STATISTIC

ANALYTIC GEOMETRY

SESSION 3

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Analytic Geometry

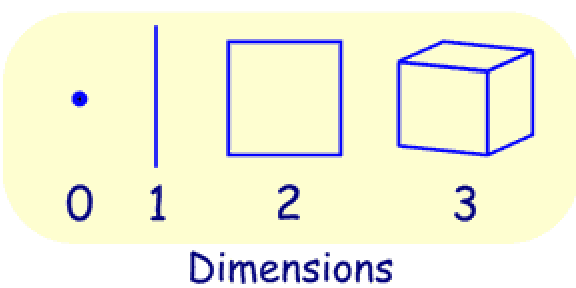
Geometry is all about **shapes** and their properties.

If you like playing with objects, or like drawing, then geometry is for you!

Geometry can be divided into:

Plane Geometry is about flat shapes like lines, circles and triangles ... shapes that can be drawn on a piece of paper

Solid Geometry is about three dimensional objects like cubes, prisms, cylinders and spheres

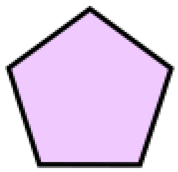


Point, Line, Plane and Solid

A Point has no dimensions, only position
A Line is one-dimensional
A Plane is two dimensional (2D)
A Solid is three-dimensional (3D)

Plane Geometry

Plane Geometry is all about shapes on a flat surface (like on an endless piece of paper).



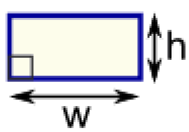
2D Shapes

Activity: Sorting Shapes

Triangles

Right Angled Triangles

Interactive Triangles



Quadrilaterals (Rhombus, Parallelogram, etc)

Rectangle, Rhombus, Square, Parallelogram, Trapezoid and Kite

Interactive Quadrilaterals

Shapes Freeplay

Perimeter

Area

Area of Plane Shapes

Area Calculation Tool

Area of Polygon by Drawing

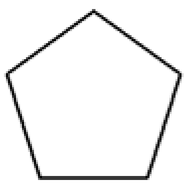
Activity: Garden Area

General Drawing Tool

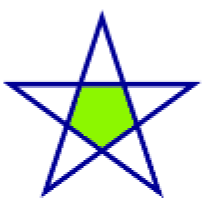
Polygons

A Polygon is a 2-dimensional shape made of straight lines. Triangles and Rectangles are polygons.

Here are some more:



Pentagon



Pentagra
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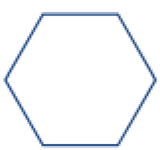


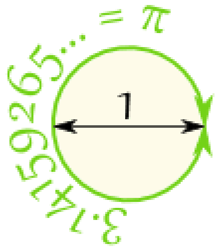
Hexagon

Properties of Regular Polygons

Diagonals of Polygons

Interactive Polygons

The Circle



[Circle]

[Pi]

[Circle Sector and Segment]

[Circle Area by Sectors]

[Annulus]

Activity: Dropping a Coin onto a Grid

Circle Theorems (Advanced Topic)

Symbols

There are many special symbols used in Geometry. Here is a short reference for you:

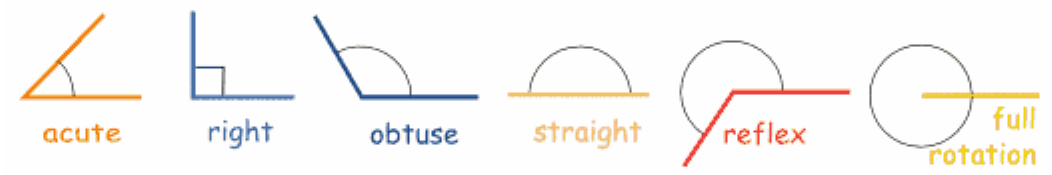
Geometric Symbols

Congruent and Similar

Congruent Shapes

Similar Shapes

Angles



Types of Angles

Acute Angles | Right Angles | Obtuse Angles | Straight Angle | Reflex Angles | Full Rotation

Degrees (Angle)

Angles Around a Point

Radians

Angles on a Straight Line

Congruent Angles

Parallel Lines and Pairs of Angles

Interior Angles

Transversal

Exterior Angles

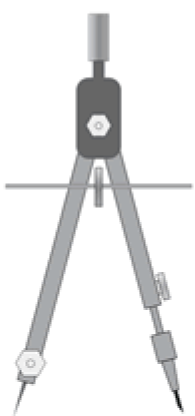
A Triangle Has 180°

Interior Angles of Polygons

Supplementary Angles

Exterior Angles of Polygons

Complementary Angles



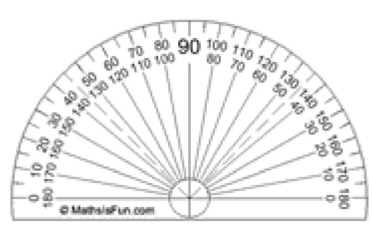
Using Drafting Tools

Geometric Constructions

Using the Protractor

Using the Drafting Triangle and Ruler

Using a Ruler and Compass



Transformations and Symmetry

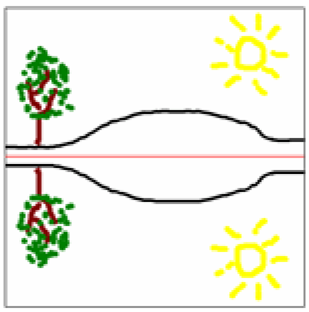
Transformations:

Rotation

Reflection

Translation

Resizing



Symmetry:

Reflection Symmetry

Rotational Symmetry

Point Symmetry

Lines of Symmetry of Plane Shapes

Symmetry Artist

Activity: Symmetry of Shapes

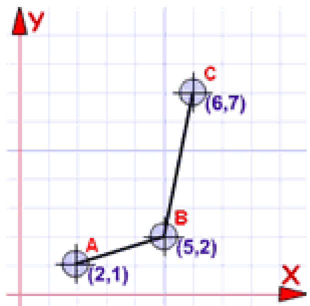
[Activity: Make a Mandala]

[Activity: Coloring (The Four Color Theorem)]

[Tessellations]

[Tessellation Artist]

Coordinates



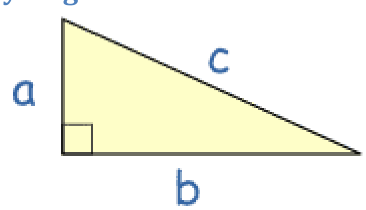
[Cartesian Coordinates]

[Interactive Cartesian Coordinates]

[Hit the Coordinate Game]

More Advanced Topics in Plane Geometry

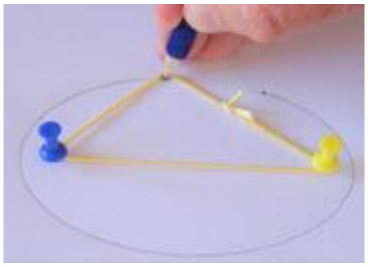
Pythagoras



[Pythagoras' Theorem]

[Pythagorean Triples]

Conic Sections



Set of all points

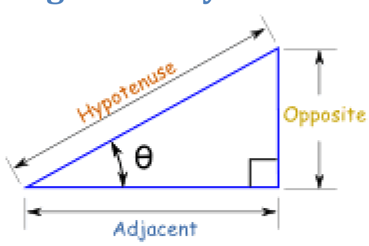
Conic Sections

Ellipse

Parabola

Hyperbola

Trigonometry



Trigonometry is a special subject of its own, so you might like to visit:

Introduction to Trigonometry

Trigonometry Index

Solid Geometry

Solid Geometry is the geometry of three-dimensional space - the kind of space we live in ...

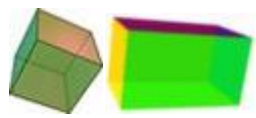
... let us start with some of the simplest shapes:

Common 3D Shapes

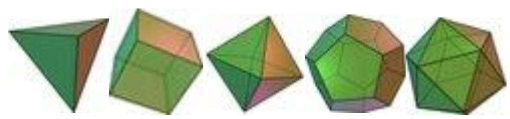
Polyhedra and Non-Polyhedra

There are two main types of solids, "Polyhedra", and "Non-Polyhedra":

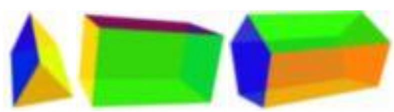
Polyhedra :
(they must have flat faces)



Cubes and Cuboids (Volume of a Cuboid)



Platonic Solids



Prisms

Pyramids

Non-Polyhedra:
(if any surface is not flat)

Sphere

Torus

Cylinder

Cone

Polyhedron Models

Vertices, Faces, and Edges

Euler's Theorem

Introduction to Euclidean geometry

Roughly 2400 years ago, Euclid of Alexandria wrote Elements which served as the world's geometry textbook until recently. Studied by Abraham Lincoln in order to sharpen his mind and truly appreciate mathematical deduction, it is still the basis of what we consider a first year course in geometry. This tutorial gives a bit of this background and then lays the conceptual foundation of points, lines, circles and planes that we will use as we journey through the world of Euclid.

Angle basics and measurement

This tutorial will define what an angle is and help us think about how to measure them. If you're new to angles, this is a great place to start.

Transformations and congruence

Two figures are congruent if you can go from one to another through some combination of translations, reflections and rotations. In this tutorial, we'll really internalize this by working through the actual transformations.

Congruence postulates

We begin to seriously channel Euclid in this tutorial to really, really (no, really) prove things--in particular, that triangles are congruents. You'll appreciate (and love) what rigorous proofs are. It will sharpen your mind and make you a better friend, relative and citizen (and make you more popular in general). Don't have too much fun.

Similarity and transformations

Two figures are similar if you can get from one to another through some combinations of translations, reflections, rotations AND DILATIONS (so you can scale up and down). This tutorial helps give us an intuition for this.

Triangle similarity

This tutorial explains a similar (but not congruent) idea to congruency (if that last sentence made sense, you might not need this tutorial). Seriously, we'll take a rigorous look at similarity and think of some reasonable postulates for it. We'll then use these to prove some results and solve some problems. The fun must not stop!

Solving problems with similar and congruent triangles

We spend a lot of time in geometry proving that triangles are congruent or similar. We now apply this ability to some really interesting problems (seriously, these are fun)!

Pythagorean theorem

Named after the Greek philosopher who lived nearly 2600 years ago, the Pythagorean theorem is as good as math theorems get (Pythagoras also started a religious movement). It's simple. It's beautiful. It's powerful. In this tutorial, we will cover what it is and how it can be used. We have another tutorial that gives you as many proofs of it as you might need.

Pythagorean theorem

The Pythagorean theorem intro

Pythagorean theorem 1

Pythagorean theorem 2

Pythagorean theorem 3

Pythagorean theorem

Pythagorean theorem word problems

Introduction to the Pythagorean theorem

Pythagorean theorem II

Pythagorean theorem proofs

The Pythagorean theorem is one of the most famous ideas in all of mathematics. This tutorial proves it. Then proves it again... and again... and again. More than just satisfying any skepticism of whether the Pythagorean theorem is really true (only one proof would be sufficient for that), it will hopefully open your mind to new and beautiful ways to prove something very powerful.

Garfield's proof of the Pythagorean theorem

Bhaskara's proof of the Pythagorean theorem

Pythagorean theorem proof using similarity

Another Pythagorean theorem proof

Pythagorean Theorem proofs

Special right triangles

We hate to pick favorites, but there really are certain right triangles that are more special than others. In this tutorial, we pick them out, show why they're special, and prove it! These include 30-60-90 and 45-45-90 triangles (the numbers refer to the measure of the angles in the triangle).

Symbols in Geometry

Common Symbols Used in Geometry

Symbols save time and space when writing. Here are the most common geometrical symbols:

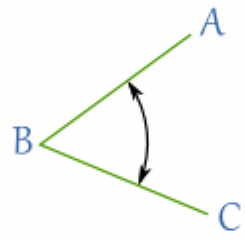
Symbol	Meaning	Example	In Words

Δ	[Triangle]()	ΔABC has 3 equal sides	*Triangle ABC has three equal sides*
∠	[Angle]()	∠ABC is 45°	*The angle formed by ABC is 45 degrees.*
⊥	[Perpendicular]()	AB⊥CD	*The line AB is perpendicular to line CD*
∥	[Parallel]()	EF∥GH	*The line EF is parallel to line GH*
°	[Degrees]()	360° makes a full circle	
∟	[Right Angle]() (90°)	∟is 90°	*A right angle is 90 degrees*
$\overline{AB}$	Line Segment "AB"	AB	*The line between A and B*
$\overleftrightarrow{AB}$	Line "AB"	$\overleftrightarrow{AB}$	*The infinite line that includes A and B*
$\overrightarrow{AB}$	Ray "AB"	$\overrightarrow{AB}$	*The line that starts at A, goes through B and continues on*
≅	[Congruent]() (same shape and size)	ΔABC ≅ΔDEF	*Triangle ABC is congruent to triangle DEF*
~	[Similar]() (same shape, different size)	ΔDEF~ΔMNO	*Triangle DEF is similar to triangle MNO*
∴	Therefore	*a=b* ∴ *b=a*	*a equals b, therefore b equals a*

Example:

When someone writes: In ΔABC, ∠BAC is ∟

They are really saying: "In triangle ABC, the angle BAC is a right angle"

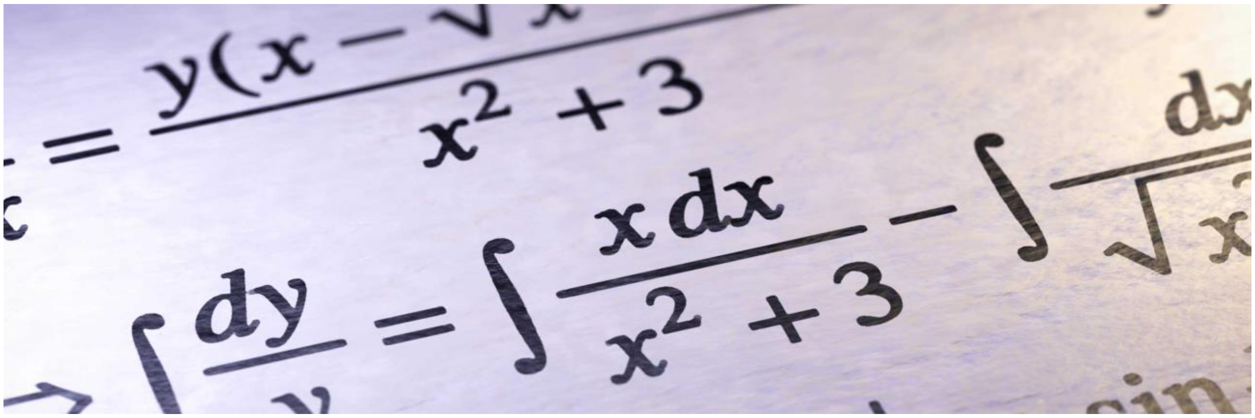


Naming Angles

For angles the central letter is where the angle is.

Example:

when we write "∠ABC is 45°", the point "B" is where the angle is.



What Is Geometry?

> *Geometry is perhaps the most elementary of the sciences that enable man, by purely intellectual processes, to make predictions (based on observation) about physical world. The power of geometry, in the sense of accuracy and utility of these deductions, is impressive, and has been a powerful motivation for the study of logic in geometry.*

H. M. S. Coxeter (1907-2003)

> *Perhaps it may be asserted, that there are no difficulties in geometry which are likely to place a serious obstacle in the way of an intelligent beginner, except the temporary embarrassment which always attends the commencement of a new study ...*
>
> ***A. De Morgan*** (1806-1871)

> *And, for geometry, till of very late times it had no place at all (at universities), as being subservient to nothing but rigid truth. And if any man by the ingenuity of his own nature had attained to any degree of perfection therein, he was commonly thought of a magician and his art diabolical.*
>
> ***Thomas Hobbes*** (1588-1679)

Geometry is a branch of mathematics that is concerned with the properties of configurations of geometric objects - ***points***, (straight) ***lines***, and ***circles*** being the most basic of these. Although the word *geometry* derives from the Greek *geo* (earth) and *metron* (measure) [***Words***], which points to its practical roots, Plato already knew to differentiate between *the art of mensuration which is used in building* and *philosophical geometry* [***Philebus*** (57)]. Earlier in the dialogue [***Philebus*** (51)], Socrates mentions the matter of beauty:

> I do not mean by beauty of form such beauty as that of animals or pictures, which many would suppose to be my meaning; but, says the argument, understand me to mean straight lines and circles, and the plane or solid figures which are formed out of them by turning-lathes and rulers and measures of angles; for these I affirm to be not only relatively beautiful, like other things, but they are eternally and absolutely beautiful, and they have peculiar pleasure, quite unlike the pleasures of scratching.

In another dialogue - ***Phaedrus*** (274) - Socrates ascribes creation of geometry, albeit in a company of other arts, to the god Theuth who resided in the Egyptian city of Naucratis. Truth be told, Phaedrus questions Socrates' account right away: "Yes, Socrates, you can easily invent tales of Egypt, or of any other country." But even if not of divine origin, the objects of geometry are not to be found in the physical world. They are pure ***abstractions***, creations of the human mind.

Around 300 BC, ***Euclid gave*** the definitions of points and lines that withstood two ***millennia of diligent study***. The mathematicians of the 19th ***found them lacking***. According to Euclid, *A point is that which has no part*. As F. Klein [***Klein***, p. 196] notes "a point is by no means determined by this property alone." According to Euclid, *A line is length without breadth*. Even if ***length and breadth*** are accepted as the basic notions, Euclid's definition conflicts with the ***existence of curves*** that cover a surface [***Klein***, p. 196]. According to Euclid, *A straight line is a line which lies evenly with respect to its points*, which Klein [***ibid***] finds completely obscure. Klein goes to considerable length to uncover and explain the deficiencies in Euclid's *Elements*. A less benevolent but still very accessible critique, was given by B. Russell and can be found in C. Pritchard's *The Changing Shape of Geometry* [***Pritchard***, pp. 486-488]. Klein, for example, notes that such a simple proposition as the statement that two circles each passing through the center of the other meet in two points is not derivable from Euclid's postulates without a leap of faith.

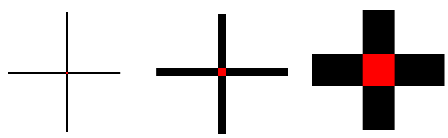
Modern mathematics found two ways to remedy the deficiencies and place geometry on a sound foundation. First, mathematicians have perfected the axiomatic approach of Euclid's *Elements*. They came to a realization that it's impossible and in fact futile to attempt to define such basic notions as *points* and *lines*. In ***analytic geometry***, on the other hand, both *points* and *lines* are perfectly definable. However, ***analytic geometry*** contains no "geometric axioms" and is built on top of the theory of sets and numbers.

The most influential work on the axiomatization of geometry is due to ***D. Hilbert*** (1862-1943). In ***Foundations of Geometry***, that appeared in 1899, he listed 21 axioms and analyzed their significance. Hilbert's axioms for plane geometry can be found in an appendix to [***Cederberg***, pp. 205-207] along with an unorthodox, but short, axiomatization by G. D. Birkhof [***Birkhof***, ***Cederberg***, pp. 208-209] and a later one, influenced by that of Birkhof, by the S.M.S.G. (School Mathematics Study Group) [***Cederberg***, pp. 210-213]. (The School Mathematics Study Group has been set up in the 1960s as a response to the success of the Soviet space program and the perceived need to improve on math education in the US. The effort led to the now defunct ***New Math program***.)

Unlike Euclid's *Elements*, modern axiomatic theories do not attempt to define their most fundamental objects, *points* and *lines* in case of geometry. The reason is nowadays obvious: all possible definitions would apparently include even more fundamental terms, which would require definitions of their own, and so on ad infinitum. Instead, the comprehension of the fundamental, i.e. *undefined*, terms builds on their use in the axioms and their properties as emerge from subsequently proved theorems. For example, the claim of existence of a straight line through any two points, that of the uniqueness of such a line or the assertion that two lines meet in at most one point, tell us something about the points and the lines without actually defining what these are. (The first two are Hilbert's axioms I.1 and I.2, while the last one is a consequence of the first two.)

The usage of the undefined terms, in the above paragraph, certainly meets our expectation and intuition of the meaning of the terms *points* and *lines*. However, depending on intuition may be misleading, as, for example, in ***projective geometry***, according to the ***Duality Principle***, all occurrences of the two terms in the axioms and theorems are interchangeable.

Modern geometry is thus a complete abstraction that crystallizes our ideas of the physical world, i.e., to start with. I say "to start with", because most of the edifice built on top of the chosen axioms, does not reflect our common experiences. Mathematicians who work with the abstract objects develop an intuition and insights into a separate world of abstraction inhabited by mathematical objects. Still, their intuition and the need to communicate their ideas are often fostered by pictorial representation of geometric configurations wherein points are usually represented by dots and straight lines are drawn using straightedge and pencil. It must be understood that, however sharpened a pencil may be, a drawing is only a representation of an abstract configuration. Under a magnifying glass, the lines in the drawing will appear less thin, and their intersection won't look even like a dot thought to represent an abstract point.



If it were at all possible, placing a magnifying glass in front of our mind's eye would not change the appearance of points and lines, regardless of how strong the magnification could be. This is probably not very different from the meaning Euclid might have meant to impute to the objects he had tried to define. The difference is not in the imaging of the geometric objects, but in the late realization that the definition is not only not always possible, it may not even be necessary for a construction of a theory.

As a word of precaution, the diagrams supply an important tool in geometric investigations, but may suggest ***wrongful facts*** if not accompanied by deductive reasoning. (***Worse yet***, faulty deductive reasoning may accidently lead to correct facts in which case you may be left oblivious of the frivolous ways in which a correct fact had been obtained.)

The second approach to resolving inconsistencies in the *Elements* came with the advent of ***analytic geometry***, a great invention of Descartes and Fermat. In plane analytic geometry, e.g., points are **defined** as ordered pairs of numbers, say, (x, y), while the straight lines are in turn **defined** as the sets of points that satisfy linear equations, see excellent expositions by ***D. Pedoe*** or ***D. Brannan et al***.

There are many geometries. All of these share some basic elements and properties. Even finite geometries deal with points and lines and universally just a single line may pass through two given points. Thus I believe that a frequently used term "Taxicab Geometry" is a misnomer. The ***taxicab metric*** is a useful mathematical concept that turns the plane into a ***metric space*** - in one way of many. Which, still, does not make it a geometry

Calculating the Area of a Square

How to find the area of a square:

•The area of a square can be found by multiplying the base times itself. This is similar to the area of a rectangle but the base is the same length as the height.

•If a square has a base of length 6 inches its area is 6*6=36 square inches

Calculating the Area of a Rectangle

Volume of a Cube

The volume of a cube is (length of side)3

How to find the area of a rectangle:

•The area of a rectangle can be found by multiplying the base times the height.

•If a rectangle has a base of length 6 inches and a height of 4 inches, its area is 6*4=24 square inches

Calculating the Perimeter of a Square

The perimeter of a square is the distance around the outside of the square. A square has four sides of equal length. The formula for finding the perimeter of a square is 4*(Length of a Side).

Topics in a Geometry Course

To learn more about a topic listed below, click the topic name to go to the corresponding *MathWorld* classroom page.

General

Congruent	(1) A property of two geometric figures if one can be transformed into the other via a distance preserving map. (2) A property of two integers whose difference is divisible by a given modulus.
Geometry	The branch of mathematics that studies figures, objects, and their relationships to each other. This contrasts with algebra, which studies numerical quantities and attempts to solve equations.
Similar	A property of two figures whose corresponding angles are all equal and whose distances are all increased by the same ratio.

High-Dimensional Solids

High-Dimensional Solid:	A generalization of a solid such as a cube or a sphere to more than three dimensions.
Hypercube:	The generalization of a cube to more than three dimensions.
Hyperplane:	The generalization of a plane to more than two dimensions.
Hypersphere:	The generalization of a sphere to more than three dimensions.
Polytope:	A generalization of a polyhedron to more than three dimensions.

Plane Geometry

Acute Angle:	An angle that measures less than 90 degrees.
Altitude:	A line segment from a vertex of a triangle which meets the opposite side at a right angle.
Angle:	The amount of rotation about the point of intersection of two lines or line segments that is required to bring one into correspondence with the other.

Area:	The amount of material that would be needed to "cover" a surface completely.
Circle:	The set of points in a plane that are equidistant from a given center point.
Circumference:	The perimeter of a circle.
Collinear:	Three or more points are said to be collinear if they lie on the same straight line.
Complementary Angles:	A pair of angles whose measures add up to 90 degrees.
Diameter:	(1) The maximum distance between two opposite points on a circle. (2) The maximum distance between two antipodal points on a sphere.
Geometric Construction:	A construction of a geometric figure using only straightedge and compass. Such constructions were studied by the ancient Greeks.
Golden Ratio:	Generally represented as φ. Given a rectangle having sides in the ratio 1:φ, partitioning the original rectangle into a square and new rectangle results in the new rectangle having sides with the ratio 1:φ. φ is approximately equal to 1.618.
Golden Rectangle:	A rectangle in which the ratio of the sides is equal to the golden ratio. Such rectangles have many visual properties and are widely used in art and architecture.
Hypotenuse:	The longest side of a right triangle (i.e., the side opposite the right angle).
Midpoint:	The point on a line segment that divides it into two segments of equal length.
Obtuse Angle:	An angle that measures greater than 90 degrees and less than 180 degrees.
Parallel:	In two-dimensional Euclidean space, two lines that do not intersect. In three-dimensional Euclidean space, parallel lines not only fail to intersect, but also maintain a constant separation between points closest to each other on the two lines.
Perimeter:	The length around the boundary of a closed two-dimensional region. The perimeter of a circle is called its circumference.

Perpendicular: Two lines, vectors, planes, etc. that intersect at a right angle.

Pi: The ratio of the circumference of a circle to its diameter. It is equal to 3.14159....

Plane Geometry: The portion of geometry dealing with figures in a plane, as opposed to solid geometry.

Point: A zero-dimensional mathematical object that can be specified in n-dimensional space using n coordinates.

Radius: The distance from the center of a circle to its perimeter, or from the center of a sphere to its surface. The radius is equal to half the diameter.

Supplementary Angles: For a given angle, the angle that when added to it totals 180 degrees.

Triangle Inequality: The sum of the lengths of any two sides of a triangle must be greater than the length of the third side.

Polygons

Equilateral Triangle: A triangle in which all three sides are of equal length. In such a triangle, the angles are all equal as well.

Isosceles Triangle: A triangle with (at least) two sides of equal length, and therefore also with (at least) two equal angles.

Parallelogram: A quadrilateral with opposite sides parallel and therefore opposite angles equal.

Polygon: A two-dimensional figure that consists of a collection of line segments, joined at their ends.

Quadrilateral: A four-sided polygon.

Rectangle: A quadrilateral with opposite sides of equal lengths, and with four right angles.

Regular Polygon: A polygon in which the sides are all the same length and the angles all have the same measure.

Right Triangle: A triangle that has a right angle. The Pythagorean Theorem is a relationship among the sides of a right triangle.

Square: A polygon with four sides of equal length and at right angles to each other.

[Trapezoid](): A quadrilateral with two sides parallel.

[Triangle](): A three-sided (and three-angled) polygon.

Solid Geometry

Cone: A pyramid with a circular cross section.

Convex Hull: For a set of points *S*, the intersection of all convex sets containing *S*.

Cross Section: The plane figure obtained by a solid's intersection with a plane.

Cube: A Platonic solid consisting of six equal square faces that meet each other at right angles. It has 8 vertices and 12 edges.

Cylinder: A solid of circular cross section in which the centers of the circles all lie on a single line.

Dodecahedron: A Platonic solid consisting of 12 pentagonal faces, 30 edges, and 20 vertices.

Icosahedron: (1) A 20-sided polyhedron. (2) The Platonic solid consisting of 20 equilateral triangles.

Octahedron: A Platonic solid consisting of eight triangular faces, eight edges, and six vertices.

Platonic Solid: A convex solid composed of identical regular polygons. There are exactly five Platonic solids.

Polyhedron: A three-dimensional solid that consists of a collection of polygons, joined at their edges.

Prism: A polyhedron with two congruent polygonal faces and with all remaining faces parallelograms.

Pyramid: A polyhedron with one face (known as the "base") a polygon and all the other faces' triangles meeting at a common polygon vertex (known as the "apex").

Solid Geometry: That portion of geometry dealing with solids, as opposed to plane geometry.

Sphere: The set of all points in three-dimensional space that are located at a fixed distance from a given point.

Surface: A two-dimensional piece of three-dimensional space.

Surface Area: The area of a surface that lies in three-dimensional space, or the total area of all surfaces that bound a solid.

Tetrahedron: A Platonic solid consisting of four equilateral triangles.

Volume: The amount of space occupied by a closed three-dimensional object.