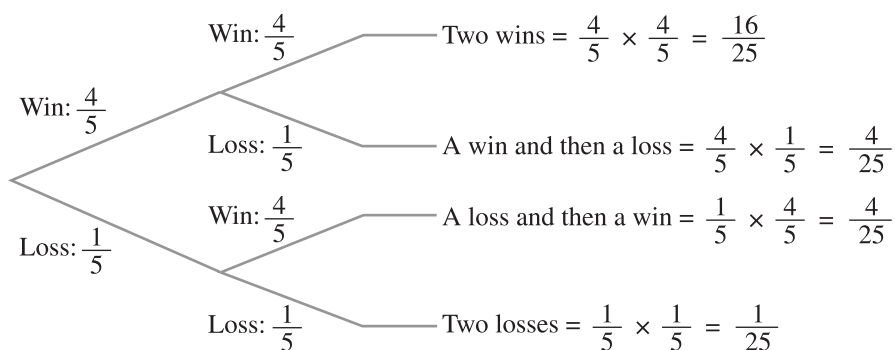
$$P(\text{blue}) \cdot P(\text{red} \mid \text{blue})$$

$$\frac{6}{10} \cdot \frac{4}{9} = 0.27$$

14. Based on previous performance, the probability of a particular baseball team winning any game is  $\frac{4}{5}$ . The probability that this team will win their next 2 games is

- A.  $\frac{1}{5}$   
 B.  $\frac{4}{5}$   
 C.  $\frac{1}{25}$   
 \*D.  $\frac{16}{25}$

**Possible Solution:**



The probability that the baseball team wins their next two games is given by two wins =  $\frac{16}{25}$ .

**SE**

- b. What is the probability that the team will win 1 game and lose 1 game during the next 2 games?

**Possible Solution:**

$$\begin{aligned}
 P(\text{Win} \cap \text{Lose}) &= P(\text{Win 1st} \cap \text{Lose 2nd}) + P(\text{Lose 1st} \cap \text{Win 2nd}) \\
 &= \frac{4}{25} + \frac{4}{25} \\
 &= \frac{8}{25}
 \end{aligned}$$



Use the following information to answer question 15.

From a particular bag that contains tiles, one tile is selected and the colour is recorded. From a second bag that contains marbles, one marble is selected and the colour is recorded. The probability of randomly selecting a blue tile from the first bag is 0.62. The probability of randomly selecting a blue tile from the first bag **and** a blue marble from the second bag is 0.46.

**SE**

15. The probability, to the nearest hundredth, of selecting a blue marble from the second bag is \_\_\_\_\_.

**Possible Solution:**

$$\begin{aligned}P(\text{Blue tile} \cap \text{Blue marble}) &= P(\text{Blue tile}) \cdot P(\text{Blue marble}) \\0.46 &= 0.62 \cdot P(\text{Blue marble}) \\0.74 &= P(\text{Blue marble})\end{aligned}$$

16. A hotel offers free breakfast to its guests. One morning the hotel has 3 different kinds of juice, 4 different kinds of cereal, and 2 different types of pastries available. If a guest can choose one kind of juice, one kind of cereal and one type of pastry, how many different possible breakfasts can be ordered?

**Possible Solution:**

$$\begin{array}{ccc} \frac{\quad}{\text{juice}} \cdot \frac{\quad}{\text{cereal}} \cdot \frac{\quad}{\text{pastry}} \\ \frac{3}{\text{juice}} \cdot \frac{4}{\text{cereal}} \cdot \frac{2}{\text{pastry}} \end{array}$$

$$= 24$$

Use the following information to answer numerical-response question 17.

A new licence plate in Alberta consists of three letters followed by four digits. Letters are chosen from a list of 23 acceptable letters that may be repeated.

Maureen wants the first letter on her licence plate to be an M, which is an acceptable letter, and she also wants the four digits to match the last four digits of her cell phone number in the same order.

### Numerical Response

**SE**

- 17.** The number of licence plates that will meet Maureen's criteria is \_\_\_\_\_.

**Possible Solution:**

$$1 \cdot 23 \cdot 23 \cdot 1 \cdot 1 \cdot 1 \cdot 1 = 529$$

- 18. a.** Determine the number of 4-digit numbers that can be created using the digits 0 to 9 without repetition.

**Possible Solution:**

$$9 \cdot 9 \cdot 8 \cdot 7 = 4\,536$$

**SE**

- b.** Determine the number of six-digit odd numbers that can be created using the digits 0 to 9 without repetition. Describe any restrictions that exist.

**Possible Solution:**

$$8 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 5 = 67\,200$$

The sixth digit is restricted if the number is going to be odd. The first digit is also restricted – it cannot be 0, nor can it be the same digit as the sixth digit.

Use the following information to answer numerical-response question 19.

Julian is planning a trip from Calgary to Denver. The map below shows the different flight options from a particular airline.



### Numerical Response

**SE**

- 19.** If Julian must fly with this airline, then how many different flight options are possible?

**Possible Solution:**

Calgary to Denver + Calgary to Seattle to Denver

$$2 + 2 \cdot 3$$

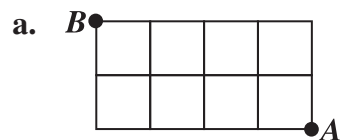
$$= 8$$

- 20.** Determine the number of distinct arrangements of all the letters in the word TATTOO.

**Possible Solutions:**

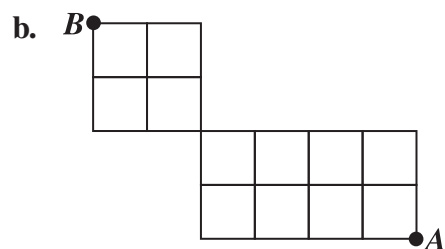
$$\frac{6!}{3!2!} = 60 \quad \text{or} \quad \frac{{}_6P_6}{3!2!} = 60 \quad \text{or} \quad \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{3!2!} = 60$$

21. Determine the number of different possible routes that Tyler can travel from Point A to Point B if he travels only north or west.



**Possible Solution:**

$$\frac{6!}{4!2!} = 15$$



**Possible Solution:**

$$\frac{6!}{4!2!} \cdot \frac{4!}{2!2!} = 90$$

22. Determine the number of 3-letter arrangements of the letters of the word **DIPLOMA**.

**Possible Solution:**

$${}_n P_r = \frac{n!}{(n-r)!}$$

$${}_7 P_3 = \frac{7!}{(7-3)!}$$

$${}_7 P_3 = 210$$

**or**

$${}_7 P_3 = 210$$

**or**

$$7 \cdot 6 \cdot 5 = 210$$

23. A 7-player volleyball team must stand in a line for a picture.

- a. Determine the number of different arrangements that can be made for the picture.

**Possible Solution:**

$$7! = 5\,040 \quad \text{or} \quad {}_7 P_7 = 5\,040 \quad \text{or} \quad 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 5\,040$$

- b. Determine the number of arrangements that can be made for the picture if the tallest player must stand in the middle.

**Possible Solution:**

$$6! = 720 \quad \text{or} \quad {}_6 P_6 = 720 \quad \text{or} \quad 6 \cdot 5 \cdot 4 \cdot 1 \cdot 3 \cdot 2 \cdot 1 = 720$$

24. Only six people, including Bill and Mary, have tickets for 2 prizes in a school draw, and each person has only one ticket. Once a ticket is drawn for a prize, it is not re-entered in the draw. What is the probability that Bill wins the first prize and Mary wins the second prize?

**Possible Solution:**

$$\frac{1}{6} \cdot \frac{1}{5} = \frac{1}{30} \quad \text{or} \quad \frac{1}{{}_6 P_2} = \frac{1}{30}$$

Use the following information to answer numerical-response question 25.

A student is classifying the following contexts as either permutations or combinations.

<b>Context A</b>	Dialing a 10-digit telephone number with distinct digits
<b>Context B</b>	Choosing 5 people for a committee
<b>Context C</b>	Selecting 4 fruits to put in a salad
<b>Context D</b>	Entering a 3-digit phone passcode

### Numerical Response

- 25.** For each context, use a **1** to indicate that the context would be classified as a **permutation** and use a **2** to indicate that the context would be classified as a **combination**.

Context A \_\_\_\_\_ (Record in the **first** column)

Context B \_\_\_\_\_ (Record in the **second** column)

Context C \_\_\_\_\_ (Record in the **third** column)

Context D \_\_\_\_\_ (Record in the **fourth** column)

**Solution:**

1221

- 26.** Triangles can be formed in an octagon by connecting any 3 of its vertices. Determine the number of different triangles that can be formed in an octagon.

**Possible Solution:**

$$\binom{n}{r} = \frac{n!}{(n-r)!r!}$$

$$\begin{aligned}\binom{8}{3} &= \frac{8!}{(8-3)!3!} \\ &= 56\end{aligned}$$

There are 56 possible triangles that can be formed within an octagon.

27. A fruit salad is to contain 1 green fruit, 2 different yellow fruits, and 3 different red fruits. The number of possible fruit salads that can be made from 2 different green fruits, 5 different yellow fruits, and 9 different red fruits is
- A. 20 160  
 B. 8 008  
 \*C. 1 680  
 D. 90

**Possible Solution:**

$${}_2C_1 \cdot {}_5C_2 \cdot {}_9C_3 = 1\,680$$

28. In a group of 9 students, there are 4 females and 5 males.
- a. How many different 4-member committees selected from this group have 2 females and 2 males?

**Possible Solution:**

$$\binom{4}{2} \cdot \binom{5}{2} = 60$$

SE

- b. How many different 2-member or 3-member committees can be formed from this group of students?

**Possible Solution:**

$$\binom{9}{2} + \binom{9}{3} = 120$$

- c. In the group of 9 students, 3 are in Grade ten, 3 are in Grade eleven and 3 are in Grade twelve. Determine the probability that 2 Grade ten students are chosen to be on a 2-member committee.

**Possible Solution:**

$$\begin{aligned} \frac{{}_3C_2}{{}_9C_2} &= \frac{3!}{(3-2)!2!} & \text{or} & \quad \frac{3}{9} \cdot \frac{2}{8} = \frac{1}{12} \\ &= \frac{1}{12} \end{aligned}$$

**Note:** This example is considered to be at the acceptable standard even though it is technically possible to consider  ${}_6C_0$  in the numerator of the solution on the left. Since this is a step that students will not have to perform in order to arrive at the correct answer, it is considered to

- d. Determine the probability that a 4-member committee chosen at random from this group will consist of at least 3 females.

**Possible Solution:**

$$\frac{{}^4C_3 \cdot {}^5C_1}{{}^9C_4} + \frac{{}^4C_4}{{}^9C_4} = \frac{1}{6}$$

29. Ralph knows that there are 15 distinguishable possibilities when 2 people are chosen to form a committee from a particular group of  $n$  people.

Describe any restrictions on the value of  $n$  in this context.

**Possible Solution:**

The value of  $n$  represents the number of people in the larger group. This must be a positive number, as mathematically you cannot have a negative number of people. Also,  $n$  must be greater than 2 because it is impossible to select two objects from a group smaller than what is needed.

*Use the following information to answer numerical-response question 30.*

A committee of 3 girls and 2 boys is to be chosen from a group of 9 girls and 7 boys. The total number of different committees that can be formed can be expressed in the form

$${}_wC_x \cdot {}_yC_z$$

where  ${}_wC_x$  represents the number of possible choices of girls for the committee and  ${}_yC_z$  represents the number of possible choices of boys for the committee.

### Numerical Response

30. The values of  $w$ ,  $x$ ,  $y$ , and  $z$  are \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_, and \_\_\_\_\_, respectively.

**Solution:**

9372