4-3 Simplifying Radical Expressions

Simplify each expression.

1. $\sqrt{24}$

SOLUTION:

$\sqrt{24} = \sqrt{2 \cdot 2 \cdot 2 \cdot 3}$ Prime factorization of 24.

$= \sqrt{2^2 \cdot 2 \cdot 3}$ Group factors.

$= \sqrt{2^2} \cdot \sqrt{2} \cdot \sqrt{3}$ Product Property of Square Roots.

$= 2 \cdot \sqrt{2} \cdot \sqrt{3}$ Simplify.

$= 2\sqrt{6}$ Product Property.

2. $3\sqrt{16}$

SOLUTION:

$3\sqrt{16} = 3\sqrt{2 \cdot 2 \cdot 2 \cdot 2}$ Prime factorization of 16.

$= 3\sqrt{2^2 \cdot 2^2}$ Group factors.

$= 3 \cdot \sqrt{2^2} \cdot \sqrt{2^2}$ Product Property

$= 3 \cdot 2 \cdot 2$ Simplify.

$= 12$ Simplify.

3. $2\sqrt{25}$

SOLUTION:

$2\sqrt{25} = 2\sqrt{5 \cdot 5}$ Prime factorization of 25.

$= 2\sqrt{5^2}$ Group factors.

$= 2 \cdot 5$ Simplify.

$= 10$ Simplify.

4. $\sqrt{10} \cdot \sqrt{14}$

SOLUTION:

$\sqrt{10} \cdot \sqrt{14} = \sqrt{2} \cdot \sqrt{5} \cdot \sqrt{2} \cdot \sqrt{7}$ Prime factorization of 10 and 14.

$= \sqrt{2^2} \cdot \sqrt{5} \cdot \sqrt{7}$ Product Property

$= 2\sqrt{35}$ Simplify.

5. $\sqrt{3} \cdot \sqrt{18}$

SOLUTION:

$\sqrt{3} \cdot \sqrt{18} = \sqrt{3} \cdot \sqrt{3 \cdot 3 \cdot 2}$ Prime factorizaiton of 18.

$= \sqrt{3} \cdot \sqrt{3^2 \cdot 2}$ Group factors.

$= \sqrt{3} \cdot \sqrt{3^2} \cdot \sqrt{2}$ Product Property

$= 3\sqrt{6}$ Simplify.

6. $3\sqrt{10} \cdot 4\sqrt{10}$

SOLUTION:

$3\sqrt{10} \cdot 4\sqrt{10} = 3 \cdot 4 \cdot \sqrt{10} \cdot \sqrt{10}$ Associative Property

$= 3 \cdot 4 \cdot \sqrt{10^2}$ Product Property

$= 3 \cdot 4 \cdot 10$ Simplify.

$= 120$ Simplify.

7. $\sqrt{60x^4y^7}$

SOLUTION:

$\sqrt{60x^4y^7} = \sqrt{2^2 \cdot 3 \cdot 5 \cdot x^4 \cdot y^7}$ Prime factorization of 60.

$= \sqrt{2^2} \cdot \sqrt{3} \cdot \sqrt{5} \cdot \sqrt{x^4} \cdot \sqrt{y^6} \cdot \sqrt{y}$ Product Property

$= 2 \cdot \sqrt{3} \cdot \sqrt{5} \cdot x^2 \cdot y^3 \cdot \sqrt{y}$ Simplify.

$= 2x^2y^3\sqrt{15y}$ Simplify.

8. $\sqrt{88m^3p^2r^5}$

SOLUTION:

$\sqrt{88m^3p^2r^5} = \sqrt{2^3 \cdot 11 \cdot m^3 \cdot p^2 \cdot r^5}$ Factor 88.

$= \sqrt{2^2} \cdot \sqrt{2} \cdot \sqrt{11} \cdot \sqrt{m^2} \cdot \sqrt{m} \cdot \sqrt{p^2} \cdot \sqrt{r^4} \cdot \sqrt{r}$ Product Property

$= 2 \cdot \sqrt{2} \cdot \sqrt{11} \cdot |m| \cdot \sqrt{m} \cdot |p| \cdot r^2 \cdot \sqrt{r}$ Simplify.

$= 2m|p|r^2\sqrt{22mr}$ Simplify.

9. $\sqrt{99ab^5c^2}$

SOLUTION:

$\sqrt{99ab^5c^2} = \sqrt{3^2 \cdot 11 \cdot a \cdot b^5 \cdot c^2}$ Factor 99.

$= \sqrt{3^2} \cdot \sqrt{11} \cdot \sqrt{a} \cdot \sqrt{b^4} \cdot \sqrt{b} \cdot \sqrt{c^2}$ Product Property

$= 3 \cdot \sqrt{11} \cdot \sqrt{a} \cdot b^2 \cdot \sqrt{b} \cdot |c|$ Simplify.

$= 3b^2|c|\sqrt{11ab}$ Simplify.

10. **MULTIPLE CHOICE** Which expression is equivalent to $\sqrt{\frac{45}{10}}$?

A $\frac{5\sqrt{2}}{10}$

B $\sqrt{\frac{45}{10}}$

C $\frac{\sqrt{50}}{10}$

D $\frac{3\sqrt{2}}{2}$

SOLUTION:

$\sqrt{\frac{45}{10}} = \frac{\sqrt{45}}{\sqrt{10}}$ Quotient Property

$= \frac{\sqrt{45}}{\sqrt{10}} \cdot \frac{\sqrt{10}}{\sqrt{10}}$ Multiply by $\frac{\sqrt{10}}{\sqrt{10}}$ to rationalize the denominator.

$= \frac{\sqrt{450}}{10}$ Simplify.

$= \frac{\sqrt{3^2 \cdot 5 \cdot 2 \cdot 5}}{10}$ Factor 10 and 45.

$= \frac{\sqrt{3^2} \cdot \sqrt{5^2} \cdot \sqrt{2}}{10}$ Product Property

$= \frac{3 \cdot 5 \cdot \sqrt{2}}{10}$ Simplify.

$= \frac{3\sqrt{2}}{2}$ Simplify.

The correct choice is D.

Simplify each expression.

11. $\frac{3}{3+\sqrt{5}}$

SOLUTION:

$\frac{3}{3+\sqrt{5}} = \frac{3}{3+\sqrt{5}} \cdot \frac{3-\sqrt{5}}{3-\sqrt{5}}$ Multiply by conjugate $3-\sqrt{5}$.

$= \frac{3(3-\sqrt{5})}{3^2 - \sqrt{5^2}}$ $(a-b)(a+b) = a^2 - b^2$.

$= \frac{9-3\sqrt{5}}{9-5}$ Simplify.

$= \frac{9-3\sqrt{5}}{4}$ Simplify.

12. $\frac{5}{2-\sqrt{6}}$

SOLUTION:

$\frac{5}{2-\sqrt{6}} = \frac{5}{2-\sqrt{6}} \cdot \frac{2+\sqrt{6}}{2+\sqrt{6}}$ Multiply by the conjugate of $2+\sqrt{6}$.

$= \frac{5(2+\sqrt{6})}{2^2 - \sqrt{6^2}}$ $(a-b)(a+b) = a^2 - b^2$.

$= \frac{10+5\sqrt{6}}{4-6}$ Simplify.

$= \frac{10+5\sqrt{6}}{-2}$ Simplify.

13. $\frac{2}{1-\sqrt{10}}$

SOLUTION:

$\frac{2}{1-\sqrt{10}} = \frac{2}{1-\sqrt{10}} \cdot \frac{1+\sqrt{10}}{1+\sqrt{10}}$ Multiply by conjugate $1+\sqrt{10}$.

$= \frac{2(1+\sqrt{10})}{1^2 - \sqrt{10^2}}$ $(a-b)(a+b) = a^2 - b^2$.

$= \frac{2+2\sqrt{10}}{1-10}$ Simplify.

$= \frac{2+2\sqrt{10}}{-9}$ Simplify.

14. $\frac{1}{4+\sqrt{12}}$

SOLUTION:

$\frac{1}{4+\sqrt{12}} = \frac{1}{4+\sqrt{12}} \cdot \frac{4-\sqrt{12}}{4-\sqrt{12}}$ Mutliply by conjugate $4-\sqrt{12}$.

$= \frac{1(4-\sqrt{12})}{4^2 - \sqrt{12^2}}$ $(a-b)(a+b) = a^2 - b^2$.

$= \frac{4-\sqrt{12}}{16-12}$ Simplify.

$= \frac{4-\sqrt{2^2 \cdot 3}}{4}$ Factor 12..

$= \frac{4-\sqrt{2^2} \cdot \sqrt{3}}{4}$ Product Property

$= \frac{4-2\sqrt{3}}{4}$ Simplify.

$= \frac{2-\sqrt{3}}{2}$ Simplify.

15. $\frac{4}{6-\sqrt{7}}$

SOLUTION:

$\frac{4}{6-\sqrt{7}} = \frac{4}{6-\sqrt{7}} \cdot \frac{6+\sqrt{7}}{6+\sqrt{7}}$ Multiply by conjugate $6+\sqrt{7}$.

$= \frac{4(6+\sqrt{7})}{6^2 - \sqrt{7^2}}$ $(a-b)(a+b) = a^2 - b^2$.

$= \frac{24+4\sqrt{7}}{36-7}$ Simplify.

$= \frac{24+4\sqrt{7}}{29}$ Simplify.

4-3 Simplifying Radical Expressions

16. $\dfrac{6}{5+\sqrt{11}}$

SOLUTION:

$$\frac{6}{5+\sqrt{11}} = \frac{6}{5+\sqrt{11}} \cdot \frac{5-\sqrt{11}}{5-\sqrt{11}}$$ Multiply by conjugate $5-\sqrt{11}$.

$$= \frac{6(5-\sqrt{11})}{5^2-\sqrt{11^2}}$$ $(a-b)(a+b)=a^2-b^2$.

$$= \frac{30-6\sqrt{11}}{25-11}$$ Simplify.

$$= \frac{30-6\sqrt{11}}{14}$$ Simplify.

$$= \frac{15-3\sqrt{11}}{7}$$ Simplify.

Simplify each expression.

17. $\sqrt{52}$

SOLUTION:

$\sqrt{52} = \sqrt{2 \cdot 2 \cdot 13}$ Prime factorization of 52.

$= \sqrt{2^2 \cdot 13}$ Group factors.

a

$= \sqrt{2^2} \cdot \sqrt{13}$ Product Property

$= 2 \cdot \sqrt{13}$ Simplify.

18. $\sqrt{56}$

SOLUTION:

$\sqrt{56} = \sqrt{2 \cdot 2 \cdot 2 \cdot 7}$ Prime factorization of 56.

$= \sqrt{2^2 \cdot 2 \cdot 7}$ Group factors.

$= \sqrt{2^2} \cdot \sqrt{2} \cdot \sqrt{7}$ Product Property

$= 2 \cdot \sqrt{14}$ Simplify.

19. $\sqrt{72}$

SOLUTION:

$\sqrt{72} = \sqrt{2 \cdot 2 \cdot 2 \cdot 3 \cdot 3}$ Prime factorization of 72.

$= \sqrt{2^2 \cdot 2 \cdot 3^2}$ Group factors.

$= \sqrt{2^2} \cdot \sqrt{2} \cdot \sqrt{3^2}$ Product Property

$= 2 \cdot 3 \cdot \sqrt{2}$ Simplify.

$= 6\sqrt{2}$ Simplify.

20. $3\sqrt{18}$

SOLUTION:

$3\sqrt{18} = 3 \cdot \sqrt{2 \cdot 3 \cdot 3}$ Prime factorization of 18.

$= 3 \cdot \sqrt{2 \cdot 3^2}$ Group factors.

$= = 3 \cdot \sqrt{2} \cdot \sqrt{3^2}$ Product Property

$= 3 \cdot 3 \cdot \sqrt{2}$ Simplify.

$= 9\sqrt{2}$ Simplify.

21. $\sqrt{243}$

SOLUTION:

$\sqrt{243} = \sqrt{3 \cdot 3 \cdot 3 \cdot 3 \cdot 3}$ Prime factorization of 243.

$= \sqrt{3^4 \cdot 3}$ Group factors.

$= \sqrt{3^4} \cdot \sqrt{3}$ Product Property.

$= 3^2 \cdot a\sqrt{3}$ Simplify.

$= 9\sqrt{3}$ Simplify.

22. $\sqrt{245}$

SOLUTION:

$\sqrt{245} = \sqrt{5 \cdot 7 \cdot 7}$ Prime factorization of 245.

$= \sqrt{5 \cdot 7^2}$ Group terms.

$= \sqrt{5} \cdot \sqrt{7^2}$ Product Property

$= 7\sqrt{5}$ Simplify.

23. $\sqrt{5} \cdot \sqrt{10}$

SOLUTION:

$\sqrt{5} \cdot \sqrt{10} = \sqrt{5} \cdot \sqrt{5} \cdot \sqrt{2}$ Prime factorization of 10.

$= \sqrt{5^2} \cdot \sqrt{2}$ Product Property

$= 5\sqrt{2}$ Simplify.

24. $\sqrt{10} \cdot \sqrt{20}$

SOLUTION:

$\sqrt{10} \cdot \sqrt{20} = \sqrt{2} \cdot \sqrt{5} \cdot \sqrt{2} \cdot \sqrt{2} \cdot \sqrt{5}$ Prime factorization of 10 and 20.

$= \sqrt{2^2} \cdot \sqrt{2} \cdot \sqrt{5^2}$ Product Property

$= 2 \cdot 5 \cdot \sqrt{2}$ Simplify.

$= 10\sqrt{2}$ Simplify.

4-3 Simplifying Radical Expressions

25. $3\sqrt{8}\cdot 2\sqrt{7}$

SOLUTION:

$3\sqrt{8}\cdot 2\sqrt{7} = 3\cdot 2\cdot \sqrt{8}\cdot\sqrt{7}$ Associative Property

$= 3\cdot 2\cdot \sqrt{2^2\cdot 2}\cdot\sqrt{7}$ Factorization of 8.

$= 3\cdot 2\cdot \sqrt{2^2}\cdot\sqrt{2}\cdot\sqrt{7}$ Product Property

$= 3\cdot 2\cdot 2\cdot\sqrt{2}\cdot\sqrt{7}$ Simplify.

$= 12\sqrt{14}$ Simplify.

26. $4\sqrt{2}\cdot 5\sqrt{8}$

SOLUTION:

$4\sqrt{2}\cdot 5\sqrt{8} = 4\cdot 5\cdot \sqrt{2}\cdot\sqrt{8}$ Associative Property

$= 4\cdot 5\cdot \sqrt{2}\cdot\sqrt{2^2}\cdot\sqrt{2}$ Factor 8.

$= 4\cdot 5\cdot \sqrt{2^2}\cdot\sqrt{2^2}$ Product Property

$= 4\cdot 5\cdot 2\cdot 2$ Simplify.

$= 80$ Simplify.

27. $3\sqrt{25t^2}$

SOLUTION:

$3\sqrt{25t^2} = 3\cdot\sqrt{25}\cdot\sqrt{t^2}$ Product Property

$= 3\cdot\sqrt{5^2}\cdot\sqrt{t^2}$ Prime factorization of 25.

$= 3\cdot 5\cdot |t|$ Simplify.

$= 15|t|$ Simplify.

28. $5\sqrt{81q^5}$

SOLUTION:

$5\sqrt{81q^5} = 5\cdot\sqrt{81}\cdot\sqrt{q^5}$ Product Property

$= 5\cdot\sqrt{9^2}\cdot\sqrt{q^4}\cdot\sqrt{q}$ Factorization of 81 and q^5.

$= 5\cdot 9\cdot q^2\cdot\sqrt{q}$ Simplify.

$= 45q^2\sqrt{q}$ Simplify.

29. $\sqrt{28a^2b^3}$

SOLUTION:

$\sqrt{28a^2b^3} = \sqrt{28}\cdot\sqrt{a^2}\cdot\sqrt{b^3}$ Product Property

$= \sqrt{2^2}\cdot\sqrt{7}\cdot\sqrt{a^2}\cdot\sqrt{b^2}\cdot\sqrt{b}$ Factor 29 and b^3.

$= 2\cdot\sqrt{7}\cdot|a|\cdot b\cdot\sqrt{b}$ Simplify.

$= 2|a|b\sqrt{7b}$ Simplify.

30. $\sqrt{75qr^3}$

SOLUTION:

$\sqrt{75qr^3} = \sqrt{75}\cdot\sqrt{q}\cdot\sqrt{r^3}$ Product Property

$= \sqrt{5^2}\cdot\sqrt{3}\cdot\sqrt{q}\cdot\sqrt{r^2}\cdot\sqrt{r}$ Factor 75 and r^3.

$= 5\cdot\sqrt{3}\cdot\sqrt{q}\cdot r\cdot\sqrt{r}$ Simplify.

$= 5r\sqrt{3qr}$ Simplify.

31. $7\sqrt{63m^3p}$

SOLUTION:

$7\sqrt{63m^3p} = 7\cdot\sqrt{63}\cdot\sqrt{m^3}\cdot\sqrt{p}$ Product Property

$= 7\cdot\sqrt{3^2}\cdot\sqrt{7}\cdot\sqrt{m^2}\cdot\sqrt{m}\cdot\sqrt{p}$ Factor 63 and m^3.

$= 7\cdot 3\cdot\sqrt{7}\cdot m\cdot\sqrt{m}\cdot\sqrt{p}$ Simplify.

$= 21m\sqrt{7mp}$ Simplify.

32. $4\sqrt{66g^2h^4}$

SOLUTION:

$4\sqrt{66g^2h^4} = 4\cdot\sqrt{66}\cdot\sqrt{g^2}\cdot\sqrt{h^4}$ Product Property

$= 4\cdot\sqrt{66}\cdot|g|\cdot h^2$ Simplify.

$= 4|g|h^2\sqrt{66}$ Simplify.

33. $\sqrt{2ab^2}\cdot\sqrt{10a^5b}$

SOLUTION:

$\sqrt{2ab^2}\sqrt{10a^5b} = \sqrt{2}\cdot\sqrt{a}\cdot\sqrt{b^2}\cdot\sqrt{10}\cdot\sqrt{a^5}\cdot\sqrt{b}$ Product Property

$= \sqrt{2}\cdot\sqrt{a}\cdot\sqrt{b^2}\cdot\sqrt{2}\cdot\sqrt{5}\cdot\sqrt{a^5}\cdot\sqrt{b}$ Factor 10.

$= \sqrt{2^2}\cdot\sqrt{5}\cdot\sqrt{a^6}\cdot\sqrt{b^2}\cdot\sqrt{b}$ Simplify.

$= 2a^3b\sqrt{5b}$ Simplify.

34. $\sqrt{4c^3d^3}\cdot\sqrt{8c^3d}$

SOLUTION:

$\sqrt{4c^3d^3}\sqrt{8c^3d} = \sqrt{4}\cdot\sqrt{c^3}\cdot\sqrt{d^3}\cdot\sqrt{8}\cdot\sqrt{c^3}\cdot\sqrt{d}$ Product Property

$= \sqrt{2^2}\cdot\sqrt{c^6}\cdot\sqrt{2^2}\cdot\sqrt{2}\cdot\sqrt{d^4}$ Factor 4 and 8.

$= 2\cdot c^3\cdot 2\cdot\sqrt{2}\cdot d^2$ Simplify.

$= 4c^3d^2\sqrt{2}$ Simplify.

35. **ROLLER COASTER** Starting from a stationary position, the velocity *v* of a roller coaster in feet per second at the bottom of a hill can be approximated by $v = \sqrt{64h}$, where *h* is the height of the hill in feet.

a. Simplify the equation.
b. Determine the velocity of a roller coaster at the bottom of a 134-foot hill.

SOLUTION:

a.

$$\begin{aligned} v &= \sqrt{64h} \\ &= \sqrt{64} \cdot \sqrt{h} \\ &= \sqrt{8^2} \cdot \sqrt{h} \\ &= 8\sqrt{h} \end{aligned}$$

b. To determine the velocity of the roller coaster at the bottom of the hill, substitute 134 for *h* in the equation $v = 8\sqrt{h}$.

$$\begin{aligned} v &= 8\sqrt{h} \\ &= 8\sqrt{134} \\ &\approx 92.6 \end{aligned}$$

The roller coaster will have a velocity of about 92.6 ft/sec at the bottom of a 134-foot hill.

36. **CCSS PRECISION** When fighting a fire, the velocity *v* of water being pumped into the air is modeled by the function $v = \sqrt{2hg}$, where *h* represents the maximum height of the water and *g* represents the acceleration due to gravity (32 ft/s^2).

a. Solve the function for *h*.
b. The Hollowville Fire Department needs a pump that will propel water 80 feet into the air. Will a pump advertised to project water with a velocity of 70 feet per second meet their needs? Explain.
c. The Jackson Fire Department must purchase a pump that will propel water 90 feet into the air. Will a pump that is advertised to project water with a velocity of 77 feet per second meet the fire department's need? Explain.

SOLUTION:

a.

$$\begin{aligned} v &= \sqrt{2hg} \\ (v)^2 &= (\sqrt{2hg})^2 \\ v^2 &= 2hg \\ \frac{v^2}{2g} &= h \end{aligned}$$

b. To determine the height of the water, substitute 70 for *v* in the function $h = \frac{v^2}{2g}$.

$$\begin{aligned} h &= \frac{v^2}{2g} \\ &= \frac{(70)^2}{2(32)} \\ &= 76.6 \end{aligned}$$

A pump with a velocity of 70 feet per second will pump water only to a maximum height of 76.6 feet. Therefore, this pump will not meet the fire department's need.

c. To determine the height of the water, substitute 77 for *v* in the function $h = \frac{v^2}{2g}$.

$$\begin{aligned} h &= \frac{v^2}{2g} \\ &= \frac{(77)^2}{2(32)} \\ &= 92.6 \end{aligned}$$

A pump with a velocity of 77 feet per second will pump water to a maximum height of 92.6 feet. Therefore, this pump will meet the fire department's need.

4-3 Simplifying Radical Expressions

Simplify each expression.

37. $\sqrt{\frac{32}{t^4}}$

SOLUTION:

$$\sqrt{\frac{32}{t^4}} = \frac{\sqrt{32}}{\sqrt{t^4}}$$ Quotient Property

$$= \frac{\sqrt{2^5}}{\sqrt{t^4}}$$ Factor 32.

$$= \frac{\sqrt{2^4}\cdot\sqrt{2}}{\sqrt{t^4}}$$ Product Property

$$= \frac{2^2\cdot\sqrt{2}}{t^2}$$ Simplify.

$$= \frac{4\sqrt{2}}{t^2}$$ Simplify.

38. $\sqrt{\frac{27}{m^5}}$

SOLUTION:

$$\sqrt{\frac{27}{m^5}} = \frac{\sqrt{27}}{\sqrt{m^5}}\cdot\frac{\sqrt{m^5}}{\sqrt{m^5}}$$ Quotient Property

$$= \frac{\sqrt{27}}{\sqrt{m^5}}\cdot\frac{\sqrt{m^5}}{\sqrt{m^5}}$$ Multiply by conjugate $\sqrt{m^5}$.

$$= \frac{\sqrt{27}\cdot\sqrt{m^5}}{m^5}$$ Simplify.

$$= \frac{\sqrt{3^2}\cdot\sqrt{3}\cdot\sqrt{m^4}\cdot\sqrt{m}}{m^5}$$ Factor 28 and m^5.

$$= \frac{3\cdot\sqrt{3}\cdot m^2\cdot\sqrt{m}}{m^5}$$ Simplify.

$$= \frac{3\sqrt{3m}}{m^3}$$ Simplify.

39. $\frac{\sqrt{68ac^3}}{\sqrt{27a^2}}$

SOLUTION:

$$\frac{\sqrt{68ac^3}}{\sqrt{27a^2}} = \frac{\sqrt{68ac^3}}{\sqrt{27a^2}}\cdot\frac{\sqrt{27a^2}}{\sqrt{27a^2}}$$ Multiply by conjugate $\sqrt{27a^2}$.

$$= \frac{\sqrt{1836a^3c^3}}{27a^2}$$ Simplify.

$$= \frac{\sqrt{1836}\cdot\sqrt{a^3}\cdot\sqrt{c^3}}{27a^2}$$ Product Property

$$= \frac{\sqrt{2^2}\cdot\sqrt{3^2}\cdot\sqrt{3}\cdot\sqrt{17}\cdot\sqrt{a^2}\cdot\sqrt{a}\cdot\sqrt{c^2}\cdot\sqrt{c}}{27a^2}$$ Factor $1836, a^3$ and c^3.

$$= \frac{2\cdot 3\cdot\sqrt{3}\cdot\sqrt{17}\cdot|a|\cdot\sqrt{a}\cdot c\cdot\sqrt{c}}{27a^2}$$ Simplify.

$$= \frac{6\cdot|a|\cdot c\cdot\sqrt{51ac}}{27a^2}$$ Simplify.

$$= \frac{2c\sqrt{51ac}}{9|a|}$$ Simplify.

40. $\frac{\sqrt{h^3}}{\sqrt{8}}$

SOLUTION:

$$\frac{\sqrt{h^3}}{\sqrt{8}} = \frac{\sqrt{h^3}}{\sqrt{8}}\cdot\frac{\sqrt{8}}{\sqrt{8}}$$ Multiply by conjugate $\sqrt{8}$.

$$= \frac{\sqrt{h^2}\cdot\sqrt{h}\cdot\sqrt{2^2}\cdot\sqrt{2}}{8}$$ Factor 8 and h^3.

$$= \frac{h\cdot\sqrt{h}\cdot 2\cdot\sqrt{2}}{8}$$ Simplify.

$$= \frac{h\sqrt{2h}}{4}$$ Simplify.

41. $\sqrt{\frac{3}{16}}\cdot\sqrt{\frac{9}{5}}$

SOLUTION:

$$\sqrt{\frac{3}{16}}\cdot\sqrt{\frac{9}{5}} = \frac{\sqrt{3}}{\sqrt{16}}\cdot\frac{\sqrt{9}}{\sqrt{5}}$$ Quotient Property

$$= \frac{\sqrt{3\cdot 9}}{\sqrt{16\cdot 5}}$$ Property Property

$$= \frac{\sqrt{27}}{\sqrt{80}}\cdot\frac{\sqrt{80}}{\sqrt{80}}$$ Simplify.

$$= \frac{\sqrt{27}}{\sqrt{80}}\cdot\frac{\sqrt{80}}{\sqrt{80}}$$ Multiply by conjugate $\sqrt{80}$.

$$= \frac{\sqrt{2160}}{80}$$ Simplify.

$$= \frac{\sqrt{2^4}\cdot\sqrt{3^2}\cdot\sqrt{3}\cdot\sqrt{5}}{80}$$ Factor 2160.

$$= \frac{2^2\cdot 3\cdot\sqrt{15}}{80}$$ Simplify.

$$= \frac{12\sqrt{15}}{80}$$ Simplify.

$$= \frac{3\sqrt{15}}{20}$$ Simplify.

4-3 Simplifying Radical Expressions

42. $\sqrt{\frac{7}{2}} \cdot \sqrt{\frac{5}{3}}$

SOLUTION:

$\sqrt{\frac{7}{2}} \cdot \sqrt{\frac{5}{3}} = \frac{\sqrt{7}}{\sqrt{2}} \cdot \frac{\sqrt{5}}{\sqrt{3}}$ Quotient Property

$= \frac{\sqrt{7 \cdot 5}}{\sqrt{2 \cdot 3}}$ Product Property

$= \frac{\sqrt{35}}{\sqrt{6}}$ Simplify.

$= \frac{\sqrt{35}}{\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}}$ Multiply by conjugate $\sqrt{6}$.

$= \frac{\sqrt{210}}{6}$ Simplify.

43. $\frac{7}{5+\sqrt{3}}$

SOLUTION:

$\frac{7}{5+\sqrt{3}} = \frac{7}{5+\sqrt{3}} \cdot \frac{5-\sqrt{3}}{5-\sqrt{3}}$ Multiply by conjugate $5-\sqrt{3}$.

$= \frac{7(5-\sqrt{3})}{5^2-\sqrt{3^2}}$ $(a-b)(a+b)=a^2-b^2$.

$= \frac{35-7\sqrt{3}}{25-3}$ Simplify.

$= \frac{35-7\sqrt{3}}{22}$ Simplify.

44. $\frac{9}{6-\sqrt{8}}$

SOLUTION:

$\frac{9}{6-\sqrt{8}} = \frac{9}{6-\sqrt{8}} \cdot \frac{6+\sqrt{8}}{6+\sqrt{8}}$ Multiply by conjugate $6+\sqrt{8}$.

$= \frac{9(6+\sqrt{8})}{6^2-\sqrt{8^2}}$ $(a-b)(a+b)=a^2-b^2$.

$= \frac{54+9\sqrt{8}}{36-8}$ Simplify.

$= \frac{54+9 \cdot 2\sqrt{2}}{28}$ Simplify.

$= \frac{54+18\sqrt{2}}{28} = \frac{27+9\sqrt{2}}{14}$ Simplify.

45. $\frac{3\sqrt{3}}{-2+\sqrt{6}}$

SOLUTION:

$\frac{3\sqrt{3}}{-2+\sqrt{6}} = \frac{3\sqrt{3}}{-2+\sqrt{6}} \cdot \frac{-2-\sqrt{6}}{-2-\sqrt{6}}$ Multiply by conjugate $-2-\sqrt{6}$.

$= \frac{3\sqrt{3}(-2-\sqrt{6})}{(-2)^2-\sqrt{6^2}}$ $(a-b)(a+b)=a^2-b^2$.

$= \frac{-6\sqrt{3}-3\sqrt{18}}{4-6}$ Simplify.

$= \frac{-6\sqrt{3}-3\sqrt{3^2 \cdot \sqrt{2}}}{-2}$ Factor 18.

$= \frac{-6\sqrt{3}-9\sqrt{2}}{-2}$ Simplify.

$= \frac{6\sqrt{3}+9\sqrt{2}}{2}$ Simplify.

46. $\frac{3}{\sqrt{7}-\sqrt{2}}$

SOLUTION:

$\frac{3}{\sqrt{7}-\sqrt{2}} = \frac{3}{\sqrt{7}-\sqrt{2}} \cdot \frac{\sqrt{7}+\sqrt{2}}{\sqrt{7}+\sqrt{2}}$ Multiply by conjugate $\sqrt{7}+\sqrt{2}$.

$= \frac{3(\sqrt{7}+\sqrt{2})}{\sqrt{7^2}-\sqrt{2^2}}$ $(a-b)(a+b)=a^2-b^2$.

$= \frac{3\sqrt{7}+3\sqrt{2}}{7-2}$ Simplify.

$= \frac{3\sqrt{7}+3\sqrt{2}}{5}$ Simplify.

47. $\frac{5}{\sqrt{6}+\sqrt{3}}$

SOLUTION:

$\frac{5}{\sqrt{6}+\sqrt{3}} = \frac{5}{\sqrt{6}+\sqrt{3}} \cdot \frac{\sqrt{6}-\sqrt{3}}{\sqrt{6}-\sqrt{3}}$ Multiply by conjugate $\sqrt{6}-\sqrt{3}$.

$= \frac{5(\sqrt{6}-\sqrt{3})}{\sqrt{6^2}-\sqrt{3^2}}$ $(a-b)(a+b)=a^2-b^2$.

$= \frac{5\sqrt{6}-5\sqrt{3}}{6-3}$ Simplify.

$= \frac{5\sqrt{6}-5\sqrt{3}}{3}$ Simplify.

48. $\frac{2\sqrt{5}}{2\sqrt{7}+3\sqrt{3}}$

SOLUTION:

$\frac{2\sqrt{5}}{2\sqrt{7}+3\sqrt{3}} = \frac{2\sqrt{5}}{2\sqrt{7}+3\sqrt{3}} \cdot \frac{2\sqrt{7}-3\sqrt{3}}{2\sqrt{7}-3\sqrt{3}}$ Multiply by conjugate $2\sqrt{7}-3\sqrt{3}$.

$= \frac{2\sqrt{5}(2\sqrt{7}-3\sqrt{3})}{2^2\sqrt{7^2}-3^2\sqrt{3^2}}$ $(a-b)(a+b)=a^2-b^2$.

$= \frac{4\sqrt{35}-6\sqrt{15}}{4 \cdot 7-9 \cdot 3}$ Simplify.

$= \frac{4\sqrt{35}-6\sqrt{15}}{28-27}$ Simplify.

$= \frac{4\sqrt{35}-6\sqrt{15}}{1}$ Simplify.

$= 4\sqrt{35}-6\sqrt{15}$ Simplify.

49. **ELECTRICITY** The amount of current in amperes *I* that an appliance uses can be calculated using the formula $I = \sqrt{\frac{P}{R}}$, where *P* is the power in watts and *R* is the resistance in ohms.

a. Simplify the formula.

b. How much current does an appliance use if the power used is 75 watts and the resistance is 5 ohms?

SOLUTION:

a.

$$I = \sqrt{\frac{P}{R}}$$

$$I = \frac{\sqrt{P}}{\sqrt{R}} \cdot \frac{\sqrt{R}}{\sqrt{R}}$$

$$I = \frac{\sqrt{PR}}{R}$$

b. Substitute $P = 75$ and $R = 5$ in the equation $I = \frac{\sqrt{PR}}{R}$.

$$I = \frac{\sqrt{PR}}{R}$$

$$= \frac{\sqrt{75 \cdot 5}}{5}$$

$$= \frac{\sqrt{375}}{5}$$

$$\approx 3.9$$

An appliance uses about 3.9 amps of current if the power used is 75 watts and the resistance is 5 ohms.

50. **KINETIC ENERGY** The speed *v* of a ball can be determined by the equation $v = \sqrt{\frac{2k}{m}}$, where *k* is the kinetic energy and *m* is the mass of the ball.

a. Simplify the formula if the mass of the ball is 3 kilograms.

b. If the ball is traveling 7 meters per second, what is the kinetic energy of the ball in Joules?

SOLUTION:

a. If the mass of the ball is 3 kilograms, substitute $m = 3$ into the equation.

$$V = \sqrt{\frac{2k}{m}}$$

$$= \sqrt{\frac{2k}{3}}$$

$$= \frac{\sqrt{2k}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{\sqrt{6k}}{3}$$

b. Substitute $V = 7$ in the equation $V = \frac{\sqrt{6k}}{3}$.

$$V = \frac{\sqrt{6k}}{3}$$

$$7 = \frac{\sqrt{6k}}{3}$$

$$3 \cdot 7 = 3 \cdot \frac{\sqrt{6k}}{3}$$

$$21 = \sqrt{6k}$$

$$(21)^2 = \left(\sqrt{6k}\right)^2$$

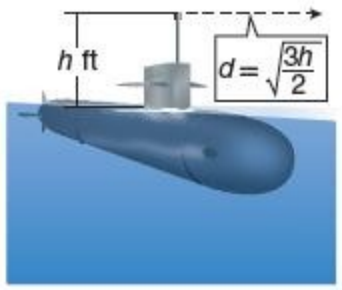
$$441 = 6k$$

$$\frac{441}{6} = \frac{6k}{6}$$

$$73.5 = k$$

The kinetic energy of the ball is 73.5 Joules.

51. **SUBMARINES** The greatest distance *d* in miles th the lookout can see on a clear day is modeled by the formula $d = \sqrt{\frac{3h}{2}}$. Determine how high the submarine would have to raise its periscope to see a ship, if the submarine is the given distances away fro a ship.



Distance	3	6	9	12	15
Height					

SOLUTION:

First solve the equation for *h*.

$$d = \sqrt{\frac{3h}{2}}$$

$$d^2 = \left(\sqrt{\frac{3h}{2}}\right)^2$$

$$d^2 = \frac{3h}{2}$$

$$\frac{2}{3} \cdot d^2 = \frac{2}{3} \cdot \frac{3h}{2}$$

$$\frac{2}{3}d^2 = h$$

Distance	3	6	9	12	15
Height = $\frac{2}{3}d^2$	$h = \frac{2}{3}(3)^2 = 6$	$h = \frac{2}{3}(6)^2 = 24$	$h = \frac{2}{3}(9)^2 = 54$	$h = \frac{2}{3}(12)^2 = 96$	$h = \frac{2}{3}(15)^2 = 150$

52. **CCSS STRUCTURE** Explain how to solve $\frac{\sqrt{3}+2}{x} = \frac{\sqrt{3}-1}{\sqrt{3}}$.

SOLUTION:

To solve an equation of equal ratios, first find the equal cross products and then solve for the variable.

$$\frac{\sqrt{3}+2}{x} = \frac{\sqrt{3}-1}{\sqrt{3}} \quad \text{Original equation}$$

$$\sqrt{3}(\sqrt{3}+2) = (\sqrt{3}-1)x \quad \text{Find the cross products}$$

$$\frac{\sqrt{3}(\sqrt{3}+2)}{\sqrt{3}-1} = \frac{(\sqrt{3}-1)x}{\sqrt{3}-1} \quad \text{Divide each side by } \sqrt{3}-1.$$

$$\frac{3+2\sqrt{3}}{\sqrt{3}-1} = x \quad \text{Simplify.}$$

Use the conjugate of $\sqrt{3}-1$ to rationalize the denominator.

$$\frac{3+2\sqrt{3}}{\sqrt{3}-1} = \frac{3+2\sqrt{3}}{\sqrt{3}-1} \cdot \frac{\sqrt{3}+1}{\sqrt{3}+1} \quad \text{The conjugate of } \sqrt{3}-1 \text{ is } \sqrt{3}+1.$$

$$= \frac{(3+2\sqrt{3})(\sqrt{3}+1)}{(\sqrt{3})^2-1^2} \quad (a-b)(a+b) = a^2-b^2$$

$$= \frac{5\sqrt{3}+9}{2} \quad \text{Simplify.}$$

So, the solution is $x = \frac{5\sqrt{3}+9}{2}$.

53. **CHALLENGE** Simplify each expression.

a. $\sqrt[3]{27}$

b. $\sqrt[3]{40}$

c. $\sqrt[3]{750}$

SOLUTION:

a. $\sqrt[3]{27} = \sqrt[3]{3^3} = 3$

b. $\sqrt[3]{40} = \sqrt[3]{8 \cdot 5} = \sqrt[3]{2^3 \cdot 5} = 2\sqrt[3]{5}$

c. $\sqrt[3]{750} = \sqrt[3]{125 \cdot 6} = \sqrt[3]{5^3 \cdot 6} = 5\sqrt[3]{6}$

54. **REASONING** Marge takes a number, subtracts 4, multiplies by 4, takes the square root, and takes the reciprocal to get $\frac{1}{2}$. What number did she start with? Write a formula to describe the process.

SOLUTION:

Let x = a number.

$$\frac{1}{\sqrt{4(x-4)}} = \frac{1}{2}$$ Original equation

$$1 \cdot 2 = 1 \cdot \sqrt{4(x-4)}$$ Cross multiply.

$$2 = \sqrt{4(x-4)}$$ Simplify.

$$(2)^2 = \left(\sqrt{4(x-4)}\right)^2$$ Square both sides.

$$4 = 4(x-4)$$ Simplify.

$$4 = 4x - 16$$ Simplify.

$$4 + 16 = 4x - 16 + 16$$ Add 16 to each side.

$$20 = 4x$$ Simplify.

$$\frac{20}{4} = \frac{4x}{4}$$ Divide each side by 4.

$$5 = x$$ Simplify.

55. **OPEN ENDED** Write two binomials of the form $a\sqrt{b} + c\sqrt{f}$ and $a\sqrt{b} - c\sqrt{f}$. Then find their product.

SOLUTION:

Two binomials of the form $a\sqrt{b} + c\sqrt{f}$ and $a\sqrt{b} - c\sqrt{f}$ are $1 + \sqrt{2}$ and $1 - \sqrt{2}$.

$$\begin{aligned}(1+\sqrt{2})(1-\sqrt{2}) &= 1(1) - 1(\sqrt{2}) + 1(\sqrt{2}) - \sqrt{2}^2 \\ &= 1 - 2 \\ &= -1\end{aligned}$$

56. **CHALLENGE** Use the Quotient Property of Square Roots to derive the Quadratic Formula by solving the quadratic equation $ax^2 + bx + c = 0$. (*Hint*: Begin by completing the square.)

SOLUTION:

$ax^2 + bx + c = 0$ Original equation

$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$ Divide each side by a.

$x^2 + \frac{b}{a}x = -\frac{c}{a}$ Subtract $\frac{c}{a}$ from each side.

$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = -\frac{c}{a} + \frac{b^2}{4a^2}$ Complete the square.

$\left(x + \frac{b}{2a}\right)^2 = \frac{-4ac + b^2}{4a^2}$ Factor $x^2 + \frac{b}{a}x + \frac{b^2}{4a^2}$.

$\left|x + \frac{b}{2a}\right| = \sqrt{\frac{b^2 - 4ac}{4a^2}}$ Take the square root of each side.

$x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$ Remove the absolute value symbols and insert $\pm$.

$x + \frac{b}{2a} = \pm\frac{\sqrt{b^2 - 4ac}}{\sqrt{4a^2}}$ Quotient Property of Square Roots

$x + \frac{b}{2a} = \pm\frac{\sqrt{b^2 - 4ac}}{2a}$ $\sqrt{4a^2} = 2a$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ Subtract $\frac{b}{2a}$ from each side.

57. **WRITING IN MATH** Summarize how to write a radical expression in simplest form.

SOLUTION:

No radicals can appear in the denominator of a fraction. So, rationalize the denominator to get rid of the radicand in the denominator. Then check if any of the radicands have perfect square factors other than 1. If so, simplify.

For example, simplify the following.

$$\frac{12\sqrt{5}}{3+\sqrt{5}} = \frac{12\sqrt{5}}{3+\sqrt{5}} \cdot \frac{3-\sqrt{5}}{3-\sqrt{5}}$$ Multiply by conjugate $3 - \sqrt{5}$.

$$= \frac{12\sqrt{5}(3-\sqrt{5})}{3^2 - \sqrt{5^2}}$$ $(a-b)(a+b) = a^2 - b^2$.

$$= \frac{36\sqrt{5} - 12\sqrt{5^2}}{9-5}$$ Distributive Property and Multiply.

$$= \frac{36\sqrt{5} - 12 \cdot 5}{4}$$ Simplify.

$$= 9\sqrt{5} - 15$$ Divide.

58. Jerry's electric bill is $23 less than his natural gas bill. The two bills are a total of $109. Which of the following equations can be used to find the amount of his natural gas bill?

A $g + g = 109$
B $23 + 2g = 109$
C $g - 23 = 109$
D $2g - 23 = 109$

SOLUTION:

Let g = Jerry's natural gas bill. Jerry's electric bill is $23 less than his natural gas bill, so the electric bill is $g - 23$.
The two bills are a total of $109.

$$g + (g - 23) = 109$$
$$2g - 23 = 109$$

So, the correct choice is D.

59. Solve $a^2 - 2a + 1 = 25$.

F −4, −6
G 4, −6
H −4, 6
J 4, 6

SOLUTION:

Solve for a.

$$a^2 - 2a + 1 = 25$$
$$a^2 - 2a - 24 = 0$$
$$a^2 - 6a + 4a - 24 = 0$$
$$(a^2 - 6a) + (4a - 24) = 0$$
$$a(a - 6) + 4(a - 6) = 0$$
$$(a + 4)(a - 6) = 0$$
$$a + 4 = 0 \quad \text{or} \quad a - 6 = 0$$
$$a = -4 \qquad x = 6$$

The roots are –4 and 6. So, the correct choice is H.

60. The expression $\sqrt{160x^2y^5}$ is equivalent to which of the following?

A $16|x|y^2\sqrt{10y}$
B $|x|y^2\sqrt{160y}$
C $4|x|y^2\sqrt{10y}$
D $10|x|y^2\sqrt{4y}$

SOLUTION:

$$\sqrt{160x^2y^5} = \sqrt{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 5 \cdot x^2 \cdot y^5}$$
$$= \sqrt{2^4} \cdot \sqrt{2} \cdot \sqrt{5} \cdot \sqrt{x^2} \cdot \sqrt{y^4} \cdot \sqrt{y}$$
$$= 2^2 \cdot \sqrt{2} \cdot \sqrt{5} \cdot |x| \cdot y^2 \cdot \sqrt{y}$$
$$= 4y^2|x|\sqrt{10y}$$

So, the correct choice is C.

61. **GRIDDED RESPONSE** Miki earns $10 an hour and 10% commission on sales. If Miki worked 38 hours and had a total sales of $1275 last week, how much did she make?

SOLUTION:

Miki's earnings are $10 per hour and 10% commission on sales.

$$\text{Total earnings} = \text{hourly pay} + \text{commission}$$
$$= 10(38) + 0.10(1275)$$
$$= 380 + 127.5$$
$$= 507.50$$

Miki made $507.50 last week.