Engineering Mechanics: ME101

Dr. Poonam Kumari

Department of Mechanical Engineering
Indian Institute of Technology Guwahati

D Block : Room No 201 : Tel: 3434

Statics: Lecture 1

05th Jan 2017

ME101: Division I–L2

Day	Timing
Tuesday	4:0-4:55
Wednesday	3:0-3:55
Thrusday	2:0-2:55

Tutorial Monday -8-8:55

Syllabus Handout ME101-2017 (Jan-May-17)

Plan of lectures and topics for ME101 (Till Mid Sem

	Topics		
1	Basic principles: Equivalent force system; Equations of equilibrium; Free body diagram; Reaction; Static indeterminacy, Stability. Center of Gravity and Moment of Inertia: First and second moment of area	5	
2	Structures: Difference between trusses, frames and beams, Assumptions followed in the analysis of structures; 2D truss; Method of joints; Method of section;	3	
3	Frame; Simple beam; types of loading and supports; Shear Force and bending Moment diagram in beams; Relation among load, shear force and bending moment	3	
4	Friction: Dry friction; Description and applications of friction in wedges, thrust bearing (disk friction), belt, screw, journal bearing (Axle friction); Rolling resistance	3	
5	Virtual work and Energy method: Virtual Displacement; Principle of virtual work; Applications of virtual work principle to machines; Mechanical efficiency; Work of a force/couple (springs etc.); Potential energy and equilibrium	3	
6	Kinematics of Particles: Rectilinear motion; Curvilinear motion; Use of Cartesian, polar and spherical coordinate system; Relative and constrained motion;	4	

Syllabus Handout ME101-2017 (Jan-May-17)

Plan of lectures and topics for ME101 (After Midsem)

It will be reframed as per requirement later

7	Radius of gyration; Parallel axis theorem; Product of inertia, Rotation of axes and principal moment of inertia; Moment of inertia of simple and composite bodies. Mass moment of inertia, Space curvilinear motion, Kinetics of Particles: Force, mass and acceleration; Work and energy;	5	
8	Impulse and momentum; Impact problems; System of particles	3	
9	Kinematics and Kinetics of Rigid Bodies: Translation; Fixed axis rotational;	3	
10	General plane motion; Coriolis acceleration; Work energy; Power; Potential energy	3	
11	Impulse-momentum and associated conservation principles;	2	
12	Euler equations of motion and its application.	2	

Texts:

[1] I. H. Shames, Engineering Mechanics: Statics and Dynamics, 4th Ed., PHI, 2002.

[2] F. P. Beer and E. R. Johnston, Vector Mechanics for Engineers, Vol I - Statics, Vol II – Dynamics, 3rd Ed., Tata McGraw Hill, 2000.

References:

[1] J. L. Meriam and L. G. Kraige, Engineering Mechanics, Vol I – Statics, Vol II – Dynamics, 5th Ed., John Wiley, 2002.

[2] R. C. Hibbler, Engineering Mechanics, Vols. I and II, Pearson Press, 2002.

[3] D Gross, W Haugher et al., Engineering Mechanics 1,11, Springer 2009

Grading Policy

Tutorial – 15 % (10+5), Quiz (both) – 15 %,

Mid-semester – 20 %, End-semester – 40 %,

Attendence – 5 %, Class participation – 5 %.

S. No.	Particulars	Date of Tutorial
1.	Tutorial No. 0	9.01.2017
2.	Tutorial No. 1	16.01.2017
3.	Tutorial No. 2	23.01.2017
4.	Tutorial No. 3 Quiz 1	30.01.2017 (Monday) 30.01.2017 (Evening)
5.	Tutorial No. 4	6.02.2017
6.	Tutorial No. 5	13.02.2017
7.	Tutorial No. 6	20.02.2017
8.	Midsemester Exam	27.02.2017 (Monday) 2PM – 4 PM
9.	Tutorial No. 7	06.03.2017
10.	Tutorial No. 8	20.03.2017
11.	Tutorial No. 9	27.03.2017
12.	Quiz 2 – 03.04.2017 (Monday)	
13.	Tutorial No. 10	10.04.2017
14.	Tutorial No. 11	17.04.2017
15.	Tutorial No. 12	24.04.2017
16.	Make up Tutorial	24.04.2017 (5 – 6 PM).
17.	Endsemester Exam	29.04.2017 (Saturday) 2 PM – 5 PM

Sr. No	Name	Div No	Hall No	Timing
1	Dr. Poonam Kumari	1	L2	Tuesday: 4-5 PM Wednesday: 3-4 PM Thursday: 2-3 PM
2	Dr. M. Pandey	3	L2	Tuesday: 9-10 AM Wednesday: 10-11 AM Thursday: 11-12 AM
3	Dr. A. K. Singh	2	L3	Tuesday: 4-5 PM Wednesday: 3-4 PM Thursday: 2-3 PM
4	Dr. B. Hazra	4	L3	Tuesday: 9-10 AM Wednesday: 10-11 AM Thursday: 11-12 AM

Sr No	Name of Tutor	Tutorial Group	Room No	Reporting Coordinator
1	Dr. Atanu Banerjee (ME)	MT 1	L1	Dr. Poonam Kumari
2	Dr. Swarup Bag (ME)	MT 2	L2	
3	Dr. Satyajit Panda (ME)	MT 3	L3	
4	Dr. Prasenjit Khanikar (ME)	MT 4	L4	
5	Dr. S. K. Dwivedy (ME)	MT 9	1G1	Dr. M Pandey
6	Dr. Dipankar Narayan Basu (ME)	MT 10	1G2	
7	Dr. Sangamesh Deepak R (ME)	MT 11	1207	
8	Dr. Amit Kumar (CL)	MT 12	2101	
9	Dr. Narayana Reddy (ME)	MT 5	1006	Dr. A.K. Singh
10	Dr. H. Sharma (CE)	MT 6	4001	
11	Dr. Pallap Ghosh (CL)	MT 7	2102	
12	Dr. S Talukdar (CE)	MT 8	4G3	
13	Dr. Sandip Das (CE)	MT 13	4G4	Dr. B. Hazra
14	Dr. K. Dasgupta (CE)	MT 14	4005	
15	Dr. Suresh Kartha (CE)	MT 15	4201	
16	Dr. G. Pattader Partho Sarathi (CL)	MT 16	3202	

Tutorial   Monday -8-8:55

Tutorial questions and answers will be sent through respective coordinator by every Friday.

Mode of conduct: First 40 minutes, Tutors shall answer the students queries and in last 15 minutes any one question (Tutor will decide randomly) will be solved by students.

75% Attendance Mandatory

Tutorials: One question will be solve on spot and rest of the question solved at hostel and submit it in the next class.

Mechanics

Oldest and the most highly developed branch of physics

Important foundation of engineering and its relevance continues to increase as its range of application grows

Mechanics is a branch of the physical sciences that is concerned with the state of rest or motion of bodies subjected to the action of forces.

The task of mechanics include the description and determination of the motion of bodies as well as the investigation of the forces associated with the motion.

Analytical Mechanics: Analytical methods of mathematics are applied with the aim of gaining principle insight into the laws of mechanics

Engineering Mechanics: Concentrate on the needs of the practicing engineer, engineer has to analyze bridges, building, machines, vehicles or components of microsystem to determine whether they are able to sustain certain loads or perform certain movements.

Classification of Mechanics

Based on the state of material

Mechanics of Solids
Hydrodynamic or gas dynamics

Based on state of rest or motion

Statics (Latin: status=standing) deals with the equilibrium of bodies subjected to forces

Dynamics (Greek: dynamic =force) is subdivided into kinematics and kinetics
Kimematics (Greek: kiness=movement) investigates the motion of bodies without referring to forces as a cause or result of the motion.

Historical Origin and contributions

Archimedes (287-212 BC): Lever and fulcrum, block, center of gravity and buoyancy

Leonardo da Vinci (1542-1519 AD) :observation of the equilibrium on inclined plane

Galileo Galilei (1564-1642 AD) :Law of gravitation

Johannes Kepler (1571-1630 AD) :Law of planetary motion

Mechanics: Fundamental Concepts

Length (Space): needed to locate position of a point in space, & describe size of the physical system Distances, Geometric Properties

Time: measure of succession of events basic quantity in Dynamics

Mass: quantity of matter in a body measure of inertia of a body (its resistance to change in velocity)

Force: represents the action of one body on another characterized by its magnitude, direction of its action, and its point of application
Force is a Vector quantity.

Mechanics: Fundamental Concepts

Newtonian Mechanics

Length, Time, and Mass are absolute concepts independent of each other

Force is a derived concept

not independent of the other fundamental concepts. Force acting on a body is related to the mass of the body and the variation of its velocity with time.

Force can also occur between bodies that are physically separated (Ex: gravitational, electrical, and magnetic forces)

Mechanics: Fundamental Concepts

Remember:

- Mass is a property of matter that does not change from one location to another.

- Weight refers to the gravitational attraction of the earth on a body or quantity of mass. Its magnitude depends upon the elevation at which the mass is located

- Weight of a body is the gravitational force acting on it

Mechanics: Idealizations

To simplify application of the theory

Particle: A body with mass but with dimensions that can be neglected.

Rigid Body: A body is called a rigid body if it does not deform under the influences of forces

The effect of a force on a rigid body is independent of the location of the point of application on the line of action.

In most cases, actual deformations occurring in structures, machines, mechanisms, etc. are relatively small, and rigid body assumption is suitable for analysis

Mechanics: Idealizations

Force:

The concept of force can be taken from our daily experience. Although forces cannot be seen or directly observed, we are familiar with their effect.

In Statics, bodies at rest are investigated.
E.g. To prevent a stone from falling, to keep it in equilibrium, we need to exert a force on it, for example our muscle force. In other words:

A force is a physical quantity that can be brought into equilibrium with gravity.

Force:

The direction of the force can be described by its line of action and its sense of direction (orientation).

A quantity determined by magnitude and direction is called a vector. In contrast to a free vector, which can be moved arbitrarily in space provided it maintains its direction, a force is tied to its line of action and has a point of application. Therefore

The Force is a bound vector

Classification of forces

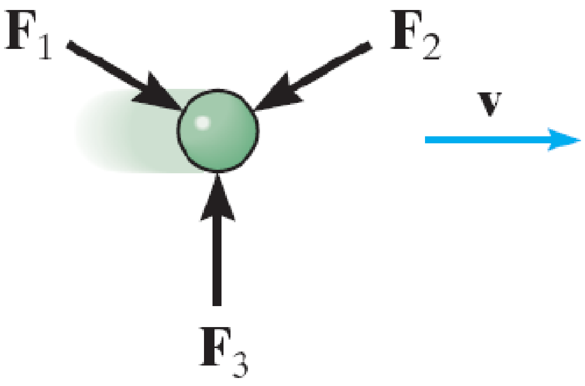
- **Concentrated force:** A single force with a line of action and a point of application.
- **Volume force:** is a force that is distributed over volume of the a body or portion.
- **Area force:** occur in the region where two bodies are in contact.
- **Active forces:** physically prescribed forces in a mechanical system
- **Reactive force:** generated if the freedom of movement of a body is constraint.

Mechanics: Newton's Three Laws of Motion

Basis of formulation of rigid body mechanics.

First Law: A particle originally at rest, or moving in a straight line with constant velocity, tends to remain in this state provided the particle is not subjected to an unbalanced force.

First law contains the principle of the equilibrium of forces → main topic of concern in Statics



Mechanics: Newton's Three Laws of Motion

Second Law: A particle of mass "m" acted upon by an unbalanced force "F" experiences an acceleration "a" that has the same direction as the force and a magnitude that is directly proportional to the force.

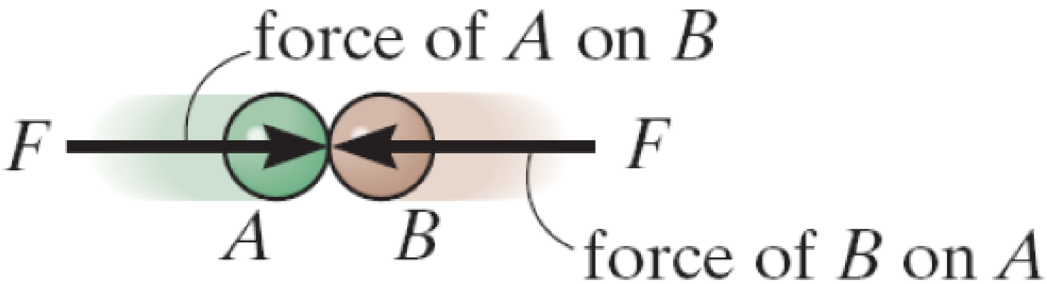
$F = ma$

Accelerated motion

Second Law forms the basis for most of the analysis in Dynamics

Mechanics: Newton's Three Laws of Motion

Third Law: The mutual forces of action and reaction between two particles are equal, opposite, and collinear.



Action – reaction

Third law is basic to our understanding of Force → Forces always occur in pairs of equal and opposite forces.

Mechanics: Units

Four Fundamental Quantities

Quantity	Dimensional Symbol	SI UNIT	
		Unit	Symbol
Mass	M	Kilogram	Kg
Length	L	Meter	M
Time	T	Second	s
Force	F	Newton	N

Basic Unit

$$F = ma$$ → N = kg.m/s^2

$$W = mg$$ → N = kg.m/s^2

1 Newton is the force required to give a mass of 1 kg an acceleration of 1 m/s^2

Mechanics: Units Prefixes

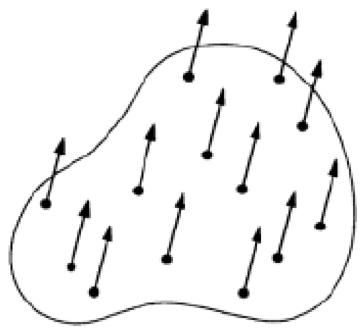
	Exponential Form	Prefix	SI Symbol
Multiple			
1 000 000 000	10^9	giga	G
1 000 000	10^6	mega	M
1 000	10^3	kilo	k
Submultiple			
0.001	10^{-3}	milli	m
0.000 001	10^{-6}	micro	μ
0.000 000 001	10^{-9}	nano	n

Solution of Static Problems

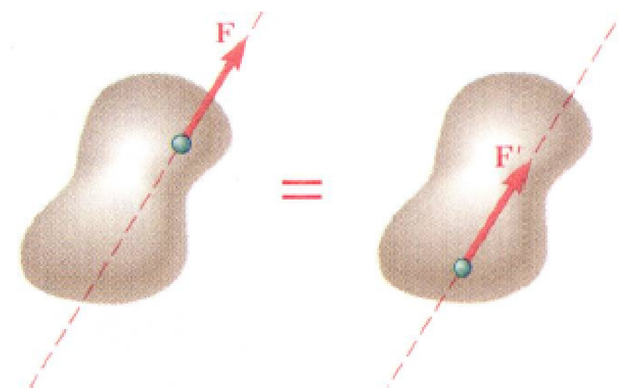
- Formulation of engineering problems
- Establishing a mechanical model that maps all of the essential characteristics of the real system.
- Solution of the mechanical problem using the established model.
- Discussion and interpretation of the solution.

L-2

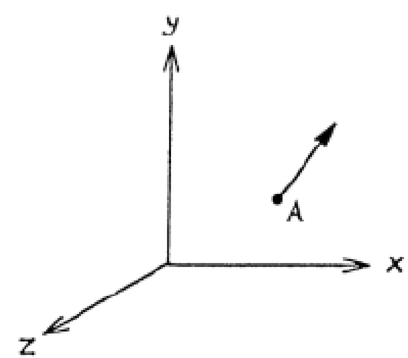
Vectors



Free Vector: whose action is not confined to or associated with a unique line in space
Ex: Movement of a body without rotation.



Sliding Vector: has a unique line of action in space but not a unique point of application
Ex: External force on a rigid body
→ Principle of Transmissibility
→ Imp in Rigid Body Mechanics



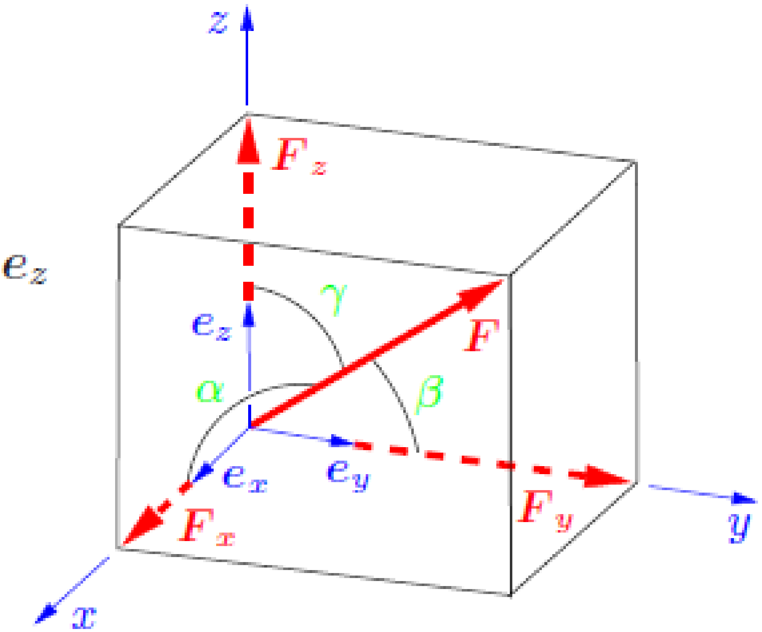
Fixed Vector: for which a unique point of application is specified
Ex: Action of a force on deformable body

Resolution of vectors in Cartesian coordinate

e = unit vector

Resultant Force vector

$$F = F_x + F_y + F_z = F_x\, e_x + F_y\, e_y + F_z\, e_z$$



Force vector magnitude

$$F = \sqrt{F_x^2 + F_y^2 + F_z^2}\,.$$

unit vectors e_x, e_y, e_z

The direction angles and therefore the direction of the force follow from

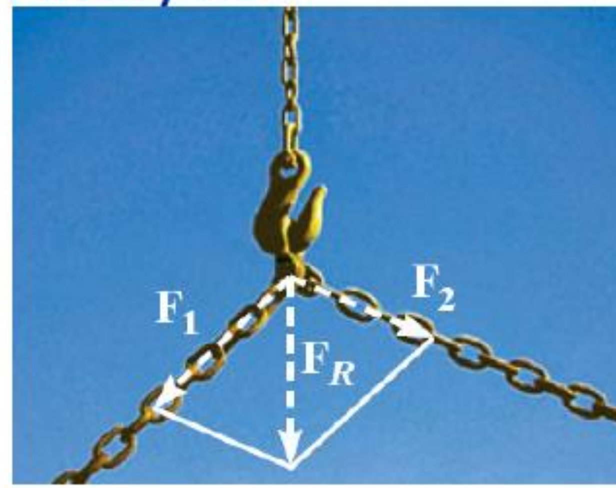
$$\cos\alpha = \frac{F_x}{F}\,, \quad \cos\beta = \frac{F_y}{F}\,, \quad \cos\gamma = \frac{F_z}{F}$$

Vector Addition: Procedure for Analysis

Parallelogram Law (Graphical)

Resultant Force (diagonal)

Components (sides of parallelogram)



Algebraic Solution

Using the coordinate system

Cosine law:

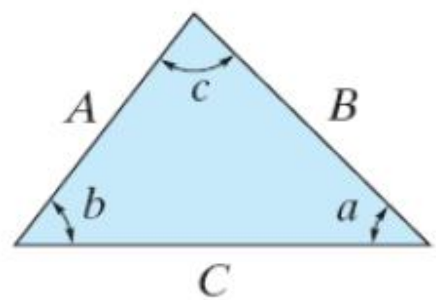
$$C = \sqrt{A^2 + B^2 - 2AB\cos c}$$

Sine law:

$$\frac{A}{\sin a} = \frac{B}{\sin b} = \frac{C}{\sin c}$$

Trigonometry (Geometry)

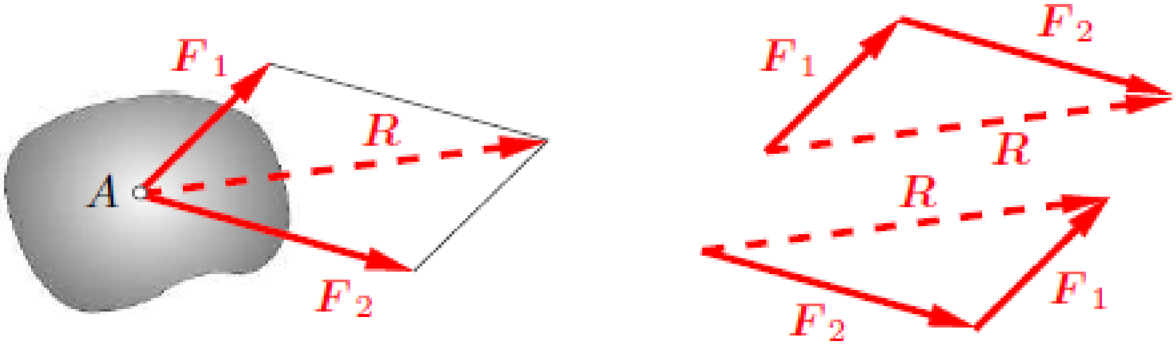
Resultant Force and Components from Law of Cosines and Law of Sines



Addition of Forces in a Plane

A systems of concentrated forces that have a common point of application are investigated. Such forces are called concurrent forces.

If all the forces acting on a body act in a plane, they are called coplanar forces

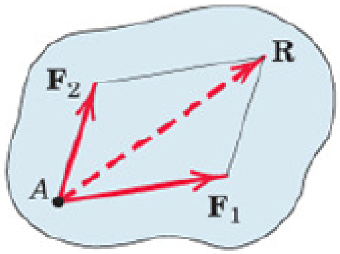


Since in this case the force vectors are sliding vectors, they may be applied at any point along their lines of action without changing their effect on the body (principle of transmissibility)

Components and Projections of Force

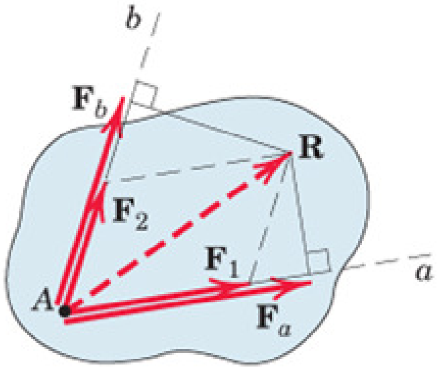
Components of a Force are not necessarily equal to the Projections of the Force unless the axes on which the forces are projected are orthogonal (perpendicular to each other).

F_1 and **F_2** are components of **R**. **$R = F_1 + F_2$**



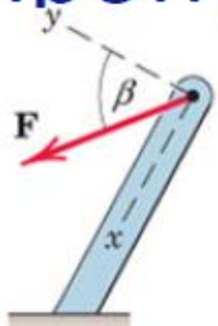
F_a and **F_b** are perpendicular projections on axes a and b, respectively.

$R \neq F_a + F_b$ unless a and b are perpendicular to each other

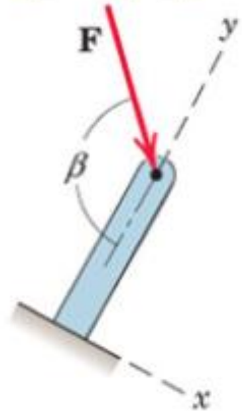


Components of Force

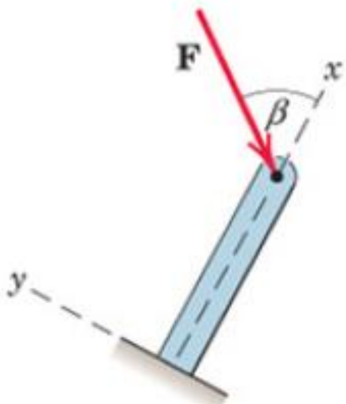
Examples



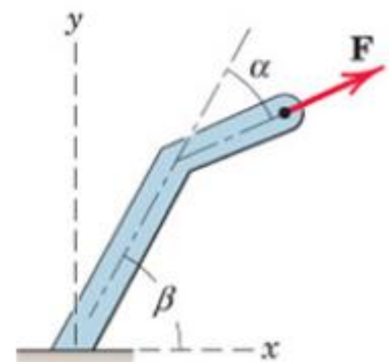
$$F_x = F \sin \beta$$
$$F_y = F \cos \beta$$



$$F_x = F \sin(\pi - \beta)$$
$$F_y = -F \cos(\pi - \beta)$$

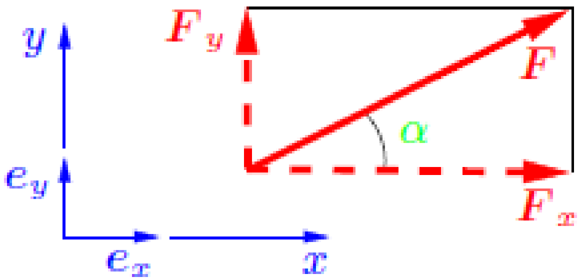


$$F_x = -F \cos \beta$$
$$F_y = -F \sin \beta$$

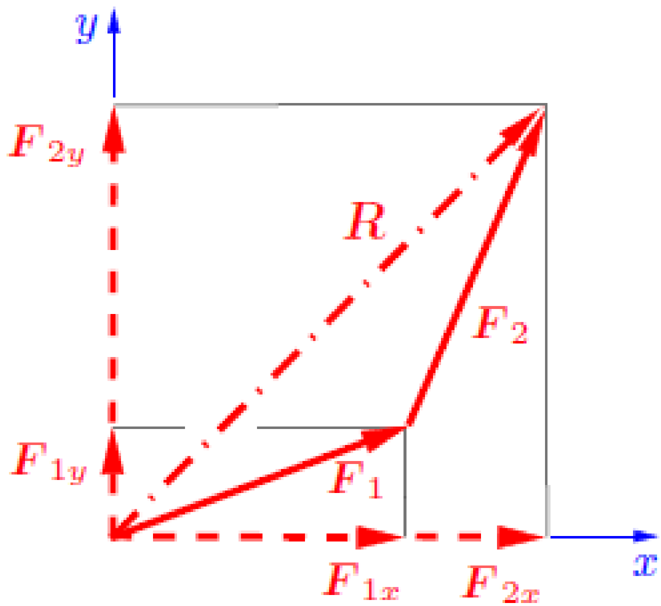


$$F_x = F \cos(\beta - \alpha)$$
$$F_y = F \sin(\beta - \alpha)$$

Addition of Forces in a Plane



$$F_x = F\cos\alpha\,, \qquad F_y = F\sin\alpha\,,$$
$$F = \sqrt{F_x^2 + F_y^2}\,, \qquad \tan\alpha = \frac{F_y}{F_x}\,.$$

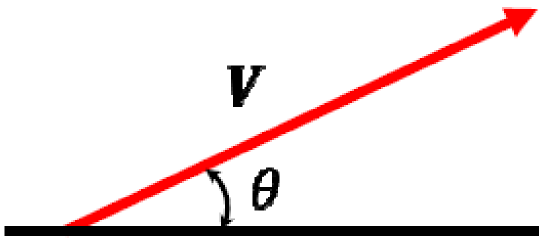


$$R_x = F_{1x} + F_{2x}\,, \quad R_y = F_{1y} + F_{2y}$$

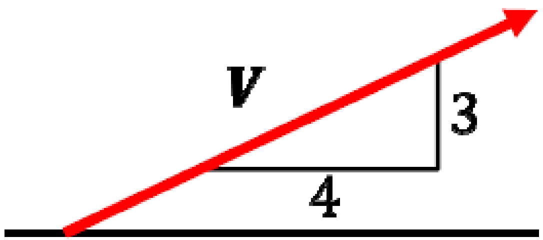
$$R_x = \sum F_{ix}\,, \quad R_y = \sum F_{iy}$$

$$R = \sqrt{R_x^2 + R_y^2}\,, \quad \tan\alpha_R = \frac{R_y}{R_x}\,.$$

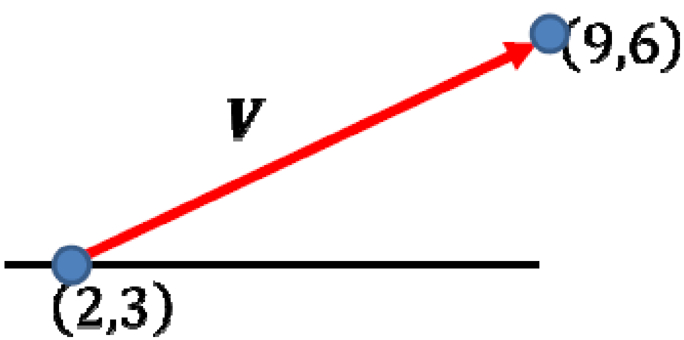
Vector



$$\boldsymbol{V} = V(\cos\theta\,\boldsymbol{i} + \sin\theta\,\boldsymbol{j})$$



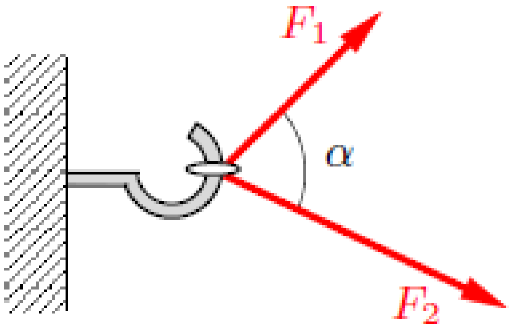
$$\boldsymbol{V} = V\left(\frac{4\boldsymbol{i} + 3\boldsymbol{j}}{\sqrt{4^2 + 3^2}}\right)$$



$$\boldsymbol{V} = V\left(\frac{(9-2)\boldsymbol{i} + (6-3)\boldsymbol{j}}{\sqrt{(9-2)^2 + (6-3)^2}}\right)$$

Ex-1

A hook carries two forces F1 and F2, as shown in figure, Determine the magnitude and direction of the resultant.

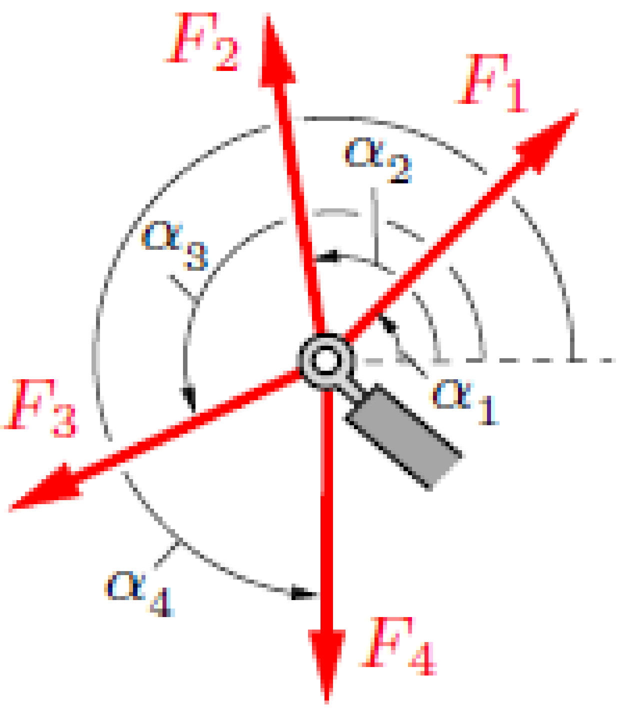


Ex-2

An eyebolt is subjected to four forces (F1 = 12 kN, F2 = 8 kN, F3 = 18 kN, F4 = 4 kN) that act under given angles

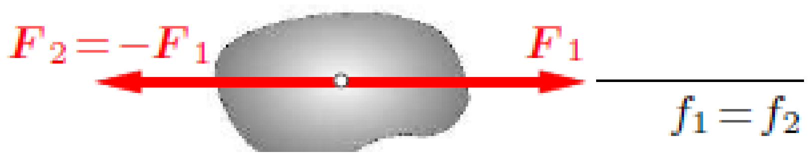
$(\alpha_1 = 45°, \alpha_2 = 100°, \alpha_3 = 205°, \alpha_4 = 270°)$

with respect to the horizontal as shown in figure. Determine the magnitude and direction of the resultant.



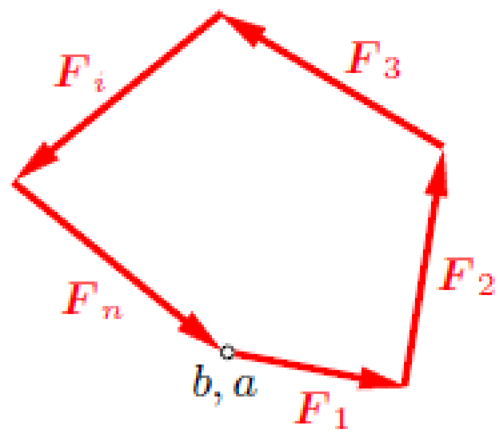
Equation of Equilibrium

Two forces are in equilibrium if they are oppositely directed on the same line of action and have the same magnitude.



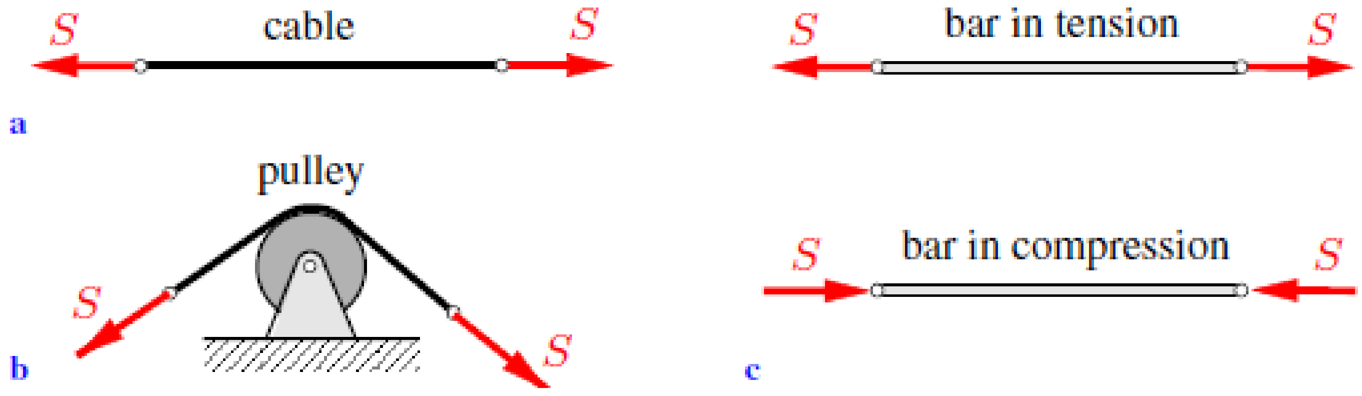
$$R = F_1 + F_2 = 0$$

A system of concurrent forces is in equilibrium if the resultant is zero



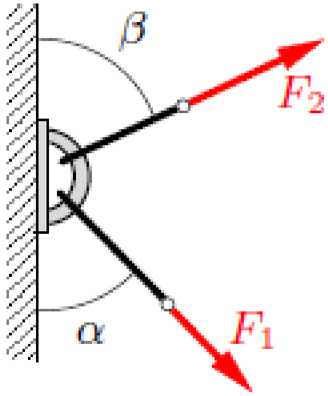
$$R = \sum F_i = 0 .$$

Example of coplanar forces



Ex-3

Two cables are attached to an eye. The directions of the forces F1 and F2 in the cables are given by the angles α and β.
Determine the magnitude of the force H exerted from the wall onto the eye

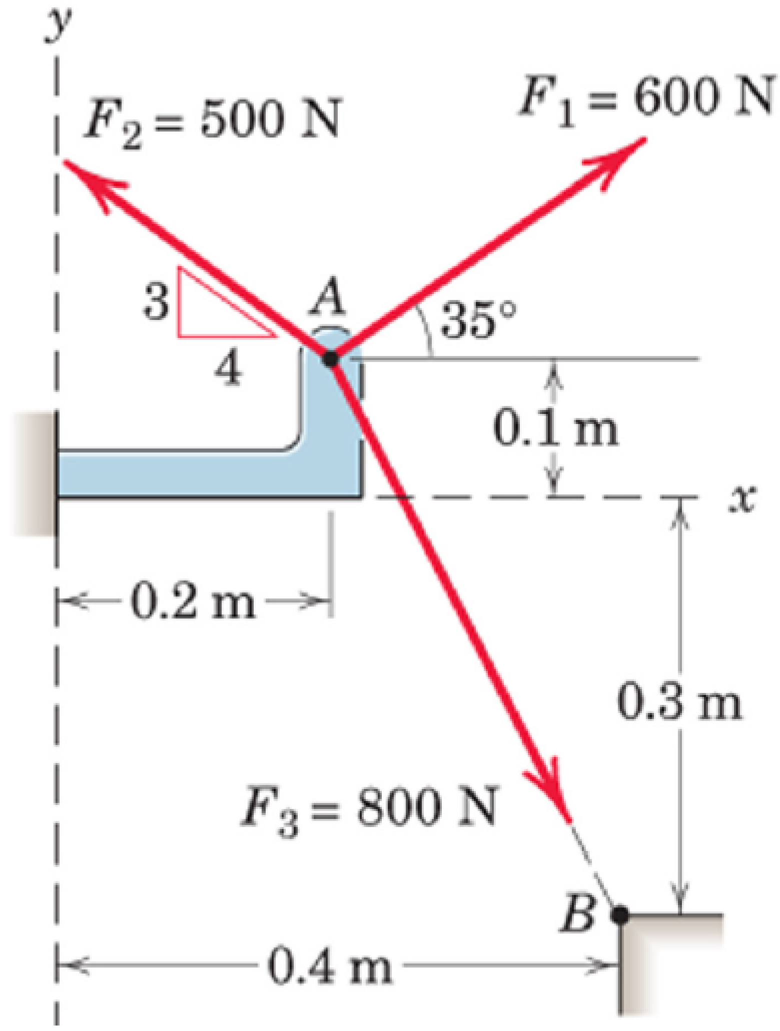


a

Components of Force

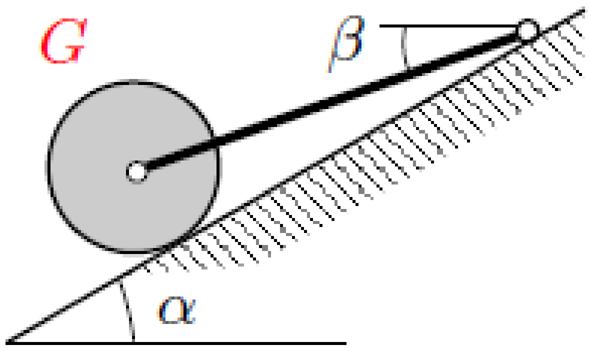
Example 1:

Determine the x and y scalar components of F_1, F_2, and F_3 acting at point A of the bracket

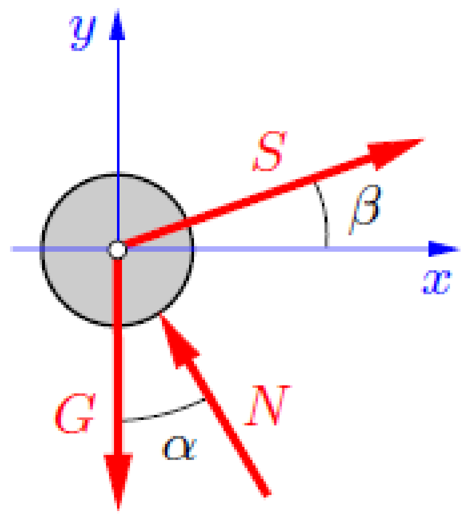


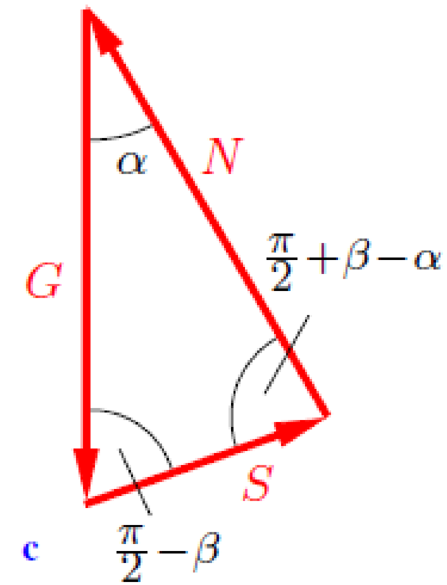
Ex-4

- A wheel with weight G is held on a smooth inclined plane by a cable (fig). Determine the force in the cable and the contact force between the plane and the wheel.



Steps





Graphical

$$\underline{\underline{S}} = G \frac{\sin\alpha}{\sin(\frac{\pi}{2} + \beta - \alpha)} = \underline{\underline{G \frac{\sin\alpha}{\cos(\alpha - \beta)}}},$$

$$\underline{\underline{N}} = G \frac{\sin(\frac{\pi}{2} - \beta)}{\sin(\frac{\pi}{2} + \beta - \alpha)} = \underline{G \frac{\cos\beta}{\cos(\alpha - \beta)}}.$$

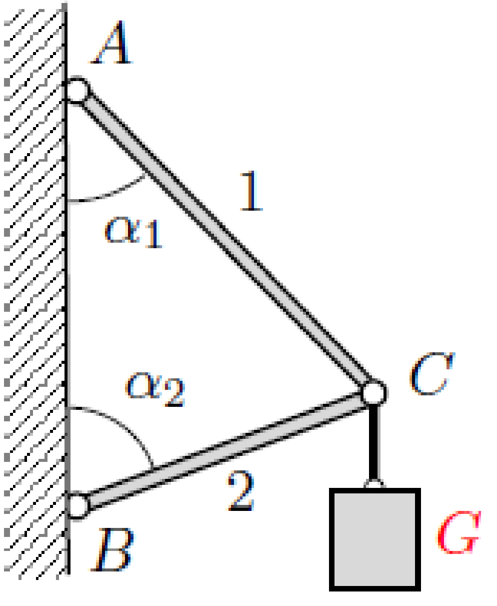
Equation of equilibrium

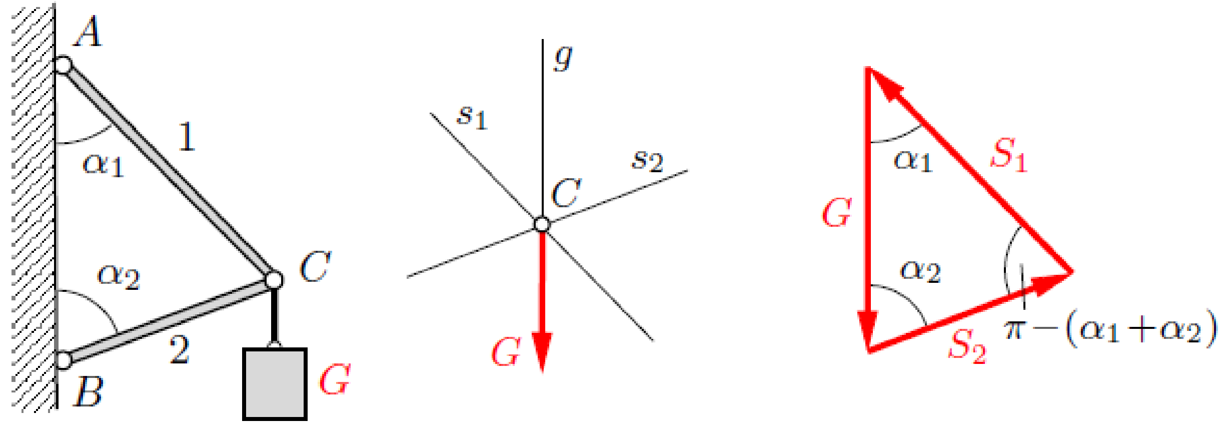
$$\sum F_{ix} = 0 \ : \qquad S\cos\beta - N\sin\alpha = 0\,,$$

$$\sum F_{iy} = 0 \ : \quad S\sin\beta + N\cos\alpha - G = 0\,.$$

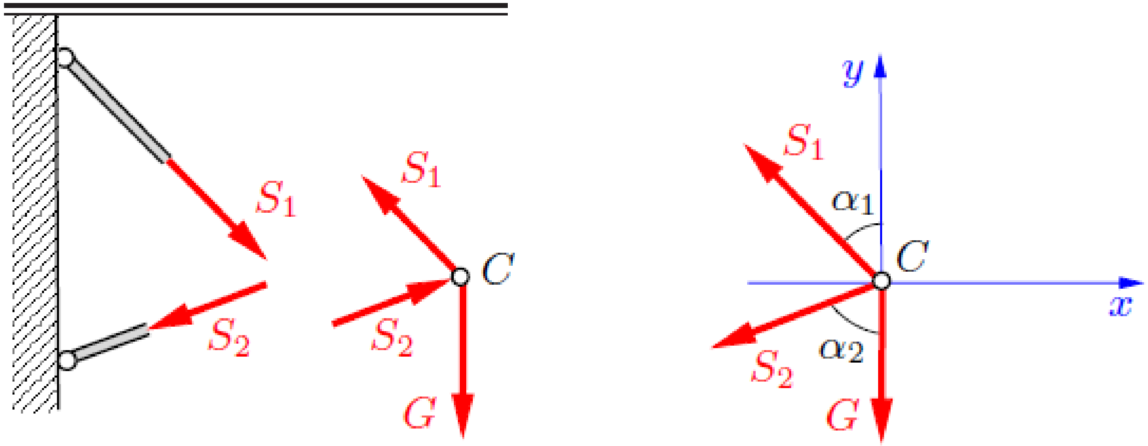
Ex-5

Two bars 1 and 2 are attached at A and B to a wall by smooth pins. They are pin-connected at C and subjected to a weight G. Calculate the forces in the bars





$$\underline{\underline{S_1 = G\,\frac{\sin\alpha_2}{\sin(\alpha_1+\alpha_2)}}}\,, \qquad \underline{\underline{S_2 = G\,\frac{\sin\alpha_1}{\sin(\alpha_1+\alpha_2)}}}\,.$$



$$\rightarrow: \qquad -S_1\sin\alpha_1 - S_2\sin\alpha_2 = 0\,,$$

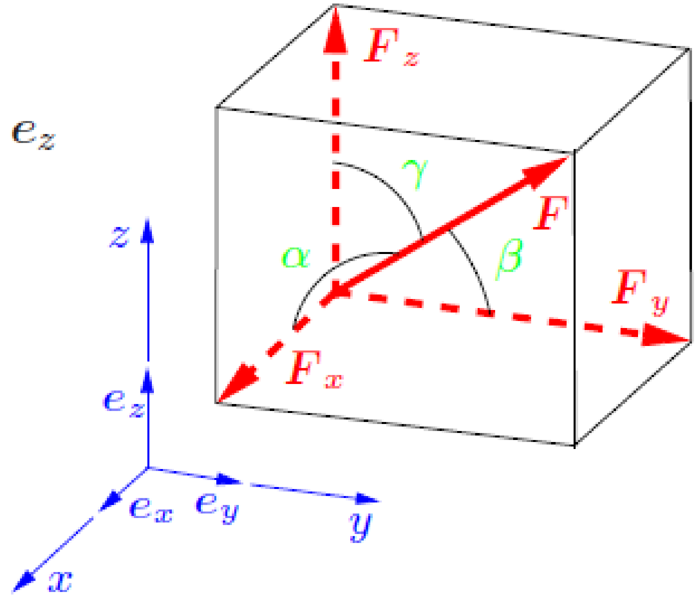
$$\uparrow: \quad S_1\cos\alpha_1 - S_2\cos\alpha_2 - G = 0$$

lead to

$$S_1 = G\,\frac{\sin\alpha_2}{\sin(\alpha_1+\alpha_2)}\,, \quad S_2 = -\,G\,\frac{\sin\alpha_1}{\sin(\alpha_1+\alpha_2)}\,.$$

Three-dimensional force system

A force can be resolved uniquely into three components in space



$$\boldsymbol{F} = \boldsymbol{F}_x + \boldsymbol{F}_y + \boldsymbol{F}_z = F_x\,\boldsymbol{e}_x + F_y\,\boldsymbol{e}_y + F_z\,\boldsymbol{e}_z$$

The magnitude and the direction of F are given by

$$F = \sqrt{F_x^2 + F_y^2 + F_z^2}$$

$$\cos\alpha = \frac{F_x}{F}\,, \quad \cos\beta = \frac{F_y}{F}\,, \quad \cos\gamma = \frac{F_z}{F}$$

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

The coordinates of the resultant in space are thus given by

$$R_x = \sum F_{ix}\,, \quad R_y = \sum F_{iy}\,, \quad R_z = \sum F_{iz}$$

Three-dimensional force system

- **The magnitude and direction of R follow as**

$$R = \sqrt{R_x^2 + R_y^2 + R_z^2}\,,$$

$$\cos\alpha_R = \frac{R_x}{R}\,, \quad \cos\beta_R = \frac{R_y}{R}\,, \quad \cos\gamma_R = \frac{R_z}{R}\,.$$

A spatial system of concurrent forces is in equilibrium if the resultant is the zero vector

$$R = \sum F_i = 0$$

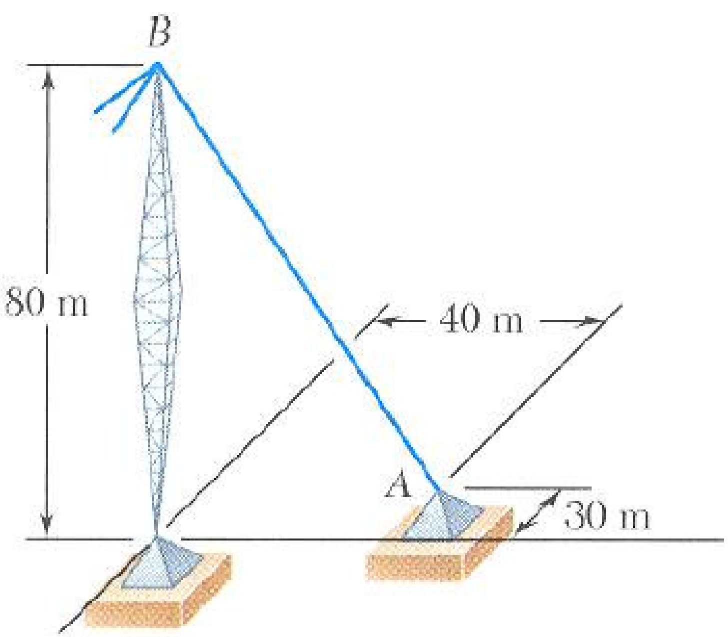
$$\sum F_{ix} = 0\,, \quad \sum F_{iy} = 0\,, \quad \sum F_{iz} = 0\,,$$

Rectangular Components in Space

Example: The tension in the guy wire is 2500 N. Determine:

a) components F_x, F_y, F_z of the force acting on the bolt at A,

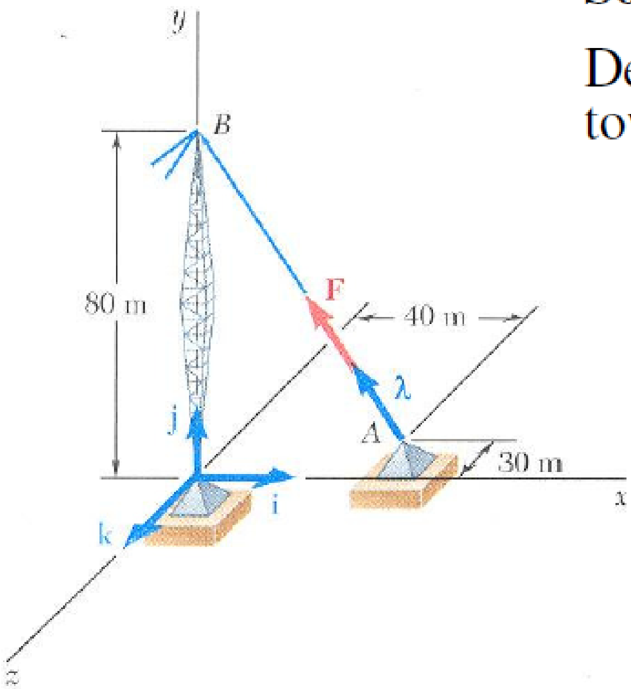
b) the angles q_x, q_y, q_z defining the direction of the force



SOLUTION:

- Based on the relative locations of the points A and B, determine the unit vector pointing from A towards B.
- Apply the unit vector to determine the components of the force acting on A.
- Noting that the components of the unit vector are the direction cosines for the vector, calculate the corresponding angles.

Rectangular Components in Space



Solution

Determine the unit vector pointing from A towards B.

$$\boldsymbol{AB} = -40\boldsymbol{i} + 80\boldsymbol{j} + 30\boldsymbol{k}$$

$$AB = \sqrt{(-40)^2 + (80)^2 + (30)^2} = 94.3$$

$$\boldsymbol{\lambda} = \frac{\boldsymbol{AB}}{AB} = \frac{-40\boldsymbol{i} + 80\boldsymbol{j} + 30\boldsymbol{k}}{94.3}$$

$$= -0.424\boldsymbol{i} + 0.848\boldsymbol{j} + 0.318\boldsymbol{k}$$

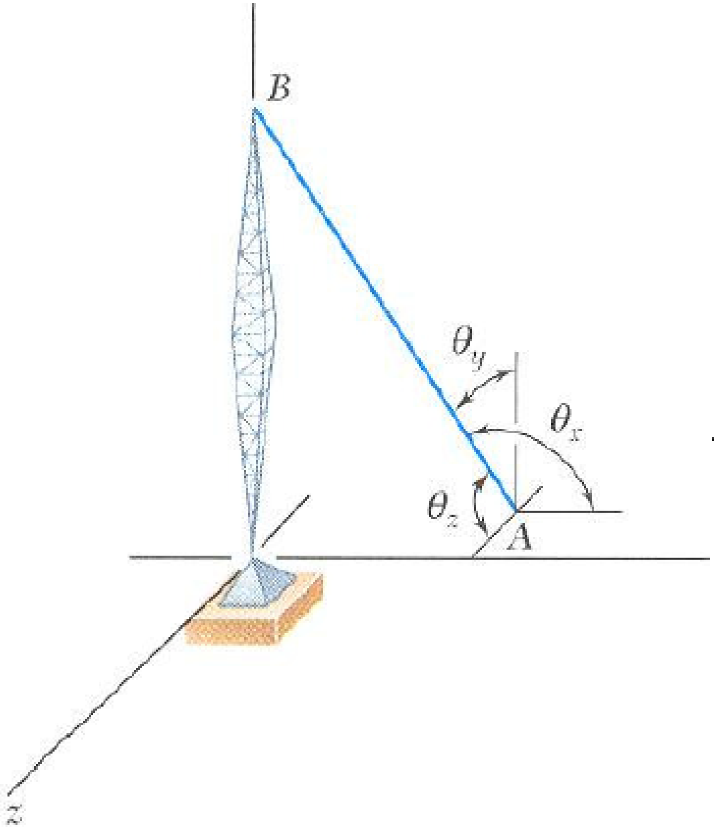
Determine the components of the force.

$$\boldsymbol{F} = F\boldsymbol{\lambda} \quad = 2500(-0.424\boldsymbol{i} + 0.848\boldsymbol{j} + 0.318\boldsymbol{k})$$

$$= -1060\boldsymbol{i} + 2120\boldsymbol{j} + 795\boldsymbol{k}$$

$$F_x = -1060\,N$$
$$F_y = 2120\,N$$
$$F_z = 795\,N$$

Rectangular Components in Space



Solution

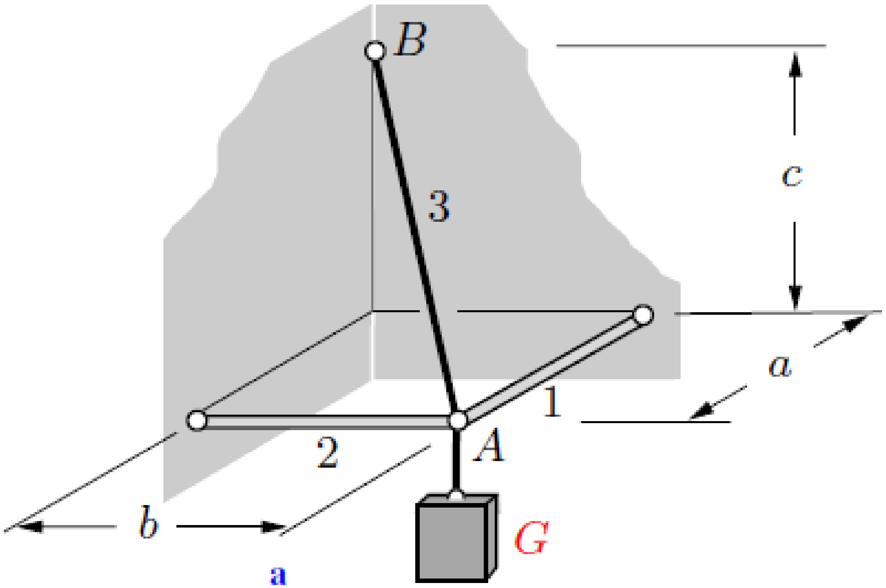
Noting that the components of the unit vector are the direction cosines for the vector, calculate the corresponding angles.

$$\boldsymbol{\lambda} = \cos\theta_x \boldsymbol{i} + \cos\theta_y \boldsymbol{j} + \cos\theta_z \boldsymbol{k}$$
$$= -0.424\boldsymbol{i} + 0.848\boldsymbol{j} + 0.318\boldsymbol{k}$$

$$\theta_x = 115.1^\circ$$
$$\theta_y = 32.0^\circ$$
$$\theta_z = 71.5^\circ$$

Ex-6

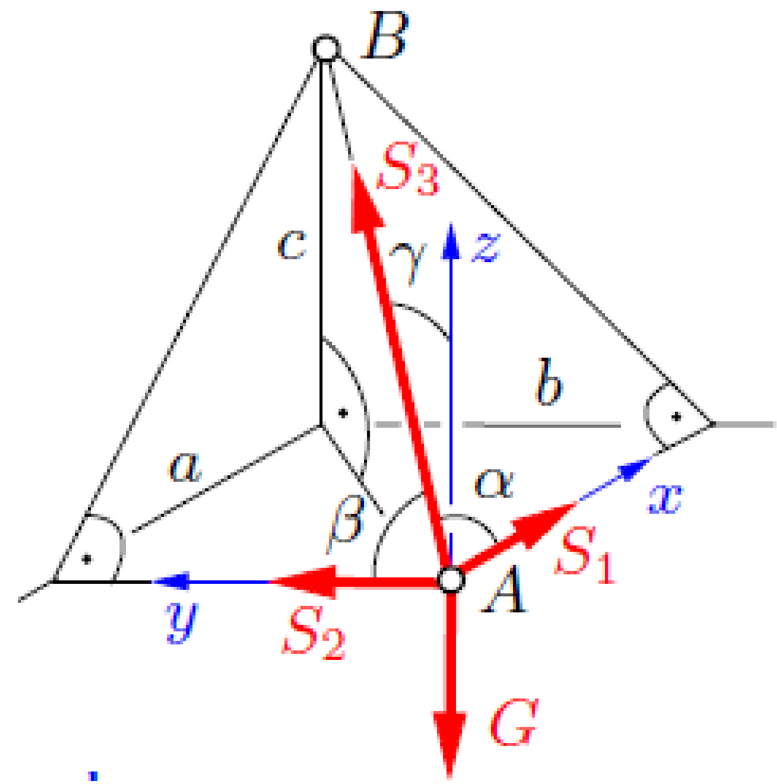
A structure consists of two bars 1 and 2 and a rope 3 (weights negligible). It is loaded in A by a box of weight G (Fig.). Determine the forces in the bars and in the rope.



a

We isolate pin A by passing imaginary sections through the bars and the rope. The internal forces are made visible in the free-body diagram and they are assumed to be tensile
forces

Solution



The equilibrium conditions are

$$\sum F_{ix} = 0 : \quad S_1 + S_3 \cos\alpha = 0\,,$$

$$\sum F_{iy} = 0 : \quad S_2 + S_3 \cos\beta = 0\,,$$

$$\sum F_{iz} = 0 : \quad S_3 \cos\gamma - G = 0\,.$$

With the diagonal $\overline{AB} = \sqrt{a^2 + b^2 + c^2}$,

The angles α, β and γ can be taken

$$\cos\alpha = \frac{a}{\sqrt{a^2+b^2+c^2}}\,, \quad \cos\beta = \frac{b}{\sqrt{a^2+b^2+c^2}}\,, \quad \cos\gamma = \frac{c}{\sqrt{a^2+b^2+c^2}}$$

$$\underline{\underline{S_3}} = \frac{G}{\cos\gamma} = \underline{\underline{G\,\frac{\sqrt{a^2+b^2+c^2}}{c}}}\,,$$

$$\underline{\underline{S_1}} = -S_3\cos\alpha = -G\,\frac{\cos\alpha}{\cos\gamma} = \underline{\underline{-G\,\frac{a}{c}}}$$

$$\underline{\underline{S_2}} = -S_3\cos\beta = -G\,\frac{\cos\beta}{\cos\gamma} = \underline{\underline{-G\,\frac{b}{c}}}\,.$$

Summary

- The lines of action of a system of concurrent forces intersect at a point.
- The resultant of a system of concurrent forces is given by the vector $\boldsymbol{R} = \sum \boldsymbol{F}_i$. In coordinates,

$$R_x = \sum F_{ix}\,, \qquad R_y = \sum F_{iy}\,, \qquad R_z = \sum F_{iz}\,.$$

  In the case of a coplanar system, the z-components vanish. Note: the coordinate system may be chosen arbitrarily; an appropriate choice may save computational work.
- The equilibrium condition for a system of concurrent forces is $\sum \boldsymbol{F}_i = \boldsymbol{0}$. In coordinates,

$$\sum F_{ix} = 0\,, \qquad \sum F_{iy} = 0\,, \qquad \sum F_{iz} = 0\,.$$

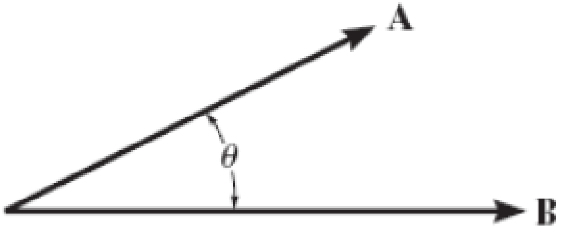
  In the case of a coplanar problem, the z-components vanish.

Summary

- In order to solve force problems the following steps are usually necessary:
  - ◇ Isolate the body (point).
  - ◇ Sketch the free-body diagram: introduce all of the forces exerted *on* the body; assume the internal forces in bars to be tensile forces.
  - ◇ Choose a coordinate system.
  - ◇ Formulate the equilibrium conditions (3 equations in spatial problems, 2 equations in coplanar problems).
  - ◇ Solve the equilibrium conditions.
- The force acting at the point of contact between two bodies can be made visible by separating the bodies. In the case of smooth surfaces, it is perpendicular to the plane of contact.

L-3

Vector Products



Dot Product $\boldsymbol{A}.\boldsymbol{B} = AB\cos\theta$

Applications:

to determine the angle between two vectors

to determine the projection of a vector in a specified direction

A.B* = *B.A (commutative)

A*.(*B*+*C*) = *A.B*+*A.C (distributive operation)

$$\boldsymbol{A}.\boldsymbol{B} = (A_x\boldsymbol{i} + A_y\boldsymbol{j} + A_z\boldsymbol{k}).(B_x\boldsymbol{i} + B_y\boldsymbol{j} + B_z\boldsymbol{k})$$

$$= A_xB_x + A_yB_y + A_zB_z$$

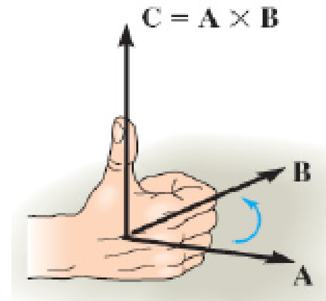
$$\boldsymbol{i}.\boldsymbol{i} = 1$$

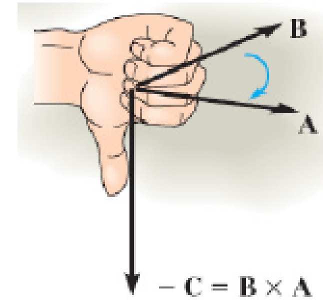
$$\boldsymbol{i}.\boldsymbol{j} = 0$$

Vector Products

Cross Product: $\boldsymbol{A} \times \boldsymbol{B} = \boldsymbol{C} = AB\sin\theta$

$$\boldsymbol{A} \times \boldsymbol{B} = -(\boldsymbol{B} \times \boldsymbol{A})$$

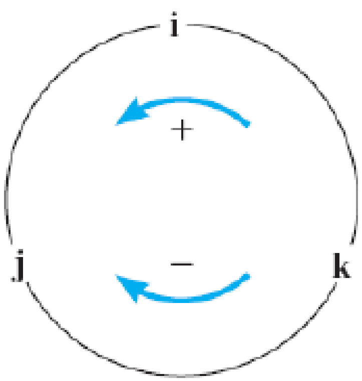


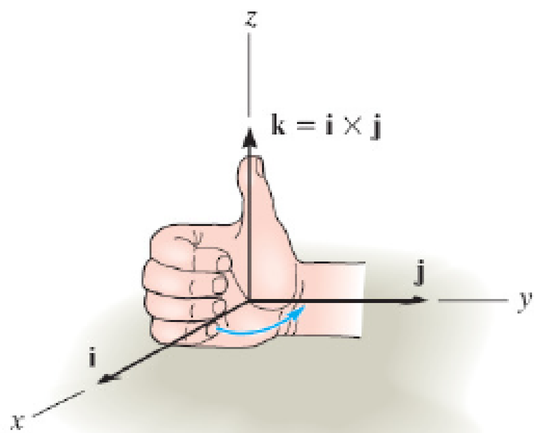


$$\boldsymbol{A} \times \boldsymbol{B} = (A_x\boldsymbol{i} + A_y\boldsymbol{j} + A_z\boldsymbol{k}) \times (B_x\boldsymbol{i} + B_y\boldsymbol{j} + B_z\boldsymbol{k})$$

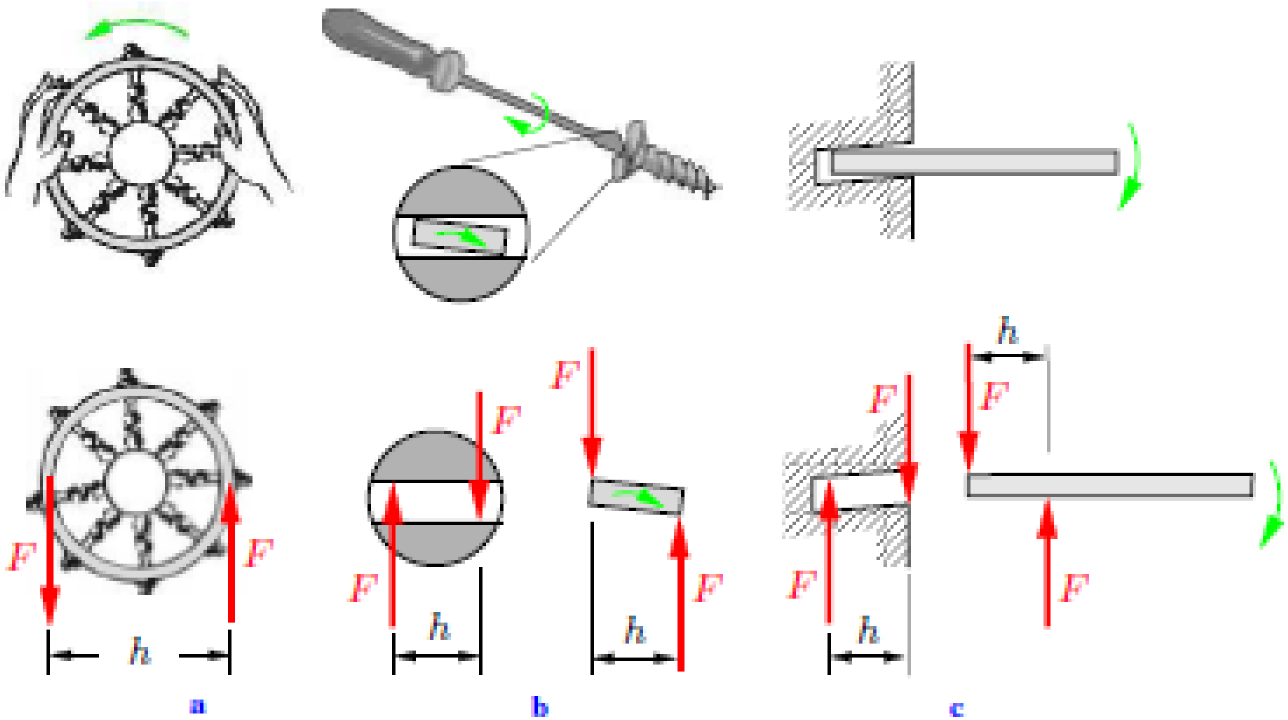
$$= \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} (A_yB_z - A_zB_y)\boldsymbol{i} + (A_zB_x - A_xB_z)\boldsymbol{j} + (A_xB_y - A_yB_x)\boldsymbol{k}$$

Cartesian Vector





$$\mathbf{i} \times \mathbf{j} = \mathbf{k} \quad \mathbf{i} \times \mathbf{k} = -\mathbf{j} \quad \mathbf{i} \times \mathbf{i} = 0$$
$$\mathbf{j} \times \mathbf{k} = \mathbf{i} \quad \mathbf{j} \times \mathbf{i} = -\mathbf{k} \quad \mathbf{j} \times \mathbf{j} = 0$$
$$\mathbf{k} \times \mathbf{i} = \mathbf{j} \quad \mathbf{k} \times \mathbf{j} = -\mathbf{i} \quad \mathbf{k} \times \mathbf{k} = 0$$



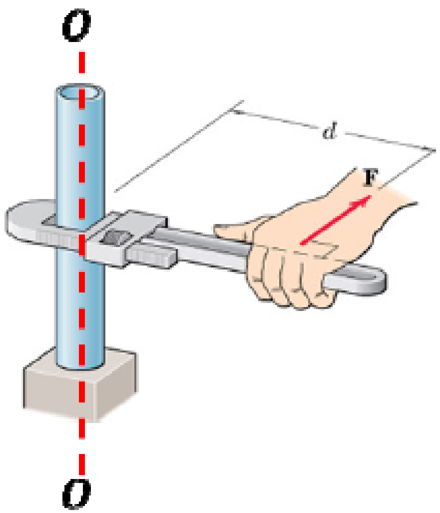
The effect of a couple on a rigid body is unambiguously determined by its moment

The moment incorporates two quantities: first, its magnitude M which is given by the product of the perpendicular distance h of the action lines (Fig.) and the magnitude F of the forces

$$M = hF$$

and, secondly, its sense of rotation.
In the figures, the sense of rotation is represented by a curved arrow.

Moment of a Force (Torque)

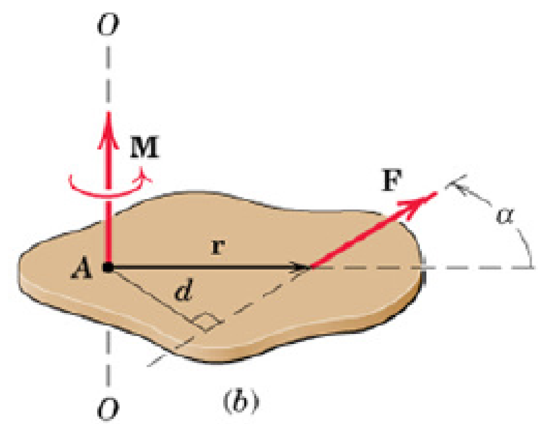


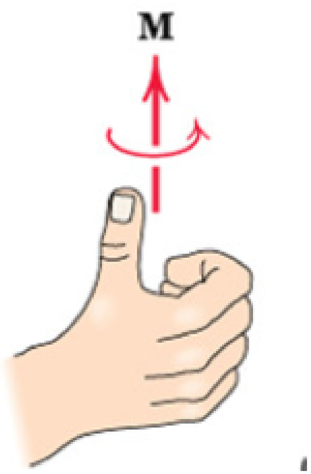
Moment about axis O-O is $\boldsymbol{M_o} = Fd$

Magnitude of $\boldsymbol{M_o}$ measures tendency of **F** to cause rotation of the body about an axis along $\boldsymbol{M_o}$.

Moment about axis O-O is $\boldsymbol{M_o} = Frsin\alpha$

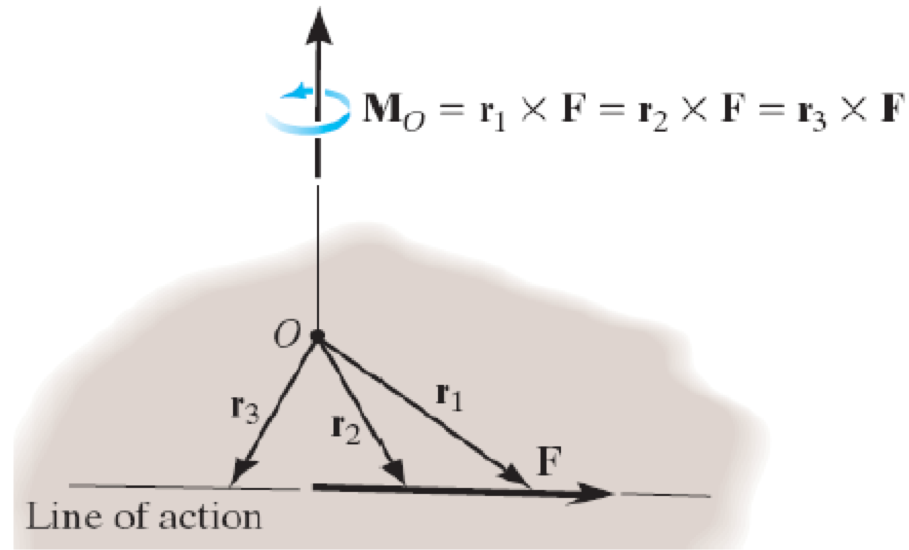
$$\boldsymbol{M_o} = \boldsymbol{r} \times \boldsymbol{F}$$





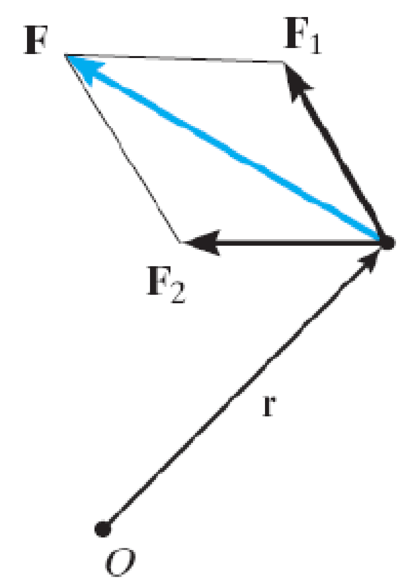
Sense of the moment may be determined by the right-hand rule

Moment of a Force



Principle of Transmissibility

Any force that has the same magnitude and direction as **F**, is *equivalent* if it also has the same line of action and therefore, produces the same moment.



Varignon's Theorem (Principle of Moments)

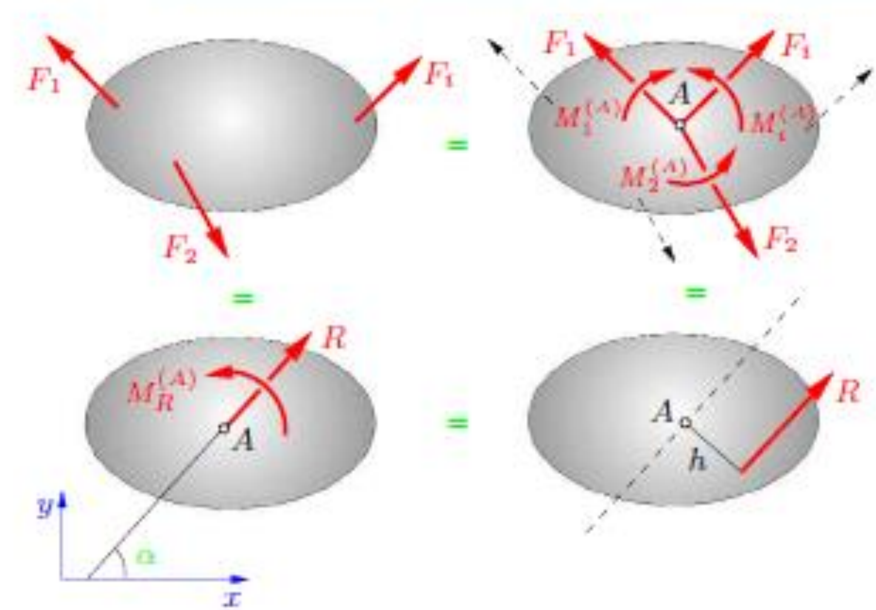
Moment of a Force about a point is equal to the sum of the moments of the force's components about the point.

$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F} = \mathbf{r} \times (\mathbf{F}_1 + \mathbf{F}_2) = \mathbf{r} \times \mathbf{F}_1 + \mathbf{r} \times \mathbf{F}_2$$

Resultant of Systems of Coplanar Forces

Consider a rigid body that is subjected to a general system of coplanar forces

The given general system of forces is replaced by a system of concurrent forces and a system of moments. These two systems can be reduced to a resultant force R with the components R_x and R_y and a resultant moment $M^{(A)}$



$$R_x = \sum F_{ix}\,, \quad R_y = \sum F_{iy}\,, \quad M_R^{(A)} = \sum M_i^{(A)}$$

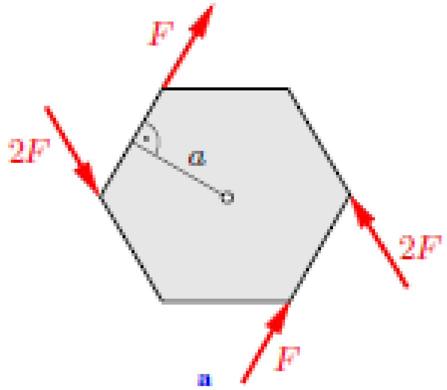
$$R = \sqrt{R_x^2 + R_y^2}\,, \quad \tan\alpha = \frac{R_y}{R_x}$$

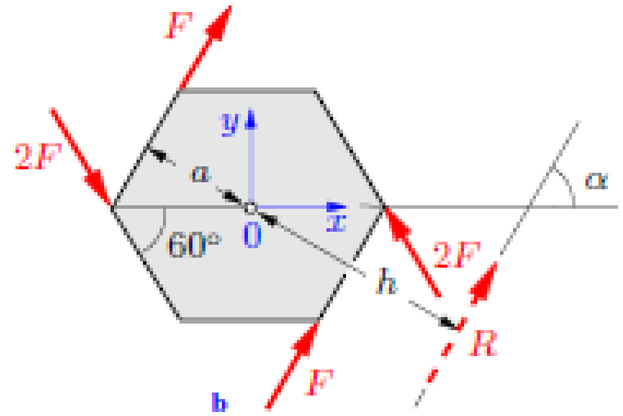
The system of the resultant R (action line through A) and the moment $M_R^{(A)}$ may be further simplified. It is equivalent to the single force R alone if the action line is moved appropriately. The perpendicular distance h (Fig.) must be chosen in such a way that the moment $M_R^{(A)}$ equals hR, i.e., $hR = M^{(A)}$

$$h = \frac{M_R^{(A)}}{R}$$

Ex-7

A disc is subjected to four forces as shown in Fig. 1. The forces have the given magnitudes F or 2 F, respectively. Determine the magnitude and direction of the resultant and the location of its line of action.





We choose a coordinate system x, y (Fig. 2), and its origin 0 is taken as the point of reference. According to the sign convention, positive moments tend to rotate the disk counterclockwise

$$R_x = \sum F_{ix} = 2\,F\cos 60^\circ + F\cos 60^\circ$$
$$\qquad + F\cos 60^\circ - 2\,F\cos 60^\circ = F\,,$$
$$R_y = \sum F_{iy} = -\,2\,F\sin 60^\circ + F\sin 60^\circ$$
$$\qquad + F\sin 60^\circ + 2\,F\sin 60^\circ = \sqrt{3}\,F\,,$$
$$M_R^{(0)} = \sum M_i^{(0)} = 2\,a\,F + a\,F + 2\,a\,F - a\,F = 4\,a\,F$$

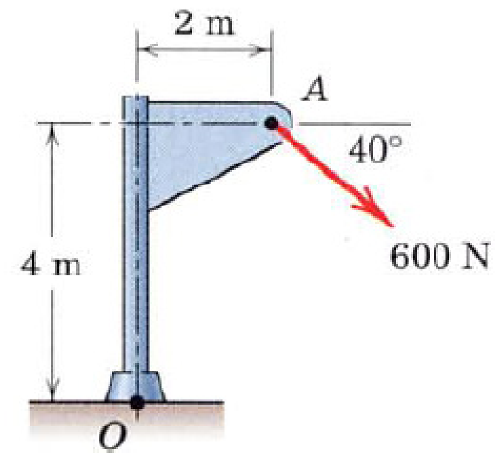
$$\underline{\underline{R}} = \sqrt{R_x^2 + R_y^2} = \underline{\underline{2\,F}}\,, \quad \tan\alpha = \frac{R_y}{R_x} = \sqrt{3} \quad \rightarrow \quad \underline{\underline{\alpha = 60^\circ}}\,.$$

The perpendicular distance of the resultant from point 0 follows from (3.11):

$$\underline{\underline{h}} = \frac{M_R^{(0)}}{R} = \frac{4\,a\,F}{2\,F} = \underline{\underline{2\,a}}\,.$$

Moment: Example

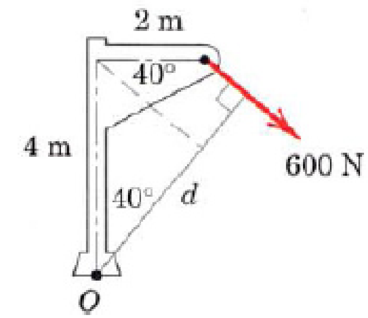
Calculate the magnitude of the moment about the base point O of the 600 N force in different ways



Solution 1.

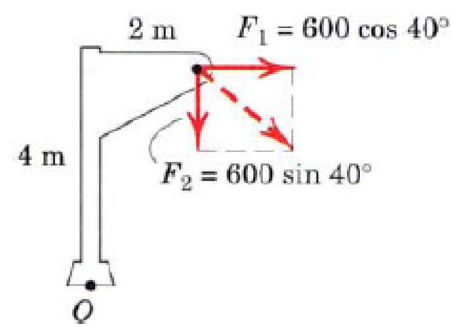
Moment about O is

$$M_o = dF \qquad d = 4cos40^o + 2sin40^o = 4.35m$$



$$M_o = 600(4.35) = 2610\ N.m \ \textbf{Ans}$$

Solution 2.



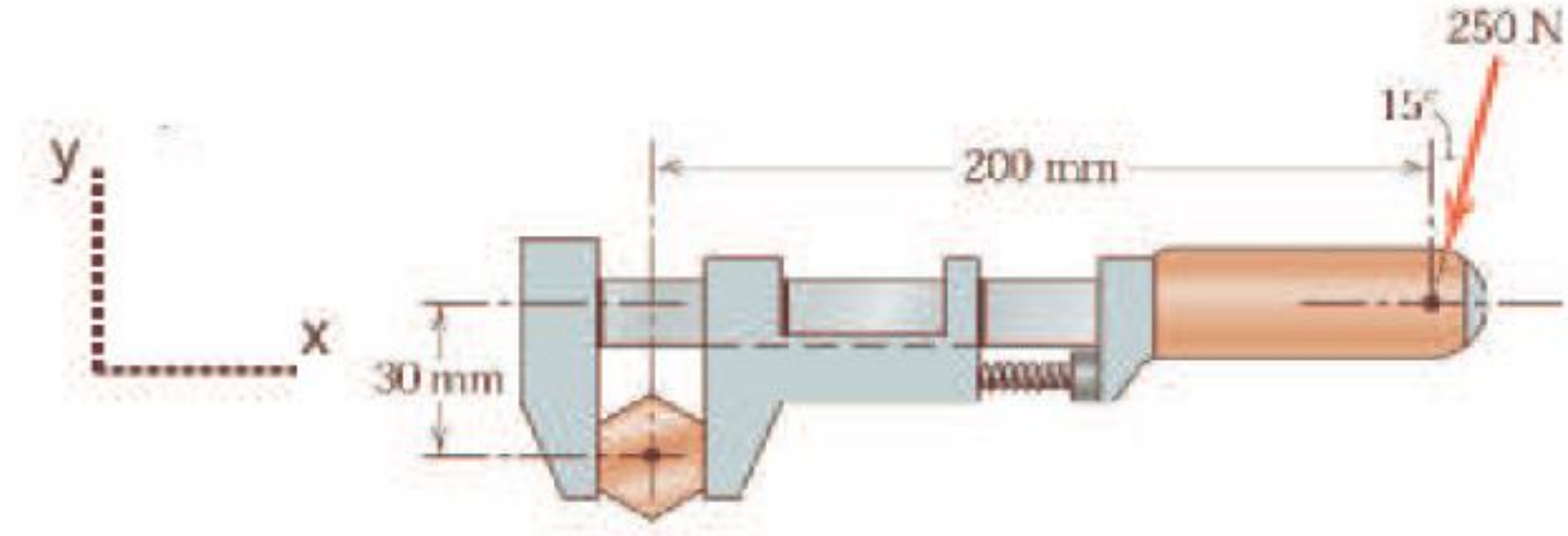
$$F_x = 600cos40^o = 460\ N$$

$$F_x = 600sin40^o = 386\ N$$

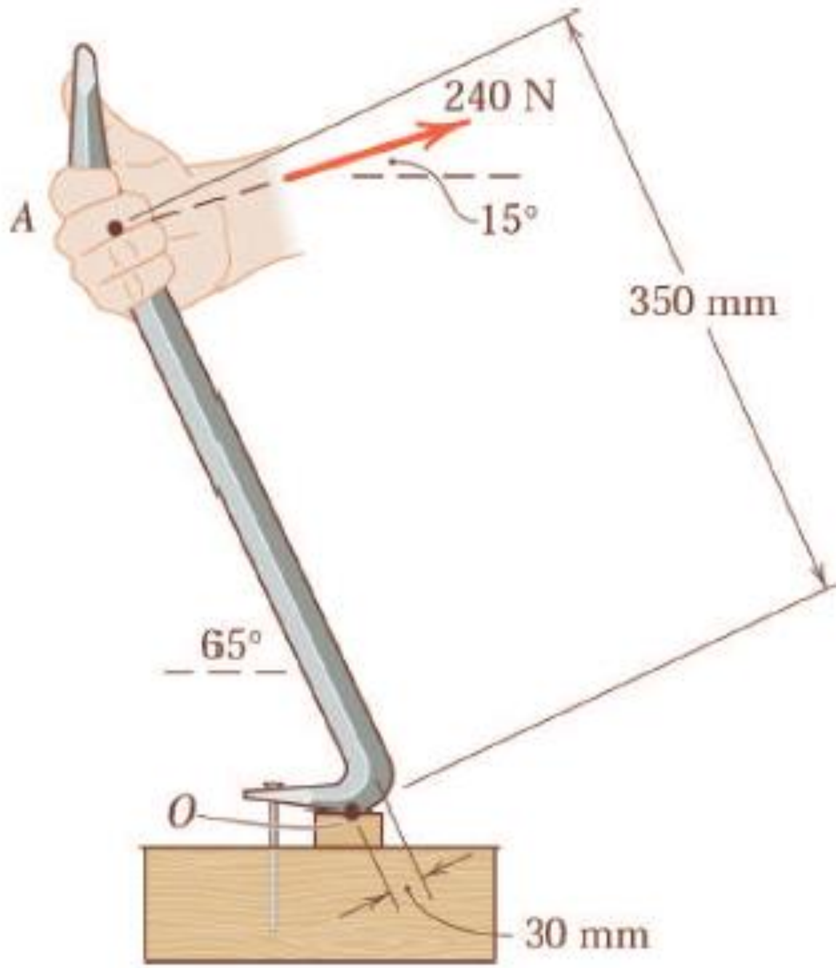
$$M_o = 460(4.00) + 386(2.00) = 2610\ N.m \ \textbf{Ans}$$

Ex-8

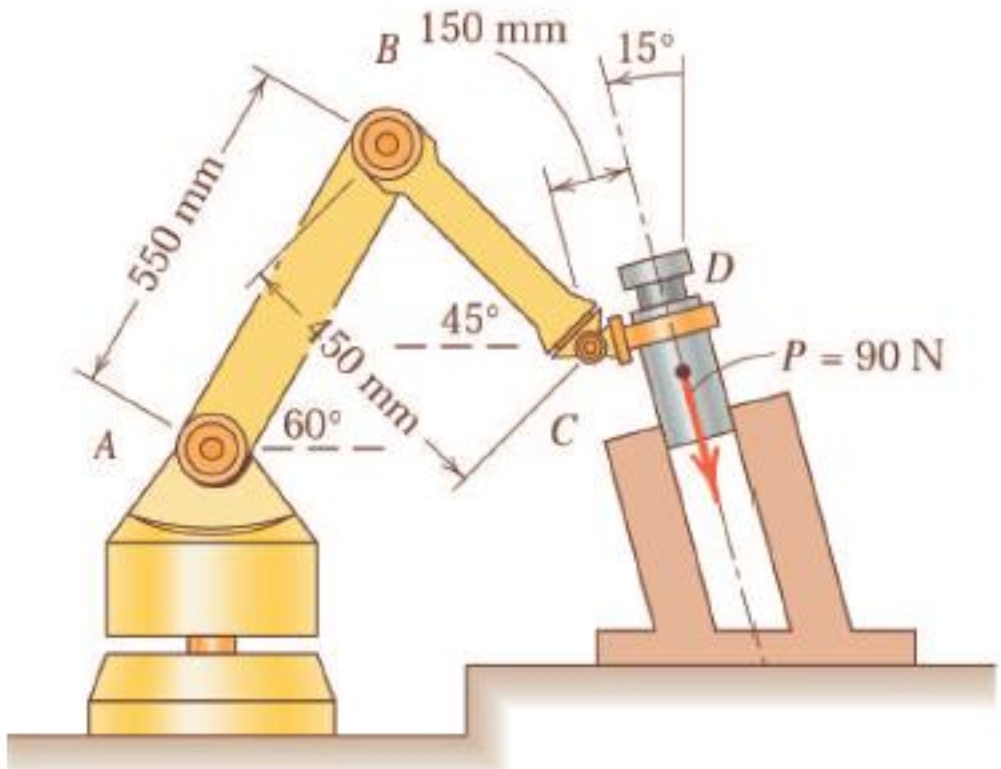
Calculate the moment of the 250 N force on the handle of the monkey wrench about the center of the bolt.



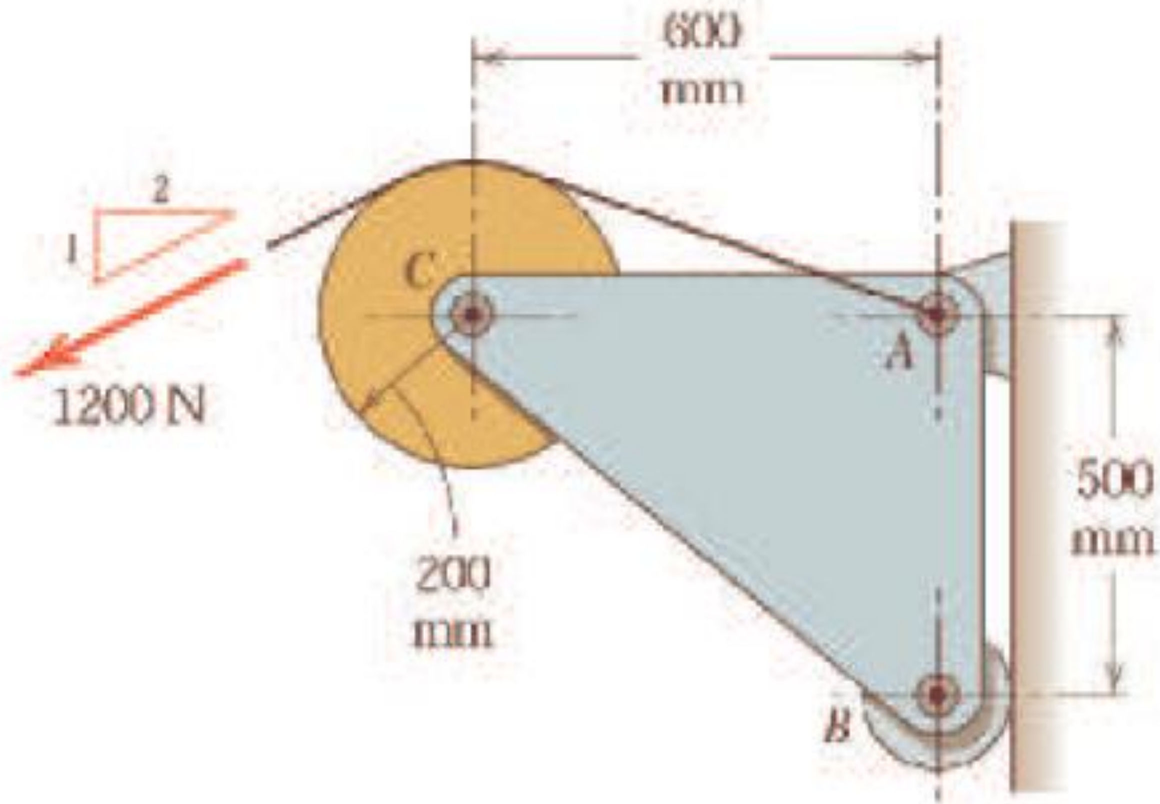
Calculate the moment of the 240 N force on the handle of the prong about the instantaneous supporting point O.



While inserting a cylindrical part into the circular hole, the robot exerts the 90 N force on the part as shown. Determine the moment about point A, B, and C of the force which the part exerts on the robot.



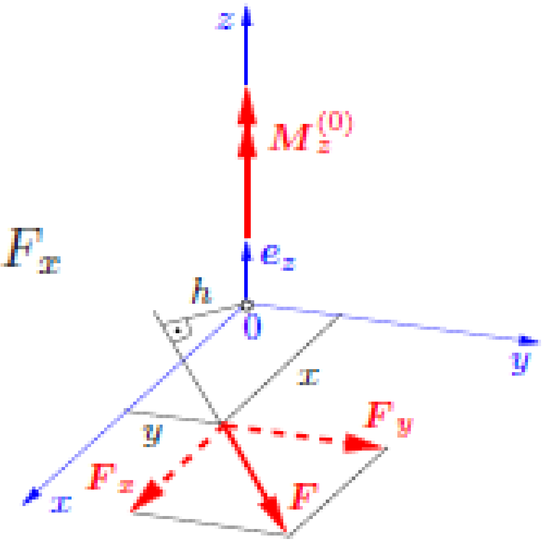
Calculate the moment of the 1200 N force about pin A of the bracket. Begin by replacing the 1200 N force by a force-couple system at point C. Calculate the moment of the 1200 N force about the pin at B.

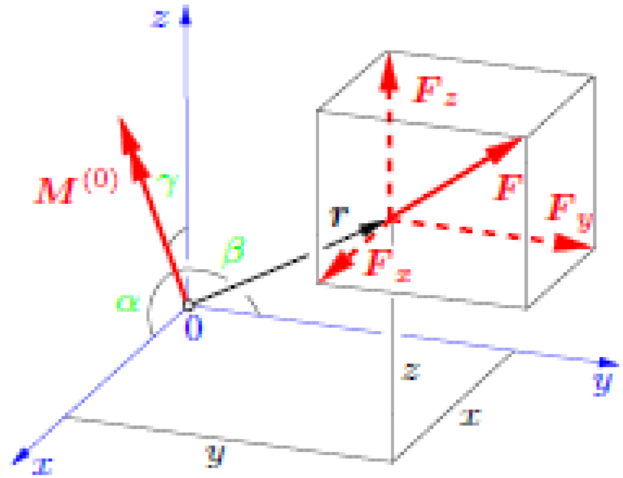


Force F, which acts in the x, y-plane, has a moment $M_{(0)}$ about point 0. With $F_x = F_x e_x$, etc., it is given by

$$M_z^{(0)} = M_z^{(0)} e_z$$

$$M_z^{(0)} = h\,F = x\,F_y - y\,F_x$$





$$\boldsymbol{M}^{(0)} = M_x^{(0)}\,\boldsymbol{e}_x + M_y^{(0)}\,\boldsymbol{e}_y + M_z^{(0)}\,\boldsymbol{e}_z$$

$$M_x^{(0)} = yF_z - zF_y,\; M_y^{(0)} = zF_x - xF_z,\; M_z^{(0)} = xF_y - yF_x.$$

The magnitude and direction of the moment vector are given by

$$|\boldsymbol{M}^{(0)}| = M^{(0)} = \sqrt{[M_x^{(0)}]^2 + [M_y^{(0)}]^2 + [M_z^{(0)}]^2}\,,$$

$$\cos\alpha = \frac{M_x^{(0)}}{M^{(0)}}\,, \quad \cos\beta = \frac{M_y^{(0)}}{M^{(0)}}\,, \quad \cos\gamma = \frac{M_z^{(0)}}{M^{(0)}}\,.$$

Alternatively

- Formally, the moment vector M(0) may be represented by the vector product

$$\boldsymbol{M}^{(0)} = \boldsymbol{r} \times \boldsymbol{F}$$

$$\boldsymbol{r} = x\,\boldsymbol{e}_x + y\,\boldsymbol{e}_y + z\,\boldsymbol{e}_z\,, \quad \boldsymbol{F} = F_x\,\boldsymbol{e}_x + F_y\,\boldsymbol{e}_y + F_z\,\boldsymbol{e}_z$$

$$\begin{aligned}\boldsymbol{M}^{(0)} &= (x\,\boldsymbol{e}_x + y\,\boldsymbol{e}_y + z\,\boldsymbol{e}_z) \times (F_x\,\boldsymbol{e}_x + F_y\,\boldsymbol{e}_y + F_z\,\boldsymbol{e}_z) \\ &= (y\,F_z - z\,F_y)\,\boldsymbol{e}_x + (z\,F_x - x\,F_z)\,\boldsymbol{e}_y + (x\,F_y - y\,F_x)\,\boldsymbol{e}_z \\ &= M_x^{(0)}\,\boldsymbol{e}_x + M_y^{(0)}\,\boldsymbol{e}_y + M_z^{(0)}\,\boldsymbol{e}_z\,.\end{aligned}$$

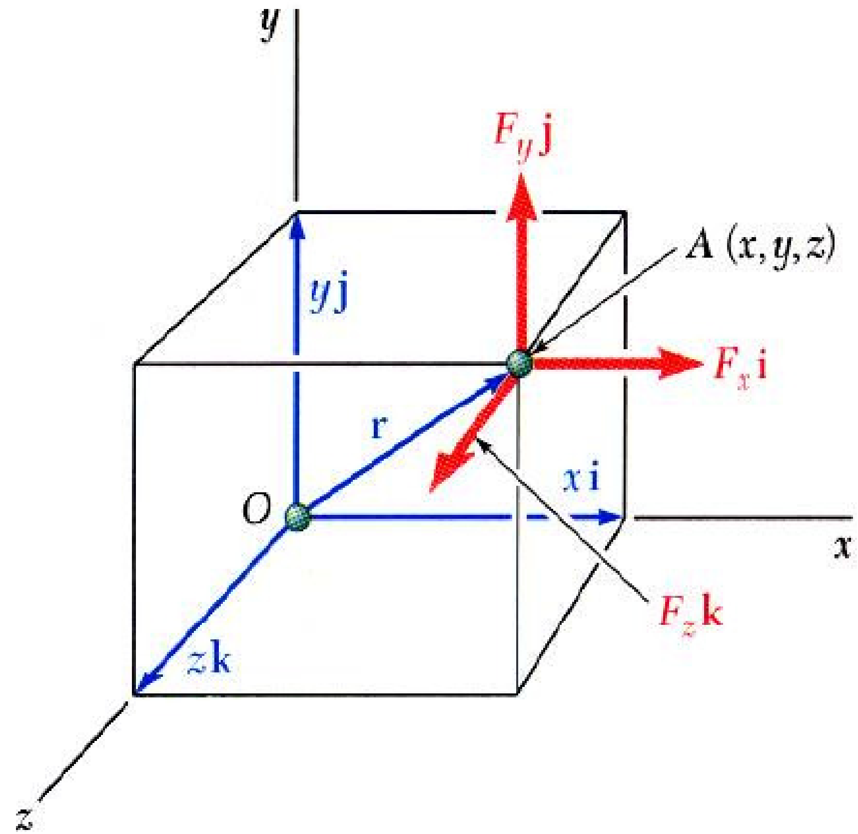
Rectangular Components of a Moment

The moment of $\boldsymbol{F}$ about O,

$$\boldsymbol{M_o} = \boldsymbol{r} \times \boldsymbol{F}$$

$$\boldsymbol{F} = F_x\boldsymbol{i} + F_y\boldsymbol{j} + F_z\boldsymbol{k}$$

$$\boldsymbol{r} = x\boldsymbol{i} + y\boldsymbol{j} + z\boldsymbol{k}$$



$$\boldsymbol{M_o} = M_x\boldsymbol{i} + M_y\boldsymbol{j} + M_z\boldsymbol{k}$$

$$= \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ x & y & z \\ F_x & F_y & F_z \end{vmatrix}$$

$$= \left(yF_z - zF_y\right)\boldsymbol{i} + (zF_x - xF_z)\boldsymbol{j} + \left(xF_y - yF_x\right)\boldsymbol{k}$$

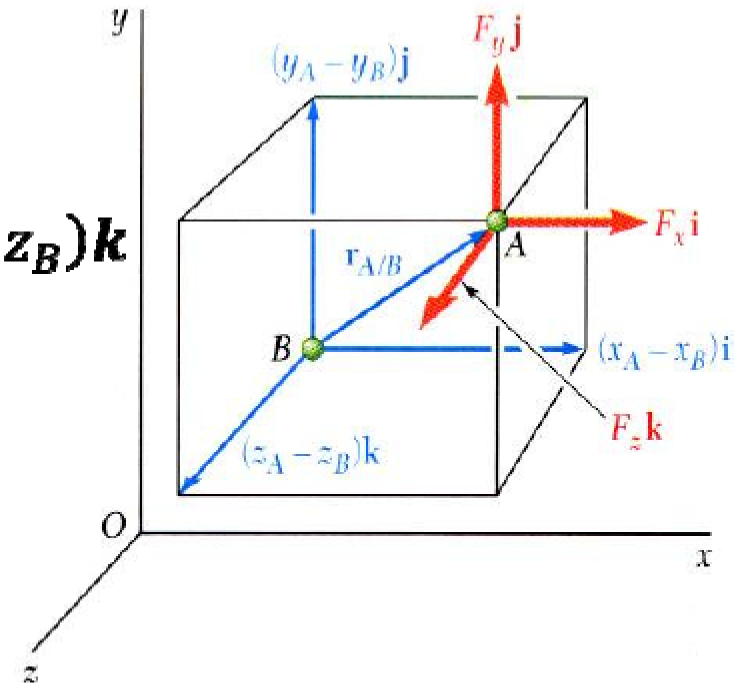
Rectangular Components of the Moment

The moment of $\boldsymbol{F}$ about B,

$$\boldsymbol{M}_B = \boldsymbol{r}_{AB} \times \boldsymbol{F}$$

$$\boldsymbol{r}_{AB} = (x_A - x_B)\boldsymbol{i} + (y_A - y_B)\boldsymbol{j} + (z_A - z_B)\boldsymbol{k}$$

$$\boldsymbol{F} = F_x\boldsymbol{i} + F_y\boldsymbol{j} + F_z\boldsymbol{k}$$



$$\boldsymbol{M}_B = M_x\boldsymbol{i} + M_y\boldsymbol{j} + M_z\boldsymbol{k}$$

$$= \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ x_A - x_B & y_A - y_B & z_A - z_B \\ F_x & F_y & F_z \end{vmatrix}$$

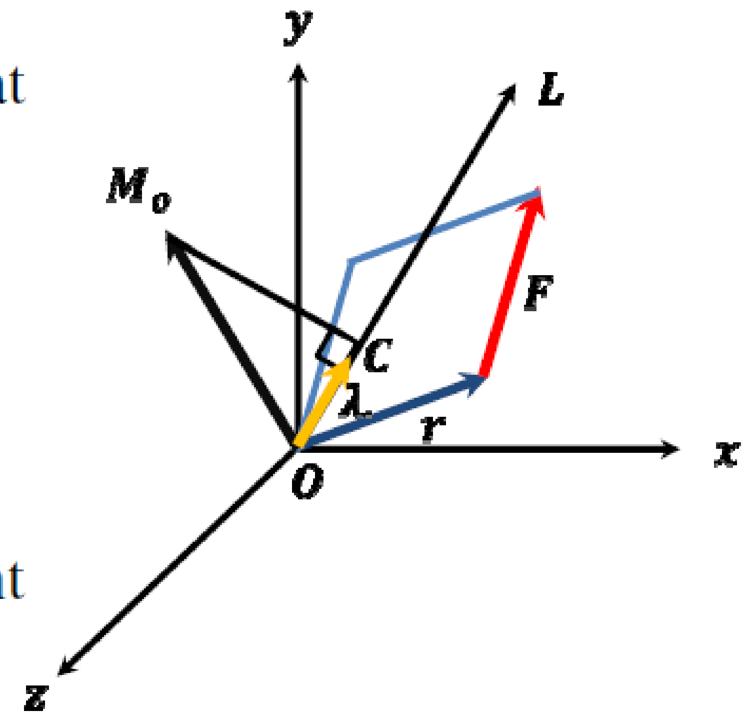
Moment of a Force About a Given Axis

Moment $\mathbf{M_O}$ of a force $\mathbf{F}$ applied at the point $\mathbf{A}$ about a point $\mathbf{O}$

$$M_o = r \times F$$

Scalar moment M_{OL} about an axis $\mathbf{OL}$ is the projection of the moment vector $\mathbf{M_O}$ onto the axis,

$$M_{OL} = \lambda . M_o = \lambda . (r \times F)$$



Moments of $\mathbf{F}$ about the coordinate axes (using previous slide)

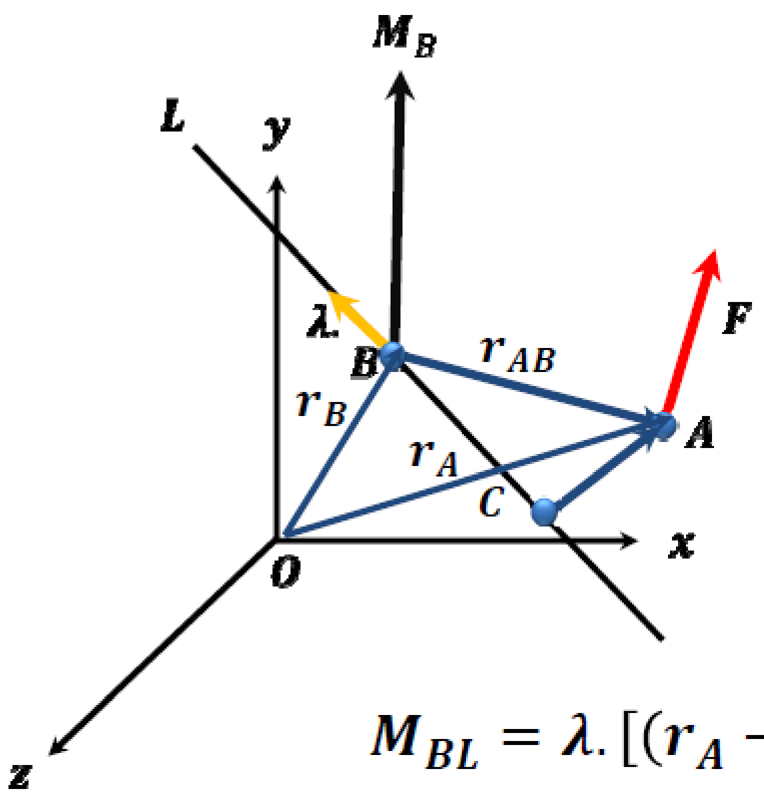
$$M_x = (yF_z - zF_y)$$

$$M_y = (zF_x - xF_z)$$

$$M_z = (xF_y - yF_x)$$

Moment of a Force About a Given Axis

Moment of a force about an arbitrary axis



$$M_B = r_{AB} \times F$$

$$M_{BL} = \lambda . M_B = \lambda . (r_{AB} \times F)$$

$$r_{AB} = r_A - r_B$$

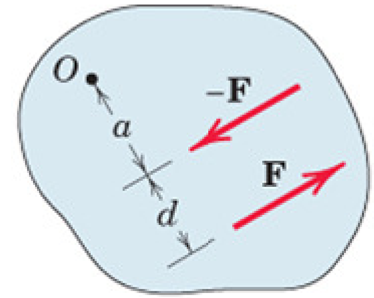
If we take point C in place of point B

$$M_{BL} = \lambda . [(r_A - r_C) \times F]$$
$$= \lambda . [(r_A - r_B) \times F] + \lambda . [(r_B - r_C) \times F]$$

0

$(r_B - r_C)$ and λ are in the same line

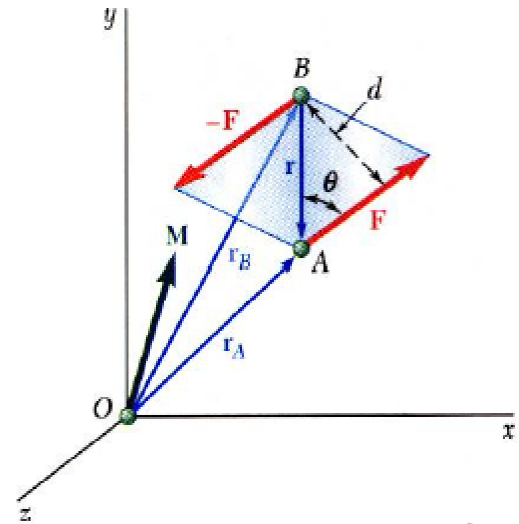
Moment of a Couple



Moment produced by two equal, opposite and non-collinear forces is called a *couple*.

Magnitude of the combined moment of the two forces about O:

$$M = F(a + d) - Fa = Fd$$



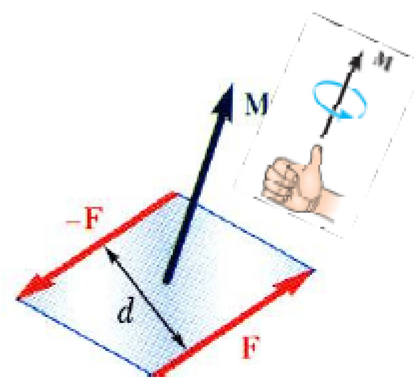
$$\boldsymbol{M} = \boldsymbol{r}_A \times \boldsymbol{F} + \boldsymbol{r}_B \times (-\boldsymbol{F})$$

$$= (\boldsymbol{r}_A - \boldsymbol{r}_B) \times \boldsymbol{F}$$

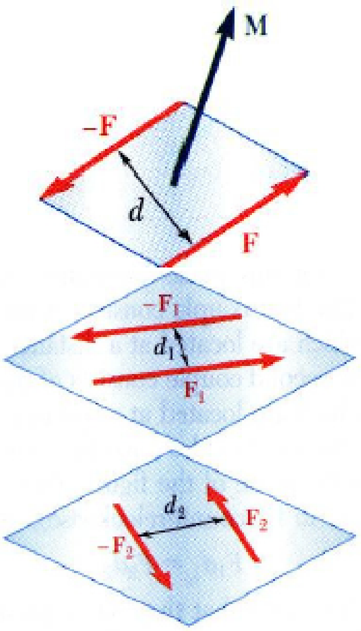
$$= \boldsymbol{r} \times \boldsymbol{F}$$

$$M = rF\sin\theta = Fd$$

The moment vector of the couple is independent of the choice of the origin of the coordinate axes, i.e., it is a *free vector* that can be applied at any point with the same effect.



Moment of a Couple

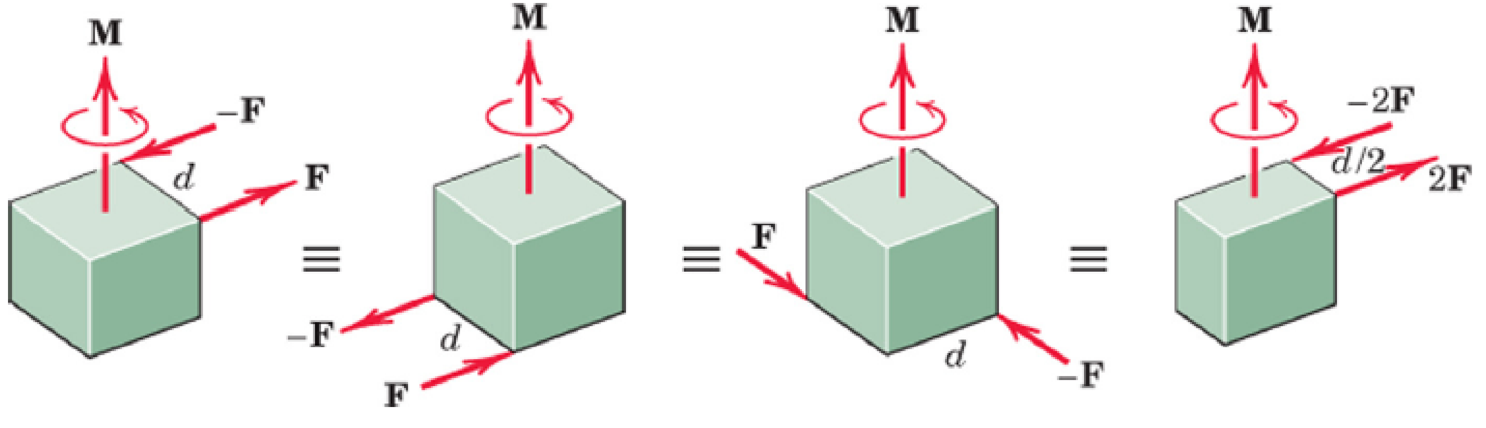


Two couples will have equal moments if $F_1 d_1 = F_2 d_2$

The two couples lie in parallel planes

The two couples have the same sense or the tendency to cause rotation in the same direction.

Examples:

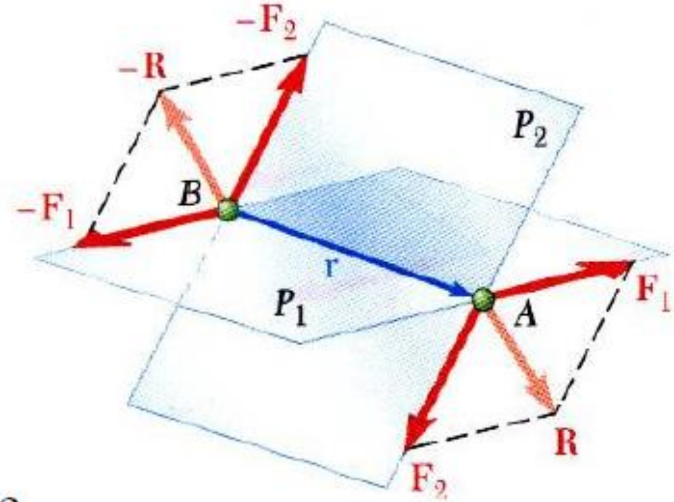


Addition of Couples

Consider two intersecting planes P_1 and P_2 with each containing a couple

$\boldsymbol{M_1} = \boldsymbol{r} \times \boldsymbol{F_1}$ in plane P_1

$\boldsymbol{M_2} = \boldsymbol{r} \times \boldsymbol{F_2}$ in plane P_2



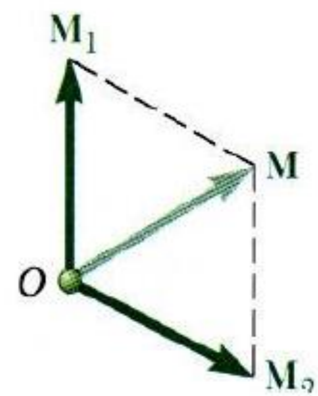
Resultants of the vectors also form a couple

$$\boldsymbol{M} = \boldsymbol{r} \times \boldsymbol{R} = \boldsymbol{r} \times (\boldsymbol{F_1} + \boldsymbol{F_2})$$

By Varigon's theorem

$$\boldsymbol{M} = \boldsymbol{r} \times \boldsymbol{F_1} + \boldsymbol{r} \times \boldsymbol{F_2}$$

$$= \boldsymbol{M_1} + \boldsymbol{M_2}$$



Sum of two couples is also a couple that is equal to the vector sum of the two couples