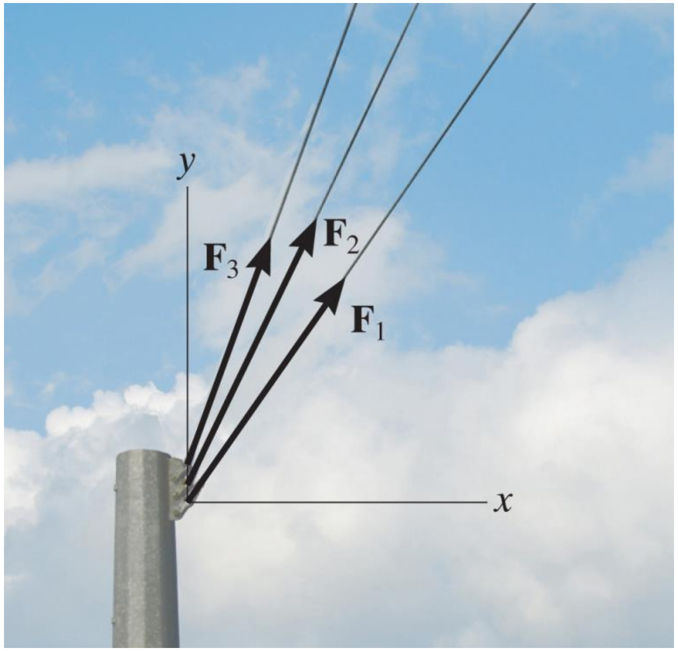
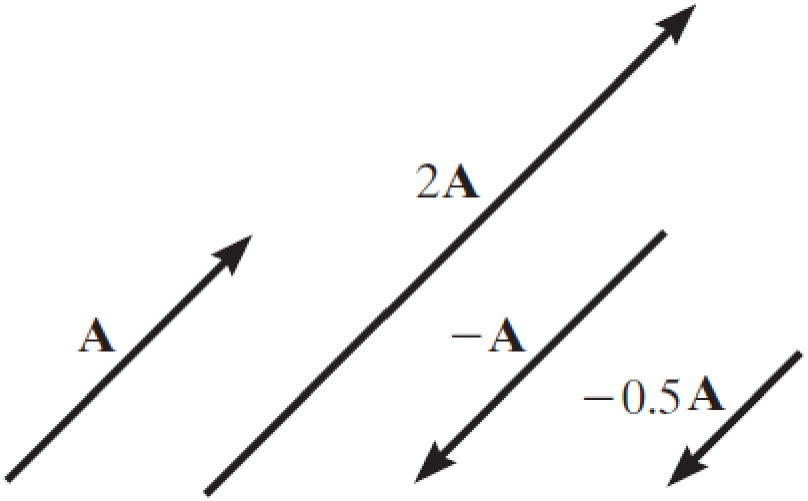
STATICS

FE Review

1. Resultants of force systems



VECTOR OPERATIONS (Section 2.2)

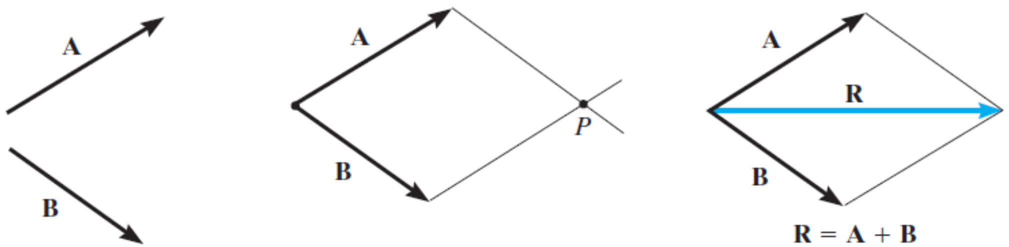


Scalar Multiplication

and Division

VECTOR ADDITION USING EITHER THE PARALLELOGRAM LAW OR TRIANGLE

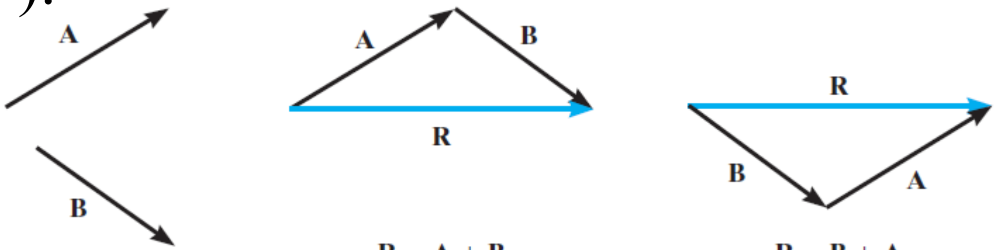
Parallelogram Law:



$\mathbf{R} = \mathbf{A} + \mathbf{B}$

Parallelogram law

Triangle method
(always 'tip to tail'):



$\mathbf{R} = \mathbf{A} + \mathbf{B}$

Triangle rule

$\mathbf{R} = \mathbf{B} + \mathbf{A}$

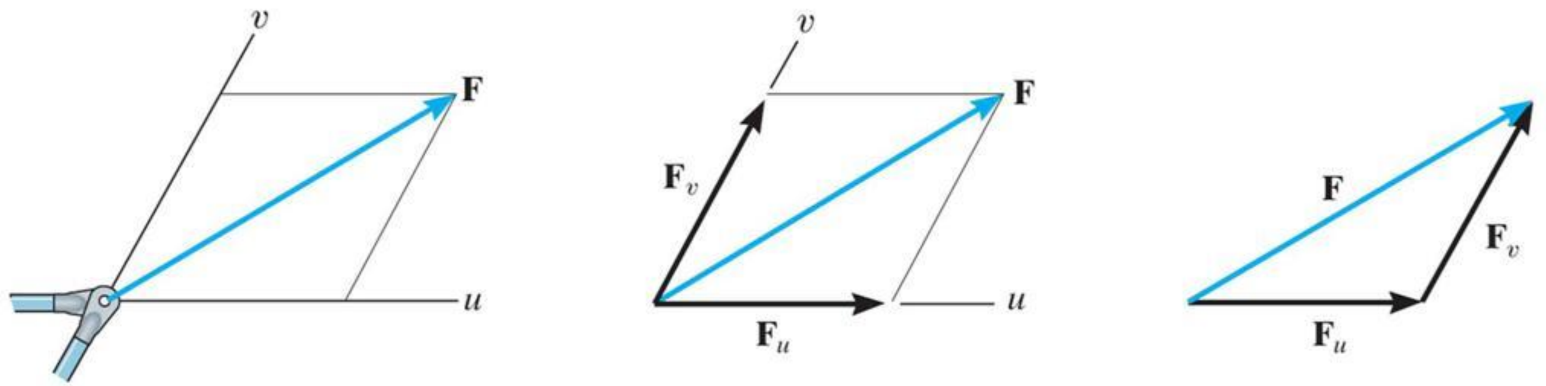
Triangle rule

How do you subtract a vector?

How can you add more than two concurrent vectors graphically?

RESOLUTION OF A VECTOR

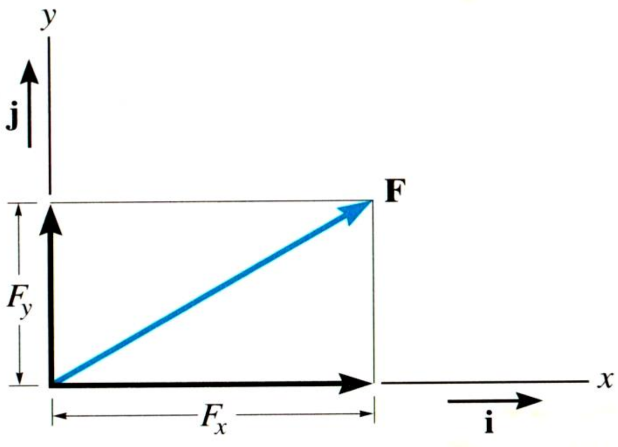
"Resolution" of a vector is breaking up a vector into components.



(a) (b) (c)

[It is kind of like using the parallelogram law in reverse.]

ADDITION OF A SYSTEM OF COPLANAR FORCES (Section 2.4)



(a)

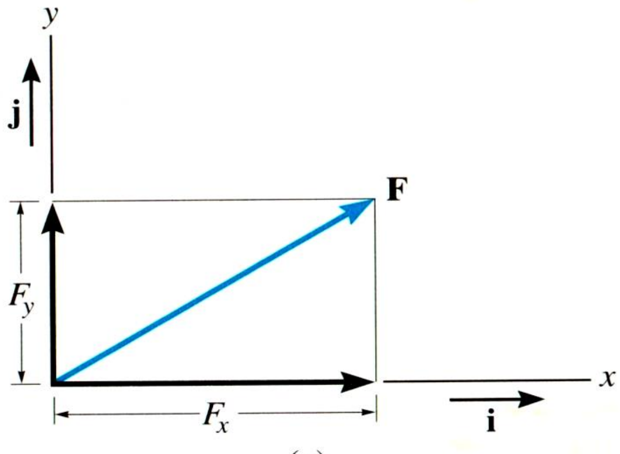
- We 'resolve' vectors into components using the x and y-axis coordinate system.

- Each component of the vector is shown as a magnitude and a direction.

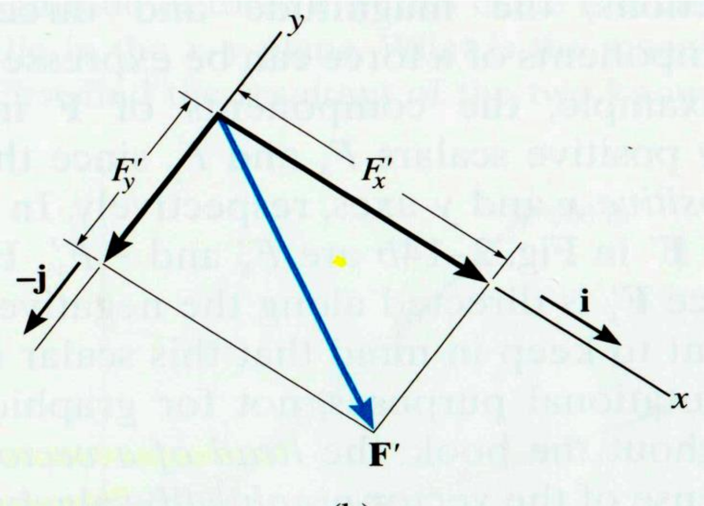
- The directions are based on the x and y axes. We use the "unit vectors" ***i*** and ***j*** to designate the x and y-axes.

For example,

$\boldsymbol{F} = F_x \boldsymbol{i} + F_y \boldsymbol{j}$ ***or*** $\boldsymbol{F'} = F'_x \boldsymbol{i} + (-F'_y)\boldsymbol{j}$



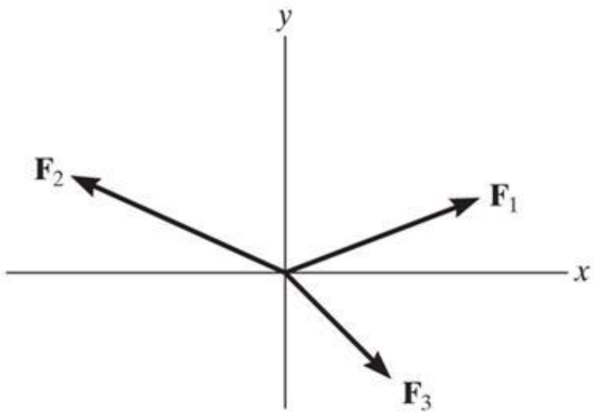
(a)

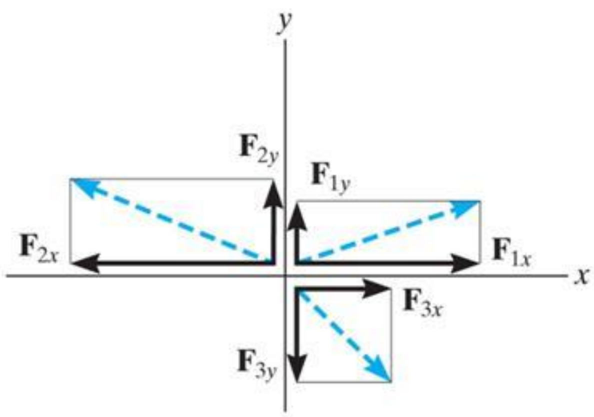


(b)

The x and y-axis are always perpendicular to each other. Together, they can be "set" at any inclination.

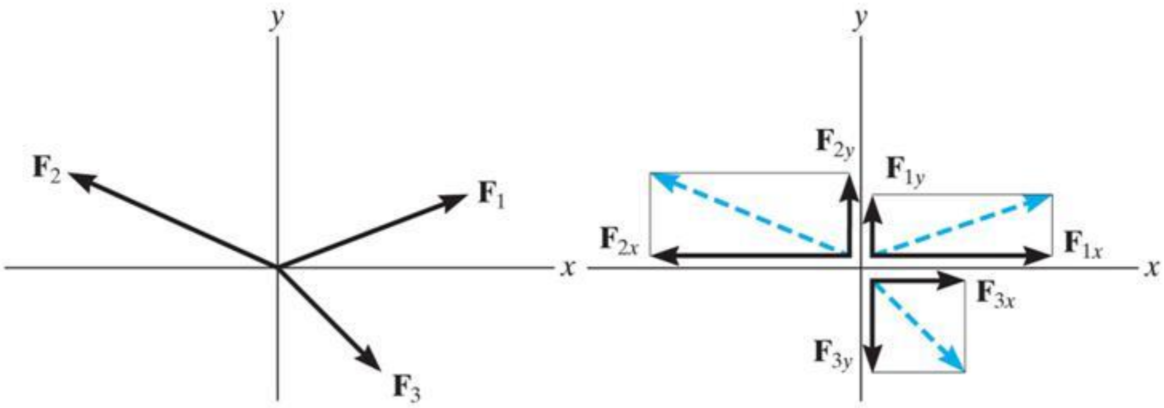
ADDITION OF SEVERAL VECTORS





- Step 1 is to resolve each force into its components.
- Step 2 is to add all the x-components together, followed by adding all the y-components together. These two totals are the x and y-components of the resultant vector.
- Step 3 is to find the magnitude and angle of the resultant vector.

An example of the process:



Break the three vectors into components, then add them.

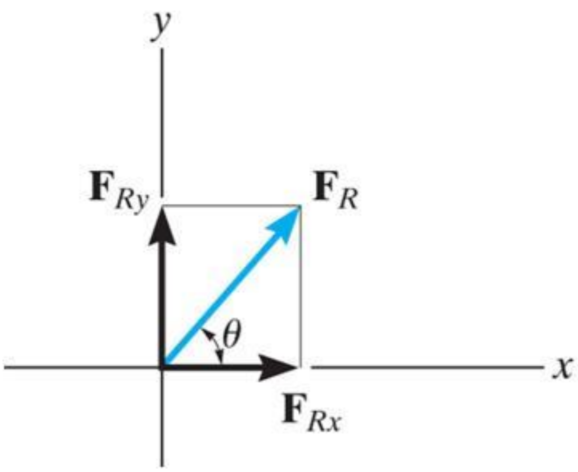
$$\boldsymbol{F_R} = \boldsymbol{F_1} + \boldsymbol{F_2} + \boldsymbol{F_3}$$

$$= F_{1x}\,\boldsymbol{i} + F_{1y}\,\boldsymbol{j} - F_{2x}\,\boldsymbol{i} + F_{2y}\,\boldsymbol{j} + F_{3x}\,\boldsymbol{i} - F_{3y}\,\boldsymbol{j}$$

$$= (F_{1x} - F_{2x} + F_{3x})\,\boldsymbol{i} + (F_{1y} + F_{2y} - F_{3y})\,\boldsymbol{j}$$

$$= (F_{Rx})\,\boldsymbol{i} + (F_{Ry})\,\boldsymbol{j}$$

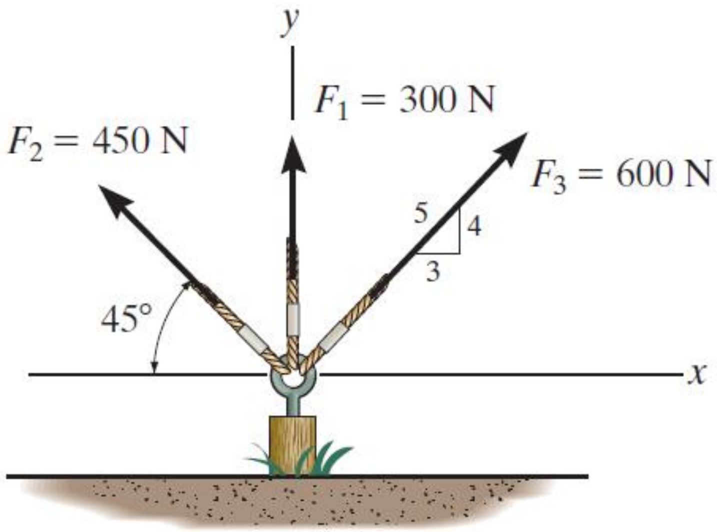
You can also represent a 2-D vector with a magnitude and angle.



$$\theta = \tan^{-1}\left|\frac{F_{Ry}}{F_{Rx}}\right|$$

$$F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2}$$

EXAMPLE I



Given: Three concurrent forces acting on a tent post.

Find: The magnitude and angle of the resultant force.

Plan:

a) Resolve the forces into their x-y components.

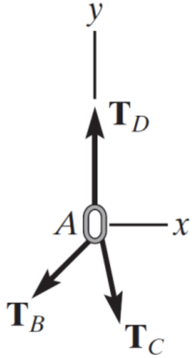
b) Add the respective components to get the resultant vector.

c) Find magnitude and angle from the resultant components.

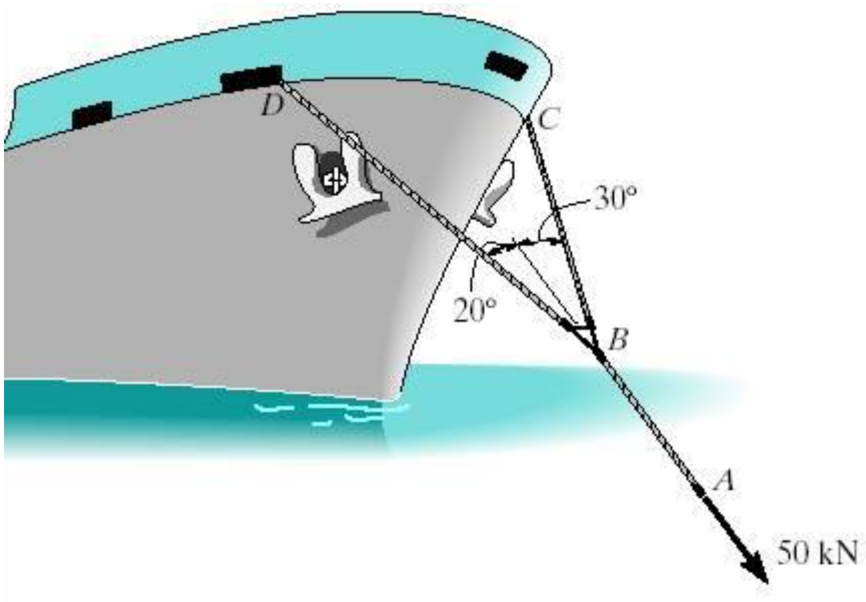
EQUILIBRIUM OF A PARTICLE, THE FREE-BODY DIAGRAM

APPLICATIONS

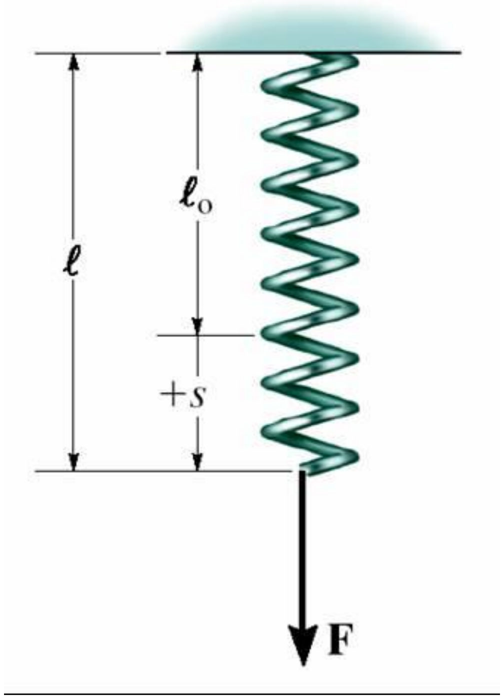
For a spool of given weight, how would you find the forces in cables AB and AC? If designing a spreader bar like the one being used here, you need to know the forces to make sure the rigging doesn't fail.



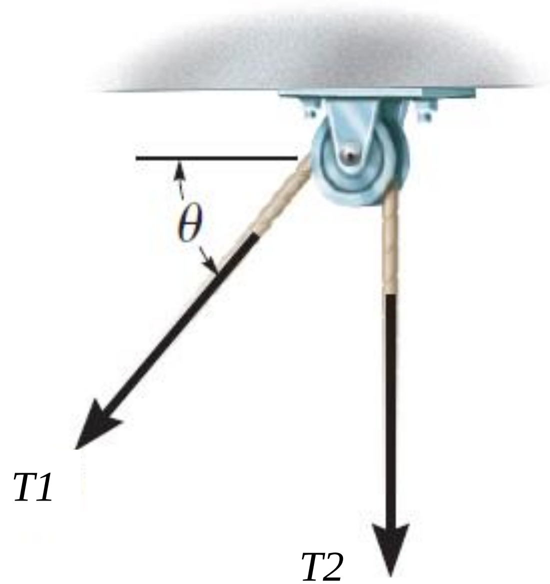
APPLICATIONS (continued)



SIMPLE SPRINGS

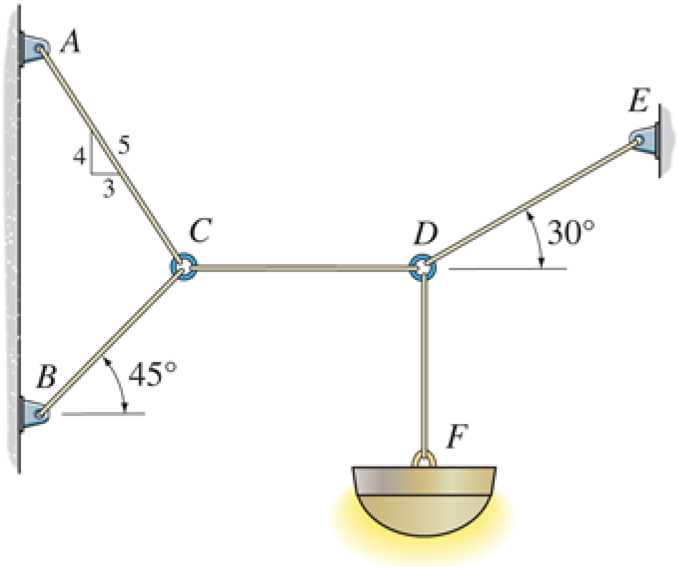


CABLES AND PULLEYS



Cable is in tension

GROUP PROBLEM SOLVING

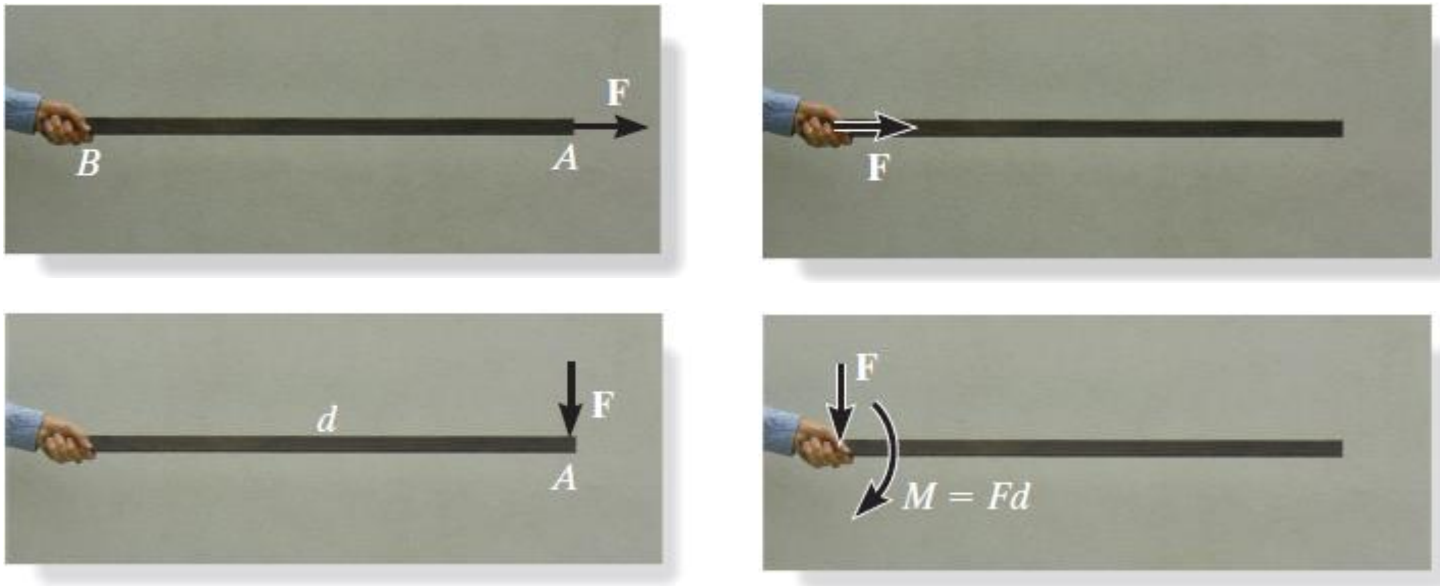


Given: The mass of lamp is 20 kg and geometry is as shown.

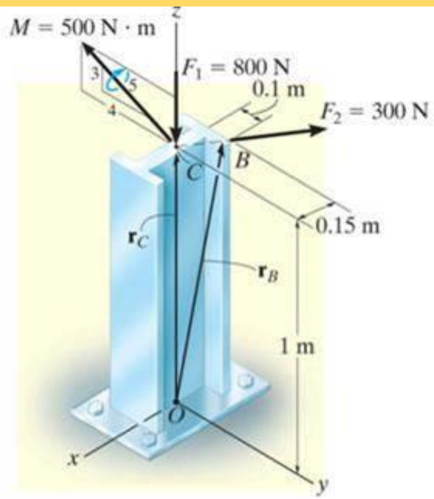
Find: The force in each cable.

Plan:

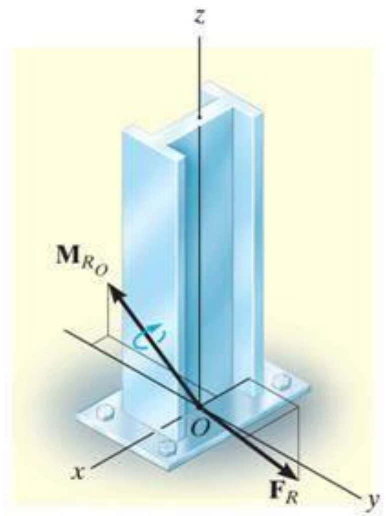
EQUIVALENT FORCE SYSTEMS



APPLICATIONS (continued)

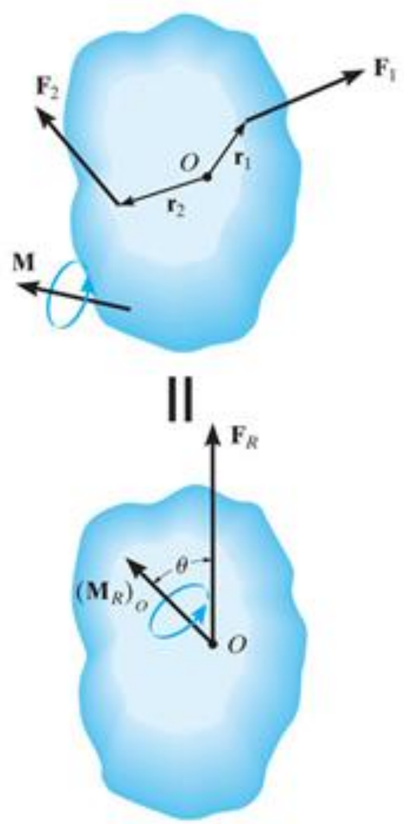


|| ??

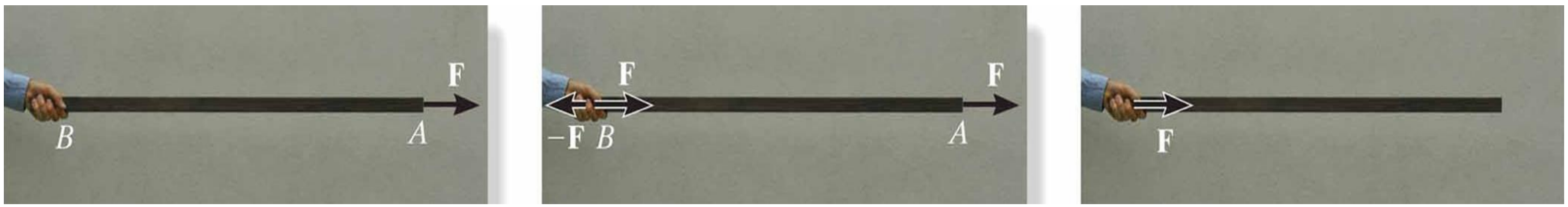


Several forces and a couple moment are acting on this vertical section of an I-beam.

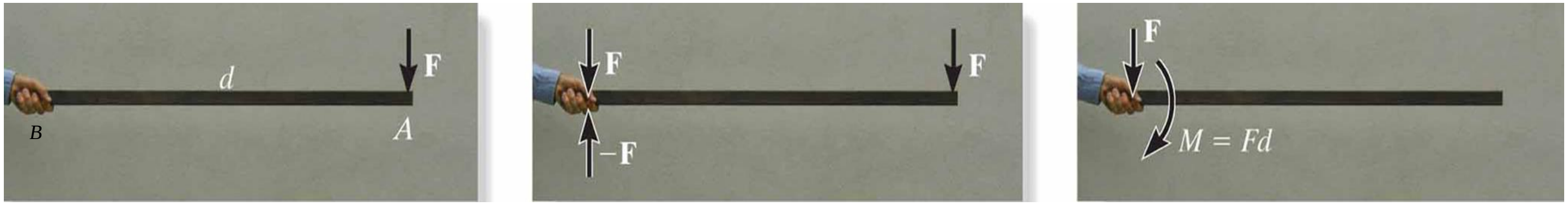
SIMPLIFICATION OF FORCE AND COUPLE SYSTEM (Section 4.7)



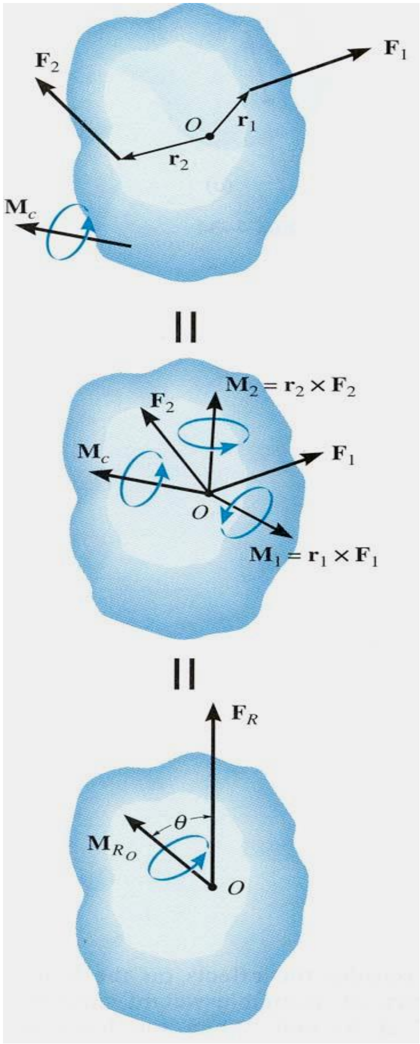
MOVING A FORCE ON ITS LINE OF ACTION



MOVING A FORCE OFF OF ITS LINE OF ACTION



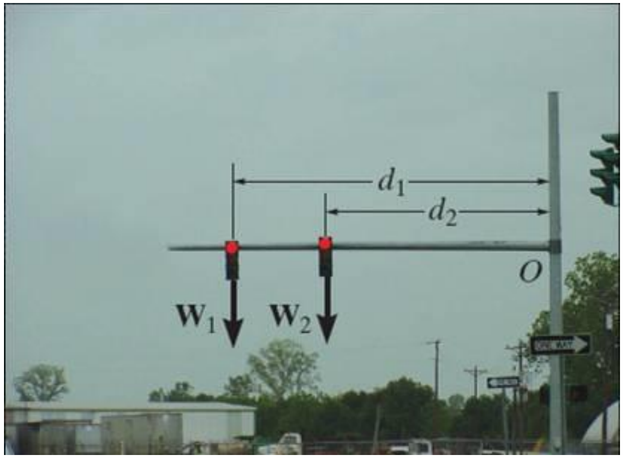
SIMPLIFICATION OF A FORCE AND COUPLE SYSTEM

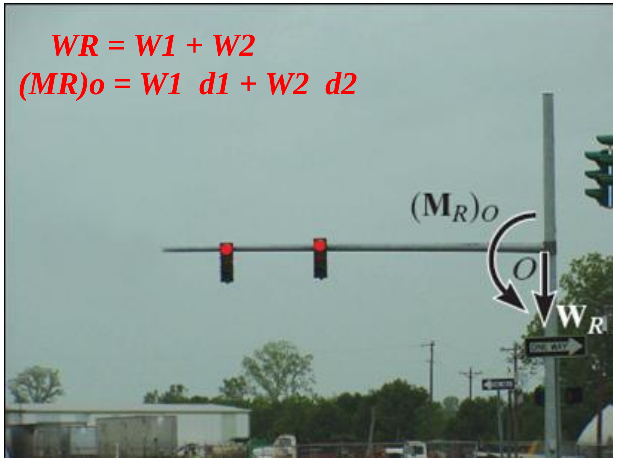


$$\mathbf{F}_R = \Sigma \mathbf{F}$$

$$\mathbf{M}_{R_O} = \Sigma \mathbf{M}_c + \Sigma \mathbf{M}_O$$

SIMPLIFICATION OF A FORCE AND COUPLE SYSTEM (continued)



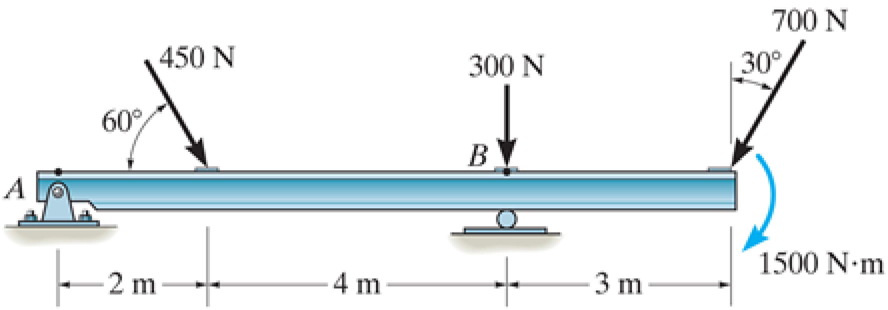


$$F_{R_x} = \Sigma F_x$$

$$F_{R_y} = \Sigma F_y$$

$$M_{R_O} = \Sigma M_c + \Sigma M_O$$

GROUP PROBLEM SOLVING I



Given: A 2-D force and couple system as shown.

Find: The equivalent resultant force and couple moment acting at A.

Plan:

$$F_{R_x} = \Sigma F_x$$

$$F_{R_y} = \Sigma F_y$$

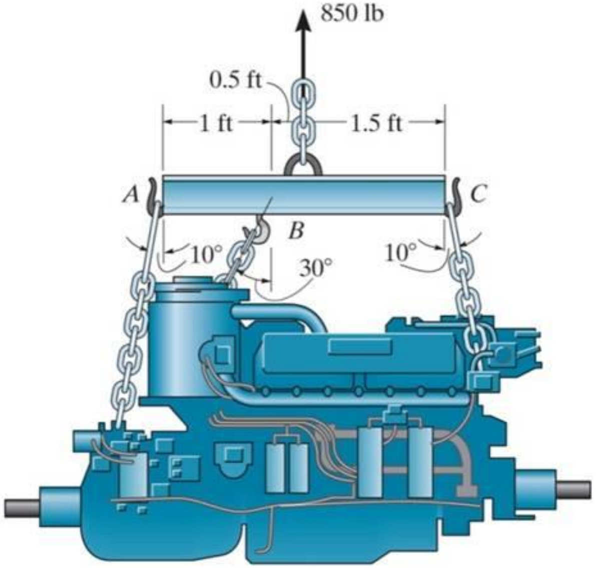
$$M_{R_O} = \Sigma M_c + \Sigma M_O$$

RIGID BODY: EQUATIONS OF EQUILIBRIUM

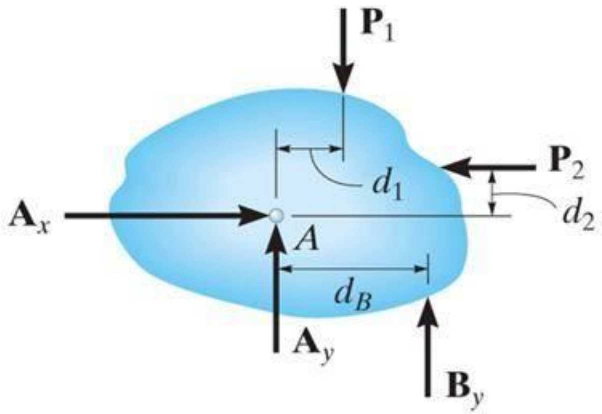
PEARSON

APPLICATIONS

APPLICATIONS (continued)



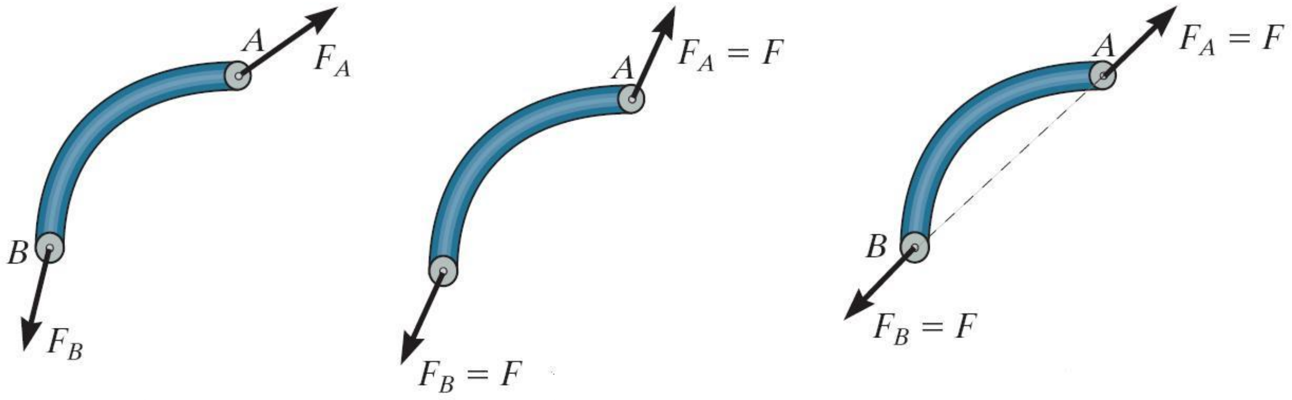
EQUATIONS OF EQUILIBRIUM (Section 5.3)



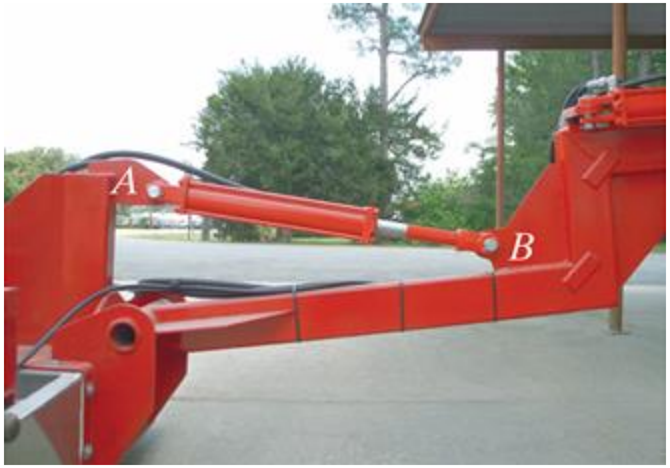
$$\sum Fx = 0 \qquad \sum Fy = 0 \qquad \sum Mo = 0$$

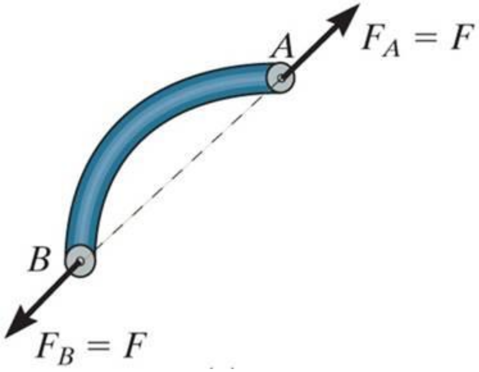
where point O is any arbitrary point.

TWO-FORCE MEMBERS & THREE FORCE-MEMBERS (Section 5.4)

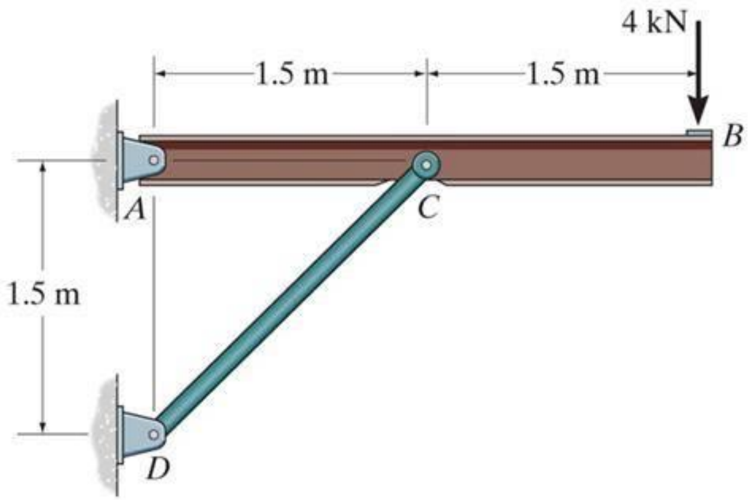


EXAMPLES OF TWO-FORCE MEMBERS





EXAMPLE



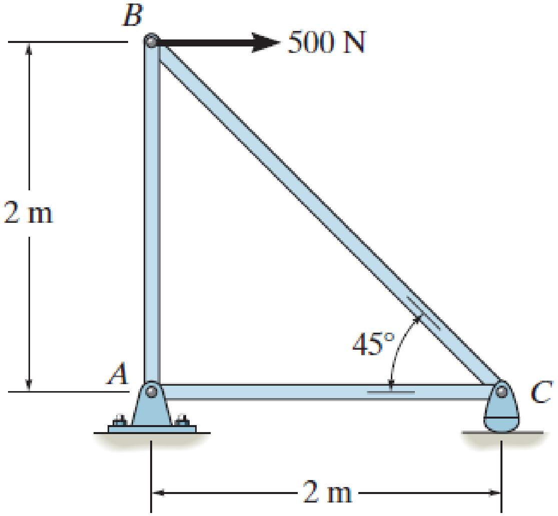
Given: The 4kN load at B of the beam is supported by pins at A and C.

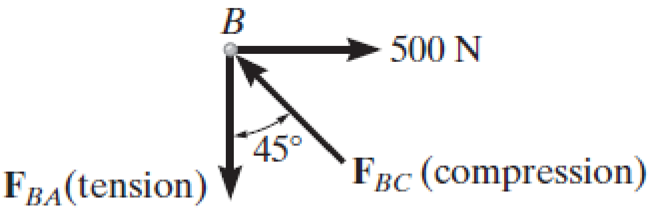
Find: The support reactions at A and C.

Plan:

SIMPLE TRUSSES, THE METHOD OF JOINTS, & ZERO-FORCE MEMBERS

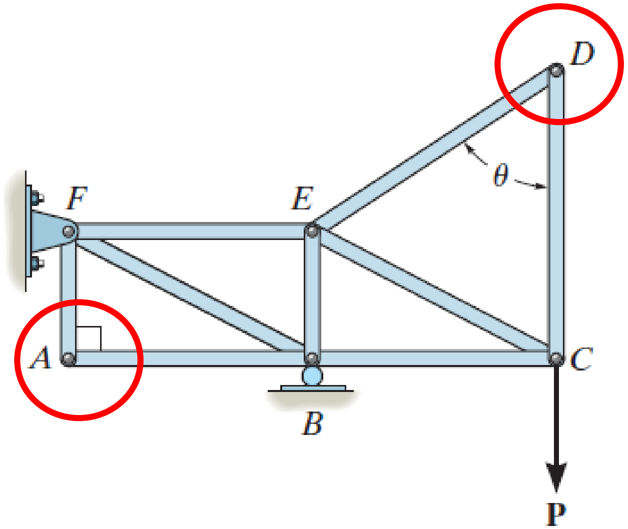
THE METHOD OF JOINTS (Section 6.2)



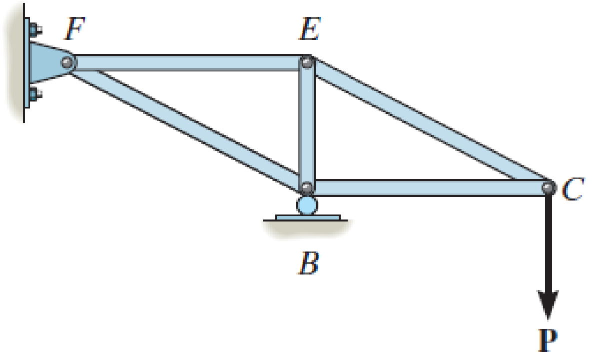


A free-body diagram of Joint B

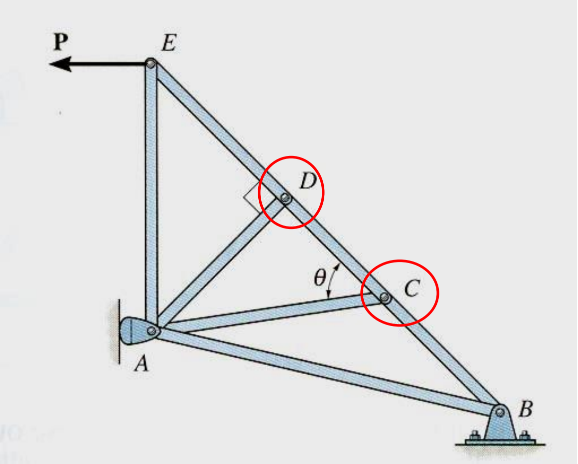
ZERO-FORCE MEMBERS (Section 6.3)



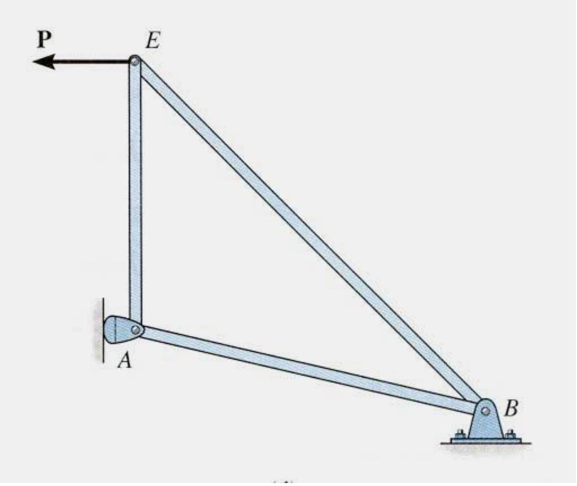
If a joint has only two non-collinear members and there is no external load or support reaction at that joint, then those two members are zero-force members.



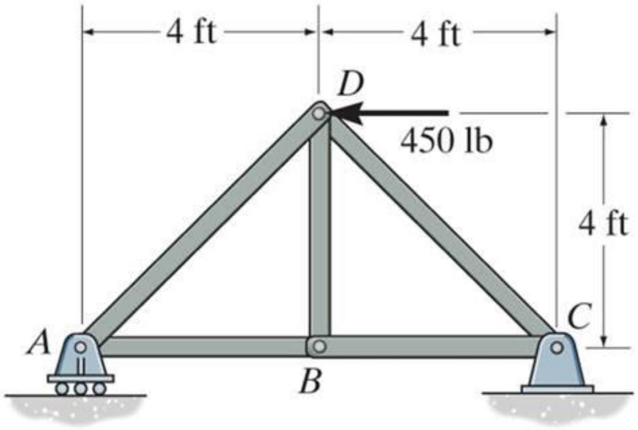
ZERO – FORCE MEMBERS (continued)



If three members form a truss joint for which two of the members are collinear and there is no external load or reaction at that joint, then the third non-collinear member is a zero force member, e.g., DA.



EXAMPLE

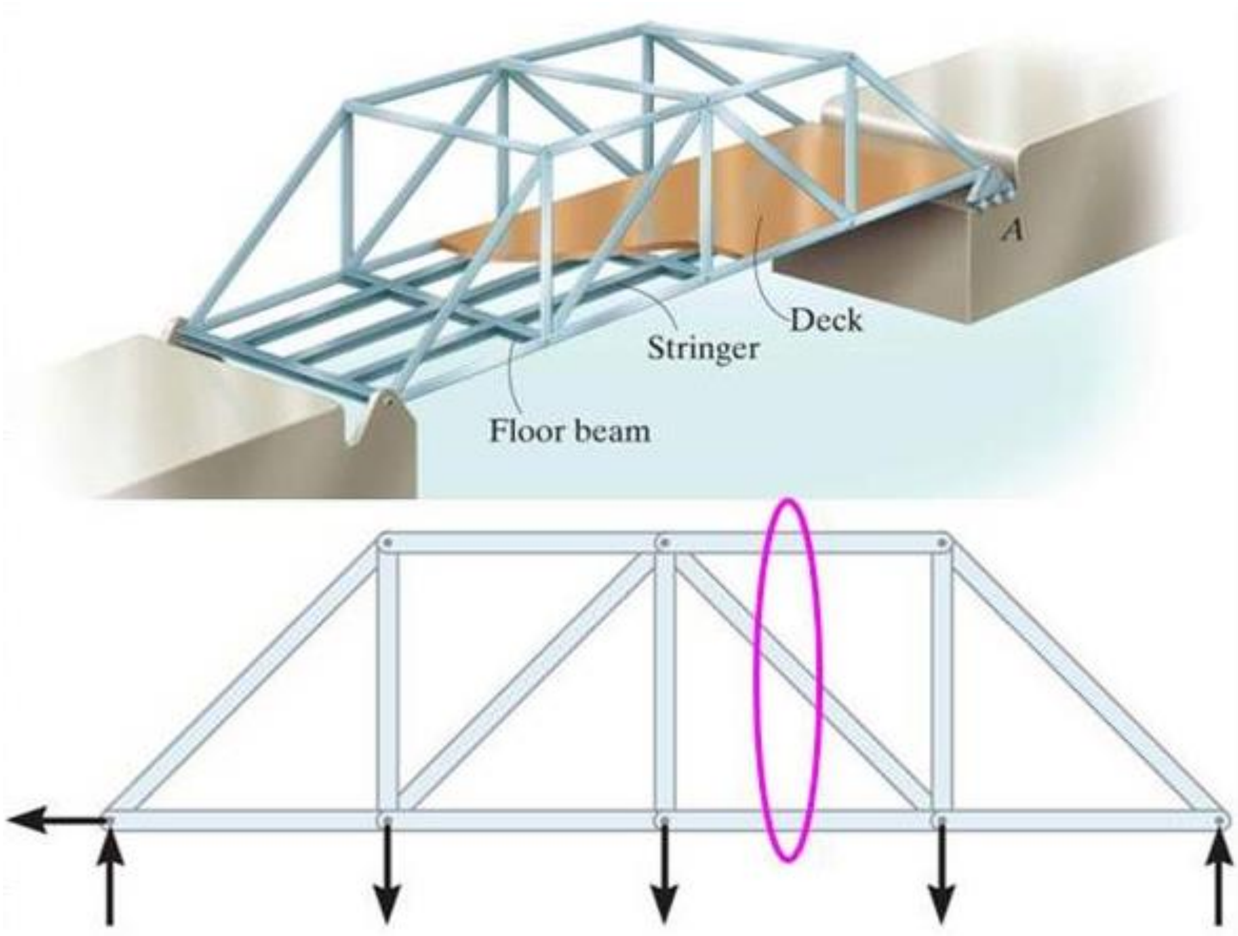


Given: Loads as shown on the truss

Find: The forces in each member of the truss.

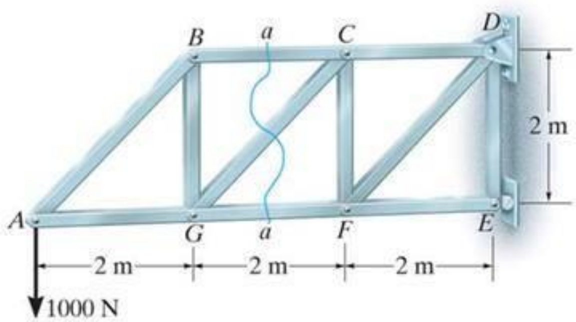
Plan:

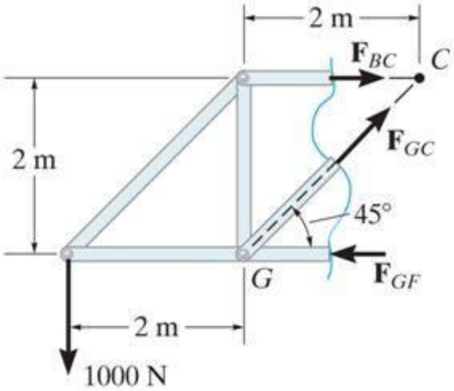
THE METHOD OF SECTIONS

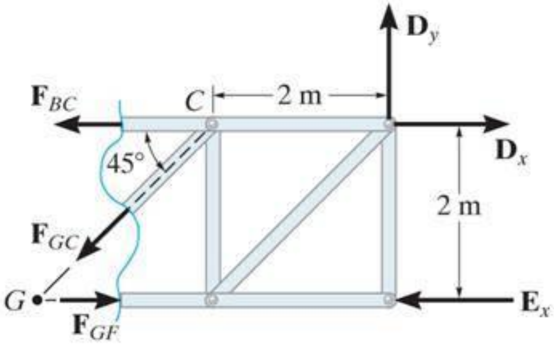


APPLICATIONS

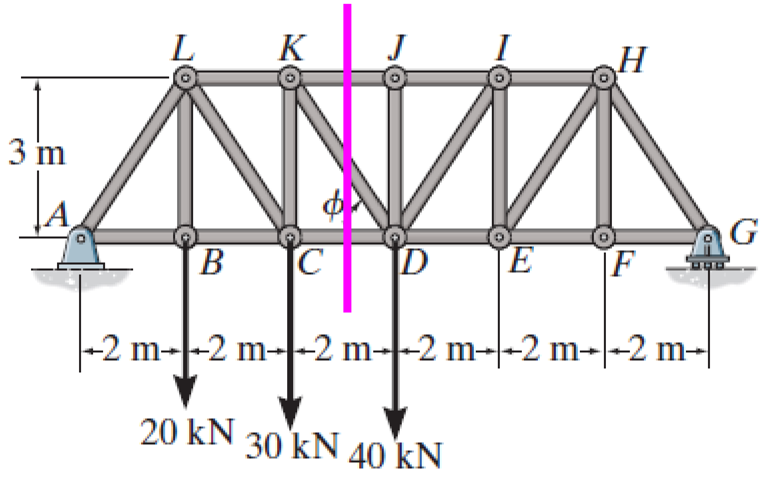
STEPS FOR ANALYSIS







EXAMPLE



Given: Loads as shown on the truss.

Find: The force in members KJ, KD, and CD.

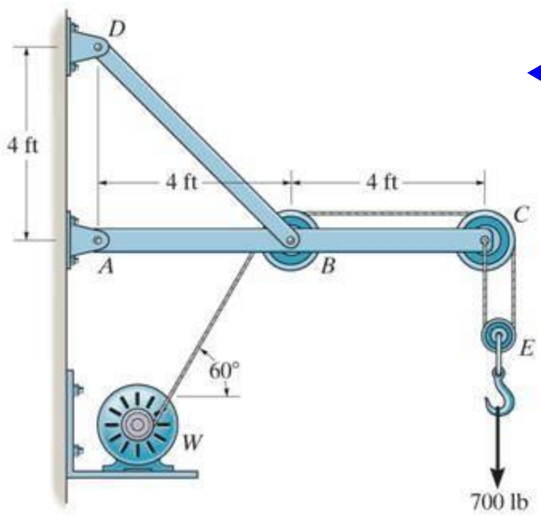
Plan:

FRAMES AND MACHINES

APPLICATIONS

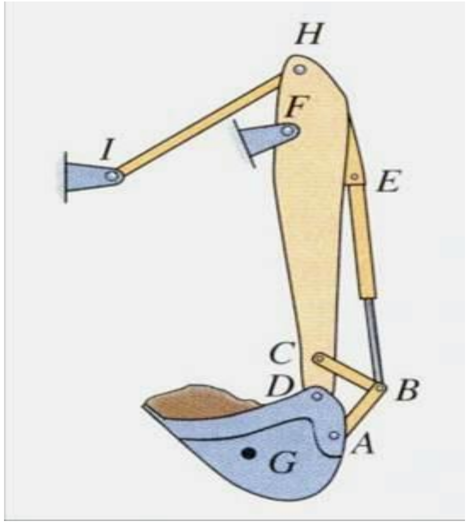
"Machines," like those above, are used in a variety of applications. How are they different from trusses and frames?

FRAMES AND MACHINES: DEFINITIONS



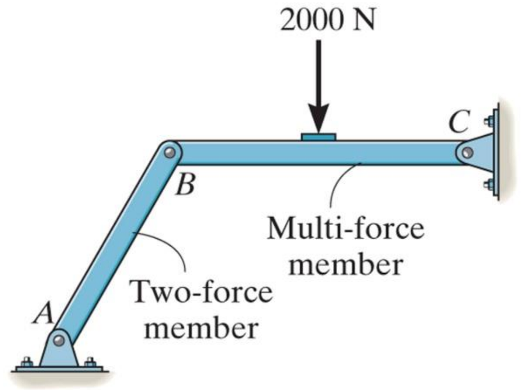
← Frame

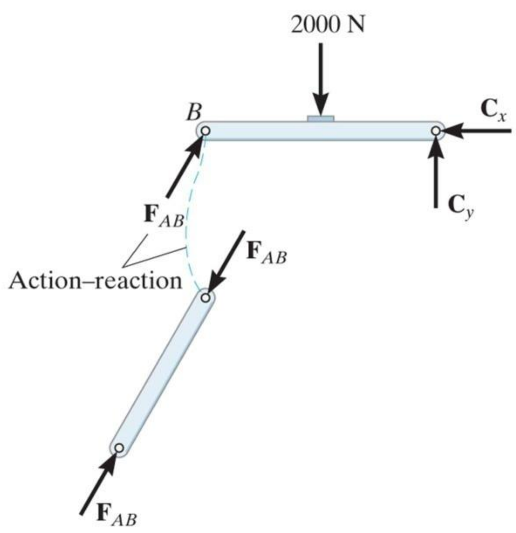
Machine →



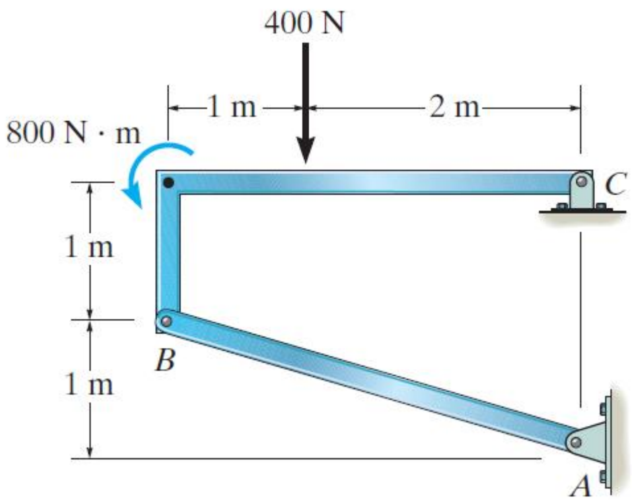
Frames are generally stationary and support external loads.

STEPS FOR ANALYZING A FRAME OR MACHINE





EXAMPLE



Given: The frame supports an external load and moment as shown.

Find: The horizontal and vertical components of the pin reactions at C and the magnitude of reaction at B.

Plan:

CHARACTERISTICS OF DRY FRICTION

APPLICATIONS

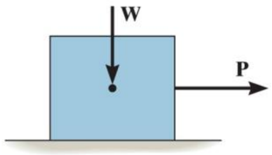
In designing a brake system for a bicycle, car, or any other vehicle, it is important to understand the frictional forces involved.

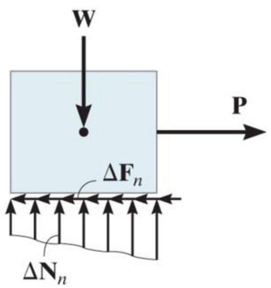
APPLICATIONS (continued)

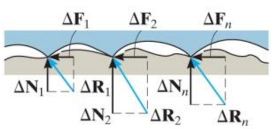
The rope is used to tow the refrigerator.

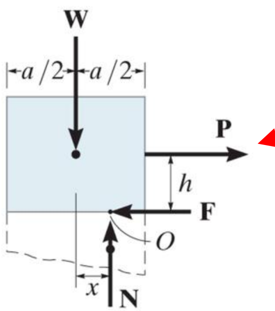
In order to move the refrigerator, is it best to pull up as shown, pull horizontally, or pull downwards on the rope?

CHARACTERISTICS OF DRY FRICTION (Section 8.1)









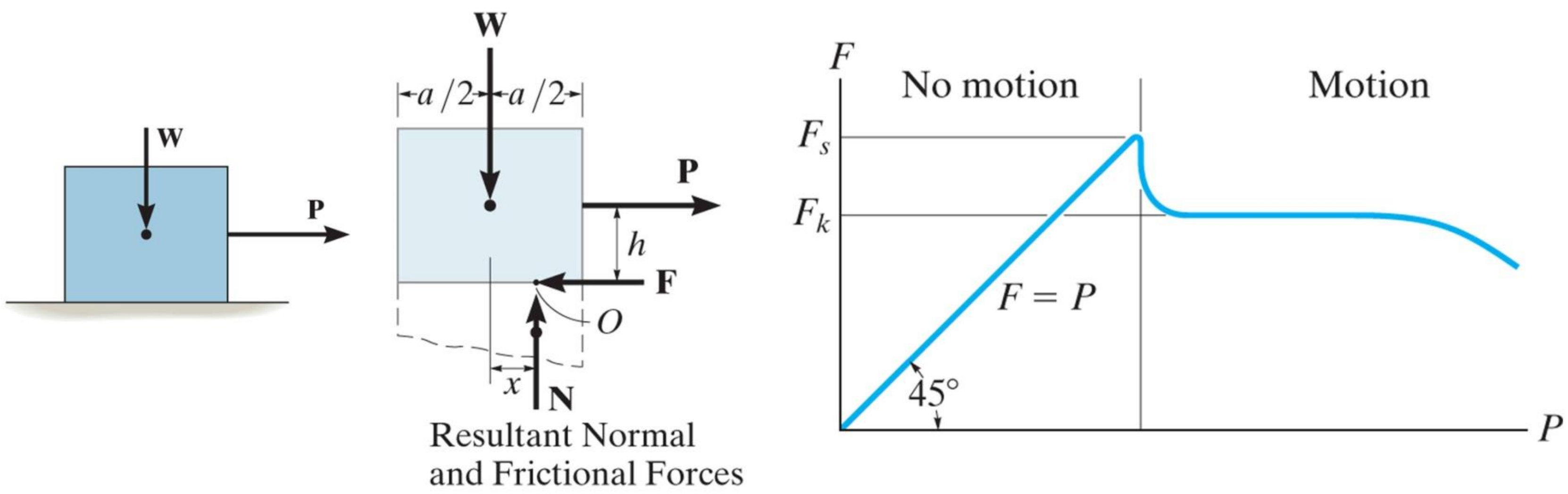
Resultant Normal and Frictional Forces

Friction is defined as a force of resistance acting on a body which prevents or resists the slipping of a body relative to a second body.

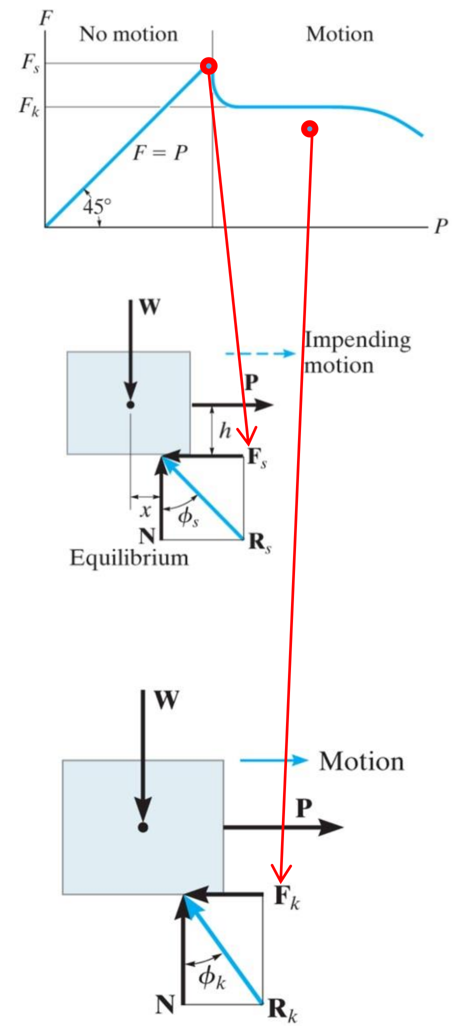
Experiments show that frictional forces act tangent (parallel) to the contacting surface in a direction opposing the relative motion or tendency for motion.

For the body shown in the figure to be in equilibrium, the following must be true:
F = P,   N = W, and W*x = P*h.

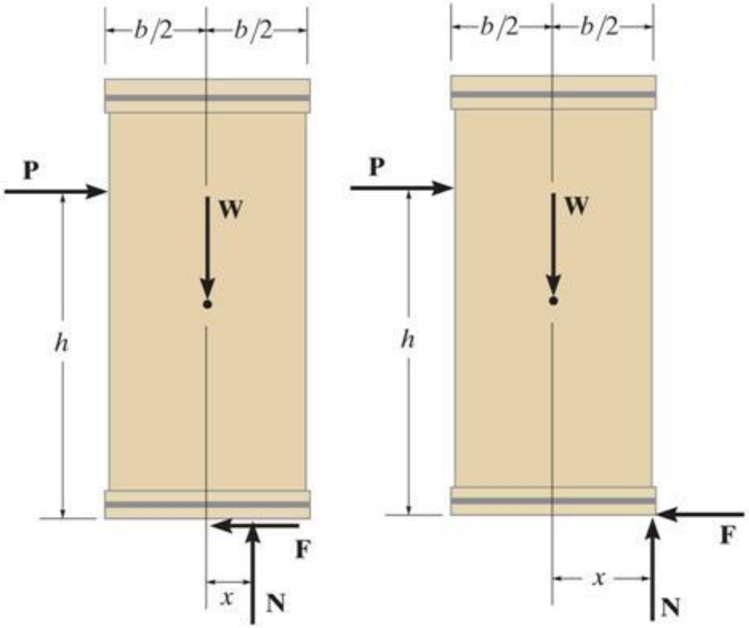
CHARACTERISTICS OF DRY FRICTION (continued)



CHARACTERISTICS OF DRY FRICTION (continued)

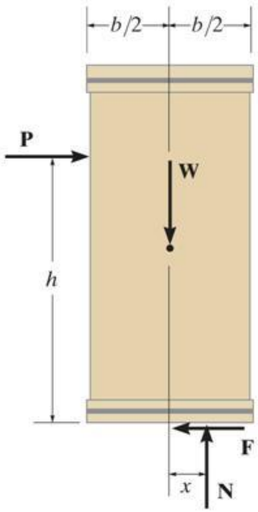


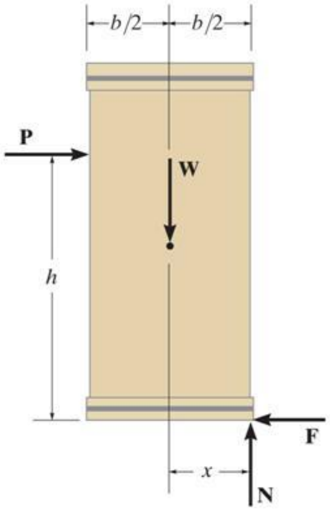
IMPENDING TIPPING versus SLIPPING



For a given W and h of the box, how can we determine if the block will slide or tip first? In this case, we have four unknowns (F, N, x, and P) and only the three E-of-E.

IMPENDING TIPPING versus SLIPPING (continued)





Assume: Slipping occurs

Known: $F = \mu_s N$

Solve: x, P, and N

Check: $0 \leq x \leq b/2$

Or

Assume: Tipping occurs

Known: $x = b/2$

Solve: P, N, and F

Check: $F \leq \mu_s N$

EXAMPLE

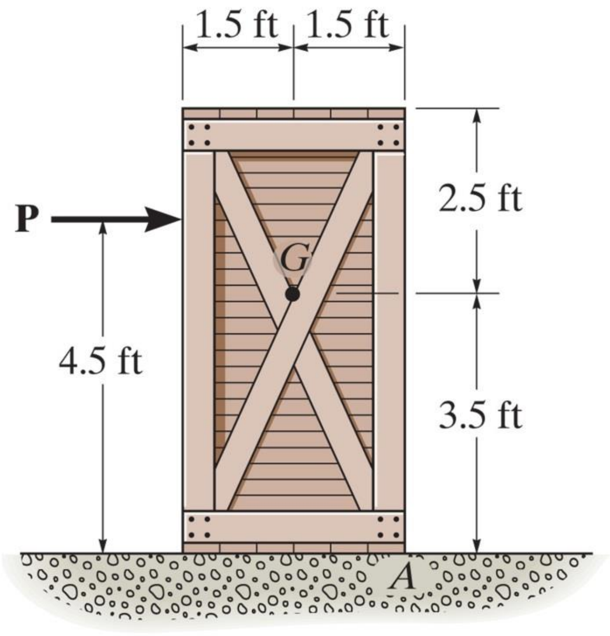
Given: Crate weight = 250 lb and

$\mu s = 0.4$

Find: The maximum force P that can be applied without causing movement of the crate.

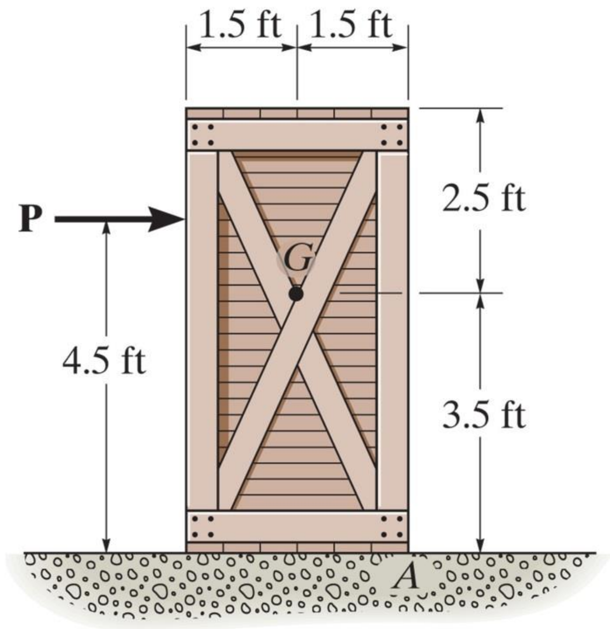
??

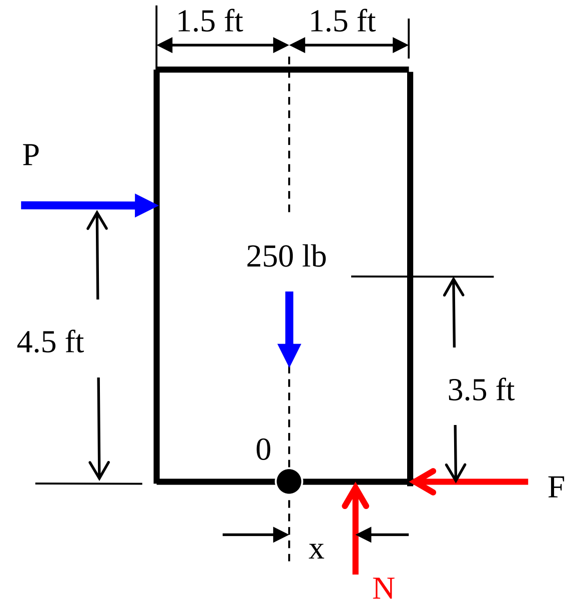
Plan:



EXAMPLE (continued)

Solution:



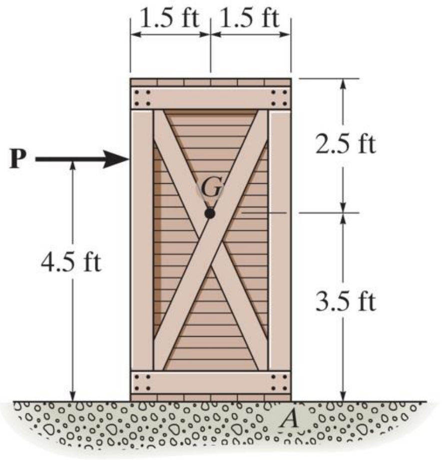


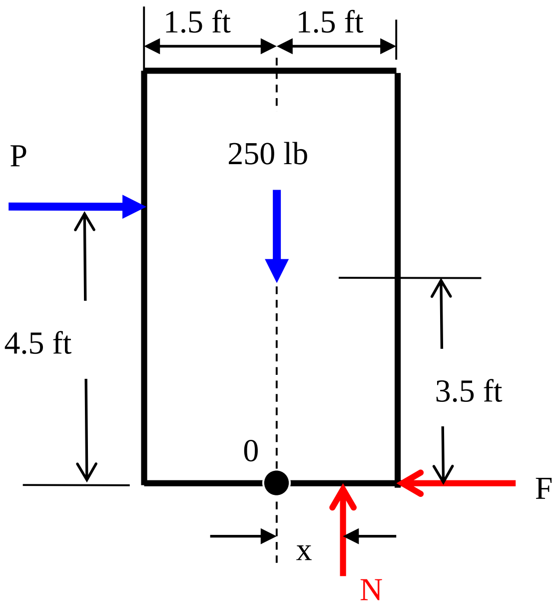
FBD of the crate

There are four unknowns: P, N, F and x.

First, let's assume the crate slips. Then the friction equation is $F = \mu s\ N = 0.4\ N$.

EXAMPLE (continued)





FBD of the crate

$$+ \rightarrow \sum FX = P - 0.4\,N = 0$$

$$+ \uparrow \sum FY = N - 250 = 0$$

Solving these two equations gives:

$P = 100$ lb and $N = 250$ lb

$$+ \sum MO = -100\,(4.5) + 250\,(x) = 0$$

Check: $x = 1.8 \geq 1.5$: No slipping will occur since $x > 1.5$

EXAMPLE (continued)

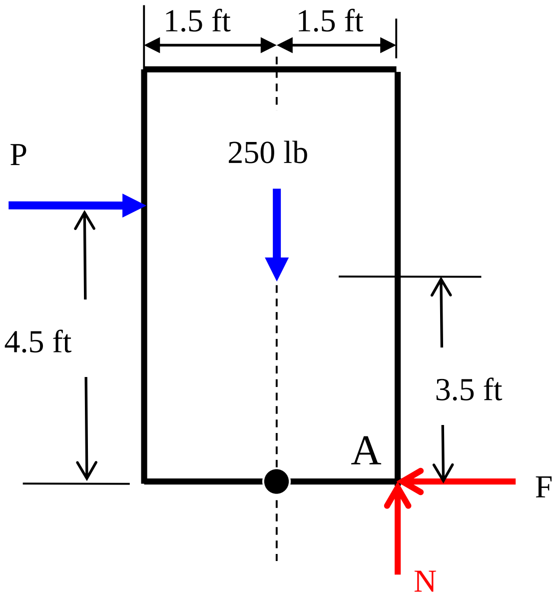
Since tipping occurs, here is the correct FBD:

$$+\rightarrow \sum F_X = P - F = 0$$

$$+\uparrow \sum F_Y = N - 250 = 0$$

These two equations give:

$P = F$ and $N = 250$ lb

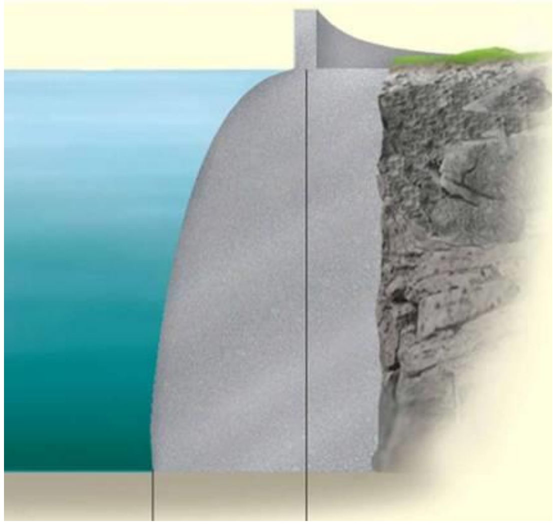


FBD of the crate

$$\circlearrowleft + \sum M_A = -P(4.5) + 250(1.5) = 0$$

$P = 83.3$ lb, and $F = 83.3 \text{ lb} < \mu_s N = 100$ lb

CENTER OF GRAVITY, CENTER OF MASS AND CENTROID OF A BODY

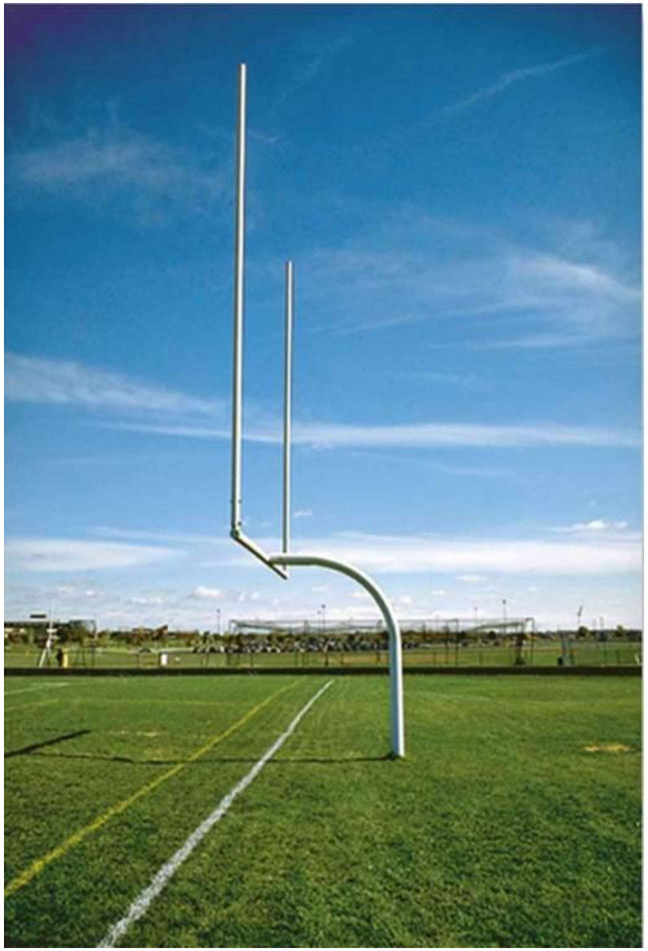


APPLICATIONS

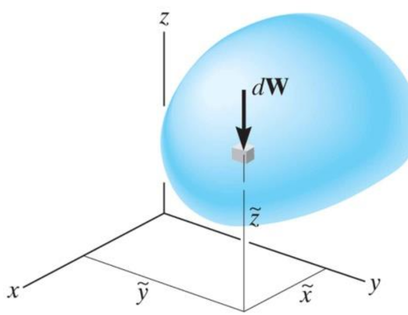
APPLICATIONS (continued)

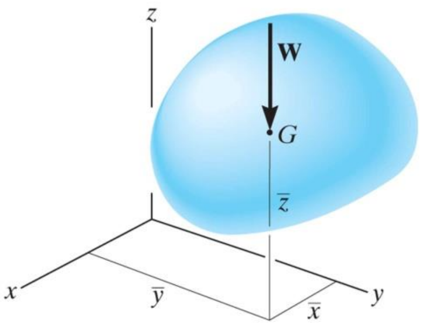
One concern about a sport utility vehicle (SUV) is that it might tip over when taking a sharp turn.

APPLICATIONS (continued)



CONCEPT OF CENTER OF GRAVITY (CG)





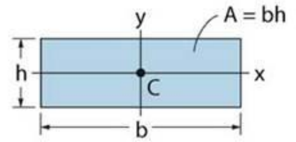
$$\bar{x} = \frac{\int \tilde{x}\, dW}{\int dW} \qquad \bar{y} = \frac{\int \tilde{y}\, dW}{\int dW} \qquad \bar{z} = \frac{\int \tilde{z}\, dW}{\int dW}$$

CM & CENTROID OF A BODY

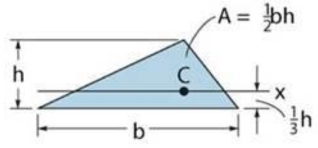
$$\bar{x} = \frac{\int \tilde{x}\, dW}{\int dW} \qquad \bar{y} = \frac{\int \tilde{y}\, dW}{\int dW} \qquad \bar{z} = \frac{\int \tilde{z}\, dW}{\int dW}$$

$$\bar{x} = \frac{\int \tilde{x}\, dm}{\int dm} \qquad \bar{y} = \frac{\int \tilde{y}\, dm}{\int dm} \qquad \bar{z} = \frac{\int \tilde{z}\, dm}{\int dm}$$

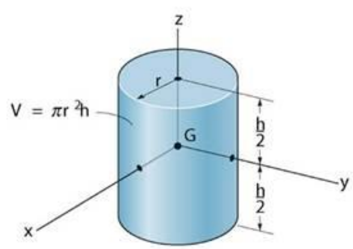
CONCEPT OF CENTROID



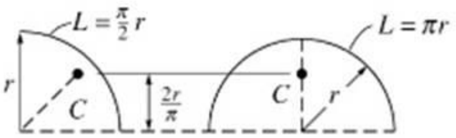
Rectangular area



Triangular area

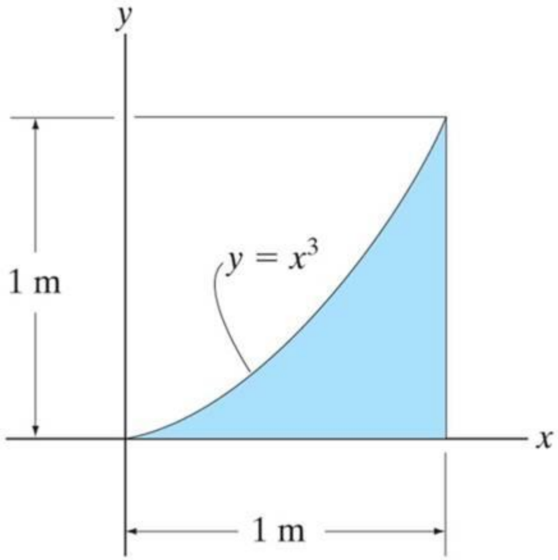


Cylinder



Quarter and semicircle arcs

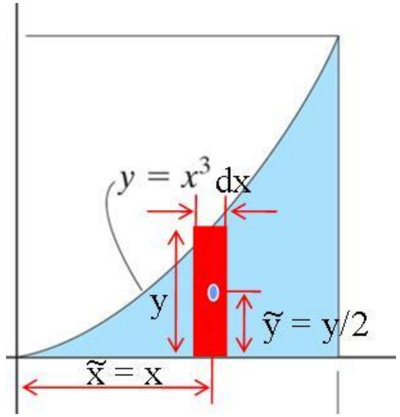
EXAMPLE I



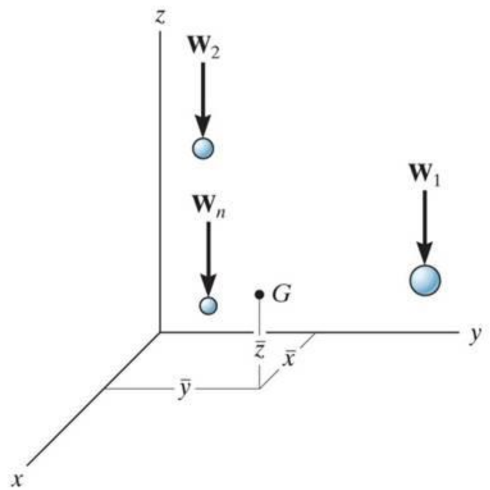
Given: The area as shown.

Find: The centroid location (x , y)

Plan: Follow the steps.

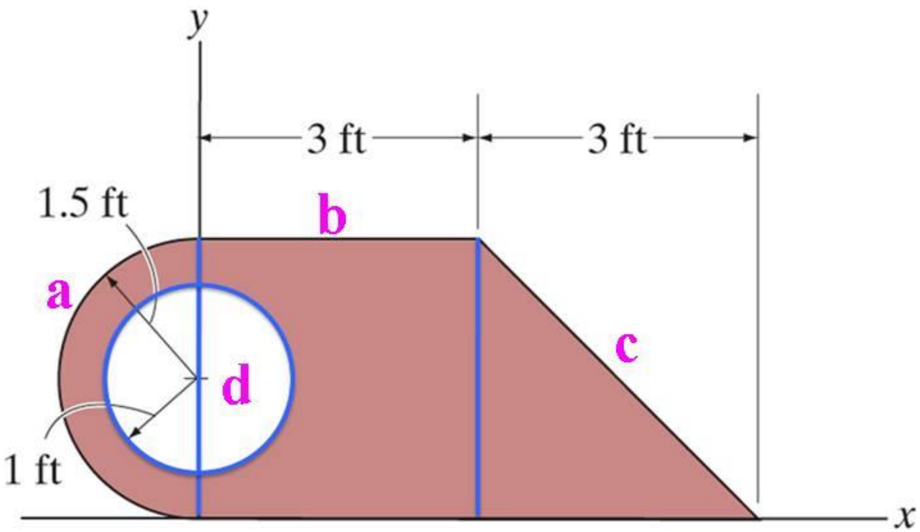


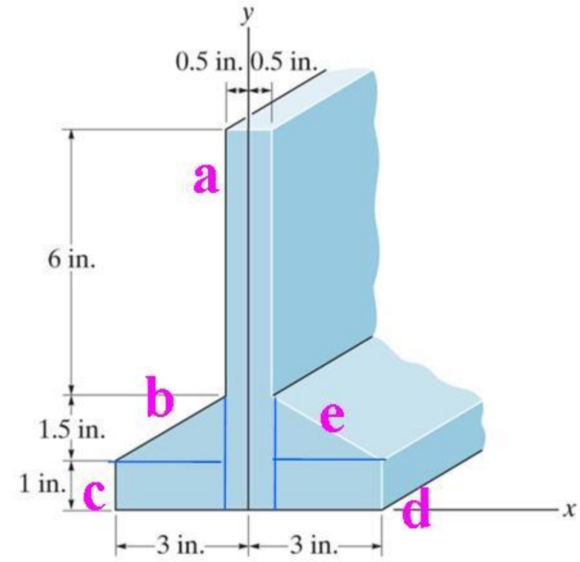
CG/CM OF A COMPOSITE BODY



$$\bar{x} = \frac{\Sigma \tilde{x} W}{\Sigma W} \qquad \bar{y} = \frac{\Sigma \tilde{y} W}{\Sigma W} \qquad \bar{z} = \frac{\Sigma \tilde{z} W}{\Sigma W}$$

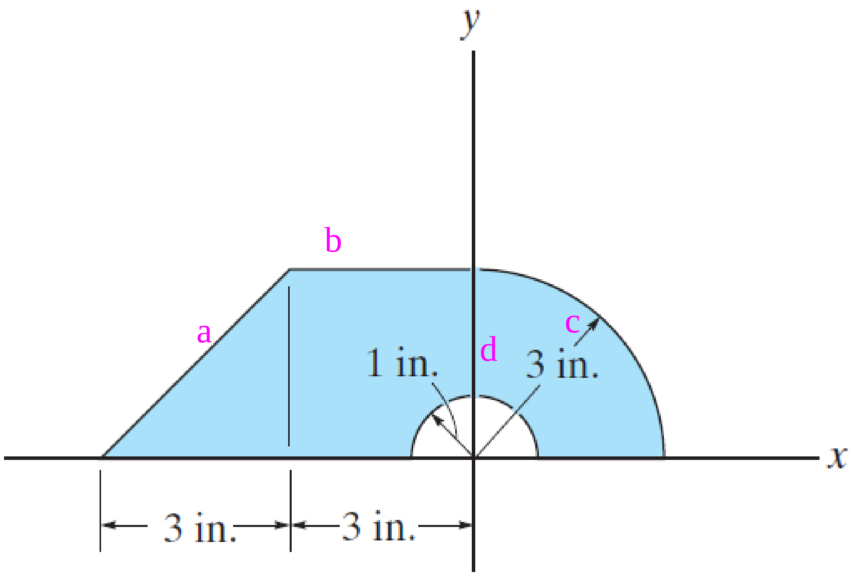
CONCEPT OF A COMPOSITE BODY





PEARSON

GROUP PROBLEM SOLVING



Given: The part shown.

Find: The centroid of the part.

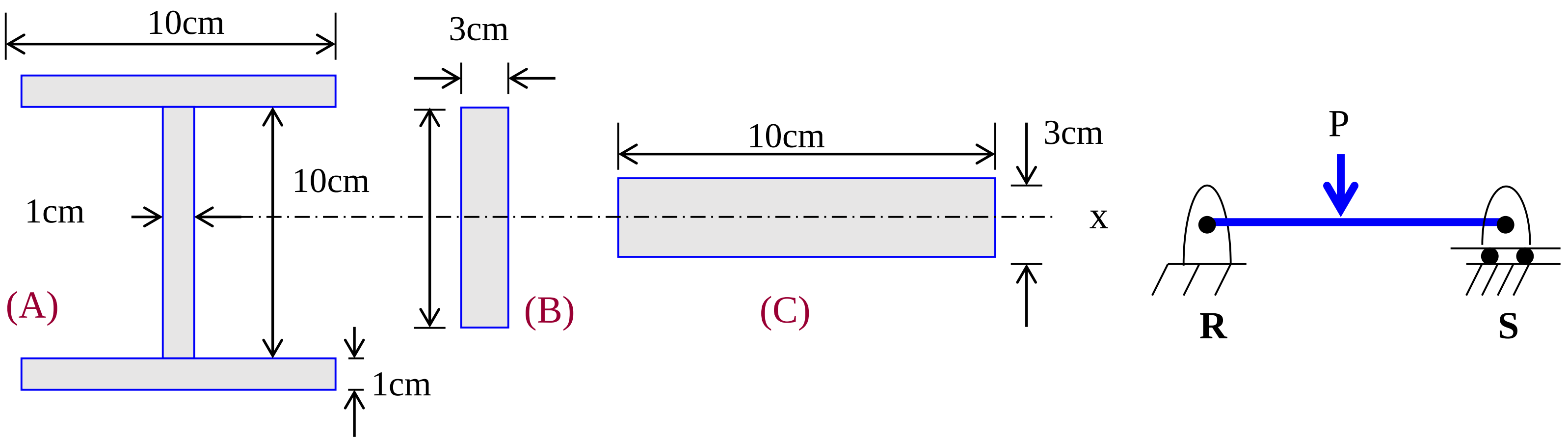
Plan: Follow the steps for analysis.

DEFINITION OF MOMENTS OF INERTIA FOR AREAS, RADIUS OF GYRATION OF AN AREA

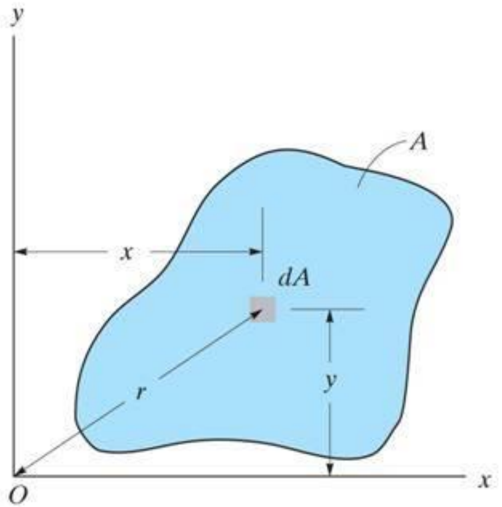
APPLICATIONS

Many structural members like beams and columns have cross sectional shapes like an I, H, C, etc..

DEFINITION OF MOMENTS OF INERTIA FOR AREAS



DEFINITION OF MOMENTS OF INERTIA FOR AREAS



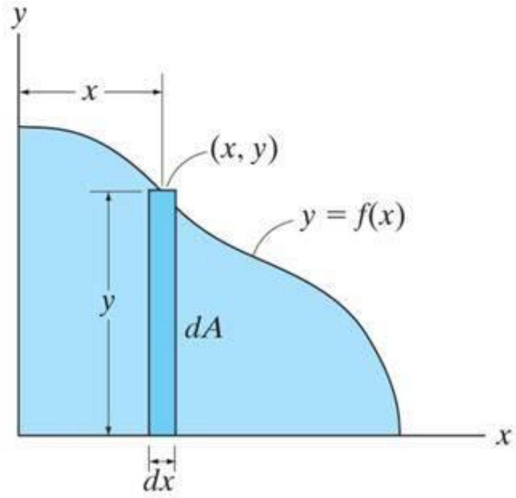
For the differential area dA, shown in the figure:

$$d\,I_x = y^2\,dA\,,$$

$$d\,I_y = x^2\,dA\,,\ \text{and,}$$

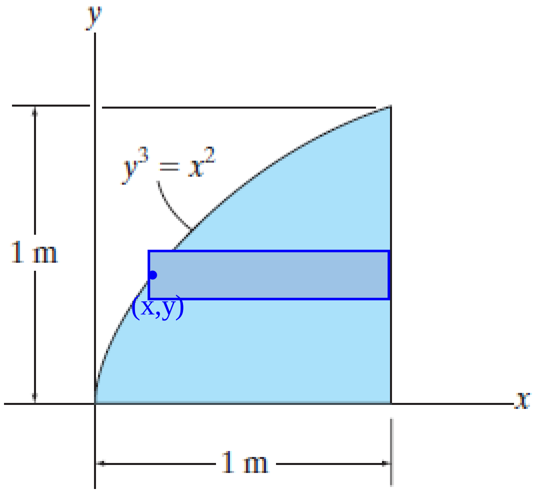
$$d\,J_O = r^2\,dA\,,$$ where J_O is the polar moment of inertia about the pole O or z axis.

MoI FOR AN AREA BY INTEGRATION



For simplicity, the area element used has a differential size in only one direction (dx or dy). This results in a single integration and is usually simpler than doing a double integration with two differentials, i.e., dx·dy.

EXAMPLE



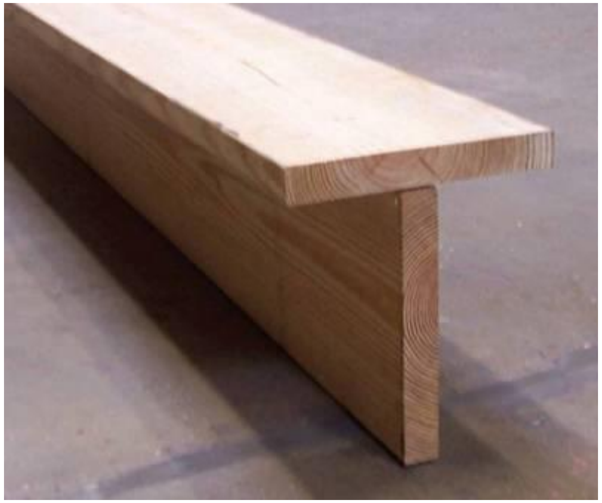
Given: The shaded area shown in the figure.

Find: The MoI of the area about the x- and y-axes.

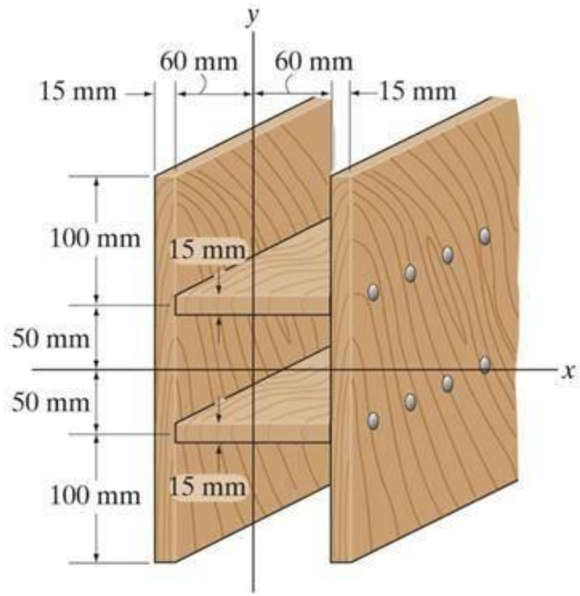
Plan: Follow the steps given earlier.

PARALLEL-AXIS THEOREM, RADIUS OF GYRATION & MOMENT OF INERTIA FOR COMPOSITE AREAS

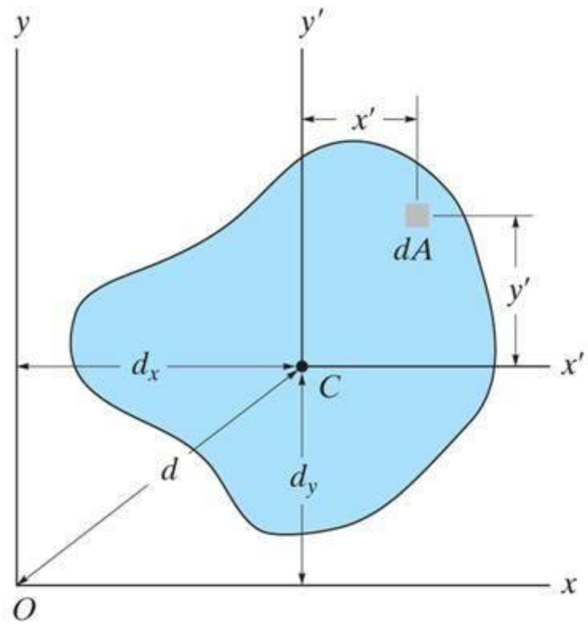
APPLICATIONS



APPLICATIONS (continued)



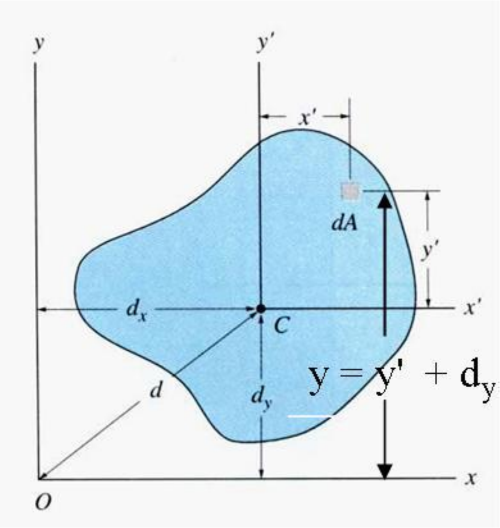
PARALLEL-AXIS THEOREM FOR AN AREA (Section 10.2)



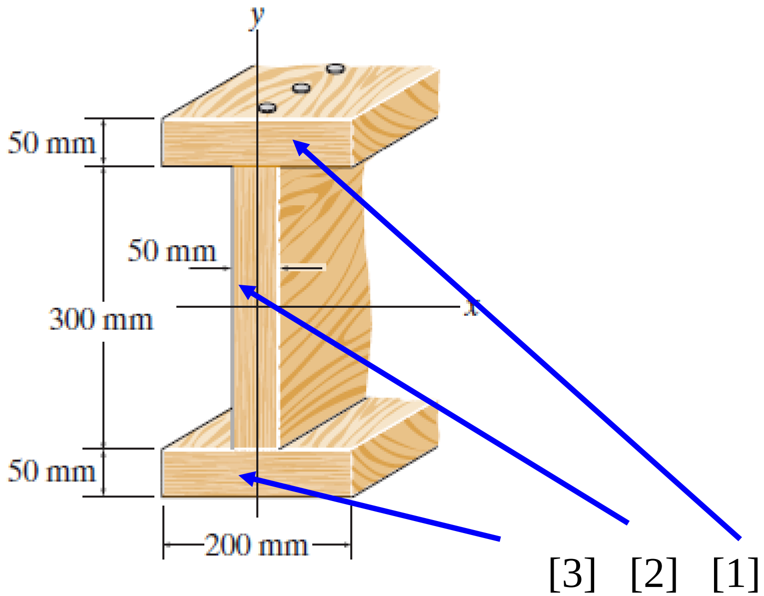
This theorem relates the moment of inertia (MoI) of an area about an axis passing through the area's centroid to the MoI of the area about a corresponding parallel axis. This theorem has many practical applications, especially when working with composite areas.

Consider an area with centroid C. The x' and y' axes pass through C. The MoI about the x-axis, which is parallel to, and distance dy from the x ' axis, is found by using the parallel-axis theorem.

PARALLEL-AXIS THEOREM (continued)



EXAMPLE



Given: The beam's cross-sectional area.

Find: The moment of inertia of the area about the x-axis

Plan: Follow the steps for analysis.