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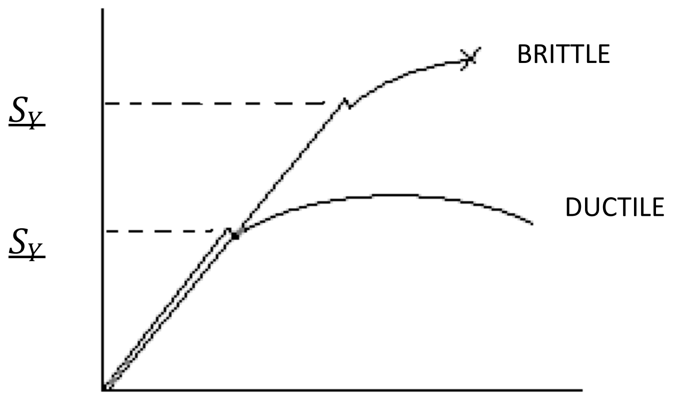
STATIC FAILURE THEORIES

- **DUCTILE**
  - **TRESCA CRITERIAN**
    - **MSST**:MAXIMUM SHEAR STRESS
  - **VON-MISES THEORY**
    - **MDET**:MAXIMUM DISTORTION ENERGY THEORY
- **BRITTLE**
  - **MOHR-COLOUMB THEORY**
  - **MAXIMUM NORMAL STRESS THEORY(MNST)**

UNIAXIAL TEST



MAXIMUM NORMAL STRESS THEORY

For maximum normal stress theory, the failure occurs when one of the principal stresses ($\sigma_1, \sigma_2 and\ \sigma_3$) equals to the yield strength.

$\sigma_1 > \sigma_2 > \sigma_3$

Failure occurs when either σ_1=S_t or $\sigma_3 = -S_c$,where S_t is strength in tension and $S_c$is strenght in compression.

MOHR-COULOMB THEORY

The Coulomb-Mohr theory or internal friction theory assumes that the critical shearing stress is related to internal friction.

MAXIMUM DISTORTION ENERGY THEORY(VON-MISES THEORY)

The maximum distortion energy theory ,also known as the Von Mises theory, was proposed by M.T.Huber in 1904 and further developed by R.von Mises(1913).In this theory failure by yielding occurs when at any point in the body ,the distortion energy per unit volume in a state of combined stress becomes equal to that associated with yielding in a simple tension test.

STRAIN ENERGY

Generally strain energy U is obtained by this equation.

$$U = \frac{1}{2}\, \sigma_{ij}\, \varepsilon_{ij}$$

$$\varepsilon_1 = \frac{1}{E}(\sigma_1 - \nu\sigma_2 - \nu\sigma_3)$$

$$\varepsilon_2 = \frac{1}{E}(-\nu\sigma_1 + \sigma_2 - \nu\sigma_3)$$

$$\varepsilon_3 = \frac{1}{E}(-\nu\sigma_1 - \nu\sigma_2 + \sigma_3)$$

Then, substituting these three equations in to general strain energy equation:

$$U=\frac{1}{2}\sigma_1 \frac{1}{E}(\sigma_1 - \nu\sigma_2 - \nu\sigma_3)+\frac{1}{2}\sigma_2 \frac{1}{E}(-\nu\sigma_1 + \sigma_2 - \nu\sigma_3) + \frac{1}{2}\sigma_3 \frac{1}{E}(-\nu\sigma_1 - \nu\sigma_2 + \sigma_3)$$

HYDROSTATIC STRESS

The hydrostatic stress(σ_h)causes a change in the volume.

$$\sigma_h = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3}$$

Strain energy associated with the hydrostatic stress:

$$U_h = \frac{1}{2E}[\sigma_h{}^2 + \sigma_h{}^2 + \sigma_h{}^2 - 2\nu(\sigma_h\sigma_h + \sigma_h\sigma_h + \sigma_h\sigma_h) = \frac{3}{2}\frac{(1-2\nu)}{E}\sigma_h{}^2$$

Then distortional energy $U_d = U - U_h$

From previous equations: $U_d = \frac{1+\nu}{3E}[\sigma_1{}^2 + \sigma_2{}^2 + \sigma_3{}^2 - \sigma_1\sigma_2 - \sigma_2\sigma_3 - \sigma_3\sigma_1]$

Then yielding will occur at this condition:

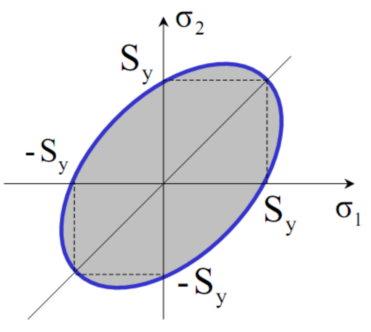
$$U_d = \frac{1+\nu}{3E} S_Y$$

$$\sigma_{eff} = S_Y = \sqrt{\sigma_1{}^2 + \sigma_2{}^2 + \sigma_3{}^2 - \sigma_1\sigma_2 - \sigma_2\sigma_3 - \sigma_3\sigma_1}$$

$$\sigma_{eff} = \sqrt{\frac{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}{2}}$$

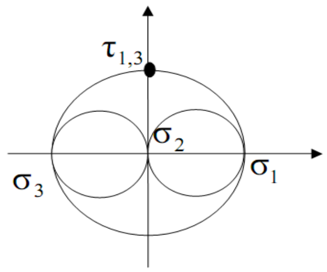
<u>For plane stress condition</u> $\sigma_3 = 0$

$$\sigma_{eff} = \sqrt{\sigma_1{}^2 + \sigma_2{}^2 + \sigma_1\sigma_2}$$

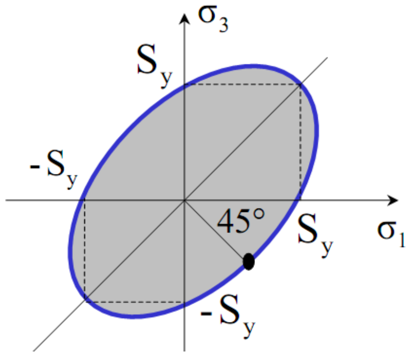


If the state stress is in this area then the material will not yield.

<u>For pure shear condition</u>



$$\sigma_3 = -\sigma_1$$



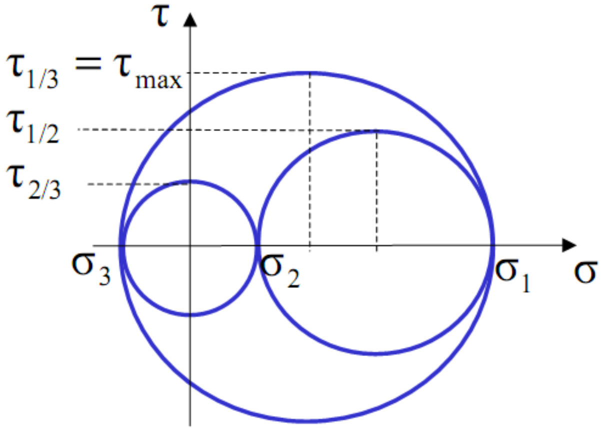
$$\sigma_{eff} = \sqrt{\sigma_1{}^2 + \sigma_3{}^2 - \sigma_1\sigma_3} = \sqrt{3\sigma_1{}^2} = \sqrt{3\tau_{max}{}^2} = S_Y$$

$$\tau_{max} = 0.577 S_Y$$

MAXIMUM SHEAR STRESS THEORY

The maximum shearing stress theory is an outgrowth of the experimental observation that a ductile material yields as a result of slip or shear along crystalline planes.

Yielding begins whenever the maximum shear stress in a part equals to the maximum shear stress in a tension test specimen that beings to yield.



$$\tau_{1/2} = \frac{\sigma_1 - \sigma_2}{2}$$

$$\tau_{2/3} = \frac{\sigma_2 - \sigma_3}{2}$$

$$\tau_{1/3} = \tau_{max} = \frac{\sigma_1 - \sigma_3}{2}$$

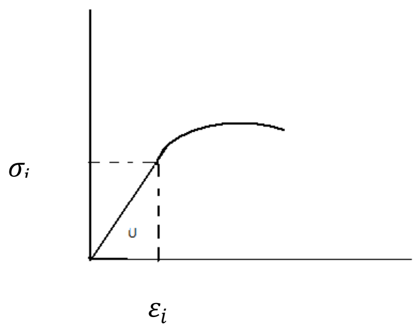
$$\tau_{max} = S_{YS} = \text{yield strength in shear} = \frac{S_Y}{2}$$

$$\tau_{max} = \frac{\sigma_1 - \sigma_3}{2}$$

Then , $S_Y = \sigma_1 - \sigma_3$

	Elasticity		Materials
-brittle	$\sigma_1(\sigma_x)$	=	$\frac{S_Y}{N}$ where N is safety factor
(MNST)			
-ductile			
(MSST)	τ_{max}	=	S_{YS}(yield strength in shear)
	$\frac{\sigma_1 - \sigma_3}{2}$	=	$\frac{S_Y}{2}$
	$\sigma_1 - \sigma_3$	=	$\frac{S_Y}{N}$
(MDET)	σ_{eff}	=	$\frac{S_Y}{N}$

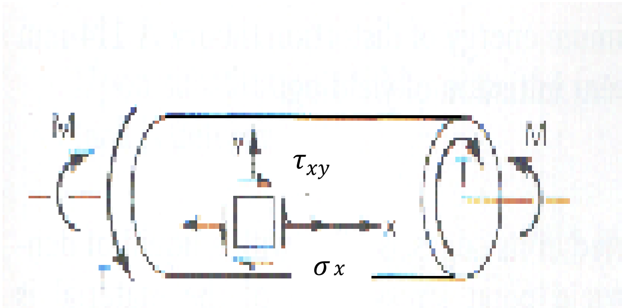
Strain Energy:

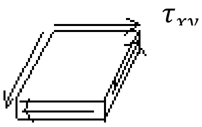


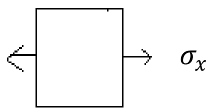
$$\frac{1}{2}\sigma_{ij}\,\varepsilon_{ij} = U$$

Design Margin=M=$\frac{S_Y - \sigma_{eff}\,N}{S_Y}$

EXAMPLE:A circular shaft of tensile strength S_Y=350 MPa is subjected to a combined state of loading defined by bending moment M=8 kN.m and torque T=24kN.m.Calculate the required shaft diameter d in order to achieve a factor of safety N=2.Use a) the maximum shearing stress theory(MSST-Tresca) b)the maximum distortion energy theory(MDET –Von Mises)



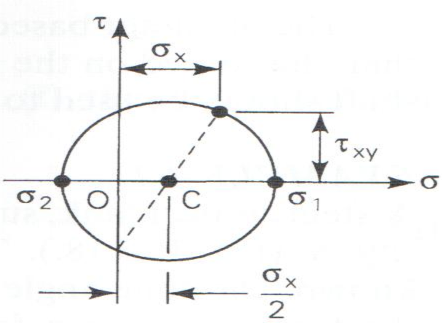




c=$\frac{d}{2}$ $\sigma_x = \frac{Mc}{I} = \frac{Md}{2I}$ and $\tau_{xy} = \frac{Tc}{J} = \frac{Td}{2J}$

Then,$\sigma_x = \frac{32M}{\pi d^3}$ and $\tau_{xy} = \frac{16T}{\pi d^3}$

We need to find principle stresses:



a) For maximum shearing stress theory

$$\sigma_{1,2} = \frac{16}{\pi d^3}(M \mp \sqrt{M^2 + T^2})$$

$$\sigma_1 - \sigma_2 = \frac{S_Y}{N} = \sqrt{\sigma_x{}^2 + 4\tau_{xy}{}^2} = \frac{32}{\pi d^3}\sqrt{M^2 + T^2}$$

Then, substituting the numerical values of M,T, S_Y and N:

d=113.8mm

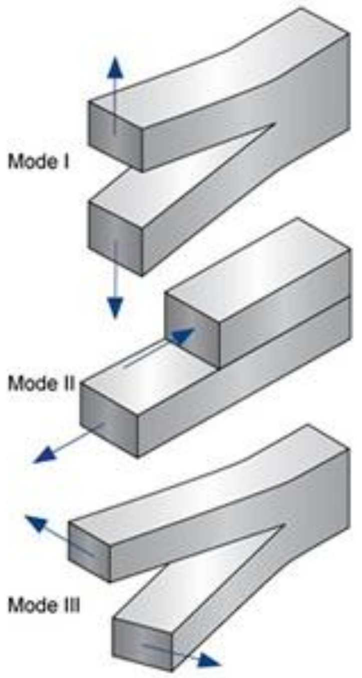
b) For maximum distortion energy theory

$$\sigma_{eff} = \sqrt{(\sigma_1 - \sigma_2)^2 + \sigma_1{}^2 + \sigma_2{}^2} = \sqrt{\sigma_x{}^2 + 3\tau_{xy}{}^2} = \frac{S_Y}{N} = \frac{16}{\pi d^3}\sqrt{4M^2 + 3T^2}$$

Then, substituting the numerical values of M,T, S_Y and N:

d=109mm

FRACTURE MODES

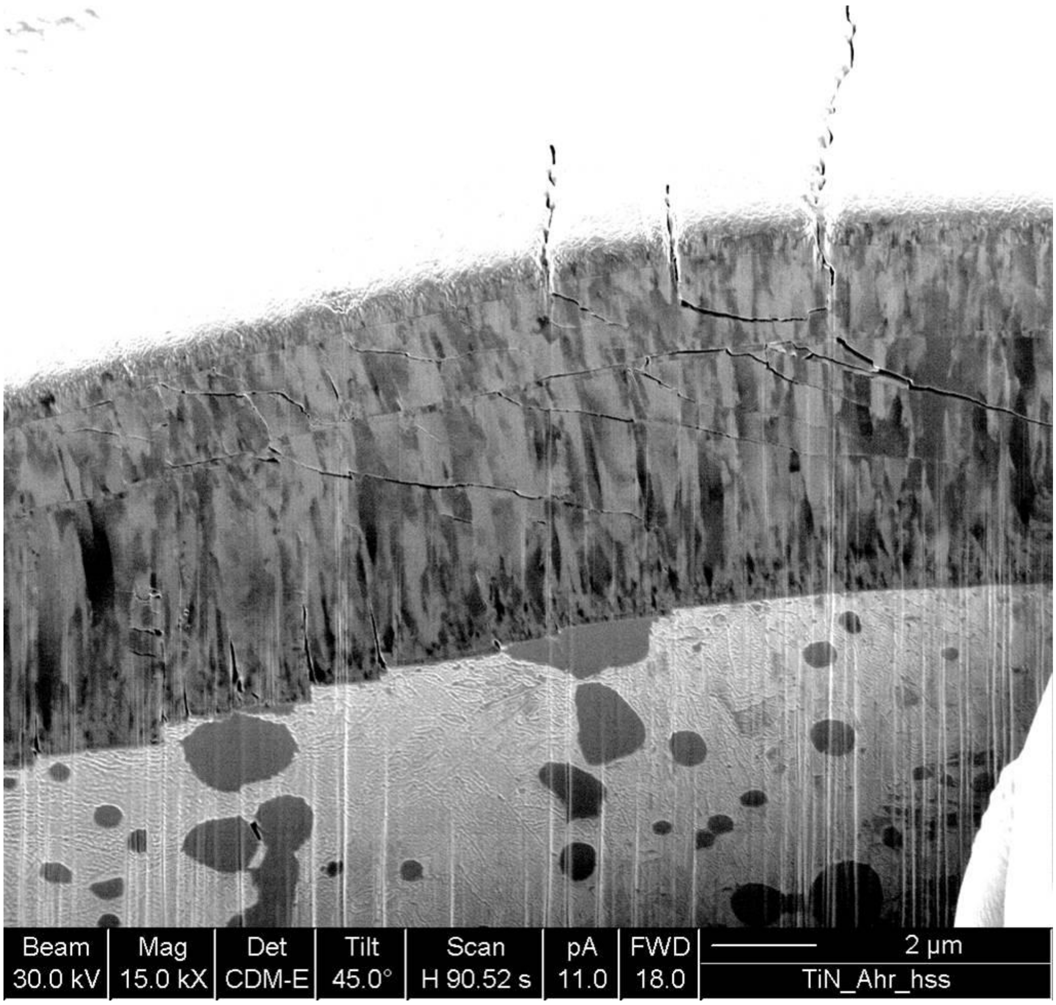


OPENING

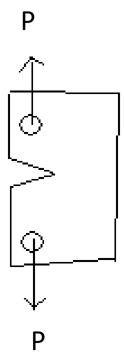
SLIDING

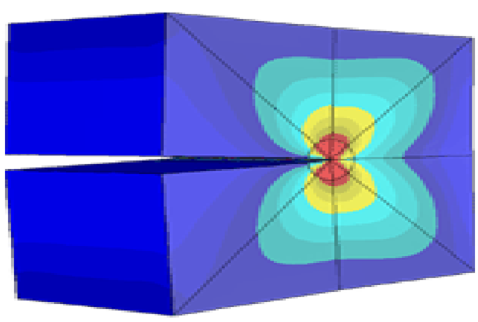
TEARING

Fracture is defined as the separation of a part into two or more pieces.The mechanisms of brittle fracture are the concern of fracture mechanics ,which is based on a stress analysis in the vicinity of a crack or defect of unknown small radius in a part.



MULTIPLE FRACTURES





Stress Intensity Factor: In the fracture mechanics approach a stress intensity factor, K_I, is evaluated. This can be thought of as a measure of the effective local stress at the crack root.

$$K_I = \beta\sigma\sqrt{\pi a}$$

where

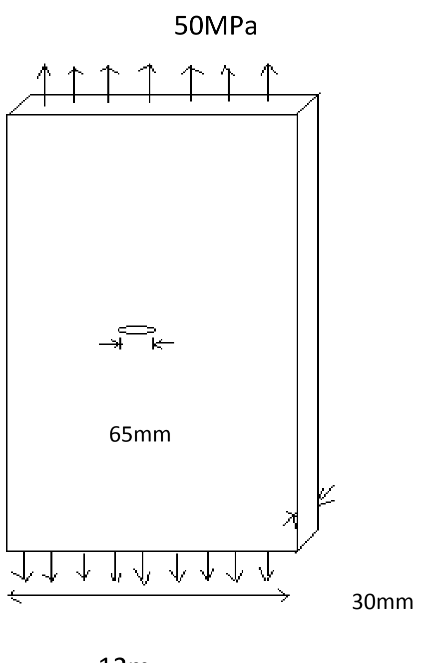
σ=normalstress,

$\beta = geometry\ factor\ \left(which\ depends\ on\ \frac{a}{w}\right), a = crack\ length$ (or half crack length) ,w=member width(or half width of member)

Fracture Toughness: In a toughness test of a given material, the stress – intensity factor at which a crack will propagate is measured. This is the critical stress intensity factor , referred to as the fracture toughness and denoted by the symbol K_{IC}.

$$N=\frac{K_{IC}}{K_I} \text{(N=factor of safety)}$$

EXAMPLE: For a shape with width 12m , crack length 65mm , thickness 30mm and applied loading 50MPa.Find the factor of safety.(K_{IC}=28.3MPa$\sqrt{m}$, S_Y=240MPa)



$N_1=\frac{240}{50} = 4.0$

From the ratio of a/w:

$$\frac{a}{w} = \frac{65 \times 10^{-3}}{12} = 0.0054$$

From table β≅ 1. $\beta = \frac{K_I}{\sigma\sqrt{\pi a}}$

$K_I = \sigma\sqrt{\pi a}$=50$\sqrt{\pi \frac{65}{2} 10^{-3}}$=15.97MPa$\sqrt{m}$

For safety factor: N_2=$\frac{K_{IC}}{K_I}$=$\frac{28.3}{15.97} = 1.77$

N_2 is better than N_1 in order to obtain a safe structure; therefore, to have a safe structure, loading should not be used to calculate safety factor.

Reference List:

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2. Static Failure theories, Ansel C. UGURAL & Saul K. Fenster, 2007, "Advanced Strength and Applied Elasticity", fourth edition.
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